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LEARNING TO UNDERSTAND INCLUSION RELATIONS OF QUADRILATERALS

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Abstract: Learning to identify geometric shapes and understand inclusive properties among these shapes is prerequisite for learning more complex concepts such as spatial reasoning or deductive thinking. Despite the importance of understanding geometric shapes and inclusion relations among these shapes, it has evidenced that pre-service teachers' subject knowledge of geometry is amongst their weakest knowledge of mathematics.This study aimed to investigate pre-service mathematics teachers' (PSMT), who are going to teach middle grade mathematics (grade 5-8), understanding of inclusion relationships of quadrilaterals. A designed questionnaire was administered to 52 PSMTs at the beginning of the semester and again by the end of the semester. The findings of this study demonstrated that the majority of the PSMTs struggled with identifying quadrilaterals and especially inclusion relations of quadrilaterals primarily. The majority of them held static view of quadrilaterals which inhibited their understanding of inclusion relations of quadrilaterals. However, the number of the PSMTs who understood hierarchical relationship between quadrilaterals increased through the end of the semester.

***Keywords*:** Geometry, hierarchical thinking, inclusion relations, quadrilaterals, pre-service teachers.

Introduction

The learning of geometrical concepts, specifically inclusion relations among quadrilaterals, is not an easy feat and demonstrates a complex process that includes both visual and property-based reasoning. Many researchers have shown that students are not very successful at identifying non-prototypical shapes (e.g. Kaur, 2015).For instance, Hershkowitz (1989) showed that young children use prototypes and imposed properties (such as orientation or the side lengths of a prototypical shape) either to accept or reject the categorization of a given geometric figure into a named class of shapes. This difficulty of correctly identifying non-prototypical shapes persists for middle school students (Clements & Battista, 1992) or even after middle school (Jones 2000; Fujita & Jones, 2006; Zilkova, 2015). Fujita and Jones (2007), in their study with pre-service teachers also found that reliance on prototypical shapes cause difficulty in their understanding of inclusion relations of geometric shapes.
De Villiers (1994) suggests that classifying shapes is closely related to defining and can be seen either as hierarchical or as partitional. According to hierarchical definitions or inclusive definitions as used in different resources (Fujita, 2012), a trapezoid is a quadrilateral with at least one pair of parallel sides, which means that a parallelogram or a rhombus are special forms of trapezoid. Using partitional or exclusive definitions, on the other hand, defines a trapezoid as a quadrilateral with only one pair of sides being parallel, which excludes parallelograms or rhombus from being classified as special forms of trapezoids.In general, in mathematics, hierarchicaldefinitions arepreferred; although, it should be stressed that patitional definitions are not incorrect mathematically, but just less useful (Jones, 2000).

This study aims to investigate pre-service mathematics teachers' (PSMT), who are going to teach middle grade mathematics (grade 5-8), understanding of inclusion relationships of quadrilaterals. Despite the importance of understanding geometric shapes and inclusion relations among these shapes, it has evidenced that pre-service teachers' subject knowledge of geometry is amongst their weakest knowledge of mathematics (Fujita & Jones, 2007; Zilkova, 2015). Thus, this study also aims to investigate how the PSMTs come to understand that some geometric shapes belong to the subset of different shapes during a geometry course. Specifically, the following questions guide this study:

1- What do PSMTs know about inclusion relations of quadrilaterals?

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a. Which types of quadrilaterals do PMSTs have the biggest problems with? Or which types of quadrilaterals do PSMTs seem to understand the inclusion relations easily?

2- How do PSMTs' understandings of inclusion relations of quadrilaterals evolve during a geometry course?

Understanding Inclusion Relations of Quadrilaterals

To document learners' understanding of quadrilateral classifications, especially understanding inclusion relations of quadrilaterals Fujita (2012) proposes a theoretical model to describe cognitive development of understanding of inclusion relations of quadrilaterals by synthesizing past theories (i.e. van Hiele theory). However, Fujita (2012) explicitly focused on the relationships between certain special quadrilaterals—parallelograms with squares, rectangles, and rhombi and the relationship between squares with rectangles. Fujita's classification levels of these specific quadrilaterals were adopted; but extended to include other quadrilaterals such as trapezoids or kite.These classification levels were used to analyze the responses of the participants.

Table 1.Levels of understanding inclusion relations of quadrilaterals

Levels	Descriptions
Level 3: Hierarchical	Learners can understand hierarchical relationships among quadrilaterals. The inclusion relationships of quadrilaterals are understood and can be used for all quadrilaterals. 'The opposing direction inclusion relationship' of definitions and attributes is understood.
Level 2: Partial Prototypical	Learners have begun to extend their figural concepts to understand inclusion relations of quadrilaterals. However, their understanding is limited and specific to some quadrilaterals.
Level 1: Prototypical	Learners have their own limited personal figural concepts. Their judgments regarding to identifying relationships of quadrilaterals are judged by their limited figural concepts.
Level 0	Learners do not have basic knowledge of quadrilaterals.

Methods

Participants

52 pre-service mathematics teachers (PSMTs), who are going to teach middle grade mathematics (grade 5-8), participated in this study. Participants enrolled in a three-credit geometry course, which was taught by the author when this study was conducted. The participants were chosen by purposeful sampling method used for qualitative research studies (Patton, 1990).

Data Sources

A geometry questionnaire with sixteen open-ended questions was used to investigate PSMTs' subject matter knowledge of geometry. The questions in the questionnaire were prepared based on existing literature (e.g. Fujita & Jones, 2006, 2007; Zilkova, 2015). In addition to the questionnaire results, the PSMTs' responses to the midterm and the final questions, which were related to quadrilaterals, were also used as secondary data sources.

Data Analysis

The framework adopted from Fujita (2012) guided the analysis of the data. PSMTs' responses to the questionnaire questions and their class work and assignments were coded in three categories (level 0 was excluded since all the participants have taken high school geometry and have basic knowledge of quadrilaterals).
Data analysis began by examining the written work of each participant and grounding it in a constant comparative method of coding (Glaser & Strauss, 1967) in which participant responses were coded with external and internal codes. Coding of the data began with a set of external codes that were derived from the framework (see Table 1).Such external coding schemes provided a lens with which to examine the data. By examining the data and reviewing the written responses, emerging themes of participants' comments on quadrilaterals, the

inclusion and transitive relations of quadrilaterals were also developed. After proposing these internal (data-grounded) codes, each written work was reexamined and recorded to incorporate these new codes.

Findings

The pre-questionnaire results of the PSMTs' showed that the pre-service mathematics teachers' struggled with identifying inclusion relations of quadrilaterals. As can be seen in the table 2 below, the majority of the PSTMs (60%) demonstrated partial prototypical reasoning. That is, they demonstrated some sorts of difficulty understanding inclusion relations of quadrilaterals. While 60% PSTMs demonstrated partial prototypical reasoning, only 2 (4%) PSTMs demonstrated hierarchical reasoning. However, even these two PSMTs struggled with opposing direction inclusion relations. Walcott and others (2009) argue that the dynamic figural concept, which consists of the visual, verbal, written, symbolic, and/or formal properties of shapes, are essential to understand opposing direction inclusion relationship since inflexible prototypes are static and non-changing. Students who hold inflexible visual prototypes are unwilling to see movement upon the shape, which resulted in not accepting that a rectangle sometimes have all equal sides. Research supports the idea of mental manipulation of shapes in the minds in order to develop such reasoning (Archavsky & Goldenberg, 2005).

Table 2. Percentages of the PSMTs' understanding of inclusion relations of quadrilaterals

	Pre-Questionnaire		**Post-Questionnaire**	
	Frequency	***Number of the PSMTs***	***Frequency***	***Number of the PSMTs***
Hierarchical Reasoning With Opposing Inclusion Relations			%62	(32)
Hierarchical Reasoning With Limited Opposing Inclusion Relations	%4	(2)	%38	(20)
Hierarchical Reasoning (With Limited Opposing Inclusion Relations)With An Exception	%10	(5)		
Partial Prototypical Reasoning	%60	(30)		
Prototypical Reasoning (With An Exception)	%14	(7)		
Prototypical Reasoning	%12	(6)		

Six PSTMs held entirely prototypical reasoning about quadrilaterals. Seven PSTMs, on the other hand, were able to extend their figural concept of one specific quadrilateral, which was either recognizing a square as a rhombus or a rhombus as a parallelogram. Since these seven PSMTs relied heavily on their prototypical examples for all quadrilaterals, except just for one specific quadrilateral, they were coded in Prototypical level. However,these PSTMs were acknowledged as in transition to Partial Prototypical level. Figure 1 below demonstrates the pathway of the PSMTs while learning inclusion relations of quadrilaterals.

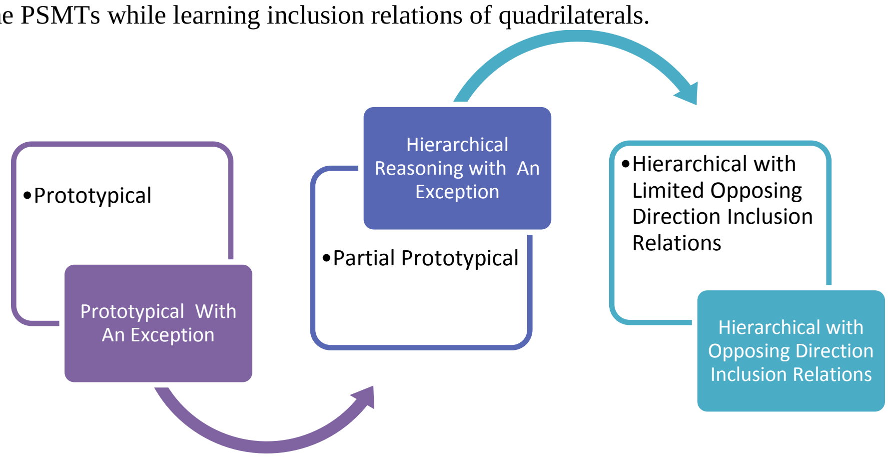


Figure 1. The pathway of understanding inclusion relationships of quadrilaterals

The study concluded that although the pre-service teachers possessed formal definitions of quadrilaterals, their prototypical images affected their personal figural concepts. The pre-service teachers found rhombus parallelogram, square rhombus, and rectangle parallelogram relationships easier and less problematic.

However, accepting square, rectangle, parallelogram and rhombus as a trapezoid was more problematic for the PSMTs. The PSMTs understand trapezoid as a disjoint class. These results aligned with the results of Zilkova (2015) who found that for pre-service teachers some inclusion relations were easier to establish such as square □parallelogramthantherelationships between other quadrilaterals such askite □quadrilateral.

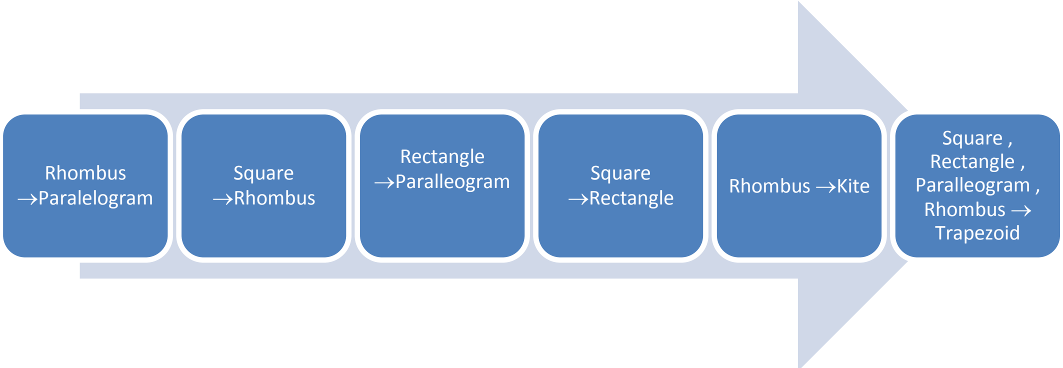
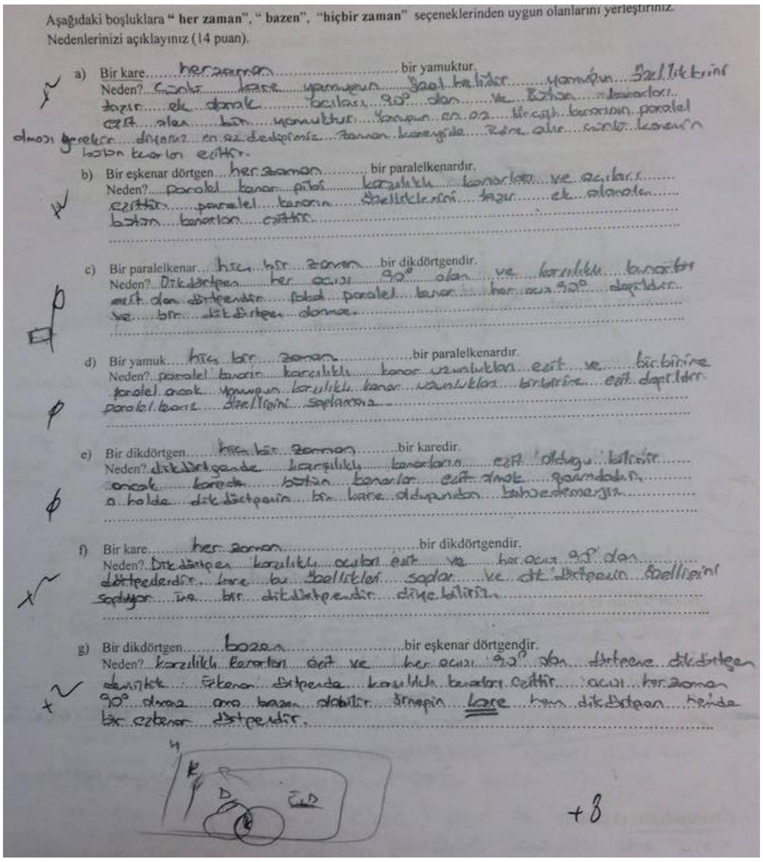


Figure 2. The pathway of extending inclusion relationships of quadrilaterals

According to the post-questionnaire results, the number of the PSMTs who could reason hierarchically increased significantly (see Table 2). Although 62% of the PSMTs were able to develop opposing direction inclusion relation, 38% of the PSMTs still struggled with opposing direction inclusion relation. That is, they were able to state that a square is always a rhombus however a rhombus can never be a square as it was evidenced in the figure below. Although the PSMT in his work in figure 3 was able to draw the Venn diagram to demonstrate the relationships among quadrilaterals correctly, he argued that a rectangle cannot have all four equal sides or a parallelogram cannot have a 90-degree angle.


Aşağıdaki boşluklara " her zaman", " bazen", "hiçbir zaman" seçeneklerinden uygun olanlarını yerleştiriniz. Nedenlerinizi açıklayınız (14 puan).

a) Bir kare her zaman bir yamuktur.
Neden?

b) Bir eşkenar dörtgen her zaman bir paralelkenardır.
Neden?

c) Bir paralelkenar hiç bir zaman bir dikdörtgendir.
Neden?

d) Bir yamuk hiç bir zaman bir paralelkenardır.
Neden?

e) Bir dikdörtgen hiç bir zaman bir karedir.
Neden?

f) Bir kare her zaman bir dikdörtgendir.
Neden?

g) Bir dikdörtgen bazen bir eşkenar dörtgendir.
Neden?

Figure 3. A sample response to demonstrate limited opposing direction inclusion relation

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