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GENERALIZING REPEATING PATTERNS: A STUDY WITH CHILDREN AGED FOUR

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ABSTRACT: This paper presents part of a study that aimed to understand how the emergence of algebraic thinking takes place in a group of four-year-old children, as well as its relationship to the exploration of children's literature. To further deepen and guide this study the following research questions were formulated: (1) How can children's literature help preschoolers identify patterns?; (2) What strategies and thinking processes do children use to create, analyze and generalize repeating and growing patterns?; (3) What strategies do children use to identify the unit of repeat of a pattern? and (4) What factors influence the identification of patterns? The paper focuses only on the strategies and thinking processes that children use to create, analyze and generalize repeating patterns. The present study was developed with a group of 14 preschoolers in a private school in Lisbon, and it was carried out with all children. In order to develop the research, a qualitative research methodology under the interpretive paradigm was chosen, emphasizing meanings and processes. The researcher took the dual role of teacher-researcher, conducting the study with her own group and in her own natural environment. Participant observation and document analysis (audio and video recordings, photos and children productions) were used as data collection methods. Data collection took place from October 2013 to April 2014. The results of the study indicate that children master the concept of repeating patterns, and they are able to identify the unit of repeat, create and analyze various repeating patterns, evolving from simpler to more complex forms.

Key words: children's literature; repeating patterns; algebraic thinking in four-year-old children.

INTRODUCTION

Mathematics is part of our life and we use it to solve different problems on a daily base. Children are no exception and they use it intuitively when they play and when they need to solve problems. Considering the children's ability to interpret stories and the association that literature makes with real life as well as our imaginary world, we think that the understanding of several basic mathematical concepts can be developed from certain children's books, recognizing the potential of children's literature in learning mathematics (Loureiro, 2006; Smole, Rocha, Cândido, & Stancanelli, 2007). When performing connections between children's literature and mathematics, children are encouraged to make use of mathematical ideas they already know and to extend mathematical concepts, bringing out new content.

The algebraic thinking, in particular, can be developed within the context of children's literature, considering both illustration and text. Several researchers reported the importance of early development of algebraic thinking (Garrick, Threlfall, & Orton, 1999; Threlfall, 1999) beginning by the study of patterns right from kindergarten (Threlfall, 1999; Borralho, Cabrita, Palhares, & Vale, 2007). Threlfall (1999) reinforces the idea that repeating patterns prepare children for future algebra learning and offer a vehicle for learning to interpret symbols. Working with repeating patterns in preschool encourages logical thinking development and helps to develop the generalization of rules about the patterns made.

This paper presents part of a study (Serra, 2014) that aimed to understand how the emergence of algebraic thinking takes place in a group of children aged 3,5 to 4,5 years, as well as its relationship with the exploration

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of children's literature. To further deepen and guide this study the following research questions were formulated: (1) How can children's literature help preschoolers identify patterns?; (2) What strategies and thinking processes do children use to create, analyze and generalize repeating and growing patterns?; (3) What strategies do children use to identify the unit of repeat of a pattern? and (4) What factors influence the identification of patterns? This paper focuses only on the repeating patterns of the 2nd and 3rd questions. The present study was developed with a group of 14 preschoolers in a private school in Lisbon and it was carried out with all the children**.**

THEORETICAL FRAMEWORK

Algebraic Thinking

The word algebra may be regarded by the common people as associated with formulas and equations, letters and symbols that are manipulated and worked only at high levels of education (Suh, 2007), leading teachers themselves into thinking that algebraic thinking should not be promoted soon. Moreover today, algebra is seen, more broadly, as a generalizing and human activity. According to Kaput (2008), there are two essential aspects of algebraic thinking: (a) the generalization and formalization of patterns, and (b) symbolic manipulation.

Blanton and Kaput (2011) define algebraic reasoning as a generalizing activity of mathematical ideas, defending its development in elementary levels, and they refer to this activity as *early algebra*. This is a kind of activity that prepares children for developing structures and mathematical generalization modes. As referred by Carraher, Schliemann, & Schwartz (2008), early algebra is not the same as algebra early. Early algebra is sought to develop algebraic thinking in a way that includes an understanding of mathematical structures represented by the language and the gestures, using concrete materials and representations, and in this sense, it does not mean beginning algebra´s study earlier than usual (Warren & Cooper, 2008). Children today need to learn a significantly different mathematics than the one their parents learned, using for this significant experiences that lead them to recognize and develop structures and mathematical relationships, using objects for mathematical reasoning (Blanton & Kaput, 2011). This will be the path for young children to become mathematically successful later on (Blanton & Kaput, 2011). Moreover current research has shown that young children can generalize mathematical ideas much earlier than previously supposed (Mulligan, 2013).

Patterning at the Preschool Level

Borralho et al. (2007) assert that the learning of patterns in preschool assist the development of logical thinking, being a way to explore other mathematical content and to create a foundation for the future learning of algebra. Papic, Mulligan and Mitchelmore (2011) consider repeating patterns, which have a cyclic structure that is being repeated, suitable for work in preschool. Thus, there are many children who would spontaneously create simple repeating patterns using different classroom materials, such as bead necklaces and other manipulatives, or representing them in the drawings and clothing decorations (Threlfall, 1999). So it is important to have materials available to children as diverse as shapes, stones, shells, bears or other objects. It is also fundamental to make children use sounds, with the body or with musical instruments, movements, and iconic and symbolic representations (colored dots, letters, numbers) to help them to generate and generalize repeating patterns (Palhares & Mamede, 2002; Threlfall, 1999). As referred by Vale et al. (2011), it is important to encourage children to see patterns using different materials, modes (colors, shapes, gestures, words) or symbols (letters or numbers) in order to identify that the structure of a pattern does not depend on the material used. Palhares and Mamede (2002) propose the exploitation of different representations of the same pattern, so that children can generalize and identify patterns in other contexts.

Palhares and Mamede (2002) report that different pattern types can be exploited in preschool and, based on the articulation of their differences and similarities, clustered as follows: (a) with an alternative component, which may be unique (the AB type); (b) with an arithmetic progression component (the ABAABAAABAAAAB type); (c) with a symmetric component (the ABABBABA type); and (d) by adding a second dimension (ABABAB BABABA ABABAB).

In curricular terms, patterns assume a higher standard as a unifying theme or as a supporter for meaningful learning (Borralho et al., 2007; Vale et al., 2011). According to NCTM (2000/2007), the patterns are the foundation of algebraic thinking and working with patterns invites students to identify relationships and to make generalizations. This document also proposes the inclusion of exploratory activities that make use of diverse materials, encouraging the ability to continue patterns and cope with different properties of algebraic relations.

Mathematics has been an important subject area within the Portuguese curricula for preschool education. The Portuguese Curriculum Guidelines for Pre-School Education (PCGPE) (DEB, 1997), in the mathematics domain,

propose the use of repeating patterns, giving as an example the days of the week, or growing patterns, namely the sequence of natural numbers. These activities aim to develop the logical reasoning in tasks in which children discern the underlying rule of a given pattern or create their own patterns. Also, in musical domain, PCGPE refers to the construction and the discovery of musical or rhythmic patterns (DEB, 1997). In the area of language, we can find patterns in rhymes or stories that have linguistic rhythms, which can be transformed into mathematical sequences.

Research on Repeating Patterns in Early Years Classrooms

According to Threlfall (1999), several empirical studies undertaken with preschoolers suggest two independent strands to development in repeating patterns: their complexity and the way children see them. Concerning the first strand, the author states that the AB pattern (the simplest alternating elements type) is more common among younger children than the ones with more complex elements or with more than one attribute (color and size and shape, for instance, instead of just color). In the second strand, the author emphasizes the importance of children's awareness that a pattern is a whole being related to a unit of repeat. The identification of the unit of repeat can occur in two ways: by a chant that emphasizes the unit of repeat by the rhythm used (eg: red blue blue, red blue blue) or by an explicit reference to the unit of repeat (eg: one red and two blues). Threlfall (1999) stresses the higher level of awareness present in this last way of describing the pattern. The level of awareness of the unit of repeat has implications for generalization (Vale et al., 2011) and mathematical understanding (Threlfall, 1999).

Rustigian (cited by Threlfall, 1999) observed how 3- to 5- year old children explore repeating patterns and concluded that finding a physical movement (enactive mode) when dealing with a pattern representation was easier than finding a pictorial representation (iconic mode), and that shape attribute was easier than color attribute. This author further identified a progression in children's procedures when asked to extend a given pattern. The responses were ordered by a hierarchy of response: (a) *random selection* of new elements, without reference to prior elements; (b) *repeating the last element* (perseverance); (c) *use of the previous elements but in any order*; (d) *a symmetrical approach* however the given sequence is inversely reproduced; and (e) *a deliberate continuation of the pattern*, looking at the start in order to check the elements to be put (Threlfall, 1999). In research undertaken by Palhares (cited by Palhares & Mamede, 2002) with 4- to 6- year old children, when working with the AB repeating patterns within the color attribute, the children were able to continue the given model and identify that the same pattern existed in objects around the room but found it difficult to make other types of patterns using the same material. Recently, Papic et al. (2011) conducted a study with 53 preschool children aged 3 years 9 months to 5 years old, implementing a 6-month intervention in only one group, focused on repeating patterns. They concluded that children intervened showed great understanding of the unit of repeat and the structure of a pattern. These authors identified five strategies that children use when they work with repeating patterns, increasing the order of sophistication: (a) *random arrangement* (the elements are placed randomly without any care about their place and orientation); (b) *direct comparison* (when copying a pattern, children make a one to one correspondence, matching item by item); (c) *alternation* (children focus on independent successive items independently of the unit of repeat; for example, green, then blue, then green; and not on the unit of repeat, e.g., green-blue); (d) *basic unit of repeat* (children identify the unit of repeat, regardless of the number, type and complexity of elements and attributes, and use it to extend the pattern); and (e) *advanced unit of repeat* (as the children develop their sense of the unit of repeat, they can transfer the same pattern in different modes or materials, reconstructing it in more creative ways).

The study undertaken by Garrick et al. (1999) points to the ease of children to identify patterns of their own creation contrasting with the difficulties in the recognition of patterns created by others. According to Vale et al. (2011), most preschoolers when creating repeating patterns, create n(A) m(B) y(C) patterns where n, m and y, may range from 0 to 3. In repeating patterns with the color attribute, these authors observed some children who invent additional spaces or in the last space overlap the colors needed to complete the unit of repeat. Relatively to the two-dimensional patterns, they refer that children tend to not respect the regularity in column.

The generalization occurs when children can determine that the pattern has a unit of repeat that is repeated cyclically, and using different materials or forms, they are able to recognize the structure pattern (Papic et al., 2011). Indeed, it is the awareness of the structure pattern that allows them to generalize. Mulligan (2013) reports studies with 4- to 8- year old students that show that the awareness of mathematical pattern and structure is a critical aspect, and simultaneously a fundamental one, to their mathematical development. According to the author, it is important to implement a pedagogical approach to promote the pattern awareness since this awareness is correlated with mathematical understanding. With appropriate designed and implemented learning experiences, young children are able to develop forms of reasoning involving the process of generalizing (Papic et al., 2011).

METHODOLOGY

A qualitative research methodology under the interpretive paradigm was chosen, emphasizing meanings and processes (Bogdan & Biklen, 1994). The researcher, second author of this paper, took on the dual role of teacher-researcher, conducting the study with her own group and in her own natural environment. The participant children were 14 preschoolers aged 3 years 6 months to 4 years 6 months, at the beginning of data collection, in a private school in Lisbon. The group was active, curious and interested in mathematics, which is one of the most sought after areas in the class. To make the data collection permission was sought from both the School Director and the parents of the participant group children. Taking into account some ethical issues (Bogdan & Biklen, 1994), fictitious names are being used. We present the ages of the children referred in this paper in table 1:

Table 1. Children Names and Ages

Fictitious names	Ages (years; months)
António	4;4
Dinis	3;11
David	4
Frederico	3;10
Fernando	4;6
Guilherme	4;2
Joaquim	4;2
Jacinto	4;5
Luísa	4;3
Mário	4;6
Matilde	4;3
Tatiana	3;6

Participant observation and document analysis (audio and video recordings, images and documents produced by the children) were used as data collection methods. All the children's drawings representations (when produced) and their recorded explanations supplemented the video recordings as well as field notes and photographs. The group was accustomed to the teacher making digital recordings and taking photos of classroom activities.

Data collection took place from October 2013 to April 2014 and were proposed eleven tasks based on two children's literature books and lasted about 30 minutes each. In this paper we will address only some tasks that are presented in table 2. Their numeration corresponds to the order they were implemented.

Table 2. Books, Tasks, and Descriptors

Book	Tasks	Descriptors
The very hungry caterpillar Eric Carl	1st task *Coloring the caterpillar*	Create repeating patterns coloring on paper the caterpillar's body with a limit of 20 rings.
	2nd task *Reading the caterpillars patterns*	Read orally the created repeating pattern.
	3rd task *Reading the caterpillar pattern by gestures*	Use gestures to represent the created pattern.
	4th task *Creating patterns using gestures*	Create repeating patterns using gestures. Color in an orange sequence the gesture pattern from memory.
The house of Fly Fosca Eva Mejuto	6 th task *Decorating Fly Fosca's House*	Copy and continue AB, ABC, ABB, AABB and ABBB patterns using a single attribute (shape). Read orally the repeating patterns. Identify the unit of repeat. Identify the pattern type.
	7 th task *Decorating Fly Fosca's House with candies*	Copy and continue AB, ABB, ABC and ABCD patterns using two attributes (color and shape). Read orally the repeating patterns. Identify the unit of repeat. Identify the pattern type.

The analytical categories related to this paper were the following: the creation, reading, copying and extending repeating patterns, the identification of the unit of repeat and the generalization process.

RESULTS

Creating Patterns

In the first task, *Coloring the caterpillar*, children were asked to color on paper a caterpillar's body with a limit of 20 rings, choosing the colors they wanted in order to create a pattern. Children started coloring right to left, from head to tail. The way of coloring was spontaneous and not suggested by the teacher. Some children claimed they could color the caterpillar with the same colors that they used to make necklaces -- a task performed on another day.

Fernando - I already know my pattern. I will do the same as I did.
David- I'm going too do it too.
Guilherme – I'll do, yellow orange, yellow orange, yellow orange ...
(...)
Fernando - One red and two blues.

Several children referred to what colors they were going to use before starting to color, identifying at the same time the number of pens that they were going to need. During this task, the teacher questioned children trying to mobilize their informal ideas about patterns but also to see if they could identify what was being repeated and similarities and differences between the patterns that were being created.

Guilherme – I'll do, orange, yellow.
Teacher - You'll need how many pens?
Guilherme - Two.
(...)
Teacher - And Joaquim? How many colors will you use? (*Joaquim shows 3 fingers*). Three colors? Good!
(...)
Mário - Me! I'm doing with two colors, red and blue. It is red, blue blue, red blue blue. (*Reading his pattern as far as he had already colored*).

The colored caterpillars were of three different patterns (figure 1):

Figure 1. AB, ABC and ABB Patterns

So the strategy that some children used was previously thinking about the colors they were going to use, simplifying the creation of a pattern. Two of the children, Joaquim and Guilherme, put the necessary pens to color the caterpillar near the drawings, removing them from the box, already showing some sense of the unit of repeat. The way Fernando verbalized the pattern created earlier in beaded necklaces and now replicated in the task of coloring the caterpillar -- "One red and two blue" -- indicates a higher level of understanding of the unit of repeat, as he referred to it explicitly (Threlfall, 1999). Children were able to relate the various types of pattern with two colors (AB or ABB) or with 3 colors (ABC).

Children, who did not put their pens outside of the box, used the strategy of looking back to the beginning of the caterpillar to check the correct order of the colors. Dinis (figure 2) used a symmetrical approach. He started by using the sequence of colors of purple, red and blue and at the 10^{th} ring, he reversed the sequence of colors, placing purple, blue and red. Probably Dinis looked at what he had already colored, from left to right, reversing the sequence and did not look at the beginning of the caterpillar, from the head to its end.

Figure 2. Dinis's Caterpillar

António tried to create a pattern using all the pens in the box, but he was not able to make a repetition (figure 3). António used the strategy of arranging a wide variety of colors, without repeating any color (the first nine rings of the caterpillar), and from the repetition of gray appears to have arranged them randomly. He was the only child that failed to make a sequence with repeated motifs.

Figure 3. António's Caterpillar

The other children claimed that it was not a pattern, but could not explain why not; the most used argument was that "it has many colors and you can not make a pattern with lots of colors". The caterpillar colored by António was looked at as a counterexample of pattern. The children identified the initial difficulties of António and tried not to make the same mistake.

The given caterpillar had 20 spaces to color. In the AB patterns, children ended them using the last element of the unit of repeat, since 20 is a multiple of 2. In the ABC or ABB patterns, it did not occur, but the majority of children did not experience any difficulty completing the caterpillar with the last color that followed in sequence, making no reference that it had to end using the last element of the unit of repeat, unlike the children observed by Vale et al. (2011). Nevertheless, Mário ended his caterpillar with a entire unit of repeat, coloring just one blue ring, after the red one in the 16th position (figure 4).

Figure 4. Mário's Caterpillar

Reading Patterns

During the first moment of the *Coloring the caterpillar* task, three children started a dialogue about the similarities of their patterns, all ABB patterns, and with the same sequence of colors in the unit of repeat- red, blue, blue. The way two of them read their pattern led to a discussion where they concluded that what they had done was the same pattern, although they read it differently:

Mário - Me! I'm doing with two colors, red and blue. It is red, blue blue, red blue blue...
Fernando - Mine is the same as my pattern (...) because look, red, two blues, red, two blues.
David - Mine is red blue blue, red blue blue.
Teacher - David says, red blue blue, Fernando says red two blues, red two blues. Are your patterns the same?
Fernando- No.

David- Yes.
Mário - Yes, because it's red, blue blue, red, blue blue.
Teacher - So let's see. You were saying one red two blues, one red two blues, David was saying red blue blue. Is it the same?
Fernando – It is. Because look, I have a red and two blues and David has a red and two blues.
Teacher- Ah! He also has two blues. I thought it was not the same because he was saying, red blue blue, but it is! You're right. It's another way of saying it. Is yours equal to them or not, Mário?
Mário – It is.
Teacher - Equal to whom? To the one David did or to the one Fernando did?
Mário - (*pause before answering*) Equal to both of them.

When children were asked to read each other their created patterns, it was identified children's use of a rhythmic chant that emphasizes the unit of repeat by the intonation they used. Also some gestures or hand movements were used, allowing the children to identify the correct sequence of colors and even some mistakes made. In the case of Dinis's caterpillar (figure 2), Mário made the following remark:

Mário – Ah….you've done it differently! (*making hands gestures*). It's purple, red blue purple, red, blue …and in the end it's….after purple it´s blue, it looks different (*using a rhythmic chant when he speaks*)
Teacher – Is it different? How do you think it's different?
Mário – Because it has purple with red next to the blue, then the blue changed next to the purple, then the red changed next to the blue (*he explains using his fingers hopping three*)
Teacher – Can you come here and explain to the teacher what you are saying?
Mário – Because Dinis placed purple here then here he broke it up and placed purple, blue and red.

Frederico suggested a way for Dinis not make any more mistakes and wrote it on a sheet:

Frederico - Oh I know! We do some squares for to not make a mistake.
Mário - I know. We can make a pattern so that Dinis doesn't make a mistake , we put the sheet in front of him and he will know.
Frederico - I'll take the pens that he used. Red, purple, blue.

Figure 5. Control Pattern Registers Made by Frederico (A) and by David (B)

Frederico also took from the box a green pen and used it to draw a closed line within he made colored "squares", according to the unit of repeat used by Dinis: purple, red and blue (A, figure 5). David suggested another way and drew a green "circle" surrounded by other circular lines of identical colors of Dinis's caterpillar:

David - A large green circle ... blue, brown ...
António - Purple, it's purple, the caterpillar begins here!
David - I can do around, the first is a turn, now the other ...
Luisa - There are many turns!
António - First was it green?
David – Because… it was ... it was a circle that was holding the colors, then Dinis came here to see what the first color was. It was this, then this, then this.

Frederico's drawing shows that he identified the unit of repeat independently on the number of items (Papic et al., 2011). It was the first time that an iconic record of the unit of repeat appeared. This drawing was as a way for children to have greater control over the pattern correction during the process of creation.
Children also identified the correct sequence of colors.

Teacher - Were you never wrong? (*Tatiana shakes her head*) How do you know that you were ever wrong?
David - Because she says brown and blue, brown and blue, brown and blue (*using a rhythmic chant*).

In this case, David realizes that no error occurred by the tone that Tatiana read her caterpillar, emphasizing the unit of repeat. The children were able to analyze their own patterns as well the patterns created by their friends.

Transferring Iconic Patterns to Gesture Patterns

In the 3rd task, *Reading the caterpillar pattern by gestures*, each child was asked to reproduce his own caterpillar pattern with gestures, touching any part of the body, and teaching it to the group, who also reproduced it using the same gestures. All the children reproduced with gestures the patterns previously made in caterpillars, identifying easily the equivalence between gestures and colors. While touching the various parts of the body, these were verbalized:

Duarte - Head feet feet, head feet feet, head feet feet.
Teacher - What is the color of the head? (...)
David - Red.
Teacher - And when do you touch your feet, what is the color that you're saying?
David - Blue.
Teacher - And why did you touch your feet twice?
David - Because there are two blues.

The teacher reinforced the idea of repetition and that it was only necessary to make a "unit" of gestures to teach the pattern to friends, and not the complete represented pattern. The teacher also stressed that if they continued to make gestures, they could go on indefinitely.

Teacher - If the teacher does not say stop, you could stay here repeating, repeating ... (...) The night would come and we would be here repeating the pattern.
(...)
Teacher - So if you want to teach your pattern to children you just have to teach ...
David - Head feet feet!
Teacher - And from there they will repeat. Is that it?
David - It is.

Children that followed just mentioned, with gestures and orally, their unit of repeat and the group repeated it to reproduce the pattern.

Teacher - Again ... you just figured it out! Jacinto said nose, feet, and everyone began to repeat. So now... Tatiana, what gestures do you need to repeat to do your caterpillar?
Tatiana - Mouth eyebrow (*just saying the unit of repeat*).

In this task, children used an advanced unit of repeat strategy, since they transferred the same pattern in different modes, reconstructing it in more creative ways (Papic et al., 2011). There is also evidence of awareness of the unit of repeat when they were able to teach to their friends just the unit and use it to continue the pattern.

Transferring Gesture Patterns to Iconic Patterns

All children, in large group, created without difficulty, gesture patterns, verbalizing the body parts they touched. They used the process described above. They taught the group just the unit of repeat that was used by everyone to continue the pattern. They did it one at a time, knowing that everyone should memorize their own created pattern to reproduce it pictorially later on. The first to create a pattern had a more difficult task than the last one as the time distance to the realization of this reproduction was higher.

After the creation of gesture patterns, each one went to their desk to color the sequence of oranges using colors to reproduce the same pattern. They adopted the orientation from left to right to color the oranges. The patterns evolved to a more complex form, being documented, in the sequences of oranges, different complex patterns: AB (4), ABC (4), ABB (1) ABCDE (2) ABBCD (1), ABCC (1).

They could associate a gesture to a color. Almost all remembered the gesture pattern created and eight children matched exactly the gesture pattern to the oranges pattern, showing the use of an advanced unit of repeat strategy (Papic et al., 2011). Five of the children colored patterns in the sequence of oranges, but without corresponding accurately to the gesture pattern invented before. Mário stated "eyes nose mouth nose feet shoulder" (ABCBDE) but he colored a figurative ABBCD pattern, both complex. Fernando was able to match his gesture pattern "head feet arm hand belly" to the figurative pattern ABCDE (figure 6).

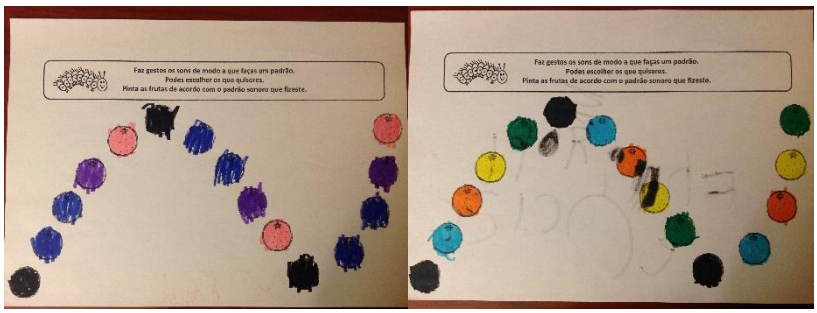

Figure 6. The Patterns of Mário and Fernando Respectively

António created a gesture pattern "eyes mouth ears feet", mentioning that he will require four colors. However when he colored he did an ABCDE pattern (figure 7), having used five colors. Although he did not fully achieve matching the gesture pattern to the pictorial one, there is a great evolution since the first task, in which he was not able to create a repeating pattern. In this task, António maintained his preference of using a large number of colors manifested in the first task of coloring the caterpillar, and he already managed to color a pattern without any mistakes.

Figure 7. António's Orange Pattern

After the initial choice of pens, eight children chose to position the pens outside the box showing to identify the unit of repeat, as they selected and put together all colors necessary for making the unit of repeat, focusing exclusively on the color sequence. Children who did not take their pens outside of the box, again used the strategy of looking at the beginning of the pattern to check the correct order to place the colors.

Coding Patterns

The teacher began to encourage children to identify the structure of a pattern, first through the use of numbers ("If your pattern were numbers, how would you read it?", and Matilde answered to an ABC pattern, "1 ... 2 ... .3, 1 2 3, 1 2 3"). After this codification, the teacher fostered the use of letters in order to not confuse with the number of times an item could be repeated in the unit of repeat of a given pattern representation, as might be the case of the use of numbers.

The use of letters to recognize the structure of the patterns allows children to realize that their patterns are similar regardless of the colors used, as we can see in the following excerpt:

Teacher - But they are different colors ... Why are they equal?
Fernando - Because they are ABABAB.
David- ABABABAB.
Teacher - Ah, because they are all AB, although not of the same color.

The need to assign a name to the unit of repeat arose spontaneously in a situation where the children were asked to make necklaces with color beads and yarn. The children were asked to identify and draw the findings on a big wall paper (figure 8).

Figure 8. Registration and Collage of Patterns and Codes

Dinis represented the sequence of colors and identified the unit of repeat, representing it in a designated box, describing it by *code*, because his older brother played computer games that had codes. Dinis explained: "It's so we don't deceive ourselves. We look at the *code"*. From this moment on, all the children referred to the unit of repeat as the *code*.

Copying and Continuing Patterns

In the task *Decorating Fly Fosca's House*, the children were asked to decorate the house for the party that Fosca Fly would give, using patterns. Children were to copy and continue repeating patterns presented in strips with patterns of various types: AB, ABC, ABB, AABB and ABBB. Each strip contained two units of repeat. Each child had a set of cards with different shapes but with the same color (yellow, red, green, orange or blue) to make the pattern represented in the strip. The strips varied in shape or position attributes.

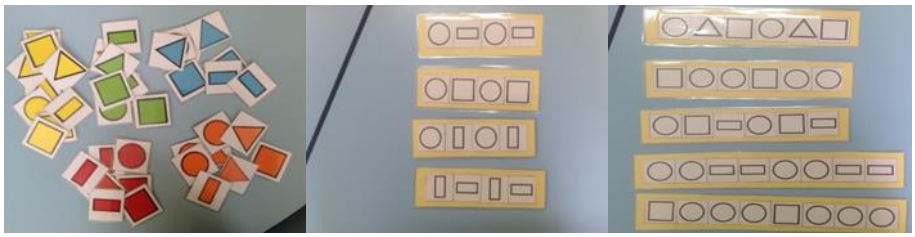

Figure 8. Material Used to Perform the Task

For the AB patterns, most children had no difficulty in copying and continuing the patterns shown and did so reproducing the pattern that was presented to them, using linearly the colored cards intended for each one. However a child, Dinis, used a two-dimensional disposition (figure 9). Dinis seems to ignore the formation law of a two-dimensional pattern, not respecting the regularity in column, since attended the alternation only on row (Vale et al., 2011). Thus, Dinis copied the strip composed of two units of repeat and repeated it twice by duplicating the unit of repeat below.

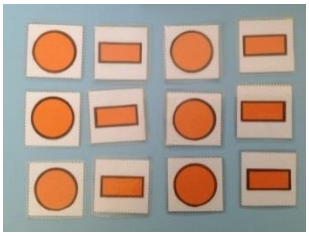

Figure 9. Dinis's Two-dimensional Disposition

The patterns continued by the children had three to six units of repeat. Most children ended their pattern, using the last element of the unit of repeat. But some children did not seem to assign this special importance, as was the case of Fernando (figure 10) who ended his AB pattern using the first item of the unit of repeat.

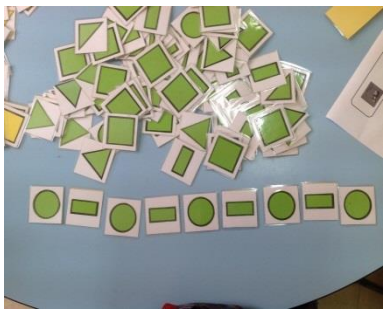

Figure 10. Fernando's Pattern

The code facilitated structure patterns awareness by the children:

Teacher - What is your pattern *code* António?
António – Circle rectangle.
(...)
Teacher - What is your code? That name that Dinis invented? (*Luísa shows with her hands the unit of repeat and separate it from the rest of the pattern)*
Luísa - ABABAB.
(...)
Teacher - What is the bit that is repeated which is always equal?
Tatiana - (*with two fingers points to the unit of repeat*) One two three (*counting the unit of repeat repetitions*).

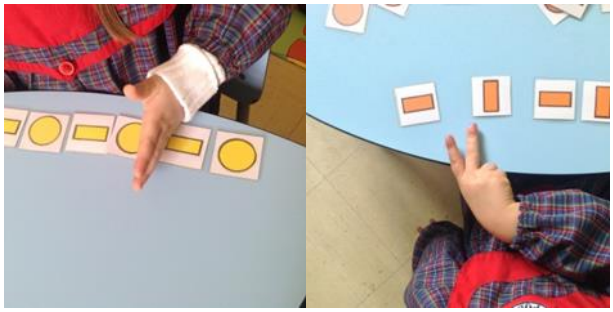

Figure 11. Luísa Identifying the Unit of Repeat and Tatiana Identifying and Counting the Units of Repeat Respectively

While António just verbalized the unit of repeat ("Circle rectangle "), Luísa read all the elements that she had built, representing them by the letters AB. Luísa used hand movements to help her separate the unit of repeat revealing an awareness of it. Like Tatiana, a large number of children could count the number of times that the unit of repeat repeats.

For the other type of patterns, the majority of children was able to copy and continue the patterns shown. Some children made them separating the respective unit of repeat. While António put the unit of repeat on the left, Guilherme and Joaquim put it over the pattern (figure 12).

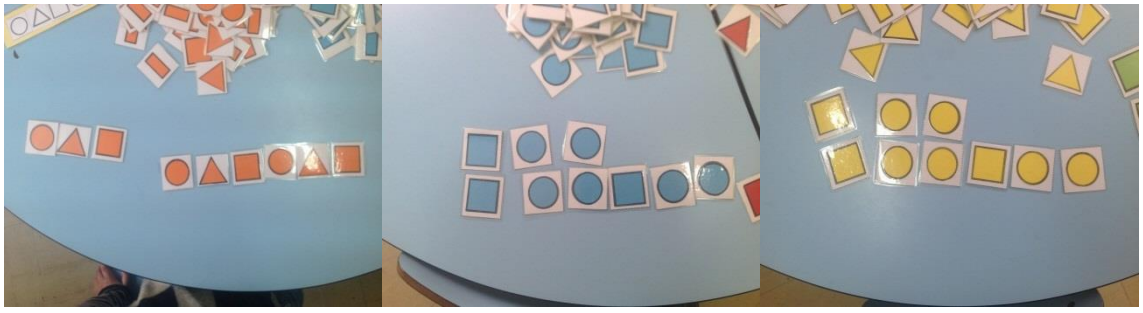

Figure 12. Isolating the Unit of Repeat in the Patterns Made by António, Guilherme and Joaquim

With these types of patterns, the teacher tried to understand if they could identify how often the different elements were repeated in a specific pattern, bearing in mind the unit of repeat. Concerning the circle triangle, square pattern, see the following excerpt which illustrates their thinking:

Teacher - And how many squares?

Matilde - (*counting*) One, two, three.
Teacher - And how many triangles?
Matilde - (*counting*) One, two, three.
David - One, two, three.
Teacher - And if you repeat your code 10 times, how many circles would you put? (*pause*) Don't you know? (*Matilde shakes her head*)
Fernando - Can I?
David - Can I?
Teacher - And you how often did you repeat the code?
David - (*with fingers*) One ... two ... three ... (*he gets a square because his pattern ended with a triangle*)
Teacher - So how many circles did you place in your pattern?
David - (*counting*)) One, two, three. Three.
Teacher - And how many triangles?
David - Three.
Teacher - And how many squares?
David - Three.
Teacher - And if you did the code five times how many circles would you place?
David - Five (*showing fingers*)
Fernando - I repeated six.
Teacher - (*to Matilde*) If you did your code six times how many circles would you place?
Matilde - Six.
Teacher - And if you repeat your code four times how many circles would you place?
David - Ahm ..... four!
Teacher - Four. Why?
Mário - So it is easier to make the numbers (*shows his pattern which has three units of repeat*).
Teacher - But why? Do you see something funny between the number of times we repeat the code and the number of shapes that we have?
David - I think it's funny because so we repeat, so the things… three we repeat and three the things. So many things.

These children seem to realize the relationship between repetitions number of the unit of repeat, its elements and the number of times that they are repeated. The children's response illustrated their thinking evolution. For example, David started by counting and after he answered a hypothetical question without difficulty, referring to the same number of times the code repeats explaining this relationship: "so we repeat, so the things… three we repeat and three the things".

In the seventh task, the children were asked to decorate a cake with sweets for Fly Fosca's party based on the book's illustrations. The cards represented the sweets. The children performed this task in small groups, having been formed into two groups. As in the previous task, the same type of material was used. A set of cards was given to each group: blue and red, for one group, and green and yellow to the other. The two groups had the same shapes available: squares, circles, triangles and rectangles. The children were asked to make patterns with the unit of repeat represented in each strip (figure 13).

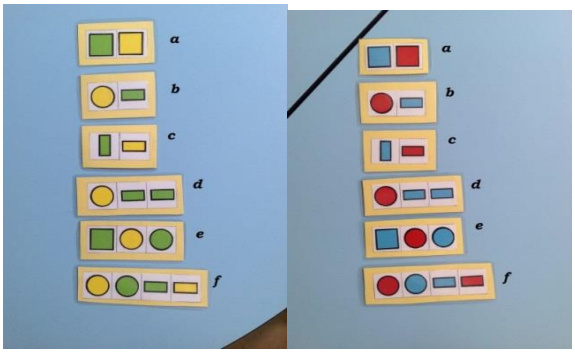

Figure 13. Strips with Units of Repeat Presented to Children

Most children had no difficulty in performing the task. Some of them used the strategy of isolating the unit of repeat to make the pattern again(figure 14). They copied the unit of repeat, the code, and then built the pattern easily.

Figure 14. - Isolating the Unit of Repeat Represented in the Strip to Make the Pattern

The given *a* pattern, which had two different colored squares (green, yellow) but maintained the shape attribute, most children read it referring to the alternation of colors. Matilde, Tatiana and David were the only children who read it referring to the name of the shape and not using any inflection to read it:

Matilde - (*long pause before starting*) Square square square square square square square square square... (*looks at me).*

Considering Matilde, the absence of a rhythmic chant seems to be due to the fact that she has focused only on the geometrical shape which does not vary throughout the pattern. When building the pattern, the children appear to have no difficulty copying and continuing it. But when these three children began reading, they assumed the pattern had identical elements, once the children only refer to as square. However some children disagreed and the following excerpt illustrates their comments:

David - Square square square square square square (*reading the pattern made*)
Teacher - Can anyone tell me what kind of pattern is this?
David - AB ... AAAAAA.
Fernando - ABAB.
Teacher - Fernando says it is ABAB and David says it's AAAAAA. Which is it?
David - AB only if it's like this. (*picks up a yellow square of its pattern and take from the pile ahead a green rectangle, putting the two together in the air*)
Mário - AAAAA? This is not true!
Teacher -Mário, explain to me how you were seeing it.
Mário –It isn't the same!
Teacher - Isn't it the same? So what's different?
Mário - Because we are using different colors, yellow ...they are the same shape ... but ... even if this has the same shape, but it's not the same ... the same colors.

Fernando said that the pattern was AB type, and Mário explained that even though the cards had the same shape, square, the colors were different and one could not assign the same letter, since the pattern elements were different. The following excerpt illustrates the reading of the unit of repeat of *f* pattern made by David when he was looking at the strip, before its construction:
David – Yellow circle green circle green rectangle yellow rectangle.

Matilde and David were the only children that made a reading considering the two attributes, color and shape. Both children appear to have evolved in their pattern awareness, since in the previous *a* pattern, they made a reading using a single attribute, not taking into account the color. The evolution of complexity of the patterns did not appear to have influenced this reading, once the pattern *a* was AB type and the pattern *f* was ABCD.

CONCLUSION

The tasks presented here began with the creation of patterns and not with copy and continue models provided by adults. According to Threlfall (1999), it is essential the children create their own patterns formerly than to copy pre-established models. The study results reinforce this idea, since the children seem to grasp easily and significantly the notion of pattern and evolve their level of awareness of the structure of the patterns, through the implementation of this approach.

After the exploitation of the tasks, the whole group achieved the pattern concept and was able to create repeating patterns. The children's evolution in the level of complexity of patterns created was shown. They first started with patterns having units of repeat with a number of elements up to 3 (Vale et al., 2011), increasing that number to a higher number of elements, up to 5. Once, Dinis used a symmetrical approach when creating a pattern. The spontaneous way as Dinis collocated a pattern, intend to be linear, assuming a two-dimensional disposition,

converges with results of Vale et al. (2011), since he did not respect the formation law in column. Nevertheless, unlike the results of Vale et al. (2011), most children did not need end the patterns with a entire unit of repeat.

The children were also able to identify errors in their pattern's construction when reading it out loud, in part due to the use of a rhythmic chant (Threlfall, 1999). Unlike the results reported by Garrick et al. (1999), the children were able to recognize the patterns of their own creation as well the patterns created by other children. Some strategies they used in the beginning seem to evidence the emergence of the awareness of the unit of repeat, such as the strategy of isolating the necessary pens to color the pattern or the strategies of helping the friends not to make mistakes in the pattern. Also important was the request of the teacher to explain the gesture patterns to friends, making only the gestures corresponding to the unit of repeat. The physical movements not only facilitate the representation of the pattern but also the perception of the unit of repeat (Threlfall, 1999). In the beginning of patterning work, there were different levels of awareness of the structure pattern: some children emphasized the unit of repeat by a rhythmic chant while others showed a higher level of awareness referring explicitly to the unit of repeat (Mulligan, 2013; Threlfall, 1999). For copying and continuing patterns more complexes, with units of repeat with 3 or more elements, some children used the strategy of isolating the unit of repeat to facilitate making the pattern.

The evolution in the level of complexity of the patterns created and recognized was due to the development of children in relation to the structure pattern awareness, seeing the pattern as a whole being related to the unit of repeat named by them as the 'code'. The "code" quickly helped understand the structure of a pattern. The use of letters to encode patterns allowed the children to recognize different structures of repeating patterns. They were not dependent on the material used in the sense they could transfer the same pattern to different modes or materials, corresponding to the strategy of advanced unit of repeat (Papic et al., 2011; Vale et al., 2011), the most sophisticated strategy.

REFERENCES

Blanton, M., & Kaput, J (2011). Funtional thinking as a route into algebra in the elementary grades. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 5-23). Berlin: Springer.

Bogdan, R., & Biklen, S. (1994). *Investigação qualitativa em educação: Uma introdução à teoria e aos métodos*. Porto: Porto Editora.

Borralho, A., Cabrita, I., Palhares, P., & Vale, I. (2007). Os padrões no ensino e aprendizagem da álgebra. In I. Vale, T. Pimentel, A. Barbosa, Fonseca, L. Santos & P. Canavarro (Orgs.), *Números e álgebra* (pp. 193-211). Lisboa: SEM-SPCE.

Carraher, D., Schliemann, A., & Schwartz, J. (2008). Early algebra is not the same as algebra early. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 235-271). New York: Lawrence Erlbaum Associates.

DEB (1997). *Orientações curriculares para a educação pré-escolar*. Lisboa: Ministério da Educação.

Garrick, R., Threlfall, J., & Orton, A. (1999). Pattern in the nursery. In A. Orton (Ed.), *Patterns in the teaching and learning of mathematics* (pp. 1-17). London: Cassell.

Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 5-17). New York: Lawrence Erlbaum Associates.

Loureiro, C. (2006). Os livros de histórias e a matemática. In APM (Ed.), *Actas do Profmat 2006*. Lisboa: APM.

Mulligan, J. (2013). Reconceptualizing early mathematics learning. In A. M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 139-142). Kiel, Germany: PME.

National Council of Teachers of Mathematics. (2007). *Princípios e normas para a matemática escolar.* (APM, Trans.) Lisboa: APM. (Original work published 2000)

Palhares, P., & Mamede, E. (2002). Os padrões na matemática do pré-escolar. *Educare-Educere, 10*(1), 107-123.

Papic, M., Mulligan, T., & Mitchelmore, M. (2011). Assessing the developing of preeschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237-268.

Serra, P. (2014). *Lendo e explorando histórias: A emergência do pensamento algébrico em crianças de 4 anos*. Lisboa: APM.

Smole, K., Rocha, G., Cândido, P., & Stancanelli, R. (2007). *Era uma vez na matemática: Uma conexão com a literatura infantil* (6th ed.). S. Paulo: CAEM.

Suh, J. M. (2007). Developing algebra: "Rithmetic" in the elementary grades. *Teaching Children Mathematics,* 246- 253.

Threlfall, J. (1999). Repeating patterns in the early primary years. In A. Orton (Ed.), *Patterns in the teaching and learning of mathematics* (pp. 18-30). London: Cassell.

Vale, I., Pimentel, T., Barbosa, A., Borralho, A., Cabrita, I., & Fonseca, L. (2011). *Padrões em matemática: Uma proposta didática do novo programa para o ensino básico*. Lisboa: Texto Editores.

Warren, E., & Cooper, T. (2008). Generalising the pattern rule for visual growth: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics, 67*(2), 171-185.