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THE DEVELOPMENT OF AN INQUIRY BASED LEARNING UNIT FOR INTEGRAL CALCULUS: THE CASE OF VOLUMES OF SOLIDS OF REVOLUTION

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ABSTRACT: The latest official national curriculum published by the Turkish Ministry of Education, now formally requires high-school mathematics teachers to actively incorporate computer software in their teaching. The primary purpose of this study is to demonstrate the development of an inquiry based learning unit especially geared for high school mathematics students and teachers for the general concept of integral calculus. The main theme chosen as a case for this proposed inquiry unit, is on volumes of solids of revolution of real life daily objects. As a result, the primary purpose will provide a report about a practical example of using pedagogically driven mathematics software, 3D digital modeling coupled with hands-on real life examples all embedded in a constructivist learning environment. A secondary goal of this study is to expose pre-service high-school mathematics teachers to this inquiry unit as a teacher and student. Finally, the shared experiences about the interconnected nature of knowledge construction through a double "lens", that is as a teacher and students through collaboratively building, testing and reflecting on their learning process will be reported.

Keywords: Mathematic Education, Inquiry, Modeling, Teacher Education

INTRODUCTION

The perspective of Turkish Ministry of National Education on technology in the new curriculum

The current official Turkish high school mathematics curriculum dictates the use of technology in mathematics classrooms (MoNE, 2013). The Turkish MoNE mathematics curriculum points out the fact that, both quality and quantity of teaching software related to mathematics education has increased as a result of the constant development of computer technology. Hence, the ministry expects mathematics teachers to make use of technology in mathematics classrooms. They emphasize that the utilization of technology could provide new learning and teaching opportunities for both teachers and students alike. Accordingly, using information and communication technology effectively, students could work on mathematical problems related to real life and they could spend more time on reasoning and creative thinking rather than time consuming computations.

The new mathematics curriculum summarizes the main information and communication technologies to be used in mathematics classrooms as; dynamic geometry software, graphing software, spreadsheet software, graphing calculators, interactive smart boards and tablets, data acquisition devices, computer algebra systems, dynamic statistics software and Internet. Correspondingly, Turkish MoNE expects students to use these technologies effectively in the new curriculum. Thus, students could explore the mathematical concepts through experiencing different types of thinking skills.

Inquiry based learning

Inquiry based learning is a concept which enables students to involve in conceptual understanding and builds students' ideas through inquiry (Chapman, 2011){Formatting Citation}. The concept of inquiry based learning has grown since Dewey (1938) supported that students can learn better when they investigate the problems according to their own experiences as cited in Barrow (2006). Several studies emphasizes that inquiry based

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learning plays a remarkable role in mathematics education (Chapman, 2011; Hähkiöniemi, 2013; National Council of teachers of Mathematics, 2000). However, there are some challenges in implementing inquiry based teaching in mathematics classrooms (Dorier & Garcia, 2013). Teacher beliefs and attitudes towards inquiry based mathematics teaching might be the main reasons of those challenges (Engeln, Euler, & Maass, 2013). Therefore, teachers should be supported to engage students in conceptual understanding through inquiry based teaching (Hähkiöniemi, 2013).

Technology enriched inquiry based learning

Many researchers support that integrating technology into mathematics classrooms enriches inquiry based learning (Hähkiöniemi, 2013; Hakverdi-Can & Sönmez, 2012; Wentworth & Monroe, 2011). According to Hahkiöniemi (2013), particularly dynamic mathematics software promotes students' investigation and exploration opportunities. Similarly, Healy and Hoyles (2001) suggest that, with the help of dynamic mathematics software, "students move from argumentation to logical deduction" (p. 235). Moreover, students could relate one geometrical representation of a formula to another and they could build hypotheses through trial and error that they apply with dynamic mathematics software.

In this study, GeoGebra was used as the dynamic mathematics software. With the help of GeoGebra, students are able to see the relationship between the theoretical formulas and their geometric meanings. Thus, students could explore the use of those formulas in their daily lives.

Teaching integral calculus

Integral calculus is perceived as challenging by many of high school students in developing conceptual understanding because of many formal definitions. According to Orton (1983), students are struggling while solving problems that they need to notice integration as a limit process of sums. Moreover, Attorps, Björk and Radic (2013) suggest that some students perceive solving integral problems as technical ability and although they do all the calculations successfully, they may not achieve conceptual understanding. As a result, the reason why students have difficulties in integral calculus is that they fail to construct a concrete meaning of the formal definitions in their mind.

The main purpose of this study is to develop a material for teachers to provide students' conceptual understanding in integral calculus through a technology integrated inquiry based lesson. Since inquiry based learning requires to work on a concept like a scientist, first of all students should understand how scientists found integral calculus. If students could understand for which need they do the integration calculations, they could build their own hypothesis, and thus, they could achieve conceptual understanding instead of solving integral calculus problems technically.

The volume of solids of revolution

In this study, particularly volume of solids of revolution was chosen as a case for this proposed inquiry based unit plan. The volume of solids of revolution is a concept of integral calculus which is thought to high school students. To get a solid of revolution, first of all, a function $y=f(x)$ is needed on an interval $[a,b]$. When we rotate this function about the x-axis, we get a three dimensional region which is called as a solid of revolution. In order to find the volume of this solid, we use the Riemann Sums approach by which we find the area under a curve by dividing the interval into finite many small subintervals as rectangles with the width Δx and length $f(x_i)$ and summing the areas of these rectangles, passing to the limit where the length of the largest subinterval goes to zero. The representation of this sum is given by the formula;

$$f(x_i)\Delta x = ¿\int_a^b f(x)dx$$

$$\sum_{i=1}^{n} ¿A = \lim_{\Delta x \to 0} ❑$$

In the case of solids of revolution we get circular disks with the length Δx and radius $f(x_i)$ instead of rectangles, and we calculate the sum of the volumes of these disks passing to the limit where the length of the largest subinterval goes zero. The volume of each circular disks are given by

$$\pi f(x_i)^2 \Delta x$$

and the sum of these circular disks passing to the limit where Δx goes zero is given by the formula;

$$V = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \pi f(x_i)^2 \Delta x$$

Finally, we use the integral formula to compute the volume V of the solid of revolution;

$$V = \pi \int_a^b f(x)^2 dx$$

This study will provide students to relate the integration formula for volumes of solids of revolution to their daily lives through hands-on activities. These activities will be done in a laboratory setting. Students will first find the volume of a solid that is obtained by revolving a curve around various lines such as glasses and bottles by a volume measuring cup. Accordingly, students will insert the image of their object on GeoGebra in correspondence with the size of real object. Then, students will find the function of the object by plotting on the borders of the object and finding the line of best fit. Thus, students could apply the integration formula for volumes of solids of revolution with that line. As a result, students will see that the result approximately corresponds with the volume that they found first. The figures below depict three examples.

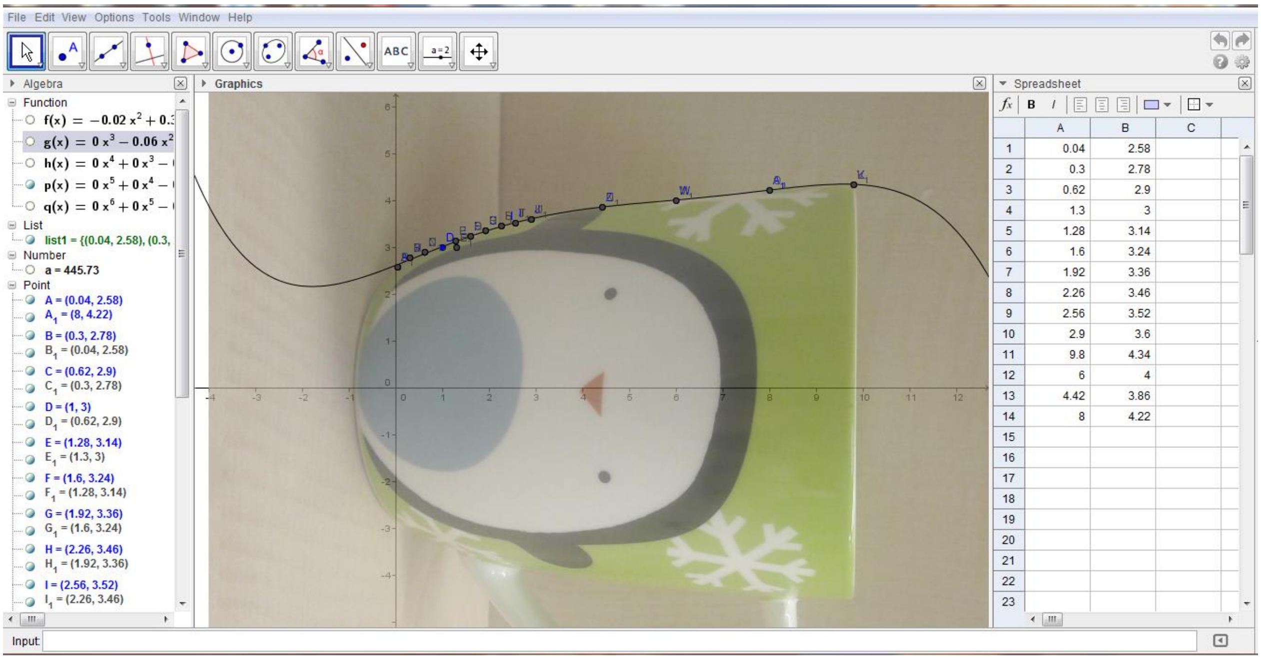

Figure 1. **Volume Of A Mug**

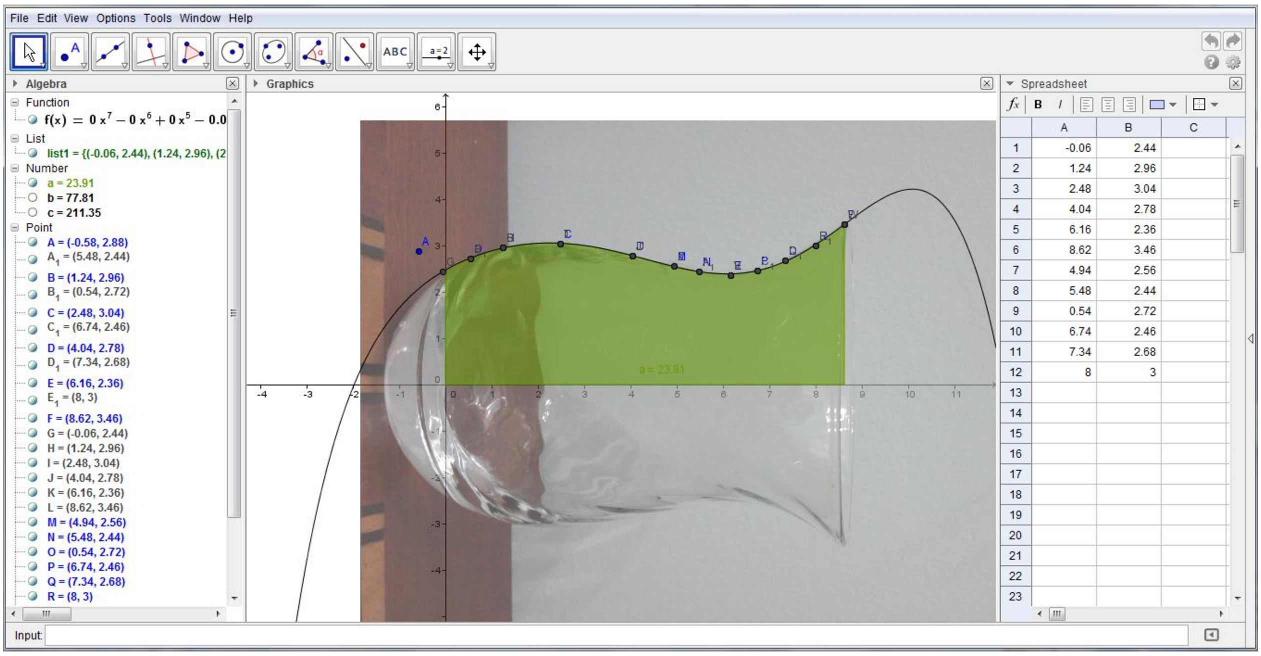

Figure 2. **Volume Of A Traditional Turkish Tea Glass.**

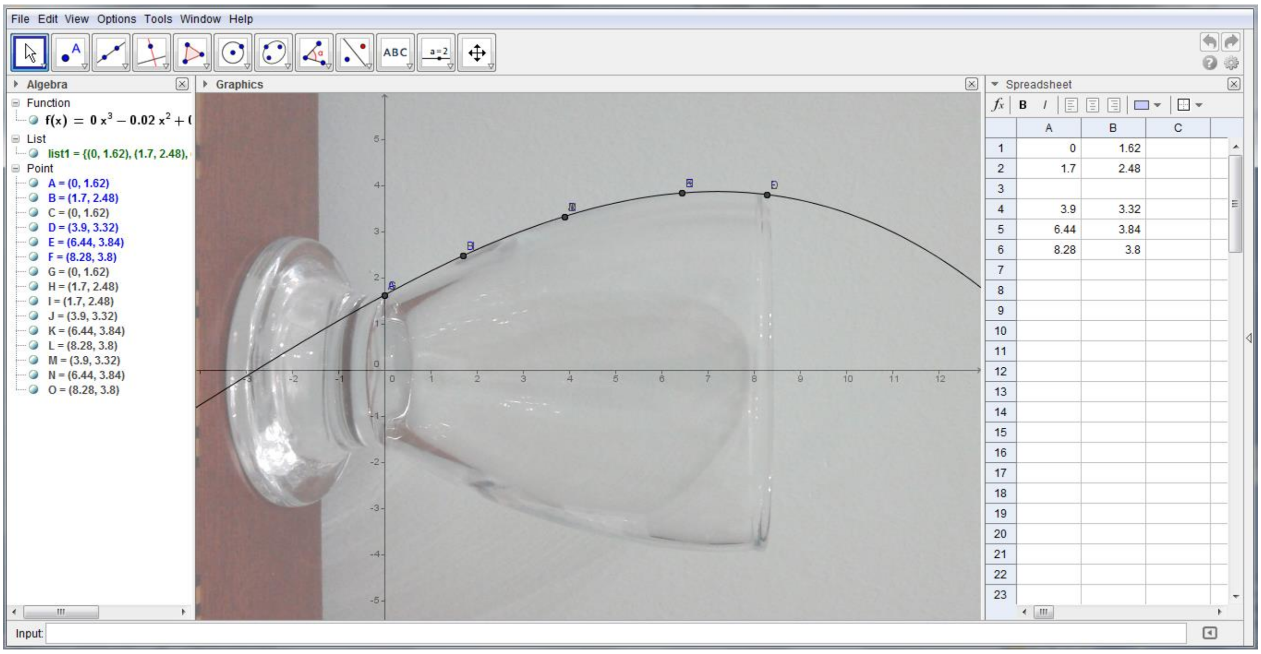

Figure 3. **Volume Of A Glass**

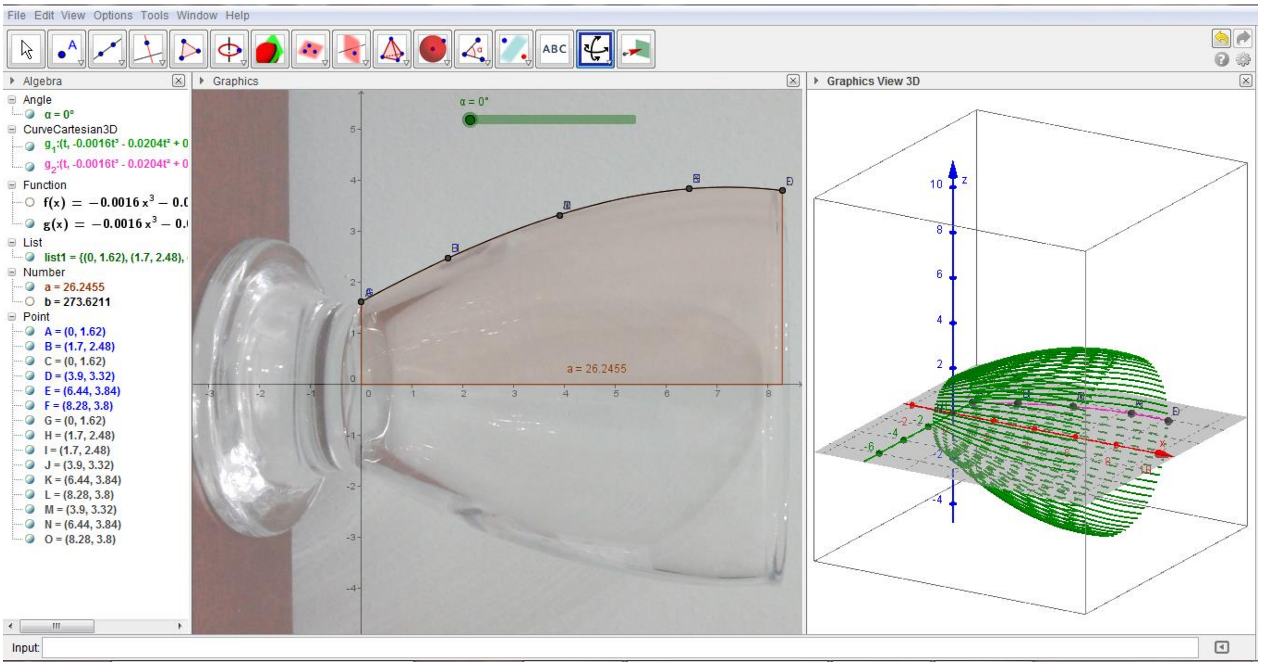

Figure4. **3D Representation Of Volume Of A Glass**

I will now explain how I had done one of these sample experiments. For example, in *Figure 3* and *Figure 4* firstly I filled the glass with water and then transferred the water into a volume measuring cup. Thus, I measured the volume of the glass as 275 ml which is equal to 275 cm^3. Then I took a photo of the glass and I inserted the image into GeoGebra as a background picture. I arranged the properties of the picture in correspondence with the length of filled part of the real glass which is approximately 8cm on the coordinate system. In order to find the curve of the border of the glass I plotted some points on the border of the glass and I entered x and y coordinates of those points on the spreadsheet view of GeoGebra. Thus, I found the line of best fit by creating a list of these points and entering them into the input box with the

FitPoly[<List of Points>, <Degree of Polynomial>]

command. I chose the degree of polynomial as three which gave the best curve of the border of the glass and I found the line of best fit function as

$$f(x) = -0.002x^3 - 0.02x^2 + 0.541x + 1.622$$

Accordingly, I calculated

$$\pi \int_0^{8.28} f(x)^2 \, dx$$

which is the formula for volume of solids of revolution by using input box command

Integral[<Function>, <Start x-Value>, <End x-Value>].

Consequently, the calculation gave the volume of the glass as 273.621 cm^3 which is an approximate value of the real volume of the glass.

My experiences as a student and as a teacher

In this section, I mentioned my experiences in terms of my lack of mathematics knowledge during my formal education and my difficulties in using dynamic mathematics software which I was not familiar with. Firstly, I mentioned how I was taught mathematics in particular integral calculus during my formal education. Then I remarked the importance of using dynamic mathematics software in mathematics education through my experiences. Finally, I reflected my experiences in terms of difficulties that I had during the process of doing modeling activities by using GeoGebra.

The major portion of my formal education was formed in a traditional way. As far as mathematics education was concerned, I had difficulties in abstract mathematical concepts such as integral calculus, because I always memorized the formal formulas and applied those formulas to challenging problems. Therefore, I always tried to improve my memorization skills; however I had forgotten the formal formulas in a short time. Thus, I had never learned the basic logic behind these concepts. For example, when I learned the integration formula as antidifferentiation, I did a lot of challenging exercises without knowing what I came up with. All those calculations were related to transforming one expression to another. I was successfully computing that;

$$\int \frac{4}{1-2x}dx = -2\ln|1-2x|+c$$

by using integration properties, but I did not know why I was doing these computations. However, I actually did not need to know the reason, as only successful computation was enough to get good marks from the exams.

Moreover, although I knew that $\int_a^b f(x)dx$ formula refers to the area under $f(x)$ curve between $[a,b]$ interval and $\pi\int_a^b f(x)^2 dx$ formula refers to the volume of revolution of $f(x)$ curve, I did not know how these formulas give the area or a volume. On the other hand, I was good at solving 'Find the area under $f(x)$ ' or ,'Find the volume of revolution of $f(x)$ ' types of problems, as I memorized which formula to be used for which situation, thus I applied the appropriate formula and did the appropriate calculations. Since I was studying exam orientated, I never wondered if these formulas really give an area or a volume.

As I experienced that dynamic mathematics software enable students to see the geometric representations of mathematical calculations, I was interested in using GeoGebra in teaching of integral calculus. By doing these GeoGebra experiments, I saw the instant visual geometrical outcomes of my calculations, thus I could analyze if they are meaningful or not. For example, firstly I found the volume of a glass by a measuring cup, but when I inserted the image of the glass and applied the integral formulas; I found a value which did not correspond with the real volume. Then, I realized that I did not place the image on the coordinate system in correspondence with the real glass. Moreover, as the notation of GeoGebra is similar to mathematical notation, it was easy to understand and use. Using GeoGebra also enabled me to consciously decide what operations to use as I saw the geometric representations of my operations. As a teacher, I thought that I could easily gain students curiosity through this kind of real life experiment and I could provide them with meaningful and conceptual understanding. For example, as a student I felt a great pleasure when I found the volume of the glass via Geogebra approximately same with the real volume of the glass.

On the other hand, I also had difficulties in doing the modeling activities both as a teacher and as a student. For example, I had never used dynamic mathematics software in my mathematics education; therefore it took time to get the basic logic behind GeoGebra. In order to do these kinds of experiments, firstly I had to learn the basic commands, notations and features of GeoGebra. However, there were plenty of sources that I could utilize through this process. For example I utilized from the GeoGebra Tube, GeoGebra forum, or YouTube videos created for teaching mathematics via GeoGebra. Nevertheless, sometimes it took a lot of time to find the appropriate feature that I need to use for my experiments. Although a lot of outcomes that I need to achieve are shown in applets or videos, I had difficulties in understanding which features of GeoGebra were used to achieve those outcomes. For instance, I had difficulties in finding the appropriate function that fits to the edges of my objects. Thus, I gave up many times. I needed to be patient in order to get the appropriate outcomes.

The views of pre-service high school mathematics teachers

The secondary goal of this study is to get the views of pre-service teachers' as both students and teachers through this inquiry based unit plan. Accordingly, a set of tutorials will be given to pre-service high school mathematics teachers and their learning process will be reported. Afterwards, their view of teaching integral calculus via a technology integrated inquiry based unit plan will be reported through several interviews.

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