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7TH GRADE STUDENTS'PROBLEM SOLVING SUCCESS RATES ON PROPORTIONAL REASONING PROBLEMS

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ABSTRACT: This research was conducted to investigate 7th grade students' problem solving success rates on proportional reasoning problems and whether these success rates change with different problem types. 331 randomly selected students of grade seven participated in this study. A problem test which contains three different types of missing value (direct proportional, inverse proportional and additive/non-proportional) word problems was designed as a data collecting tool for the research. Descriptive data analysis methods were used in this study. Analysis has shown that 7th grade students solved different problem types with different success rates. The findings of the study also indicate that problem types affect students' problem solving performances.

Key words: problem solving success rate, proportional reasoning, problem types

INTRODUCTION

Proportional Reasoning

Students' first experiences with mathematics are based on natural numbers in their school life. The first years of primary school includes addition and subtraction that is based on the first-order relationships between countable objects. In the middle school years, students introduce with rational numbers as well as natural numbers. During these years, students must make several major transitions in their mathematical thinking. A central change in thinking is required in a shift from natural number to rational numbers and from additive concepts to multiplicative concepts (McIntosh, 2013, p. 6). This is an important and difficult conceptual leap for students; mathematical experiences in elementary school focus primarily on countable objects and first-order relationships. In proportional situations students must replace additive reasoning and notions of change in absolute sense with multiplicative reasoning and notions of change in a relative sense (Baxter & Junker, 2001). This second-order relationship is difficult for students because it requires more complicated mental structures than simple multiplication and division. Piaget considered the development of proportional reasoning to be a turning point in the development of higher order reasoning (Aleman, 2007, p. 22). In this sense, the proportional reasoning ability merits whatever time and effort that must be expended to assure its careful development (NCTM, 2000; Ben-Chaim, Fey, Fitzgerald, Benedetto, Miller, 1988; Lesh, Post, Behr, 1988; Lamon, 1993; Baykul, 2009).

Smith (2002) described the importance and complexity of proportionality in this way: "No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as proportionality (Johnson, 2010, p. 3). Many important concepts at the foundation level of elementary mathematics are often linked to proportional reasoning (NCTM, 2000, p. 212). Proportional reasoning is both capstone of elementary arithmetic and the cornerstone of all that is to follow. It therefore occupies a pivotal position in school mathematics programs (Lesh et al., 1988). Using proportional reasoning, students consolidate their knowledge of elementary school mathematics and build a foundation for high school mathematics. Students who fail to develop proportional reasoning are likely to encounter obstacles in understanding higher-level mathematics (Langrall & Swafford, 2000).

Problem types

Cramer & Post (1993) categorized proportional tasks as missing-value problems, numerical comparison problems and qualitative prediction and comparison problems. In missing-value problems three pieces of numerical information are given and one piece is unknown. In numerical comparison problems, two complete rates are given. A numerical answer is not required, however the rates are to be compared. Qualitative prediction and comparison problems require comparisons not dependent on specific numerical values. Van Dooren, De Bock, Hessels, Janssens, Verschaffel, (2005) categorized non-proportional tasks (i.e., problems for which a proportional solution was manifestly incorrect but for which another method could be applied to find the correct answer) as additive problems, constant problems and linear problems. In linear problems, the linear function underlying the problem situation is of the form $f(x) = ax + b$ with $b \neq 0$. Additive problems have a constant difference between the two variables, so a correct approach is to add this difference to a third value. Constant problems have no relationship at all between the two variables. The value of the second variable does not change, so the correct answer is mentioned in the word problem.

According to Lesh et al., (1988) proportional reasoning encompasses not only reasoning about the holistic relationship between two rational expressions but wider and more complex spectra of cognitive abilities which includes distinguishing proportional and non-proportional situations. Studies on proportional reasoning has shown that additive strategy is the most frequently used error strategy while students solve proportional problems (Tourniaire, 1986;Karplus, Pulos, Stage, 1983; Bart, Post, Behr, Lesh, 1994; Singh, 2000; Misailidou & Williams, 2003; Duatepe, Akkuş, Kayhan, 2005). Similarly, students give proportional responses to non-proportional problems (Duatepe et al., 2005;Van Dooren, De Bock, Vleugels, Verschaffel, 2010; Van Dooren, De Bock, Verschaffel, 2010; De Bock, Van Dooren, Janssens, Verschaffel, 2002; De Bock, De Bolle, Van Dooren, Janssens, Verschaffel, 2003). This shows that students have difficulty in distinguishing proportional and non-proportional problem statements.

The middle school mathematics curricula also include inverse proportional relations but in related literature this relations has not studied deeply so far. Thus it could be beneficial to study whether students can distinguish this kind of relations with other relations.

Statement of the problem

This research was conducted to investigate 7th grade students' problem solving success rates and whether these success rates change with different problem types. Depending on this aim, the research problem was determined as "What are the success rates of 7th grade students in solving missing value problems with different types?"

METHOD

Research design

Since survey studies collect data from a group of people in order to describe some aspects or characteristics (such as abilities, opinions, attitudes, beliefs or knowledge) of the population of which that group is a part (Fraenkel & Wallen, 2005), this research was carried out by using survey method.

Sample

A total of 331 (162 boys and 169 girls) randomly selected students of grade seven from five different public middle schools in 2014-2015 education year participated in this study.

Instrument

A problem test which contains three different types of missing value (direct proportional, inverse proportional and additive/non-proportional) word problems was designed as a data collecting tool for the research. Problem test consisted of 24 open ended items and these items were developed in parallel with the objectives of renewed elementary mathematics curriculum (MEB, 2013).

Data analysis

Descriptive data analysis methods were used in this study. Pupils' responses to the problems in the solution task were scored in order to determine their problem solving success rates on different problem types. To check the internal consistency of the instrument, Kuder Richardson-20 coefficient was calculated and was found to be 0,823.

RESULTS AND FINDINGS

Table 1 shows the mean scores on different types of problems. Analysis of the mean scores showed that students showed the best performance on solving direct proportional problems while the worst performance on solving non-proportional problems.

Table 1.Mean Scores on Different Types Problems

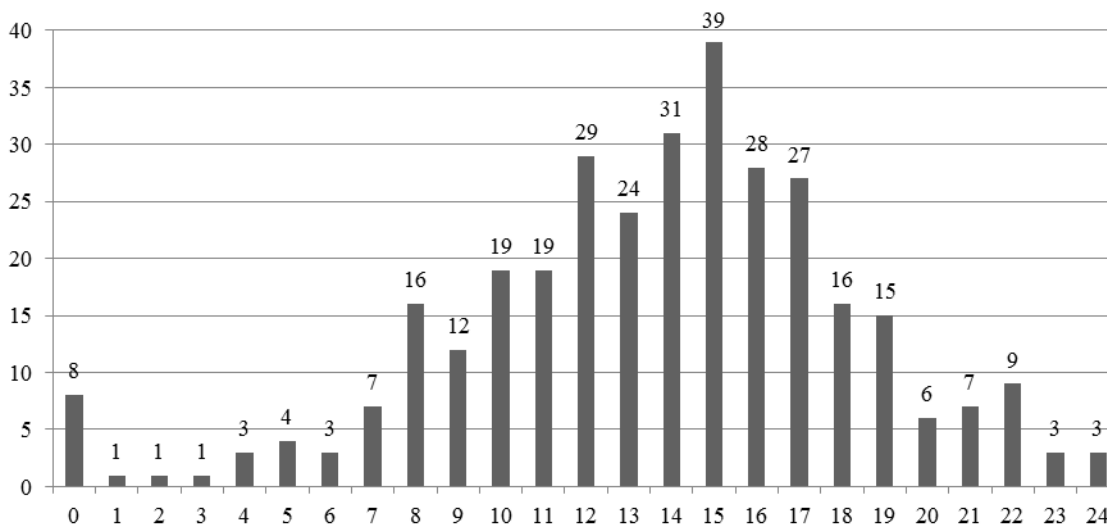| Problem Types	Direct Proportional	Inverse Proportional	Non-Proportional (Additive)	Total
Means	6,20	4,84	2,57	13,61

Figure 1. Frequency Histogram of The Total Scores

Figure 1 shows the distribution of the scores. The frequency histogram shows that the distribution of the scores is normal. Most of the students' scores concentrate between 8 and 19 points.
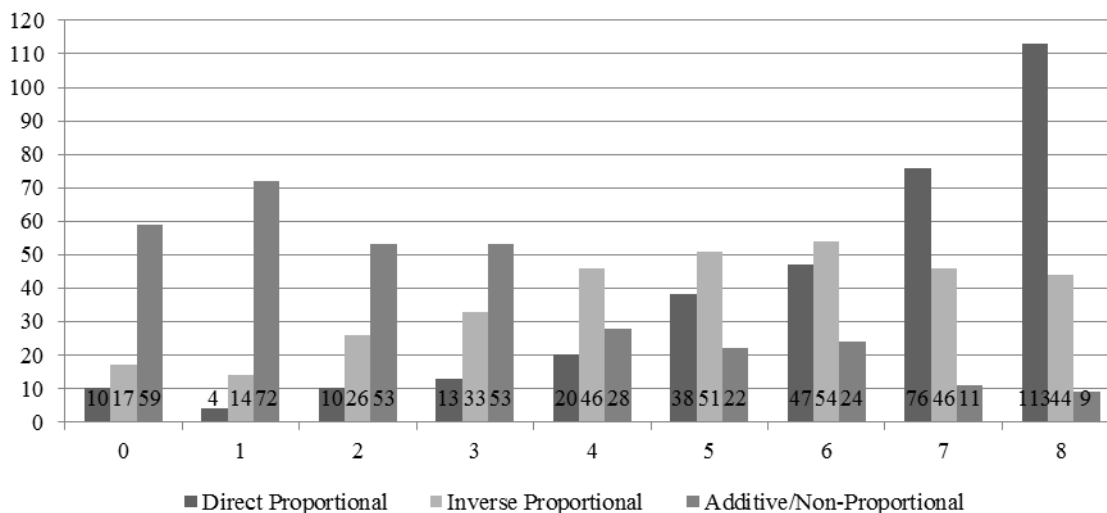

■Direct Proportional ■Inverse Proportional ■Additive/Non-Proportional

Figure 2. Frequency Histogram of The Scores on Different Types of Problems

Figure 2 shows the distribution of the scores on different types of problems. Students showed the best performance on direct proportional problems and worst performance on non-proportional (additive) problems.

Analysis taken from the data collecting tool has shown that 7th grade students solved different problem types with different success rates. The findings of the study also indicate that problem types affect students' problem solving performances. In detail, additive/non-proportional problems were solved with the lowest success rate, while direct proportional problems with the highest success rate. The tendency to overuse proportional responses in inverse proportional and additive/non-proportional situations was observed. Study showed that students have difficulty on distinguishing direct proportional, inverse proportional and non-proportional (additive) problem statements.

CONCLUSION AND RECOMMANDATIONS

The findings of the study revealed that problem affect students' success rate while solving missing value problems. Study also showed that students have difficulty on distinguishing direct proportional, inverse proportional and non-proportional (additive) problem statements. Students should encourage to realize the mathematical structures underlying the problems so that they can be more successful to distinguish direct proportional, inverse proportional and non-proportional (additive) problems and develop better conceptual understandings. In this sense, students should simultaneously be faced to both proportional (direct and inverse) and non-proportional (additive) problems in order to comprehend the mathematical structures underlying the problems. For further studies, it can be suggested to make clinical interviews with pupils in order to explore deeper understanding on how and why students make different success rates on different types of missing value problems.

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