IDENTIFYING STUDENTS' POSSIBLE SOLUTION STRATEGIES WHILE SOLVING QUESTIONS REGARDING THE CONCEPT OF MEAN

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Abstract: The purpose of this study was to investigate solution strategies used by seventh grade students regarding the concept of mean given in bar graph representations. Participants were 233 seventh grade students from two public middle schools in Gelibolu district of Çanakkale. Data were collected via a questionnaire. Students' possible solution strategies regarding the concept of mean were identified through item based in-depth analysis. The results of the study indicated that students used two different solution strategies to solve questions regarding the concept of mean. More specifically, the study indicated that the balance model and the average formula were identified as two solution strategies to solve the questions regarding the mean concept given in bar graph representations.

Keywords: Mean, solution strategies, bar graph representations

Introduction

In recent years, the emphasis laid upon statistics in the mathematics curriculum has increased by means of international movements (Jacobbe & Carvalho, 2011). One of the reasons underlying this increase in emphasis may be that statistics is an essential part of daily life. Another reason may be that statistics establishes links between other areas, such as history, science and geography (Konold & Higgins, 2003). Moreover, *The Principles and Standards for School Mathematics* (NCTM, 2000) set out of 5 content areas, one of which is Data Analysis and Probability (Pratt, 2005). Similarly, in Turkey, with the changes made in the mathematics program in 1990 and 1992, numerous concepts related to probability and statistics were added, and the new textbooks included basic concepts, such as measures of central tendency, measures of spread and probabilistic events (Bulut, Ekici, & Iseri, 1999). In addition, the elementary mathematics curriculum in Turkey included real life contexts for the purpose of making interpretations and making decisions by resorting to statistical knowledge (MoNE, 2005, 2013).

In accessible literature, although there are several studies that examine the problem solution strategies used by students (Becker, 1992; Cai 1995; 1998), there is a limited number of studies on the solution strategies used by students while solving questions regarding statistical concepts (Cai, 2000). Identifying students' possible solution strategies is significant because if a student have conceptual understanding regarding the averaging algorithm of the mean, the student can correctly and flexibly apply the averaging algorithm to solve questions regarding the concept (Cai, 1998; Watson & Moritz, 2000). However, the number of studies in Turkey is not sufficient regarding students' solution strategies while solving questions regarding the measures of central tendency concepts (Uçar & Akdoğan, 2009). In order to reveal important information about students' understanding of the average concepts, students' solution strategy preferences while trying to solve questions related to these concepts can be focused (Cai, 1998). Therefore, the purpose of the present study is to analyze the solution strategies utilized by students while solving questions regarding the concept of mean.

Method

The Research Design

In this study, the survey research design was preferred because surveys are conducted to define some aspects and characteristics of a population or a sample (Fraenkel & Wallen, 2006). More specifically, the cross-sectional survey design was used since it requires collecting data at one point of time from a selected sample from a predetermined population to describe the characteristics of the population (Fraenkel & Wallen, 2006).

Participants

Data was collected from 233 seventh grade students including 106 girls and 127 boys from 2 public middle school students in the Gelibolu district, Çanakkale, Turkey.
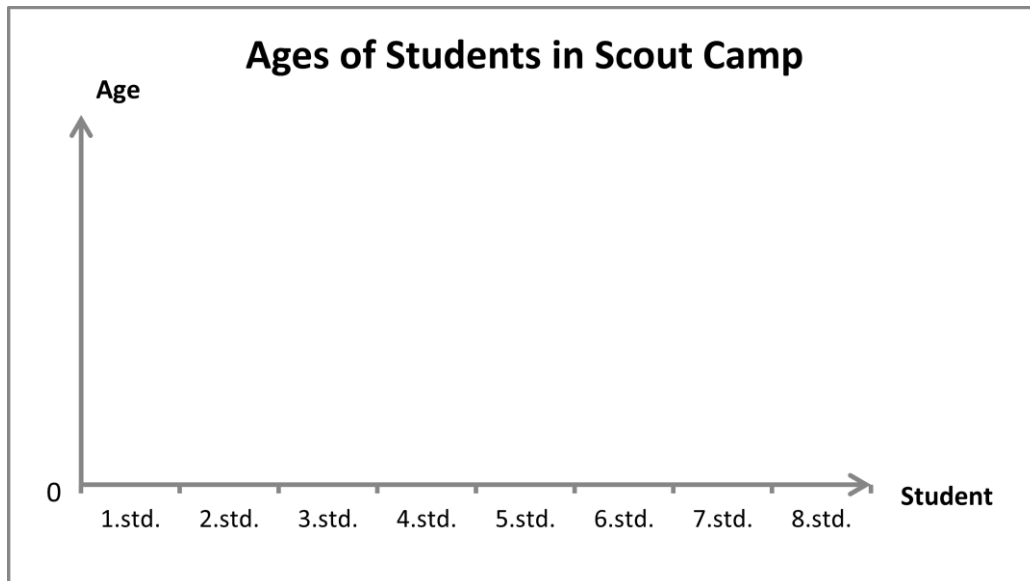
Instrument and Data Collection

In the research, a questionnaire was used as a data collection tool. The questionnaire consists of two open-ended questions in order to investigate solution strategies used by 7th grade students regarding the concept of mean given in bar graph representations. Two questions and their details were given below:

Question 1, which was adapted from study of Mokros and Russell (1995), consists of two parts. The question was asked to investigate students' solution strategies used to solve questions regarding the mean through construction of a data set for the given average.
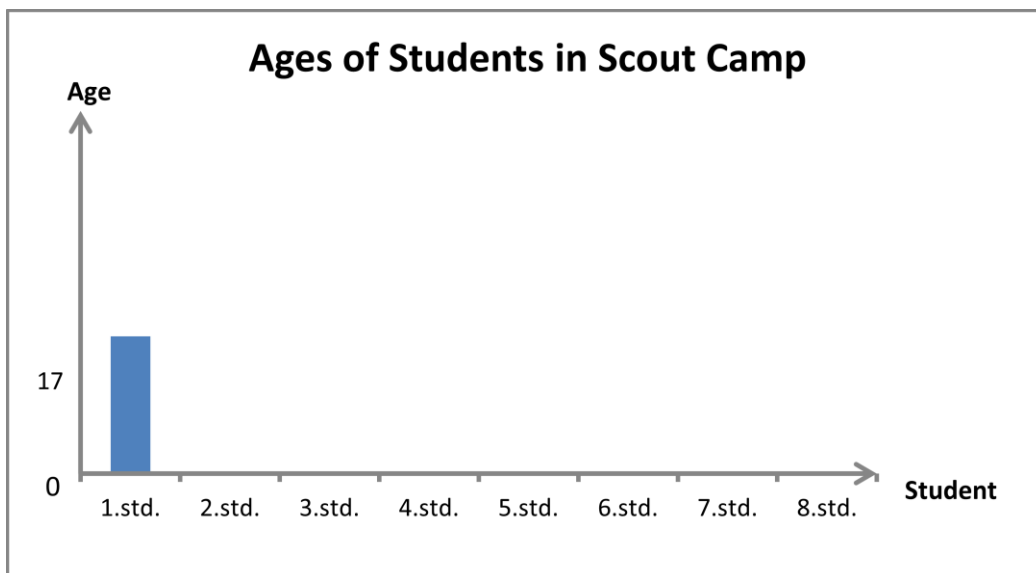
Question 1:

a) In a scout camp, there are 8 students whose ages are different from each other. Average age of the students is 15. Based on the information, what could their ages be? Draw a bar graph in order to show a possible data set for students' ages in the camp.

Ages of Students in Scout Camp

Age

0   1.std.   2.std.   3.std.   4.std.   5.std.   6.std.   7.std.   8.std.   Student

How did you give your answer? Explain.

…………………………………………………………..

b) In a scout camp, average age of 8 students is 15 and if age of a student is 17, what could be ages of other students so the average still comes out to 15? Draw a bar graph to show a possible data set for students' ages of the camp.

Ages of Students in Scout Camp

Age

17

0   1.std.   2.std.   3.std.   4.std.   5.std.   6.std.   7.std.   8.std.   Student

How did you give your answer? Explain.

....

Figure 1. The first and second part of question 1 in the questionnaire

Question 2 was developed by the researchers. The question was asked in order to identify used solution strategies to solve questions which were required calculation of the mean given in bar graph representations.

Question 2: Burning times of candles which A and B companies produced are wanted to be compared. In order to compare burning times of candles of A Company and B Company, the same size 8 candles from each company were chosen and the candles. Then, the candles were burned and the number of minutes that each candles burned were recorded.
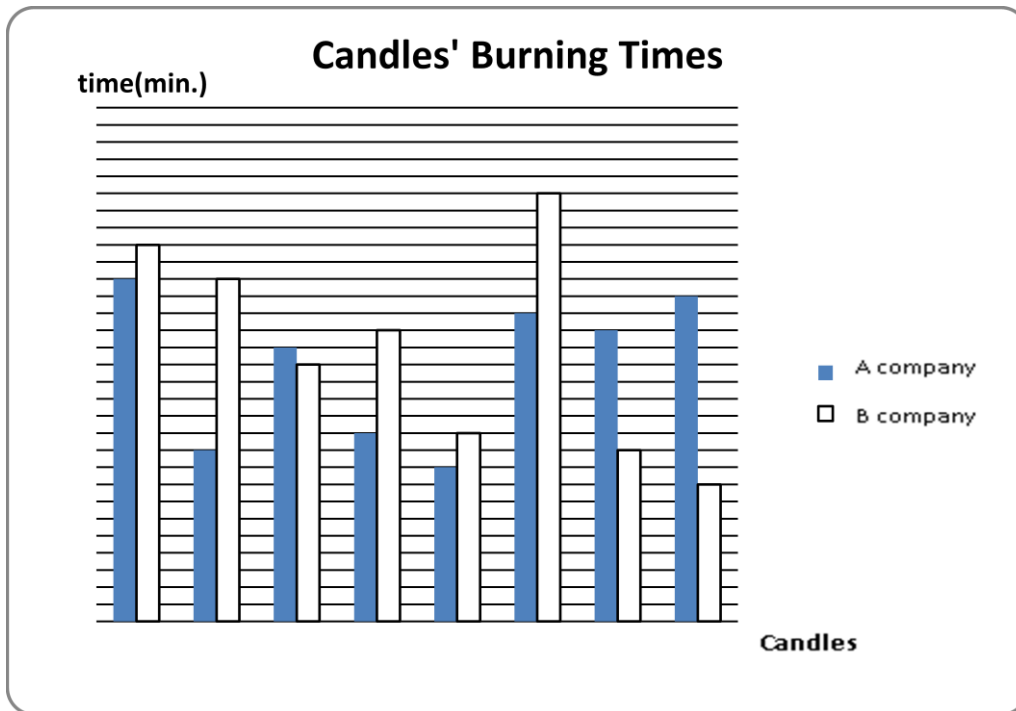
Determine the mean of burning times of candles for each company.
Mean
A Company …..........
B Company ………

b) Determine the median of burning times of candles for each company.
Median
A Company …..........
B Company ………

c) Ahmet claims that candles of A Company have longer burning times than candles of B Company. Do you agree with Ahmet?
....
- How did you decide whether Ahmet is right or not? Explain your reasoning.
....
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………………………………………………………………

Figure 2. Question 2 in the questionnaire

In general, the aim of asking these questions was to get knowledge about participants' alternative solution strategies while solving questions regarding the concept of mean. The items were reviewed by three experts from the Elementary Mathematics Education Department of different universities to provide content related evidence of validity of the instrument. As part of reliability, two researchers analyzed students' answers. A correlation of 98% was found between the two scorings. In order to identify students' solution strategies while solving questions regarding the concept of mean, item based in-depth analysis was conducted.

Findings

Solution Strategies

In accordance with purpose of the study, students' solution strategies were categorized for the mean concept. More specifically, *balance model* and *average formula* strategies were identified as solution strategies in order to

solve questions regarding the mean concept. Table 1 presents frequencies of the solution strategies employed by the students providing correct responses, for each item.

Table 1. Frequencies (and percentages) of solution strategies employed by students providing correct responses

STRATEGIES / ITEMS	MEAN	
	Balance Model	Average Formula
1-a	23 (31.1%)	51 (68.9%)
1-b	19 (30.2%)	44 (69.8%)
2	0	94 (100%)

As can be observed in Table 1, findings indicated that the majority of the students who solved the questions correctly used *average formula* as a solution method. For example, 100% of the participants used the averaging solution strategy to solve item 2. On the other hand, *balance model* and *average formula* solution strategies were used for items 1-a and 1-b as solution strategies. Examples of students' appropriate solution strategies for each identified strategy to solve questions regarding the mean are presented below.

1. Balance Model

The students, who used the balance model as a solution strategy, accepted the mean as a point of balance. For example, in one question, the mean of a data set is 5, and three of four data are 2, 4, and 6. While solving the question, student says that the mean of 4 and 6 is 5, then other data is 8 because 2 is balanced with 8 to obtain 5 as mean of the data set. Therefore, in this solution strategy, a value from a data set is balanced with the other value in order to obtain the mean of the data set. According to Table 1, the balance model was most frequently used solution strategy for item 1-a. It was seen that 23 students (31.1%) among 74 students who solved the item correctly used the balance model solution strategy to solve the item. On the other hand, 19 students (30.2%) among 63 students who solved item 1-b correctly also used the strategy to solve the item.

To illustrate, the solution strategy employed by Participant 19 for item 1-a is presented below:
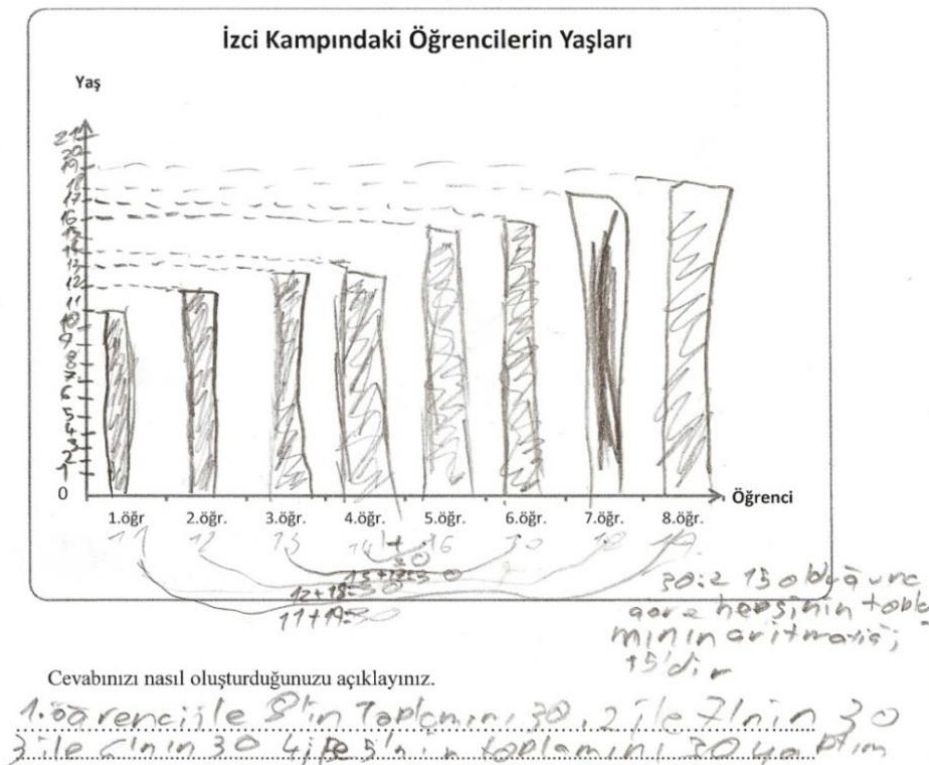
Participant 19:

Figure 3. Answer of participant 19 to item 1-a

Participant 19 stated that each value in the data set was balanced with the other value in the data set while constructing a possible data set for the given mean. Age of student 1 was accepted as 11 and the value was balanced with age of student 8 as 19 since he believed that the mean of 11 and 19 was 15. In addition, the age of student 2 was accepted as 12 and it was balanced with the 7th student's age which is decided as 18 since the mean of 12 and 18 was 15. Participant 19 identified the ages of the other students using the same procedure.

2. Average Formula: Students who used the average formula solution strategy while solving the questions regarding the mean concept used the "add and then divide "algorithm to solve the given questions. Thus, when the average of a data set was asked, the students added all the values in a data set and then divided it by the number of values. On the other hand, when a question was asked with a missing value from a data set, first the total number of values was found and then the total number of the given values in the data set was subtracted.

As can be seen in Table 1, the solution strategy was most frequently used in solving item 2. More specifically, it was observed that 94 students (100%) had used this solution strategy. Furthermore, 51 students (68.9%) among 74 students used average formula solution strategy to solve item 1-a and 44 students (69.8%) among 63 used strategy to solve item 1-b.

For example, Participant 154 used the solution strategy to solve item 2 as presented below:

Participant 154:

Figure 4. Answer of participant 154 to item 2-a

As can be observed in the solution of Participant 154, to find arithmetic average of the burning times she added all the burning times of the candles for each company and then divided the sum by the number of candles in the companies. Thus, the participant used the average formula solution strategy.

In general, based on the results of the study, the average formula solution strategy was most frequently used in the questions requiring the calculation of the mean when values of a data set were given. On the other hand, the strategy was also used in the questions requiring construction of a data set for the given mean of a date set.

Conclusion

Findings of this study revealed that the seventh grade students used two different solution strategies to solve the questions regarding the concept of mean. The solution strategies were named as the balance model and the average formula.

In the present study, while finding the mean of a data set or a missing value of a data set for the given mean, the average formula solution strategy was used mostly. This finding was in agreement with the study of Mokros and Russell (1995), where average of a data set was given and the construction of a possible data set was required. In other words, one of the five different constructions of representativeness was algorithm (arithmetic average) in the study of Mokros and Russell (1995). More precisely, most of the students used the average formula solution strategy while constructing a data set for the given mean (Mokros & Russell, 1995). This finding was also consistent with the results of studies conducted by Cai (1998; 2000) who stated that most of the participants also used average formula solution strategy to solve averaging problems. Students' tendency to use the average formula solution strategy to solve the items could be attributed to the fact that only this strategy might have been used by the mathematics teacher to find the mean of a data in the mathematics classes.

On the other hand, some of the students used the balance model solution strategy to solve these kinds of questions. The balance model solution strategy was generally used while constructing a possible data set for the given mean compared with the solution of the questions which required finding the mean of a data set. This finding was also in agreement with the study of Mokros and Russell (1995) since the findings of the study indicated that one of the constructions of representativeness of average was accepted as the balance point. Furthermore, the results of the studies of Cai (1998; 2000) showed that participants used the levelling solution strategy to solve questions regarding the mean, which was similar to the balance model solution strategy used in

the present study. The balance model solution strategy could be considered as an invented strategy since the mathematics' textbooks in Turkey do not include this model as a solution strategy to solve the mean questions. Thus, this strategy could be included into the textbooks as an alternative strategy to solve the questions related to the concept of mean.

Recommendations

Findings of the present study were limited with two questions of the questionnaire however when different questions were asked related to the concept of mean, different findings could be reached. Furthermore, a similar study might be conducted in private schools to investigate private middle school students' understandings regarding the concept of mean. Moreover, a longitudinal study could be conducted to investigate the development of middle school students' procedural and conceptual knowledge regarding the mean concept based on their grade level.

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