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CAUSAL SEM OF MATHEMATICAL COMPETENCES IN ELEMENTARY EDUCATION

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ABSTRACT: In this paper, authors defined mathematical competences for 7th year elementary school. The basic objective was to measure the mathematical competence or mathematical knowledge, skills and abilities in mathematical educations. Mathematical competences were grouped in following areas: Algebra and functions, Space and shapes, Measurements and Data. Statistical set for the research consisted of 48 children from the Elementary school Dr. Ivan Merz in Zagreb, Croatia. Authors had 15 measuring variables together with the evaluated results of described tasks. With statistical set with variables as measured mathematical competences the authors make the causal structural equation model (SEM) of mathematical competences. The authors use free software Tetrad 5.2.1-3 (Tetrad project 2015). In the results we describe structural equations between the mathematical competitions for 7th year elementary school children. This paper is a result of our previous research on causal modeling of mathematical competences in kindergarten (Tepeš at. all. 2013, 2014 and 2015)

Keywords: mathematical competences, structural equation model and causal model

INTRODUCTION

Mathematical Competences in Elementary School

Mathematical competence as a term is being defined as knowledge, ability and skills of applying the mathematical way of thinking or concluding with the purpose of solving of all kinds of mathematical or interdisciplinary problems as well as understanding mathematics as a cultural value, with the aim of understanding and creating the perception about things that surround us in our everyday life. It is important to emphasize that intuitive ways of solving tasks are supported by rules of logical opinion in the form of thinking and making adequate conclusions, using arguments, modeling, formulating and solving the problem.

Mathematical competences defined by lesson plan or curriculum with the acceptance of historical development and can be divided into four basic areas: Algebra and functions, Space and shapes, Measurements and Data. Each of these areas contain specific knowledge, abilities and skills which are presented.

Algebra and functions competences are:

- Apply percentages and percentage calculations in correct tasks and situations.
- Present simple dependence between two variables using words and tables of joined values, formula and graphic presentation, and explain the procedure by translating the variables in both forms.

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- Recognize and apply proportionality and reversed proportionality in simple tasks Solve linear equations and simple systems of two linear equations with two unknowns. Apply solutions with the goal of verifying the results.
- Translate the problem by using algebra signs (like number sentences, linear equation, and the system of two linear equations with two unknowns). Explain the procedure of solving, get the solution and determine the main points in final solutions.
- Recognize the process of joining which represents the function.

Shape and space competences are:
- Draw a specific point within the rectangle coordinate system determined by specific coordinates and a direction proposed by an equation. Recognize, draw, outline, and classify a geometrical figure in a plane (angle, triangle, polygons, circle, ring and its parts). Apply the similarity and compliance by solving geometrical problems.

Measuring competences are:
- Compare, estimate and measure the length and size of the angle and the average speed.
- Convert measuring units for length, surface, size of the angle and average speed.
- Apply the formula concerning the volume and size of geometrical figures and the formula used for summation of angles and polygons with the example of simple and practical problem solving.

Data competences are:
- The main goal is to classify, gather and organize the collected data and present them by using plain tables, tables of frequency, lined and circled diagrams. Study, analyze, explain, and interpret the named information which have been presented in various ways. Recognize accidental, impossible, probable and possible outcome. Recognize which outcomes of an accidental experiment are favorable considering the event and calculate the probability of such final outcome.

Statistical Set and Measuring Variable

Elements of our statistical set were 48 children from the Elementary school Dr. Ivan Merz. Testing was performed as a part of an ordinary testing of the children's competences, which is part of the school education's curriculum. [4] The gender structure of children examined is shown in Table 1:

Table 1. Children's gender

Children's gender	Number of children
male	27
female	21
Total	**48**

The measuring variables for algebra and functions competences are:
- Ratio and proportions (RATPRO)
- Proportionality (PROPOR)
- Inverse proportionality (INVPRO)
- Method of substitution (MRTSUB)
- Method of elimination (METELI)
- Graphing linear function (GRLIFU)

The measuring variables for shape and space competences are:
- Coordinate system (COORSY)
- Similarity of triangles (SIMTRI)
- Polygons (POLYGO)

The measuring variables for measuring competences are:
- Perimeter and an area of similar triangles (ARSITR)
- Peripheral and central angle (PECEAN)
- Perimeter and area of circle (ARECIR)

The measuring variables for data competences are:
- Percentage (PERCEN)
- Statistics (STATIS)
- Probability (PROBAB)

Every variable on this list was described separately through the tasks. Every task was evaluated with 1(needs improvement), 2(satisfactory), 3(good), 4(very good) and 5(excellent).

Task for measuring variables is:
Ratio and proportion (RATPRO). Determining any unknown participant of the ratio and applying normal sizes in reinforcement of the task. Example: In what ratio are the turtles? (Figure 1)

Proportionality (PROPOR). Recognizing and graphical presentation of proportionality and applying it in tasks concerning everyday existence. Example: Using this table, determine how many kunas (HRK) will you get for 200 Euros? (Figure 2)

Inverse proportionality (INVPRO). Recognizing and graphical presentation of reversed proportionality and determining the unknown value using practical tasks. Example: Two workers will redecorate an apartment in 15 days if they work 8 hour per day. In how many days will the apartment be redecorated if there are 3 workers working 10 hours per day?

Method of substitution (METSUB) and Method of elimination (METELI). Example: Find the solution for the system of two linear equations with two variables using: a) Method of substitution, b) Method of elimination. (Figure 3)

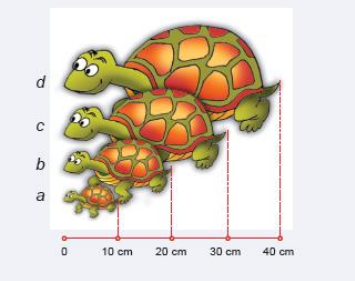
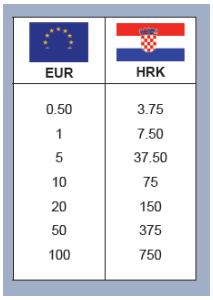


Figure 1



EUR	HRK
0.50	3.75
1	7.50
5	37.50
10	75
20	150
50	375
100	750

Figure 2

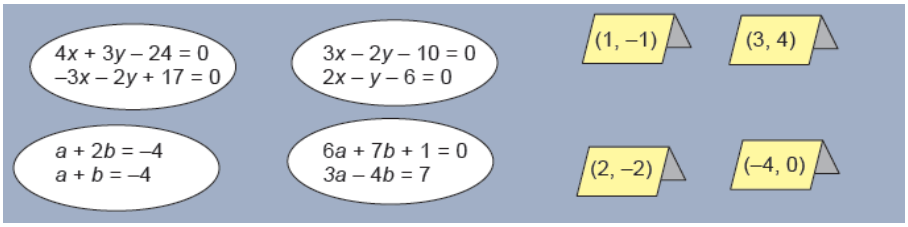


Figure 3

Graphing linear function (GRLIFU). Recognizing a chart presenting a function. Drawing a chart of a linear function emphasizing the dependence between two values using the chart and various tables. Example: Using the given chart, determine which day in a week was the hottest. (Figure 4)

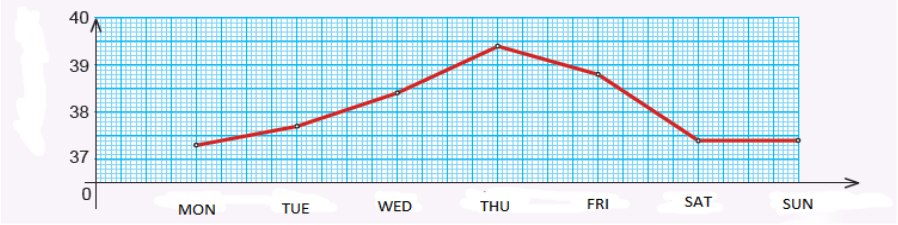


Figure 4

Coordinate system (COORSY). Pupils were asked to determine the position of a point in a plane using coordinates within the rectangle coordinate system. Each point was assigned to a rational number directed by rules. Example: Find the coordinates of marked points on a boat. (Figure 5)

Similarity of triangles (SIMTRI). Applying rules about triangle resemblances and determining the coefficient of similarity. Calculating the unknown lengths of triangle sides of different sorts of triangles. Example: Help the painter measure the man's height in ratio. (Figure 6)

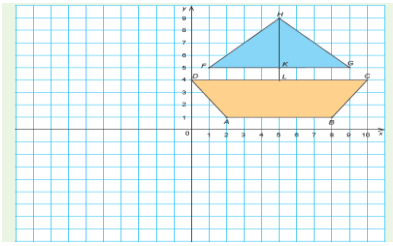


Figure 5

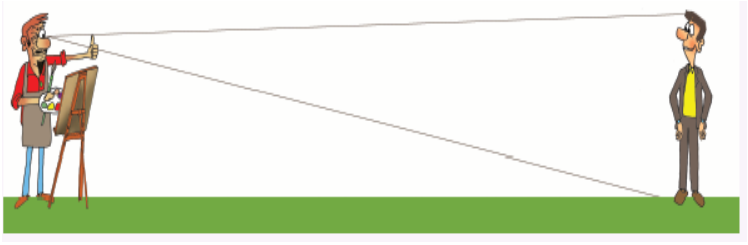


Figure 6

Polygons (POLYGO). Drawing and constructing polygons. Determining the total number of diagonals within the polygon. Calculating the total summing of all inner polygon angles and all angles of the specific triangles within the regular polygon. Example: Using the given data, find out the area of the polygon. (Figure 7)

Perimeter and an area of similar triangles (ARSITR). Comparison and calculating of volumes and sizes of similar triangles. Example: Using the given data, find out how wide is the lake? (Figure 8)

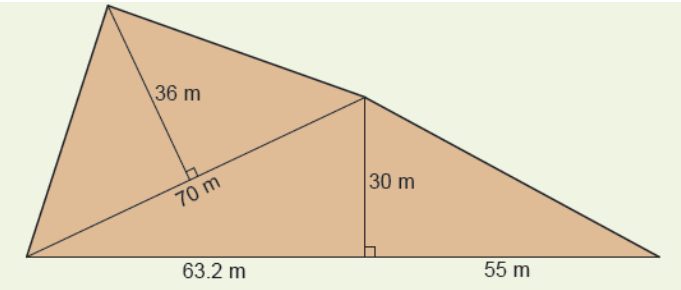


Figure 7

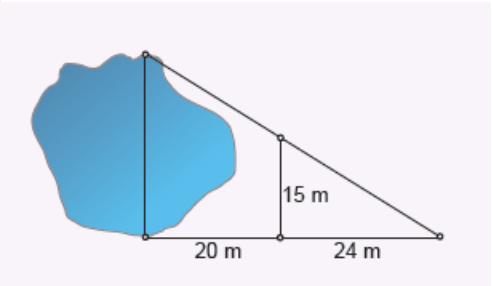


Figure 8

Peripheral and central angle (PECEAN). Determining the relation between the two sizes of two different angles and applying of Pythagoras rule in constructive tasks. Example: Determine the size of unknown angles. (Figure 9)

Perimeter and area of circle (ARECIR). The required understanding of how circle volume is proportional to its diameter. Calculating the area and size of a circle with having an area for his radius. Example: Determine an area of given circles. (Figure 10)

Percentage (PERCEN). Expressing the percentage using a rational number related to everyday situations. Example: Write down the percentage of colored parts in these shapes. (Figure 11)

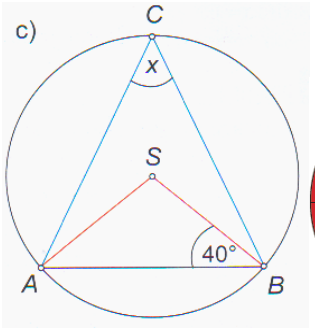


Figure 9

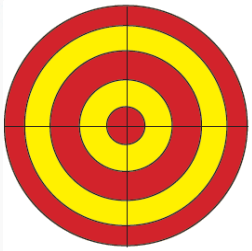


Figure 10

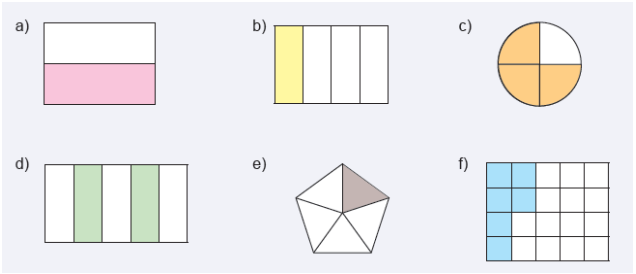
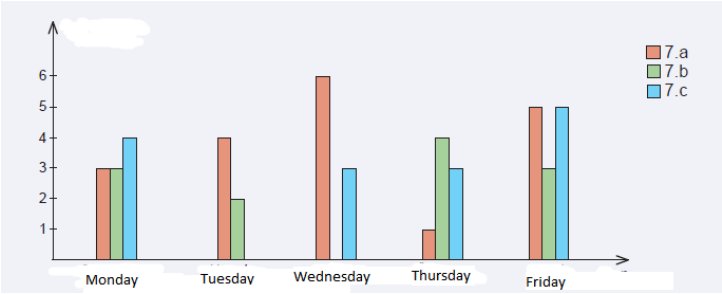


Figure 11

Statistics (STATIS). Recognizing the features of a certain information group, showing, sorting, and organizing the data from everyday situations. Presenting the information by using simple tables, charts or circled diagram. Example: Which class has the most absent students on Monday, and which the least absent students on Wednesday? (Figure 12)

Probability (PROBAB). Recognizing the impossible, probable and safe event. Calculating the probability of happening of an event (result). Example: Who do you think will win a 100 m race? (Figure 13)



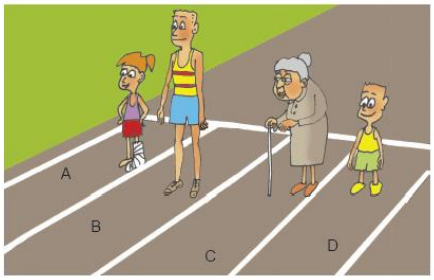


Figure 12
Figure 13

METHODS

Structure Equation Model (SEM)

At the beginning we have to describe structural equations model (SEM) (Boolen 2007). The variable of our model are measured mathematical competences or x_1, $x_{2,\ldots}$, $x_{15} \in$ {COORSY, RATPRO, PROPOR, INVPRO, PERCEN, STATIS, PROBAB, SIMTRI, ARSITR, POLYGO, PECEAN, ARECIR, METSUB, METELI, GRLIFU}. Structural equations are:

$$x_i = \sum_{j=1}^{17} b_{ij} x_j + e_i \qquad (i = 1,\ldots,17) \qquad (1)$$

where e_i are exogenous variables. In the matrix notation we can write equations.

$$x = Bx + e \qquad (2)$$

The causal structure model (CS) (Pearl 2000) can be represented with directed acyclic graph (DAG). In DAG we have vertices and edges between vertices. The vertices are mathematical competences in our SEM and the edges are causal relation between edges. For example edge $x_j \to x_i$ represent cause x_j and effect of cause x_i. Acyclic graph change structural equation (1) in new structural equations:

$$x_i = \sum_{j<i} b_{ij} x_j + e_i \qquad (i = 1,\ldots,17) \qquad (3)$$

And in the matrix equation notation (2) B is strictly lower triangular matrix. In the equation (3), if the coefficient $b_{ij} \neq 0$ we have edge $x_j \to x_i$ in directed acyclic graph (DAG) for causal structure model (CS).

In our paper we use free software Tetrad 5.2.1-3 (Tetrad project 2015). Main part of this software is Linear Non-Gaussian Acyclic Model (LiNGAM) (Simizu at al. 2006). LiNGAM work with independent component analysis (ICA) with estimation of coefficients with maximize log likelihood together with all the possible causal ordering. In the software Tetrad 5.2.1-3 we use program Linear Non-Gaussian Orientation Fixed Structure (LOFS). This program generates many different DAG.

RESULTS

The authors of this paper, teacher on Elementary School Dr. Ivan Merz in Zagreb (Paić at al. 2014a, 2014b, 2014c) together with professor (Tepeš at. all. 2009, 2013, 2014,2015) and methodologist on Faculty of Teacher Education at the University of Zagreb, choose the most appropriate model in mathematic on 7th year Elementary school represented in this paper.

Directed acyclic graph (DAG) mathematical competences on 7th year elementary school is shown in Figure 14:

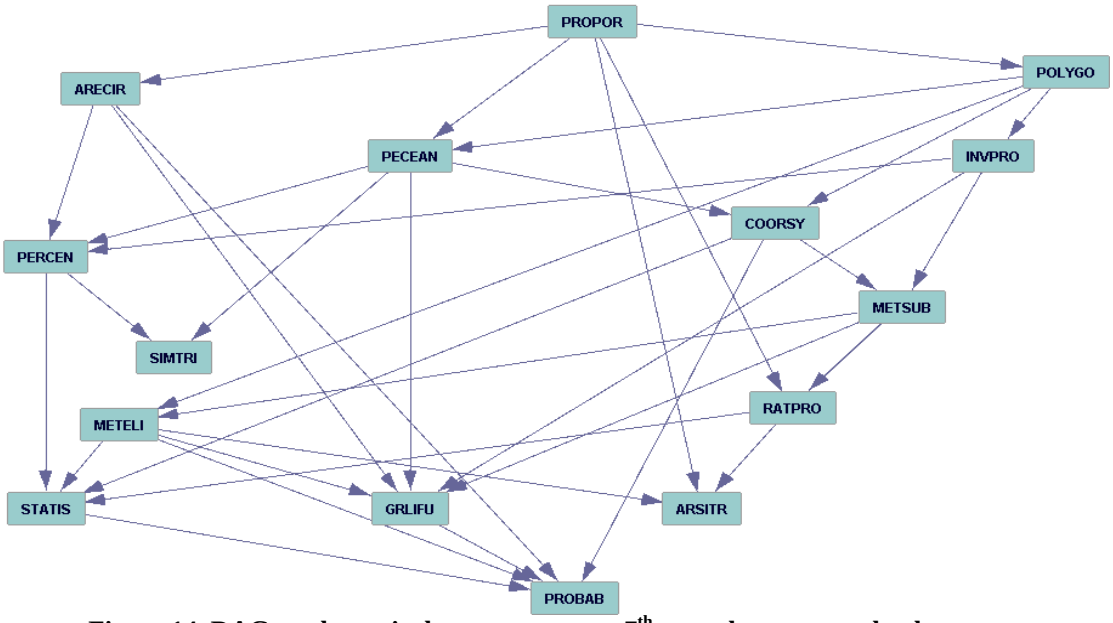


Figure 14. DAG mathematical competences on 7th year elementary school

From Figure 14 we can see eight levels of causal structure. On the first level there are causal mathematical competences PROPOR. This competences are fundamental and cause all other mathematical competences. On the second level there are mathematical competences ARECIR and POLYGO. This competences are effect of competences from the first level. On the third level there are competences effect causes from the first and the second level. Third level competences are PECEAN and INVPRO. On the fourth level there are PERCEN and COORSY. On the fifth level there are competences SIMTRI and METSUB or effect competences from previous levels. On the sixth level there is mathematical competence METELI and RATPRO. On the seventh level there is competence STATIS, GRAFLI and ARSITR. The last eighth level is competence PROBAB.

Using estimator from software Tetrad 5.2.1-3 we can describe causal structure equations (3) in our research:

$$
\begin{aligned}
&PROPOR = e(PROPOR)\\
&ARECIR = 0{,}8768PROPOR + e(ARECIR)\\
&POLYGO = 0.8393PROPOR + e(POLYGO)\\
&PECEAN = 0.8124PROPOR + 0.1044POLYGO + e(PECEAN)\\
&INVPRO = 0.8574POLYGO + e(INVPRO)\\
&PERCEN = 0.0349PECEAN + 0.4089ARECIR + 0.4455INVPRO + e(PERCEN)\\
&COORSY = 1.7644PECEAN + 0.1101POLYGO + e(COORSY)\\
&SIMTRI = 0.5042PECEAN + 0.4930PERCEN + e(SIMTRI)\\
&METSUB = 0.4872COORSY + 0.4933INVPRO + e(METSUB)\\
&METELI = 0.2541POLIGO + 0.7068METSUB + e(METELI)\\
&RATPRO = 0.6679PROPOR + 0.2924METSUB + e(RATPRO)\\
&STATIS = 0.0985RATPRO + 0.3957METELI + 0.3677PERCEN + 0.0707COORSY + e(STATIS)\\
&GRAFLI = 0.0886METELI + 0.1529ARECIR + 0.2080METELI + 0.2161INVPRO + 0.3397PECEAN + e(GRAFLI)\\
&ARSITR = 0.2589METELI + 0.5608RATPRO + 0.2016PROPOR + e(ARSITR)\\
&PROBAB = 0.3870METELI + 0.1997ARECIR + 0.1758GRLIFU + 0.3087STATIS + 0.3318COORSY + e(PROBAB)
\end{aligned}
\qquad (4)
$$

Exogenous competences are estimate with standard normal distribution $normal(0, s^2)$:

$First level$:
$e(PROPOR) \approx normal(0, 3.4221)$
$Second level$:
$e(ARECIR) \approx normal(0, 1.6750), e(POLYGO) \approx normal(0, 1.6103)$
$Third level$:
$e(PECEAN) \approx normal(0, 1.3849), e(INVPRO) \approx normal(0, 1.6369)$
$Fourth level$:
$e(PERCEN) \approx normal(0, 1.2442), e(COORSY) \approx normal(0, 3.4949)$
$Fifth level$:
$e(SIMTRI) \approx normal(0, 1.1550), e(METSUB) \approx normal(0, 1.2060)$
$Sixth level$:
$e(METELI) \approx normal(0, 1.2270), e(RATPRO) \approx normal(0, 1.2315)$
$Seventh level$:
$e(PROBAB) \approx normal(0, 1.0584)$

Coefficients in causal structure equations (4) are the average causal effect cause competence to competence on the left side of the equation. For example, competence PROPOR has greater average causal effect to competence PERCEAN than competence POLYGO because coefficient with PROPOR is 0.8124 and coefficient with POLYGO is 0.1044. All coefficients in equations are nonnegative. It means that every case competence has positive effect to competences on the second, third, fourth, fifth, sixth and seventh level.

CONCLUSION

Paper demonstrates the causal structure of mathematical competences in school education. For the purposes of adopting mathematical competences, causal model refers to the order of adopting of mathematical competences. For the purpose of further research, it is necessary to increase the statistical set or the number of children examined. Test materials must be standardized and must allow for higher gradation of results. The study should include part of mathematical competence relating to the data that have both numeric and descriptive characteristics expressed by words and letters. The curriculum for children in elementary schools in Croatia is based on social relations, and should have the educational structure of mathematical competences accustomed to the age and level of competences that children acquire by using information and communication technologies of contemporary society.

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