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PRESCHOOLERS LEARN PROPORTIONALITY AND INTEGRATION THROUGH ICON-COUNTING AND NEXT-TO ADDITION

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ABSTRACT: Preschool allows rethinking mathematics outside the tradition of ordinary school. Seeing schooling as adapting the child to the outside world containing many examples of Many, we can ask: How will mathematics look like if built as a natural science about physical fact Many? To deal with Many we count and add. The school counts in tens, but preschool allows counting in icons also. Once counted, totals can be added. To add on-top the units are made the same through recounting, also called proportionality. Adding next-to means adding areas, also called integration. So icon-counting and next-to addition offers golden learning opportunities in preschool that is lost in ordinary school allowing only ten-counting to take place.

Key words: count, add, proportionality, integration, preschool

MATH IN PRESCHOOL – A GREAT IDEA

Mathematics is considered one of the school's most important subjects. So it seems a good idea to introduce mathematics in preschool - provided we can agree upon what we mean by mathematics.

As to its etymology Wikipedia writes that the word mathematics comes from the Greek máthēma, which, in the ancient Greek language, means "that which is learnt". Later Wikipedia writes:

In Latin, and in English until around 1700, the term mathematics more commonly meant "astrology" (or sometimes "astronomy") rather than "mathematics"; the meaning gradually changed to its present one from about 1500 to 1800. (http://en.wikipedia.org/wiki/Mathematics)

This meaning resonates with Freudenthal writing:

Among Pythagoras' adepts there was a group that called themselves mathematicians, since they cultivated the four "mathemata", that is geometry, arithmetic, musical theory and astronomy. (Freudenthal 1973: 7)

Thus originally mathematics was a common word for knowledge present as separate disciplines as astronomy, music, geometry and arithmetic.

This again resonates with the educational system in the North American republics offering courses, not in mathematics, but in its separate disciplines algebra, geometry, etc.

In contrast to this, in Europe with its autocratic past the separate disciplines called Rechnung, Arithmetik und Geomtrie in German were integrated to mathematics from grade one with the arrival of the 'new math' wanting to revive the rigor of Greek geometry by defining mathematics as a collection of well-proven statements about well-defined concepts all being examples of the mother concept set.

Kline sees two golden periods, the Renaissance and the Enlightenment that both created and applied mathematics by disregarding Greek geometry:

Classical Greek geometry had not only imposed restrictions on the domain of mathematics but had impressed a level of rigor for acceptable mathematics that hampered creativity. Progress in mathematics almost demands a

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complete disregard of logical scruples; and, fortunately, the mathematicians now dared to place their confidence in intuitions and physical insights. (Kline 1972: 399)

Furthermore, Gödel has proven that the concept of being well-proven is but a dream. And Russell's set-paradox questions the set-based definitions of modern mathematics by showing that talking about sets of sets will lead to self-reference and contradiction as in the classical liar-paradox 'this sentence is false' being false if true and true if false: If M = □A | A□A)□ then M□M □ M□M.

With no general agreement as to what mathematics is and with the negative effects of imposing rigor, preschool mathematics should disintegrate into its main ingredients, algebra meaning reuniting numbers in Arabic, and geometry meaning measuring earth in Greek; and both should be grounded in their common root, the physical fact Many. To see how, we turn to skeptical research.

POSTMODERN CONTINGENCY RESEARCH

Ancient Greece saw a controversy between two different forms of knowledge represented by the sophists and the philosophers. The sophists warned that in a republic people must be enlightened about choice and nature to prevent being patronized by choices presented as nature. In contrast to this philosophers saw everything physical as examples of meta-physical forms only visible to the philosophers educated at Plato's academy, who then should be allowed to patronize.

Enlightenment later had its own century that created two republics, an American and a French. Today the sophist warning against hidden patronization is kept alive in the French republic in the postmodern skeptical thinking of Derrida, Lyotard, Foucault and Bourdieu warning against patronizing categories, discourses, institutions and education presenting their choices as nature (Tarp 2004).

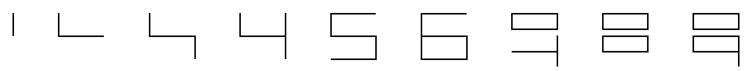
Thus postmodern skeptical research discovers contingency, i.e. hidden alternatives to choices presented as nature. To make categories, discourses and institutions non patronizing they are grounded in nature using Grounded Theory (Glaser et al 1967), the method of natural research developed in the other Enlightenment republic, the American; and resonating with Piaget's principles of natural learning (Piaget 1970) and with the Enlightenment principles for research: observe, abstract and test predictions.

With only little agreement as to what mathematics is we ask: How will mathematics look like if built as a natural science about the physical fact Many, and how can this affect early childhood education?

BUILDING A NATURAL SCIENCE ABOUT MANY

To deal with the physical fact Many, first we iconize, then we count by bundling. With 'first order counting' we rearrange sticks in icons. Thus five ones becomes one five-icon 5 with five sticks if written in a less sloppy way. In this way we create icons for numbers until ten since we do not need an icon for the bundle-number as show when counting in e.g. fives: one, two, three, four, bundle, one bundle and one, one bundle and two etc..

I II III IIII IIIII IIIIII IIIIIII IIIIIIII IIIIIIIII



1 2 3 4 5 6 7 8 9

Figure 1. Icons Contain As Many Sticks As They Represent

With 'second order counting' we bundle a total in icon-bundles. Here a total T of 7 1s can be bundled in 3s as T = 2 3s and 1. The unbundled can be placed in a right single-cup, and in a left bundle-cup we place first the bundles to be traded, first with a thick stick representing a bundle glued together, then with a normal stick representing the bundle. The cup-contents is described by icons, first using 'cup-writing' 2)1), then using 'decimal-writing' with a decimal point to separate the bundles from the unbundled, and including the unit 3s, T = 2.1 3s. Alternatively, we can also use plastic letters as B, C or D for the bundles.

IIIIIII → III III I → III III) I) → ▌▌) I) → II) I) → 2)1) → 2.1 3s or BBI → 2BI

Using squares or LEGO blocks or an abacus, we can stack the 3-bundles on-top of each other with an additional stack of unbundled 1s next-to, thus showing the total as a double stack described by a decimal number.

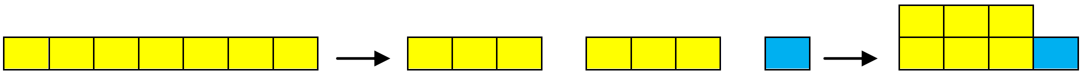


Figure 2: Seven 1s First Becomes 2 3s & 1, Then 2x3 + 1 Or 2.1 3s

We live in space and in time. To include both when counting, we can introduce two different ways of counting: counting in space, geometry-counting, and counting in time, algebra-counting. Counting in space, we count blocks and report the result on a ten-by-ten abacus in geometry-mode, or with squares. Counting in time, we count sticks and report the result on a ten-by-ten abacus in algebra-mode, or with strokes.

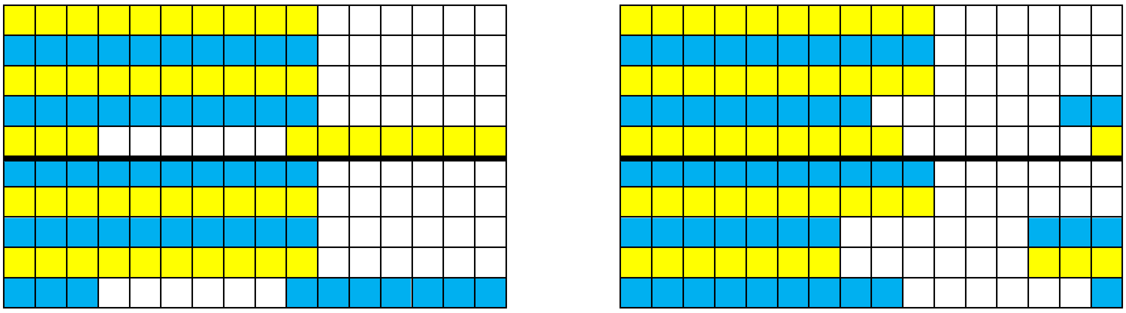


Figure 3: Counting 7 1s As 2.1 3s On An Abacus With Geometry Mode Below And Algebra Mode Above

To predict the counting result we can use a calculator. Building a stack of 2 3s is iconized as 2x3 showing a jack used 2 times to lift the 3s. As to the two icons for taking away, division shows the broom wiping away several times, and subtraction shows the trace left when taking away just once. Thus by entering '7/3' we ask the calculator 'from 7 we can take away 3s how many times?' The answer is '2.some'. To find the leftovers we take away the 2 3s by asking '7 – 2x3'. From the answer '1' we conclude that T = 7 = 2.1 3s. Showing '7 – 2x3 = 1', a display indirectly predicts that 7 can be recounted as 2 3s and 1.

7 / 3	2.some
7 – 2 x 3	1

Figure 4: A Calculator Predicts That 7 1s Can Be Recounted As 2.1 3s

Re-counting In The Same Unit And In A Different Unit

Once counted, totals can be re-counted in the same unit, or in a different unit. Recounting in the same unit, changing a bundle to singles allows recounting a total of 4 2s as 3.2 2s, 2.4 2s. Likewise 4.2s can be recounted as 5 2s less or short of 2; or as 6 2s less 4 thus leading to negative numbers:

Letters	Sticks	Calculator		T =
B B B B	Ⱨ Ⱨ Ⱨ Ⱨ			4.0 2s
B B B I I	Ⱨ Ⱨ Ⱨ I I	4x2 – 3x2	2	3.2 2s
B B I I I I	Ⱨ Ⱨ I I I I	4x2 – 2x2	4	2.4 2s
B I I I I I I	Ⱨ I I I I I I	4x2 – 1x2	6	1.6 2s
I I I I I I I I	I I I I I I I I	4x2 – 0x2	8	0.8 2s
B B B B B	Ⱨ Ⱨ Ⱨ Ⱨ Ⱨ	4x2 – 5x2	-2	5.2 2s
B B B B B B	Ⱨ Ⱨ Ⱨ Ⱨ Ⱨ Ⱨ	4x2 – 6x2	-4	6.4 2s

Figure 5: Recounting 4 2s In The Same Unit

To recount in a different unit means changing unit, called proportionality or linearity also. Asking '3 4s is how many 5s?' we can use sticks or letters to see that 3 4s becomes 2.2 5s.

IIII IIII IIII → IIIII IIIII I I → 2) 2) 5s → 2.2 5s, or with C = BI, BBB → BBIIII → CCII

Using geometry-counting on an abacus, reserving the bottom line for the single 1s, a stack of 3 4s is moved from left to right on an abacus. The top bundle is changed to 1s in the single line and twice a stick is removed to enlarge the two 4-bundles to 5-bundles. This shows that '3 4s can be recounted as 2.2 5s.'

Using algebra-counting, 3 beads are moved to the right on the bundle-line. Then one 4-bundle is changed to 4 1s on the single-line. Moving 2 beads to the left on the single-line allows enlarging the 4s to 5s thus showing that 3 4s = 2.2 5s

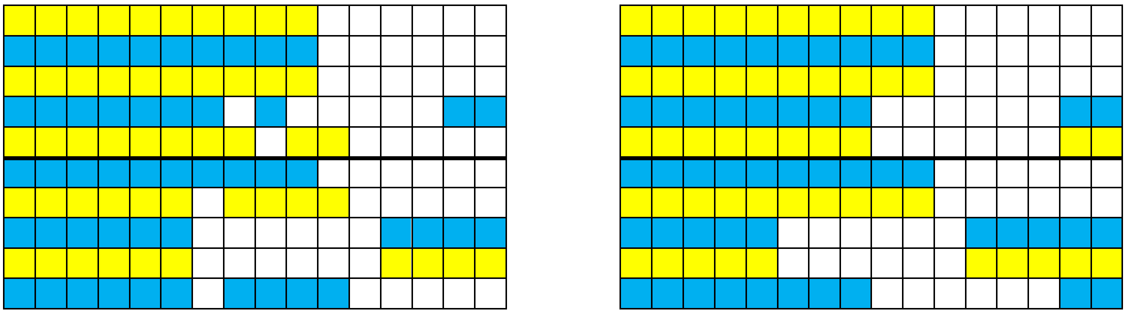


Figure 6: Re-counting 3 4s As 2.2 5s On An Abacus With Geometry And Algebra Mode

Using a calculator to predict the result we enter '3x4/5' to ask 'from 3 4s we take away 5s how many times?' The calculator gives the answer '2.some'. To find the leftovers we take away the 2 5s and ask '3x4 – 2x5'. Receiving the answer '2' we conclude that T = 3 4s = 2.2 5s.

3 x 4 / 5	2.some
3 x 4 – 2 x 5	2

Figure 7: A Calculator Predicts That 3 4s Can Be Recounted As 2.2 5s

Adding On-top And Next-to

Once counted, totals can be added on-top or next-to. Asking '3 5s and 2 3s total how many 5s?' we see that to be added on-top, the units must be the same, so the 2 3s must be recounted in 5s giving 1.1 s that added to the 3 5s gives a grand total of 4.1 5s.

IIIII IIIII IIIII III III → IIIII IIIII IIIII IIIII I → 4) 1) 5s → 4.1 5s, or 3B + 2C = 3B III III = 4BI

On an abacus in geometry mode a stack of 3 5s is moved to the right and a stack of 2 3s is moved to the middle. Now, the 2 3s is changed to 6 1s on the bottom line allowing one additional 5s to be moved to the top of the stack of 5s to show the grand total is 4.1 5s. Using algebra mode, the 3 5s become 3 beads on the bundle line and the 2 3s become 2 beads on the line above. Again the 2 3s is changed to 6 1s on the bottom line allowing one additional bead to be added to the bundle-line to give the result 4.1 5s

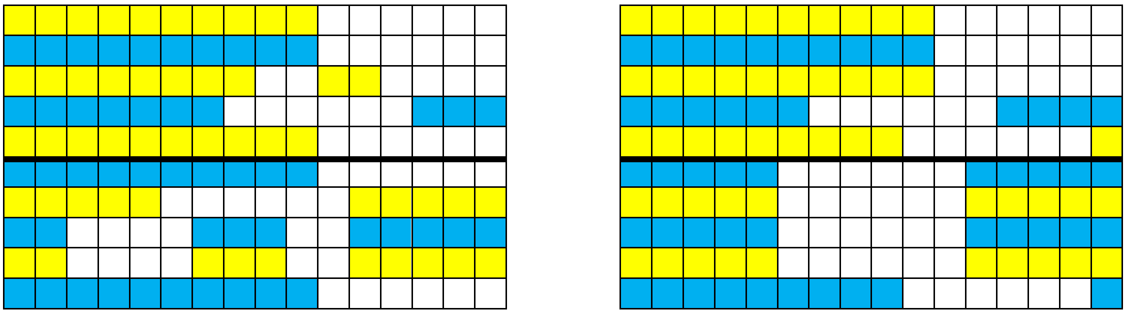


Figure 8: On-top Addition Of 3 5s And 2 3s As 5s On An Abacus With Geometry And Algebra Mode

Using a calculator to predict the result we include the two totals in a bracket before counting in 5s: Asking '(3x5 + 2x3)/5' gives the answer 4.some. Taking away 4 5s leaves 1. So the answer is 4.1 5s.

(3 x 5 + 2 x 3)/ 5	4.some
(3 x 5 + 2 x 3) – 4 x 5	1

Figure 9: A Calculator Predicts That On-Top Addition Of 3 5s And 2 3s Gives 4.1 5s

To add next-to means adding areas called integration also. Asking '3 5s and 2 3s total how many 8s?' we can use sticks or letters to see that the answer is 2.5 8s.

IIIII IIIII IIIII III III → IIIII III IIIII III IIIII → 2) 5) 8s → 2.5 8s, or 3B + 2C = 2BC + B

On an abacus in geometry mode a stack of 3 5s is moved to the right and a stack of 2 3s is moved to the middle. Now a 5-bundle is moved to the single line allowing the two stacks to be integrated as 8s, showing that the grand total is 2.5 8s. Likewise when using algebra mode.

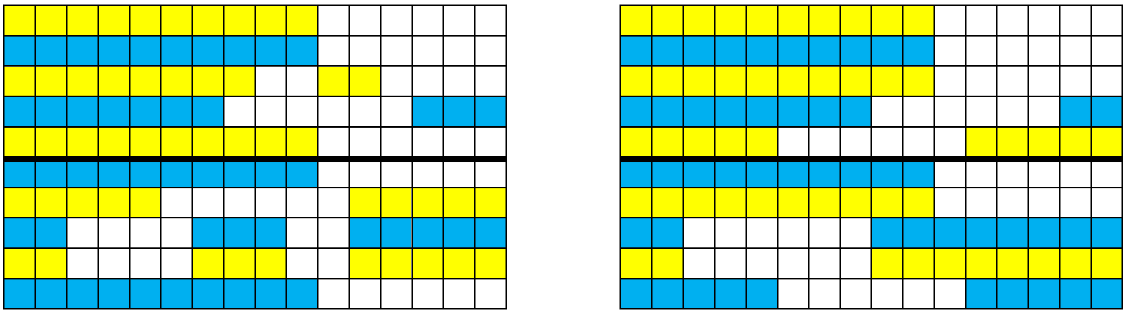


Figure 10: Next-to Addition Of 3 5s And 2 3s As 8s On An Abacus With Geometry And Algebra Mode

Using a calculator to predict the result we include the two totals in a bracket before counting in 8s: Asking '(3x5 + 2x3)/8' gives the answer 2.some. Taking away the 2 8s leaves 5. So the answer is 2.5 8s.

(3 x 5 + 2 x 3)/ 8	2.some
(4 x 5 + 2 x 3) – 2 x 8	5

Figure 11: A Calculator Predicts That Next-To Addition Of 3 5s And 2 3s Gives 2.5 8s

Reversing Adding On-top And Next-to

To reverse addition is called backward calculation or solving equations also. Asking '3 5s and how many 3s total 4.1 5s?' we can use sticks or letters to see that the answer is 2 3s:

IIIII IIIII IIIII III III ← IIIII IIIII IIIII IIIII I ← 4) 1) 5s ← 4.1 5s, or 4BI = 3B IIIII I = 3B + 2C

On an abacus in geometry mode a stack of 4 5s and 1 is moved to the right and a stack of 3 5s is moved back to the left. Now the remaining is recounted in 3s as 2 3s. Using algebra mode, after moving 3 bundle-beads to the left, the last is changed to 1s, allowing the 1s to be recounted as 2 3s.

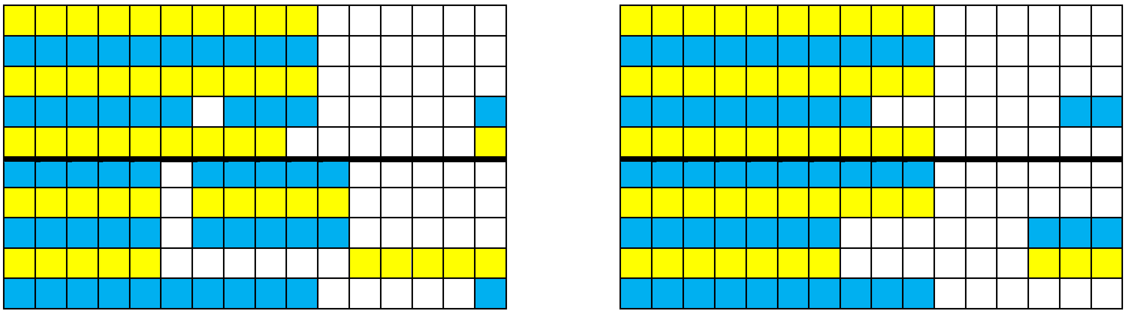


Figure 12: Reversed On-top Addition Of 3 5s And Some 3s to 4.1 5s On An Abacus

Using a calculator to predict the result we include the two totals in a bracket before counting in 3s: Asking '(4x5 +1 – 2x3)/3' gives the answer 2. Taking away the 2 3s leaves 0. So the answer is 2.0 3s or 2 3s.

(4 x 5 + 1 – 3 x 5)/ 3	2
(4 x 5 + 1 – 3 x 5) – 2 x 3	0

Figure 13: A Calculator Predicts That 2 3s Is What Must Be Added To 3 5s To Give 4.1 5s

To reverse next-to addition is called reversed integration or differentiation also. Asking '3 5s and how many 3s total 2.5 8s?' we can use sticks or letters to see that the answer is 2 3s:

IIIII IIIII IIIII III III ← IIIII III IIIII III IIIII ← 2) 5) 8s ← 2.5 8s, or = 2BC + B = BCBC B = 3B + 2C

On an abacus in geometry mode a stack of 2 8s and 5 is moved to the right and a stack of 3 5s is moved back to the left. Now the remaining is recounted in 3s as 2 3s. Using algebra mode, each 8-bundle can be changed to a 5-bundle and 3 1s. So moving 3 5s to left leaves 2 3s.

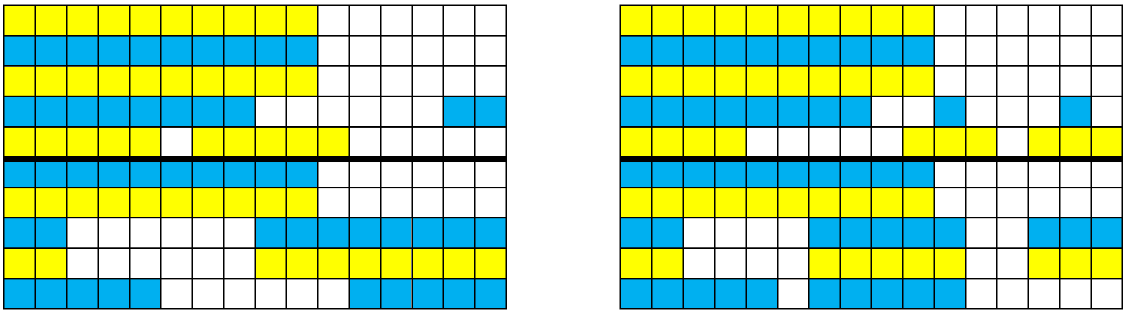


Figure 14: Reversed Next-to Addition Of 3 5s And Some 3s To 2 8s On An Abacus

Using a calculator to predict the result we include the two totals in a bracket before counting in 3s: Thus asking '(4x5 +1 – 2x3)/3' gives the answer 2. Taking away the 2 3s leaves 0. So the answer is 2.0 3s or 2 3s.

(2 x 8 + 5 – 3 x 5)/ 3	2
(2 x 8 + 5 – 3 x 5) – 2 x 3	0

Figure 15: A Calculator Predicts That 2 3s Is What Must Be Added To 3 5s To Give 2.5 8s

We notice that adding the two stacks 2 3s and 4 5s next-to each other means performing multiplication before adding; and that reversing integration means performing subtraction before division, as in the gradient formula y' = dy/t = (y2 – y1)/t.

Overloads And Extra Cups

With overloads also bundles can be bundled and placed in a new cup to the left. Thus in 6.2 3s, the 6 3-bundles can be re-bundled into two 3-bundles of 3-bundles, i.e. as 2))2 or 2)0)2), leading to the decimal number 20.2 3s:

III III) II) → II)) II), or 6)2) = 2))2) = 2)0)2, or 6.2 3s = 20.2 3s.

Adding an extra cup to the right shows that multiplying with the bundle-size just moves the decimal point:

T = 2.1 3s = 2)1) → 2)1)) = 21.0 3s

Traditional Counting

Traditional mathematics counts in tens only, which can be called 'third order counting'. Written in its full form, 354 = 3*10^2 + 5*10 + 4*1 becomes a sum of areas placed next-to each other, thus showing the four ways to unite numbers: Power unites bundles of bundles into a new bundle-size, multiplication unites like bundles into stacks, integration unites stacks with different bundle-size, and addition unites singles. Reversing uniting is predicted by the inverse operations called root, division, differentiation and subtraction. Thus it makes good sense that algebra means to reunite in Arabic.

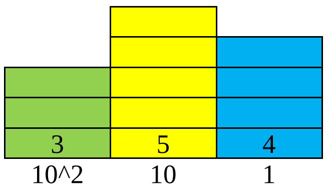


Figure 16: The Number 354 = 3*10^2 + 5*10 + 4*1 Shown As Three Integrated Stacks

The Two Counting Formulas

We have seen that to count a total of 7 in 3s, first we take away as many 3-byndles as possible, symbolized as 7/3, to be arranged as a stack of 2 3-bunles, symbolized as 2x3, that then is taken away to look for leftovers, symbolized as 7 – 2x3. Thus the counting process involves division, multiplication and subtraction, but no addition.

Counting a total T in 3s gives the counting result that T can be counted in T/3 3s. Thus we can set up a recounting formula T = (T/b) x b saying that a total T can be counted as (T/b) bs. This formula is also called a proportionality formula showing that proportionality is just another word for shifting units.

To look for leftovers, the stack is taken away and placed next-to what is left of the original total T. Again we can set up a restacking formula T = (T – b) + b saying that a total can be split into a stack and leftovers.

COMPARING MANYMATICS AND MATHEMATICS

Using postmodern contingency research we have discovered a natural science about the physical fact Many that can be called 'ManyMatics' and that allows us to deal with Many by counting and adding: First we count in icons, then in icon-bundles allowing a total to be written in a natural way as a decimal number with a unit where the decimal point separates the bundles from the unbundled. To add on-top and next-to we change the unit by recounting, predicted by a recount- and a restack-formula. Written out fully as stacked bundles, numbers show the four ways to unite: on-top and next-to addition, multiplication, and power. And to reverse addition we need inverse operations (Zybartas et al 2005), (YouTube), (Tarp 2014).

Counting Many by cup-writing and as stacked bundles contains the core of the mathematical sub-disciplines algebra and geometry. However there are fundamental differences between ManyMatics and Mathematics.

In the first an icon contains as many sticks or strokes as it represents, in the second an icon is just a symbol. In the first a natural number is a decimal number with a unit using the decimal point to separate bundles and unbundled; in the second a natural number hides the unit and misplaces the decimal point one place to the right.

The first presents operations as icons with the natural order division, multiplication, subtraction and two kinds of addition, on-top and next-to; the second presents operations as symbols; the order is the opposite; and next-to addition is neglected. The first uses a calculator for number prediction. The second neglects it. The first allows counting in icons, the second only allows counting in tens.

With ten as THE bundle-size, recounting becomes irrelevant and impossible to predict by a calculator since asking '3 8s = ? tens' leads to T = (3x8/ten) tens that cannot be entered. Now the answer is given by multiplication, 3x8 = 24 = 2 tens + 4 1s, thus transforming multiplication into division. Likewise adding next-to is neglected and adding on-top becomes THE way to add.

Furthermore the tradition changes mathematics into 'metamatism', a combination of 'meta-matics' and 'mathema-tism' where metamatics turns mathematics upside down by presenting concepts as examples of abstractions instead of as abstractions from examples, thus insisting that numbers are examples of sets in one-to-one correspondence; and where mathematism allows addition without units, thus presenting '1+2=3' as a natural fact in spite of its many counterexamples as 1 week + 2 days = 9 days, 1 m + 2 cm = 102 cm etc.

Thus the goal of a preschool curriculum should be the golden learning opportunities coming from icon-counting and next-to addition since they both disappear when traditional metamatism suppresses ManyMatics from day one in school. So ManyMatics is an example of postmodern 'paralogy' described by Lyotard to be a dissension to the ruling consensus (Lyotard 1984, 61).

THE TRADITIONAL PRESCHOOL MATHEMATICS

At the twelfth International Congress on Mathematical Education, ICME 12, the topic study group on Mathematics education at preschool level contains two interesting contributions from Sweden (http://www.icme12.org/sub/tsg/ tsg_last_ view.asp?tsg_param=1). The second discusses the content knowledge needed for preschool teachers to guide mathematical learning; and the first discusses the difficulties trying to categorize children behavior according to the revised preschool curriculum in Sweden from 2011, inspired by five categories claimed by Bishop to constitute mathematics (Bishop 1988).

The five categories are counting, i.e. the use of a systematic way to compare and order discrete phenomena; locating, i.e. exploring one's spatial environment and conceptualizing and symbolizing that environment, with models, diagrams, drawings, words or other means; measuring, i.e. quantifying qualities for the purposes of comparison and ordering; designing, i.e. creating a shape or design for an object or for any part of one's spatial environment; and playing, i.e. devising, and engaging in, games and pastimes, with more or less formalised rules that all players must abide by.

Bishop's five activities reminds of Niss' eight competencies: thinking mathematically; posing and solving mathematical problem; modelling mathematically ; reasoning mathematically; representing mathematical entities; handling mathematical symbols and formalisms; communicating in, with, and about mathematics; and making use of aids and tools (Niss 2003). Both define mathematics with action words. Bishop uses general words whereas Niss is caught in self-reference by including the term mathematics in its own definition.

However, both exceed in numbers vastly the two activities of ManyMatics, counting and adding, so skeptical thinking could ask: Since the numbers of activities alone makes it almost impossible for teachers and children to learn, is there a hidden patronizing agenda in these longs lists since just two activities or competences are needed to deal with the physical fact Many? And is it mathematics or metamatism these lists define?

To illustrate the issue we now look at the web-based training of in-service teachers at the MATHeCADEMY.net using 'pyramid-education'.

MICRO-CURRICULA AT THE MATHECADEMY.NET

The MATHeCADEMY.net sees mathematics as ManyMatics, the natural science about the physical fact Many. It teaches teachers to teach this natural science about Many to learners by allowing both teachers and learners to learn mathematics through investigations guided by educational questions and answers.

Seeing counting and adding as the two basic competences needed to deal with Many, it uses a CATS method, Count & Add in Time & Space, in a Count & Add laboratory where addition predicts counting-results, thus making mathematics a language for number-prediction. The website contains 2x4 study units with CATS1 for primary school and CATS2 for secondary school.

In pyramid-education 8 in-service teachers are organized in 2 teams of 4 teachers, choosing 3 pairs and 2 instructors by turn. The Academy coach helps the instructors instructing the rest of their team. Each pair works together to solve count & add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation. The coach helps the instructors to correct the count & add problems. In each pair each teacher corrects the other teacher's routine-assignment. Each pair is the opponent on the essay of another pair. Having finished the course, each in-service teacher will 'pay' by coaching a new group of 8 in-service teachers.

FIVE PLUS TWO LEARNING STEPS

The in-service teachers learn in the same way as their students by carrying out five learning steps: to do, to name, to write, to reflect and to communicate. For a teacher two additional steps are added: to design and to carry out a learning experiment, while looking for examples of cognition, both existing recognition and new cognition. To give an example, wanting children to learn that 5 is an icon with five sticks, the steps could be:

Do: take 5 sticks and arrange them next to each other, then as the icon 5.

Say: a total of five sticks is rearranged as the number icon 5, written as T=5.

Reflect. That five sticks is called five is old cognition. It is new cognition that five sticks can be rearranged as a 5-icon and that this contains the number of sticks it represents.

Communicate. Write a postcard: 'Dear Paul. Today I was asked to take out five sticks and rearrange them as a 5-icon. All of a sudden I realized the difference between the icon 5 and the word five, the first representing what it describes and the second representing just a sound. Best wishes'.

Design an experiment: I will help Michael, who has problems understanding 2digit numbers. Once he tries to build a number symbol for ten, eleven and twelve, he will realize how smart it is to stop inventing new symbols and instead begin to double-count bundles and unbundled. So I design an experiment asking the children to build the first twelve number-icons by rearranging sticks.

Carry out the experiment: It is my impression that constructing the number icon for ten was what broke the ice for Michael. It seems as if it enabled Michael to separate number-names from number-icons, since it made him later ask 'Why don't we say one-ten-seven instead of seventeen? It would make things much easier.' This resonates with what Piaget writes:

Intellectual adaptation is thus a process of achieving a state of balance between the assimilation of experience into the deductive structures and the accommodation of those structures to the data of experience (Piaget 1970: 153-154).

DESIGNING A MICRO-CURRICULUM SO MICHAEL LEARNS TO COUNT

This 5-lesson micro-curriculum uses activities with concrete material to obtain its learning goals. In lesson 1 Michael learns to use sticks to build the number icons up to twelve, and to use strokes to draw them, thus realizing there are as many sticks and strokes in the icon as the number it represents, if written less sloppy.

In lesson 2 Michael learns to count a given total in 1s and in 4s; and to count up a given total containing a specified numbers of 1s or of 4s.

Lesson 3 repeats lesson 2, now counting in 3s.

Lesson 4 combines lesson 2 and 3, now counting in 1s, 3s and 4s.

In lesson 5 Michael learns to recount in 4s a total already counted in 3s, both manually and by using a calculator; and vice versa.

As concrete materials anything goes in lesson 1. The other lessons will use fingers, sticks, pegs on a pegboard, beads on an abacus, and LEGO blocks.

Another 5-lesson micro-curriculum could make Michael learn to add on-top and next-to to be able to answer questions like 2 3s + 4 5s = ? 3s = ?5s = ?8s. This will not be discussed further here.

Lesson 1: Building And Drawing Number Icons

On the floor the children place six hula hoop rings next to each other as six different lands: empty-land, 1-land, 2-land, 3-land, 4-land and 5-land shown by the corresponding number of chopsticks on a piece of paper outside the ring.

Each child is asked to find a thing to place in 1-land, and to explain why. Then they are asked to turn their thing so it has the same direction as the chopstick. Finally the group walks around the room and points out examples of ‘one thing’ always including the unit, e.g. 1 chair, 1 ball, etc.

In the same way each child is asked to find a thing to place in 2-land. The instructor shows how the two chopsticks can be rearranged to form one 2-icon. The children are asked to pick up two sticks and do the same; and to draw many examples of the 2-icon on a paper discussing with the instructor why the 2-icon on the wall is slightly different from the ones they draw. Now the children are asked to rearrange their 2s in 2-land so they have the same form as the 2-icon. And again the group walks around the room and points out examples of ‘two things’ that is also called ‘one pair of things’.

This is now repeated with 3-land where three things are called one triplet. Before going on to 4-land the instructor asks the children to do the same with empty-land. Since the empty-icon cannot be made by chopsticks the instructor ask for proposals for an empty-icon hoping that one or more will suggest the form of the ring, i.e. a circle. And again the group walks around the room to try to locate examples of ‘no things’ or zero things.

Now the activity is repeated with 4-land. Here the instructor asks the children to suggest an icon for four made by four sticks. When summing up the teacher explains that the adults have rejected the square since it reminds too much of a zero, so the top stick is turned and placed below the square to the right. Here the children are asked to rearrange their 4s in 4-land so they have the same form as a square, and as the 4-icon. And again the group walks around the room and points out examples of ‘four things’ that is also called ‘a double pair’.

Now the activity is repeated with 5-land. Here the instructor asks the children to suggest an icon for five made by 5 sticks. When summing up the teacher explains that the adults have decided to place the five stick in an s-form. When walking around the room to point out examples a discussion is initiated if ‘five things’ is the same as a pair plus a triplet, and as a double pair plus one.

This activity can carry on designing icons for the numbers from six to twelve realizing that the existing icons can be recycled if bundling in tens.

Observing And Reflecting On Lesson 1

Having designed a micro-curriculum, the in-service teacher now carries it out in a classroom looking for examples of recognition and new cognition.

One teacher noticed the confusion created by asking the children to bring things to empty-land. It disappeared when one child was asked what he had just put into the ring and answered no elephant. Now all of the children were eager to put no cars, no planes etc. into the ring. Later the teacher witnessed children discussing why the 3-icon was not a triangle, and later used the word four-angle for the square. Also this teacher noticed that some children began to use their fingers instead of the chopsticks.

Under the walk around the room a fierce discussion about cheating broke out when a child suggested that clapping his hand three times was also an example of three things. It is not, another child responded. It is. No its not! Why not? Because you cannot bring it to 3-land! Let's ask the teacher!

After telling about space and time, children produced other examples as three knocks, three steps, three rounds around a table, and three notes. Other children began to look at examples of threes at their own body soon finding three fingers, three parts on a finger, and three hands twice when three children stood side by side and the middle one lent out his two hands to his neighbors.

CONCLUSION

To find how mathematics would look like if built as a natural science about the physical fact Many, and how this could affect early childhood education, postmodern contingency research has uncovered ManyMatics as a hidden alternative to the ruling tradition in mathematics. Dealing with Many means bundling and counting in icons, and recounting when adding on-top or next-to thus introducing proportionality and calculus. Likewise reversing on-top or next-to addition leads to solving equations and differentiation. Furthermore, the fact that totals must be counted before being added means that the operations division, multiplication, subtraction must be introduced before addition. However, these golden learning opportunities are lost when entering grade one, where the monopoly of ten-counting and the opposite order of operations prevents both from happening. Furthermore grounded ManyMatics is replaced with metamatism true inside a classroom but not outside when introducing one-to-one corresponding sets and when teaching that 1+2 IS 3.

RECOMMENDATIONS

Besides commenting on internal research papers meant for themselves only, researchers should also produce papers telling governments that to enlighten and to prepare the learners for the outside world, the educational system must stop presenting its choices as nature. Instead it should be forced to accept the historic fact that long, long ago the antique collective name mathematics was split up into independent disciplines.

So instead of teaching mathematics, schools should teach the two competences needed to deal with the physical fact Many, to count and to add. Consequently, the golden learning opportunities in preschool mathematics should enter ordinary school that should be forced to accept icon-counting and next-to addition instead of suppressing it. Calculators should be included to allow predicting counting results. Likewise, an abacus should be reintroduced to primary school and used both in geometry and algebra mode. This means a need for a full scale re-education of pre-service and in-service teachers. The MATHeCADEMY.net using PRAMIDeDUCATION is designed to meet exactly this need in an effective, user-friendly and inexpensive way.

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