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CAUSAL SEM OF MATHEMATICAL COMPETENCES IN TEACHER EDUCATION

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ABSTRACT: In this paper, authors defined mathematical competences in teacher education. The basic objective was to measure the mathematical competence or mathematical knowledge, skills and abilities in mathematical education. Mathematical competences were grouped in following areas: Basic mathematical competences, Arithmetic competences, Functions competences, Combinatory competences, Geometry competences. Statistical set for the research consisted of 80 students from the Faculty of Teacher Education, University of Zagreb, Croatia. Authors had 17 measuring variables together with the evaluated results of described tasks. With statistical set with variables as measured mathematical competences the authors make the causal structural equation model (SEM) of mathematical competences. The authors use free software Tetrad 5.2.1-3 (Tetrad project 2015). In the results we describe structural equations between the mathematical competitions for students in teacher education. This paper is a result of our previous research on causal modeling of mathematical competences in kindergarten (Tepeš at. all. 2013, 2014 and 2015) and in elementary education.

Keywords: mathematical competences, structural equation model and causal model

INTRODUCTION

Mathematical Competences in Teacher Education

Mathematical competence in teacher education is defined as knowledge, ability and skills for future teachers for elementary schools. At the same time Faculty of Teacher Education, University of Zagreb have curriculum and lesson plan together with strategy, mission and vision of Teacher Faculty. The basic areas of mathematical competences are: Basic mathematical competences, Arithmetic competences, Functions competences, Combinatory competences and Geometry competences. The most important part in teacher education is to learn mathematical concepts, relations and arithmetic operations. Each of these areas contain specific knowledge, abilities and skills which are presented.

The basic mathematical competences include:

Mathematical logic,
Total mathematical induction,
Basics of set theory,
Relations,
Algebraic structures.

The arithmetic competences include:

Natural numbers and integers,
Number systems,
Rational numbers,

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Real numbers,
Complex numbers.
The functions competences include:
Basics of functions,
Linear function and linear equation,
Quadratic function and quadratic equation,
Word problems.

The combinatory competences include:
Basics of combinatorics.
The geometry competences include:
Plane geometry,
Solid geometry.

Statistical Set and Measuring Variable

Elements of our statistical set were 80 students from the Faculty of Teacher Education. During the 1st year students have six preliminary exams. That exams were performed as a part of the student's mathematical competences, which is part of the teacher education's curriculum. [4] The gender structure of students examined is shown in Table 1:

Table 1. Student's gender

Student's gender	Number of students
male	4
female	76
Total	**80**

The measuring variables for the basic mathematical competences were:
Mathematical logic (LOGIC)
Mathematical induction (INDUC)
Basics of set theory (SETTH)
Relations (RELAT)
Algebraic structures (ALGST)
Measuring variables for arithmetic were:
Natural numbers and integers (NATIN)
Number systems (NUSYS)
Rational numbers (RATIO)
Real numbers (REALN)
Complex numbers (COMPL)
Measuring variables for functions were:
Basics of functions (FUNCT)
Linear function and linear equation (LINEA)
Quadratic function and quadratic equation (QUADR)
Word problems (WORDP)
Measuring variables for combinatorics were:
Basics of combinatorics (COMBI)
Measuring variables for geometry were:
Plane geometry (PLANE)
Solid geometry (SOLID)

Every variable on this list was described separately through the questions in preliminary exams. Every question was evaluated with 0 (nothing), 1(needs improvement), 2(satisfactory), 3(good), 4(very good) and 5(excellent).

Task for measuring variables is:
Mathematical logic (LOGIC). Students were asked to compute the value of a logic formula or to prove a statement dealing with logic operations. Example: 'Prove: $(a \Rightarrow b)\ \&\ (a \Rightarrow c) = a \Rightarrow (b\ \&\ c)$.'
Mathematical induction (INDUC). Students were asked to prove a simple mathematical statement using the principal of mathematical induction. Example: 'Using the principal of mathematical induction prove that expression $7^n - 1$ is divisible by 6 for all natural numbers n.'

Basics of set theory (SETTH). Students were asked to prove a statement / answer a question / draw Venn's diagram for expressions dealing with set operations. Example: 'Using Venn's diagram prove: $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$.'

Relations (RELAT). Students were determining the elements of given relations and drawing the graph in coordinate plane. Example: ‘Determine the elements of relation $\rho = \{(x, y) \in \mathbb{N} \times \mathbb{N}: x + y = 10\}$ and draw the graph.’
Algebraic structures (ALGST). Students were asked to recognize algebraic operation or structure or to examine the properties of given algebraic operation. Example: ‘Examine if operation $\circ$ defined by $a \circ b = 2a + 2b - ab + 2$ is associative.’

Natural numbers and integers (NATIN). Students were solving arithmetic problems with integers, finding least common multiple (LCM) and greatest common divisor (GCD) or decomposing numbers into prime factors. Example: ‘Decompose number 67392 into its prime factors.’

Number systems (NUSYS). Students were recalculating given number from one number system into some other number system. Example: ‘Number $(2DC)_{16}$ write in number system with base 4.’

Rational numbers (RATIO). Students were solving arithmetic problems with rational numbers, computing given fraction into decimal number or vice versa, or determining percent or ratio in given word problems. Example: ‘Write number $-0.12\dot{3}5\dot{7}$ as a fraction and then reduce.’

Real numbers (REALN). Students were solving arithmetic problems with real numbers. Example: ‘Compute $\sqrt{3} \cdot \sqrt{\sqrt{7} + \sqrt{2}} \cdot \sqrt[4]{9 - 2\sqrt{14}}$.’

Complex numbers (COMPL). Students were solving arithmetic problems with complex numbers or simple complex equations. Example: ‘Find complex number z so that $2z + 4\bar{z} = 7 + i^{153}$.’

Basics of functions (FUNCT). Students were asked to solve a problem dealing with function, draw a graph, find inverse function or compute composition of functions. Example: ‘For given functions $f, g: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = \frac{1}{2}x - 3$, $g(x) = 2x^2 - x$, determine the compositions $f \circ g$ and $g \circ f$, and compute $g(2 + f(0))$.’

Linear function and linear equation (LINEA). Students were solving a problem dealing with linear equation. Example: ‘Compute the area of the triangle bounded with line $2x - 13y + 7 = 0$ and coordinate axes.’

Quadratic function and quadratic equation (QUADR). Students were solving a problem dealing with graph of quadratic function in coordinate system or quadratic equation / inequality. Example: ‘For which values of parameter m equation $mx^2 - 2mx + 3m - 2 = 0$ has two different real solutions?’

Word problems (WORDP). Students were asked to solve a word problem that leads to solving linear equation or system of two linear equations. Example: ‘The numerator of the fraction is smaller than denominator for 18. If we increase both the numerator and denominator by 8, we get number $\frac{1}{3}$. What is the value of the fraction?’

Measuring variables for combinatorics were:
Basics of combinatorics (COMBI). Students were asked to recognize and solve simple combinatory problem. Example: ‘How many 4-digit numbers has at least one digit 5?’

Measuring variables for geometry were:

Plane geometry (PLANE). Students were solving geometric problems dealing with angles, polygons and circle. Example: ‘Interior angle of a regular polygon equals $165°$. What is the number of its sides?’

Solid geometry (SOLID). Students were solving geometric problems dealing with the measurements of surface areas and volumes of various three-dimensional figures - prisms, pyramids, cylinders, cones and spheres. Example: ‘Calculate the volume and surface area of the circumscribed sphere of the cube which surface area is 216 cm^2.’

METHODS

Structure Equation Model (SEM)

At the beginning we have to describe structural equations model (SEM) (Boolen 2007). The variable of our model are measured mathematical competences or $x_1, x_2, \ldots, x_{17} \in$ { LOGIC, INDUC, SETTH, RELAT, ALGST,

NATIN, NUSYS, RATIO, REALN, COMPL, FUNCT, LINEA, QUADR, WORDP, COMBI, PLANE, SOLID}. Structural equations are:

$$x_i = \sum_{j=1}^{17} b_{ij} x_j + e_i \qquad (i = 1,\ldots,17) \qquad (1)$$

where e_i are exogenous variables. In the matrix notation we can write equations.

$$x = Bx + e \qquad (2)$$

The causal structure model (CS) (Pearl 2000) can be represented with directed acyclic graph (DAG). In DAG we have vertices and edges between vertices. The vertices are mathematical competences in our SEM and the edges are causal relation between edges. For example edge $x_j \to x_i$ represent cause x_j and effect of cause x_i. Acyclic graph change structural equation (1) in new structural equations:

$$x_i = \sum_{j<i} b_{ij} x_j + e_i \qquad (i = 1,\ldots,17) \qquad (3)$$

And in the matrix equation notation (2) B is strictly lower triangular matrix. In the equation (3), if the coefficient $b_{ij} \neq 0$ we have edge $x_j \to x_i$ in directed acyclic graph (DAG) for causal structure model (CS).

In our paper we use free software Tetrad 5.2.1-3 (Tetrad project 2015). Main part of this software is Linear Non-Gaussian Acyclic Model (LiNGAM) (Simizu at al. 2006). LiNGAM work with independent component analysis (ICA) with estimation of coefficients with maximize log likelihood together with all the possible causal ordering. In the software Tetrad 5.2.1-3 we use program Linear Non-Gaussian Orientation Fixed Structure (LOFS). This program generates many different DAG.

RESULTS

The authors of this paper are lecturers together with professor (Tepeš at. all. 2009, 2013, 2014,2015) on Faculty of Teacher Education at the University of Zagreb, choose the most appropriate model in mathematic on 1st year students studying mathematics on Faculty of Teacher Education.

Directed acyclic graph (DAG) mathematical competences on teachers education is shown in Figure 1:

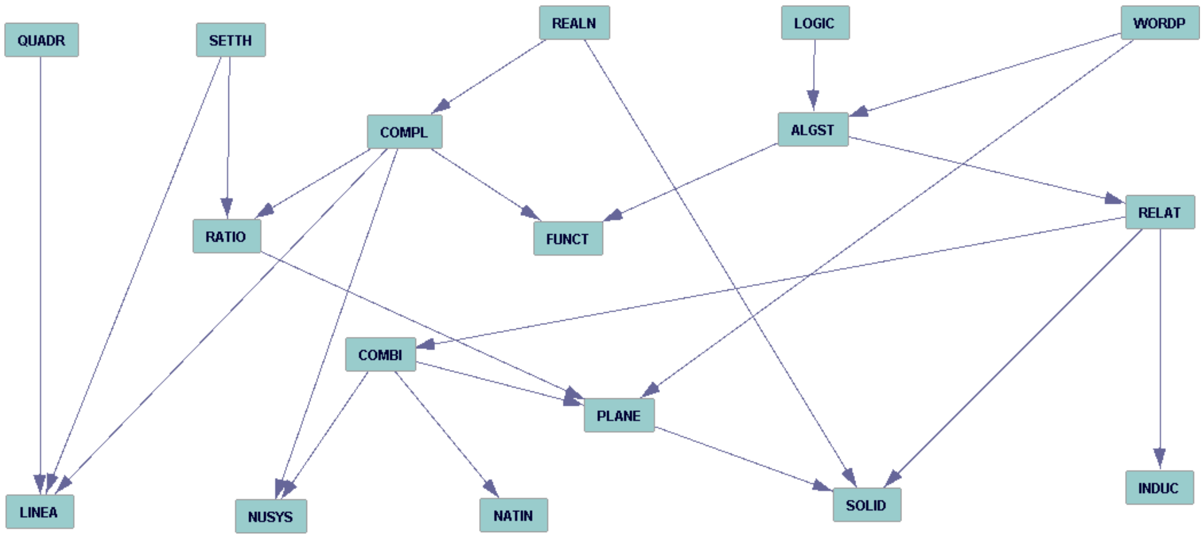

Figure 1. DAG mathematical competences on teacher education

From Figure 1 we can see six levels of causal structure. On the first level are causal mathematical competences QUADR, SEATH, REALN, LOGIC and WORDP. This competences are fundamental and cause for all other mathematical competences. On the second level are mathematical competences COMPL and ALGST. This competences are effect of competences from the first level. On the third level are competences effect causes from the first and the second level. Third level competences are LINEA, RATIO, FUNCT, REALT and INDUC. On the fourth level are competences COMBI or effect competences from previous levels. On the fifth level is mathematical competence NUSYS, NATIN and PLANE. On the sixth level is competence SOLID.

Using estimator from software Tetrad 5.2.1-3 we can describe causal structure equations (3) in our research:

$$
\begin{aligned}
&QUADR = e(QUADR)\\
&SEATH = e(SEATH)\\
&REALN = e(REALN)\\
&LOGIC = e(LOGIC)\\
&WORDP = e(WORDP)\\
&COMPL = 0.7776*REALN + e(COMPL)\\
&ALGST = 0.5833*LOGIC + 0.1694*WORDP + e(ALGST)\\
&LINEA = 0.3066*SETH + 0.2916*QUADR + 0.3686*COMPL + e(LINEA)\\
&RATIO = 0.5805*COMPL + 0.5187*SETH + e(RATIO)\\
&FUNCT = 0.6474*COMPL + 0.4953*ALGST + e(FUNCT)\\
&REALT = 0.5282*ALGST + e(REALT)\\
&COMBI = 0.6754*REALT + e(COMBI)\\
&NUSYS = 0.0042*COMBI + 1.0732*COMPL + e(NUSYS)\\
&NATIN = 0.5752*COMBI + e(NATIN)\\
&PLANE = 0.5437*RATUIO + 0.2616*WORDP + 0.3035*COMBI + e(PLANE)\\
&SOLID = 0.4018*PLANE + 0.2221*RELAT + 0.2150*REALN + e(SOLID)
\end{aligned} \tag{4}
$$

Exogenous competences are estimate with standard normal distribution $normal(0, s^2)$:

$$
\begin{aligned}
&First.level:\\
&e(QUADR) \approx normal(0, 2.6296), e(SETTH) \approx normal(0, 4.0000), e(REALN) \approx normal(0, 3.2942),\\
&e(LOGIC) \approx normal(0, 4.6524), e(WORDP) \approx normal(0, 3.6042)\\
&Second\,level:\\
&e(COMPL) \approx normal(0, 1.7610), e(COMPL) \approx normal(0, 1.7610)\\
&Third\,level:\\
&e(LINEA) \approx normal(0, 1.8375), e(RATIO) \approx normal(0, 1.8156), e(FUNCT) \approx normal(0, 1.9114),\\
&e(RELAT) \approx normal(0, 2.5653), e(INDUC) \approx normal(0, 2.0053)\\
&Fourthlevel:\\
&e(COMBI) \approx normal(0, 2.2793)\\
&Fifthlevel:\\
&e(NUSYS) \approx normal(0, 2.2035), e(NATIN) \approx normal(0, 3.3799), e(PLANE) \approx normal(0, 1.6953)\\
&Sixthlevel:\\
&e(SOLID) \approx normal(0, 1.5146)
\end{aligned}
$$

Coefficients in causal structure equations (4) are the average causal effect cause competence to competence on the left side of equation. For example competence LOGIC have greater average causal effect to competence ALGST than competence WORDP because coefficient with LOGIC is 0.5833 and coefficient with WORDP is 0.1694. All coefficients in equations are nonnegative. It means that every case competence has positive effect to competences on second, third, fourth, fifth and sixth level.

CONCLUSION

Paper demonstrates the causal structure of mathematical competences in teacher education. For the purposes of adopting mathematical competences, causal model refers to the order of adopting of mathematical competences. For the purpose of further research, it is necessary to increase the statistical set or the number of faculties and students examined. Preliminary exams materials and questions must be standardized. The curriculum for students on Faculty of Teacher Education at the University of Zagreb in faculty in Croatia is mainly based on pedagogy and should have the educational structure of mathematics for future job of students or teachers in elementary schools. It means that we have to look together mathematical competences in kindergarten, elementary and teacher education. Students on Faculty of Teacher Education at the University of Zagreb will be teachers in kindergarten and elementary schools.

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