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A Model Suggestion for In-service Teacher Training to Develop Mathematical Literacy

Murat Altun [a], ***Recai Akkaya*** [b], ***Tuğçe Kozaklı Ülger*** [c*], ***Işıl Bozkurt*** [d], ***Çiğdem Arslan*** [e], ***Furkan Demir*** [f]

a Prof. Dr., Bursa Uludag University, https://orcid.org/0000-0001-8853-8523,
b Assoc. Dr., Bolu Abant Izzet Baysal University, https://orcid.org/0000-0001-5369-7612
c Assoc. Dr., Bursa Uludag University, https://orcid.org/0000-0001-8413-8290 *tkozakli@uludag.edu.tr
d Assoc. Dr., Harran University, https://orcid.org/0000-0002-0720-7413
e Assoc. Dr., Bursa Uludag University, https://orcid.org/0000-0001-7354-8155
f Assoc. Dr., Kutahya Duplupinar University, https://orcid.org/0000-0003-3740-8088

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Abstract

This study discusses an in-service teacher training seminar planned to train teachers for a teaching process in which mathematical literacy is integrated into teaching. The seminar's content was created according to the Dual Focus Teaching Model. This study, which is design-based research, was planned and implemented for a total of 28 hours in 14 weeks, including what mathematical literacy is, making the content of the teaching suitable for mathematical literacy, choosing and writing a mathematical literacy question, introducing the Dual-Focus Teaching Model, and module development, planning and implementation for the teaching process in which the model will be used. The data of the study were collected through semi-structured interviews. The findings are presented under the headings of mathematical literacy and views on the planning and implementation of integrated courses. The results showed that the seminar's content and methodology followed in the seminar were suitable for teaching mathematical literacy and revealed that interactive studies and instant feedback in the courses held within the seminar's scope were important for the training of teachers.

Keywords: Mathematical literacy, middle school mathematics teachers, professional development, Dual-Focus Teaching Model, teaching planning.

Matematik Okuryazarlığını Geliştirmeye Yönelik Hizmet İçi Öğretmen Eğitimi İçin Bir Model Önerisi

Öz

Bu çalışmada, matematik okuryazarlığının öğretimle bütünleştirildiği bir öğretim süreci için öğretmenleri yetiştirmek üzere planlanan bir hizmet içi öğretmen yetiştirme eğitimi ele alınmaktadır. Eğitim içeriği Çift Odaklı Öğretim Modeline göre oluşturulmuştur. Tasarım tabanlı bir araştırma olan bu çalışma, matematik okuryazarlığının ne olduğu, öğretim içeriğinin matematik okuryazarlığına uygun hale getirilmesi, matematik okuryazarlığı sorusunun seçilmesi ve yazılması, Çift Odaklı Öğretim Modelinin tanıtılması ve modelin kullanılacağı öğretim süreci için modül geliştirme, planlama ve uygulamayı içeren 14 haftada toplam 28 saat olarak planlanmış ve uygulanmıştır. Araştırmanın verileri yarı yapılandırılmış görüşmeler yoluyla toplanmıştır. Bulgular, matematik okuryazarlığı ve bütünleştirilmiş derslerin planlanması ve uygulanmasına ilişkin görüşler başlıkları altında sunulmuştur. Sonuçlar, seminer içeriğinin ve seminerde izlenen metodolojinin matematik okuryazarlığı öğretimi için uygun olduğunu ve seminer kapsamında gerçekleştirilen derslerde etkileşimli çalışmaların ve anında geri bildirimin öğretmenlerin eğitimi için önemli olduğunu ortaya koymuştur.

Anahtar Sözcükler: Matematik okuryazarlığı, ortaokul matematik öğretmenleri, mesleki gelişim, Çift Odaklı Öğretim Modeli, öğretim planlama.

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INTRODUCTION

In recent years, the concept of "mathematical literacy (ML)" has come to the fore in secondary school mathematics teaching (Niss et al., 2016; 2017). The importance of ML in the agenda of secondary school mathematics teaching can be attributed to two main reasons. The *first* of these is *the incompatibility observed between school mathematics and life*. While explaining this situation, Steen et al. (2007) stated that "ML focuses on the use of basic mathematical knowledge and skills, while school mathematics focuses on the basic skills of advanced mathematics". The said disintegration and the problem it caused became more known all over the world when the Organization for Economic Co-Operation and Development (OECD) organized the Program for International Student Assessment (PISA), the primary purpose of which was to measure the level of ML achievement (OECD, 2019). For example, despite the time and effort spent on mathematics across the country, according to the latest PISA report, 61% of students in Turkey can only minimally recognize the mathematical representation of a simple situation (such as comparing the length of two different roads) (Ministry of National Education – MoNE, 2023). This shows that more than half of our students have low levels of mathematical proficiency (OECD, 2023). The *second reason is the reflections of access to information, which has become facilitated with technological developments, on teaching*. With the unrestricted use of the internet in 1985, it became easier to access information, and it became sufficient to spend less energy and time acquiring knowledge at school. When both situations are evaluated together, as and when the opportunities created by the facilitation of access to information are used to gain competencies that will be functional in the use of information in life, it can be thought that it is possible to reflect school mathematics more on human life. This situation, that is, the reflection of the learned knowledge to life, is expressed by the concept of ML, which has become well known with PISA assessments, and has become the main target of education mathematics programs today (see, MoNE, Secondary Education Mathematics Teaching Program, 2009, p:5).

The most influential factors on the emphasis on increasing ML achievement are teacher, student, and learning environment (Boesen et al., 2014; Niss et al., 2016; 2017). Niss et al. (2016; 2017) stated that teachers' knowledge of competencies strongly determines students' competencies. Teachers play an important role in scaffolding students' mathematical experiences so that they are more able to integrate mathematics into their real lives (Sumirattana et al., 2017). Botha et al. (2013) sought the answer to the question, “Is there a way, a method to teach ML effectively?”. They investigated teachers' teaching practices and how much they brought them into the classroom. In addition to being knowledgeable about practices, they determined that teaching experience plays an essential role in the productivity of teaching practices. Martin (2007) also stated that ML does not stem from the content of teaching but from the teaching methods performed by teachers. Traditional teaching methods, which involve memorising mathematical rules or formulae that are not relevant to students' real life or experiences, have failed to develop students' ML (Sumirattana et al., 2017).

These results suggest that a profound revision of the teacher education program or a special mathematics teacher education program is needed to develop ML. The primary purpose of our study, which focuses on this idea, is to reveal how "teacher education" should be, which can provide qualified ML teaching. To achieve this aim, the in-service teacher training part of the project named "Developing Mathematics Literacy with Dual-Focus Teaching Model", which was carried out and completed to improve ML teaching in the process, is discussed in terms of content and method, and its adequacy is discussed.

THEORETICAL FRAMEWORK AND LITERATURE REVİEW

Since it is closely related to the subject, information about ML, mathematical competencies, and Dual-Focus Teaching Model is given in the following section.

Mathematical Literacy and Its Integration into Teaching

The definition of ML in OECD resources is: “It is the capacity of individuals to formulate, use and interpret mathematics for various scopes and contents. It includes mathematical reasoning and the ability to use mathematical concepts, processing steps, verified information, and tools to describe, explain, and predict events. It helps individuals realize mathematics’ role in the world and make well-founded judgments and decisions that constructive, sensitive, and reflective citizens need." (OECD, 2013; 2023).

ML was seen as a solution to eliminating the gap between school mathematics and life, and the issue of how to teach has gained importance. Some studies on its teaching are as follows: Bansilal et al. (2015) analyzed teacher thoughts to identify the critical elements of a teacher training program for ML teaching. They concluded

that teacher education programs should be aligned with the subject's objectives, and competencies should be met within the curriculum. In a similar study, Frith and Prince (2006) introduced a program called "Advanced Certificate in Education" to increase the quality of ML. This program was conducted on 33 teachers, and it was reported how teachers perceived themselves as practitioners of ML. It was ensured that students had a share in determining the course content, critical thinking and communication were given importance in teaching, and collaborative studies were encouraged. This study concluded that the ML course carried out with these principles provides an effective environment for ML. Lengnink (2005) stated that it could be possible by developing a unique action style in his processes and called for a mathematics education that teaches this.

Mathematical Competencies

Mathematical competencies are concrete skills required to carry out certain mathematical activities (Jankvist & Kjeldsen, 2011). Different mathematical competencies constitute mathematical competency (Niss & Højgaard, 2019).

Competencies in mathematics teaching by different academic circles are defined under proficiency frameworks (mainly PISA, KOM, NCTM, MCRF, and NEPS proficiency frameworks). Although partial differences exist, almost all frameworks include problem-solving, reasoning, representation, communication, and modeling competencies.

To what extent an individual is mathematically literate can be decided by looking at the degree to which he reflects the mathematics he has learned to life situations. The reflection here is (i) Using mathematics to clarify a situation, (ii) Being able to model, (iii) Using mathematics as much as possible in the organization of social life, (iv) Handling events with a mathematical understanding, and (v) Appreciating the value of mathematics as a result of all these reflections. (Lengnink, 2005) All these forms of reflection can be revealed through mathematical competencies. In this respect, ML teaching requires the development of mathematical competencies. There are many studies on developing competencies in the teaching process or how to shape teaching to develop competencies (e.g., Blomhøj & Jensen, 2007; Niss et al., 2016).

Under the "Recommendations for Teachers for the Development of Competencies" title, OECD (2016) explained what teachers can do in their classrooms to develop student competencies and how to do them. Some of the suggestions made here are summarized as follows: i) Although not every student is expected to be a mathematician, the teaching of competencies is a necessity as they are necessary for all kinds of vital activities. (ii) Project assignments that are started in the classroom and completed outside should be included, and such tasks should not be abandoned by taking refuge in the well-known limitations of the curriculum. (iii) The questions should be varied, and the competencies should be allowed to be demonstrated. (iv) Cognitive activation of students should be kept active by summarizing, questioning, and guessing activities. (iv) Activities that allow students to work together or use new tools, such as technology and games, should be designed to reinforce mathematical concepts.

Palmer et al. (2018) conducted a study revealing the potential of mathematical competencies in teaching in primary schools. In this study, it was stated that for the development of entrepreneurship and mathematical competencies in the teaching process, changes in the role of the teacher are necessary, and there is a need for a "teacher who speaks less" and "a teacher who gives more room to leave the control to the students". In another study, Gresalfi et al. (2009) examined the development of competencies based on discourse analysis between teachers and students in mathematics courses in two secondary schools. This study revealed the importance of student's participation and learning opportunities in mathematics lessons for the development of ML. In the study of Blomhøj and Jensen (2007), it was stated that some mathematical competencies should be included in mathematics education for ML. Some other research; for example, Frith and Prince (2006), and Bansilal et al. (2015), emphasize that the development of ML should be met within the curriculum. Mbekwa (2006) and Bansilal et al. (2015) stated that there is a partial resistance to learning ML; Brown and Schafer (2006) stated that ML develops late, and it is possible to get good results over time.

Dual Focus Teaching Model

Today, the expectation of more reflection on knowledge in life has changed the form and content of teaching. Learning methods and techniques (5E, learning by discovery, teaching with the help of definitions, etc.) developed for learning theories focused on "knowledge creation" were insufficient to gain skills related to reflecting knowledge in life. This is because in traditional teaching, competencies remain insignificant next to learning, and they are contented as much as they can while acquiring subject knowledge. To meet this need in teaching, that is, to develop competencies within the teaching process, a Dual Focused Teaching Model (Cift

Odaklı Öğretim Modeli - COM) has been proposed (Altun, et al., 2022a.; Altun, et al., 2022b). These developments can be schematized as in Figure 1 regarding the fact that the concept of ML became more known and included in the curriculum with the PISA assessments that started in 2000:

Looking at the historical course of the developments in teaching (Figure 1), behavioral approaches left their place to cognitive approaches in the 20th century, and constructivist teaching has been more widely accepted among them (Fosnet, 2007). Constructivism is a theory of knowledge and deals with how knowledge is formed. The reflections of constructivism on teaching have been in the direction of increasing the quality of gaining knowledge (Jaworski, 1998). Another cognitive approach that affects mathematics teaching is "Realistic Mathematics Education (RME)". While RME also has a constructivist philosophy at its core, the difference from the constructivist theory emerges in reaching knowledge (Gravemaijer, 1999). As a result, the main goal of both theories is to gain understanding and increase quality in this process. In Figure 1, it is seen that the development of competencies in teaching has gained importance at the knowledge level and has taken its place in the teaching content as of the 2000s.

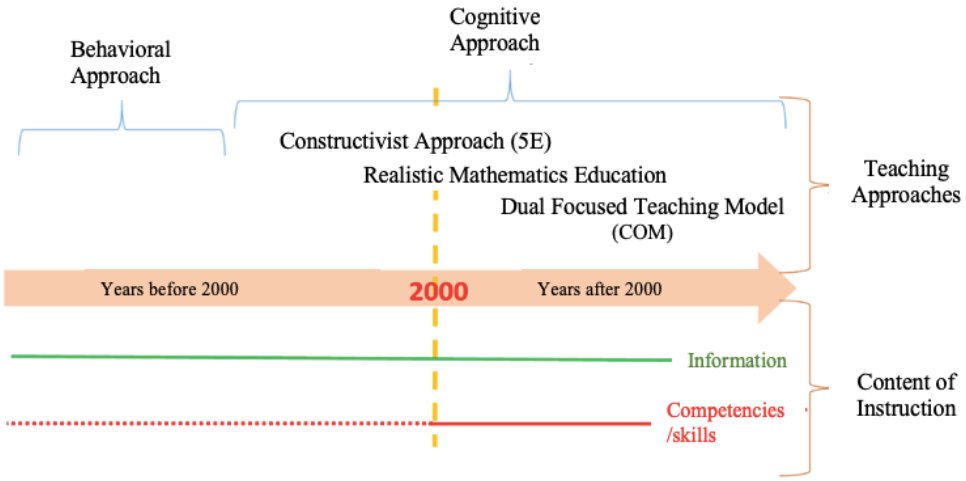

Figure 1. Historical process of involving knowledge and competencies in teaching

COM, which enables the programmatic development of competencies as well as gaining knowledge, teaches a mathematics subject. This approach deals with teaching at two focal points and their complementary work. The first focus is "work to gain knowledge". The constructivist approach gains knowledge, and as a complementary work, the deepening of the meaning is included. The second focus is "Teaching studies to ensure the reflection of knowledge in life": ML questions, vital applications of knowledge, etc. It is a simple approach and does not burden the teacher extra. As the lesson planner, the teacher needs to organize appropriate activity(s) to reach knowledge in the first focus and find ML questions and vital application example(s) of knowledge for the second focus. The two basic principles of COM are: "Mathematics is a matter of reflection and discussion" and "Teacher who speaks little". Both principles allow students to produce knowledge and defend their thoughts freely during the lesson. Exercises and verbal problems related to the subject remain in COM process (For detailed information, see Altun, et al., 2022b).

METHOD

The method of the research is "design-based research". Waug and Hanefun (2005) describe design-based research as "the investigation of cyclical analysis, design, development, and implementation processes. It is a systematic research method conducted in collaboration with the participants and in a real practice environment to develop context-sensitive design principles and theories and improve educational practices. In this study, a 28-hour seminar program for 14 weeks was prepared and applied to a selected group of teachers. The seminar content was constantly revised, considering the expectations of the participating teachers and the interim evaluations during the implementation. In this way, the study has a structure that overlaps design-based research. Specific forms of design-based research are tailored to how they are conducted in some specific areas. This study is suitable for "formative research," which aims to improve teaching systems from these. Information on the seminar's content, the participants' determination, the data collection tools, and the data analysis are given below.

Content of Teacher Training Seminar Integrated into ML

COM was used to determine the content of the teacher training seminar because it includes the literature and the development of competencies (Altun, 2021; Altun, et al., 2022a; Altun, et al., 2022b). The teacher training seminar consists of five main parts: (i) What is ML (2 hours), (ii) Choosing and writing an ML question (8 hours), (iii) Adapting the teaching content to ML (8 hours), (iv) Introducing the COM (4 hours) and (v) Module development, planning, and implementation (6 hours) suitable for COM.

The first chapter introduces how the term ML was conceptualized, its meaning and scope, its competencies, the historical development of PISA applications, and the reflections of the results on teaching. In the second part about writing ML questions, the differences between ML questions from traditional exam questions, contextuality, and references in writing ML questions are introduced. Then, examples of questions from numbers, geometry, data, change, and relations, the primary subjects of secondary school mathematics, were examined, and how the solutions were evaluated was discussed. In the third part, within the scope of making the teaching content suitable for ML teaching, the concept of activity, the use of activities to reveal concept knowledge and generalization information, and the teaching design ideal for the deepening step were studied. In the fourth chapter, the introduction of COM is given. Finally, it has been learned how to develop the competencies in writing, applying, and applying the module or lesson plan content according to COM possible opportunities, and difficulties to be encountered. For detailed information, examples, and activities regarding the seminar's content, the book Altun et al. (2022b) prepared can be reviewed.

In the planning of the seminar, the following principles were taken into account to *preserve the constructivist character of the teaching and to ensure the development of competencies*:

(i) Interactive studies should be adopted to implement the seminar program. It is important to keep individual discussions and discussions open during the teaching, check the daily homework given, and convey the corrections/feedback on the homework papers to the participants.

(ii) Instructional content should be a quality that will contribute to developing a sense of value toward mathematics.

(iii) Teachers should be encouraged to join or oppose the seminar's organizer by referring to the principle that "mathematics is a matter of thinking and discussion" during the seminar process (Altun, et al., 2022b).

Selection of Participants

To determine the teachers who will participate in teacher training, a call was made to the mathematics teachers working in secondary schools through the Bursa Provincial Directorate of National Education, and 49 teachers applied for this call. To determine those who will participate in the training, the applicants were asked to fill in an information form that included questions about their educational status, their experiences in ML, and the schools they worked at, and 32 teachers were determined by evaluating the answers given.

The demographic characteristics of the participants are as follows: According to gender, education level, and institution: 27 female, five male, 28 undergraduate, and four graduate. Twenty-eight of them teach in public schools, and 4 in private schools. Their professional experience ranges from 1 to 18 years.

Their accumulations on ML are; 3 teachers stated that they would encounter the concept of ML for the first time; most of the others indicated that they had encountered the concept before, but their knowledge was limited to the discourses about PISA and TIMSS assessments, and seven teachers said that they participated in an in-service course or seminar-like study on ML.

Data Collection Instrument and Process

At the end of the seminar, the research data were obtained through the interview form developed to determine the participants' thoughts about the seminar's effectiveness, the level of reaching the goals and satisfying their expectations, and the issues that need to be improved. This interview form was designed as semi-structured (Uslu & Demir, 2023) and consisted of open-ended questions. Models used in the evaluation of in-service training programs (Guskey, 2000; İnaltun, 2019; Yermeydan-Uğur, 2017; Yıldız, 2017) were used in the development of the interview form.

Firstly, a draft interview form was created. The draft form was then sent to three faculty members who are experts in the field of mathematics education. When the expert opinions were examined, it was seen that the feedbacks were generally for the intelligibility of the items forming the form, and there was no need for a comprehensive change in the content. After the expert's opinion, necessary adjustments were made to the draft form, and 15 interview items were included under five main headings in the state. Finally, the researchers applied

the form to the teachers who had received similar training before. As a result of these studies, some corrections were made to the items that were found to be partially overlapped. Through the interview form, which was given its final, online interviews were conducted in the week following the completion of the teacher training. Each interview, conducted by one of the authors of the present study, lasted 30-35 minutes. All online interviews were recorded with the permission of the teachers.

To check the reliability of interview instrument, participants were asked, “Do you recommend that other mathematics teachers take this training as well? If your answer is yes, explain why.” The question was asked, and the consistency between the answers given to this question and those offered for the standard evaluation questions was examined. The degree of consistency was evaluated as a measure of the reliability of the interviews.

Data Analysis

First, to analyze the qualitative data obtained from the interviews, transcripts of the recorded interviews were extracted. The data obtained from the interviews are based on the teachers' comments and statements about the training's content, applicability, and effectiveness. The data obtained from the interviews were subjected to content analysis. Based on the expressions here, categories and themes were determined, respectively.

Research Ethics

Ethical principles and rules were followed during the planning, data collection, analysis, and reporting of the research. The study was carried out after obtaining the permission of the ethics committee of ... University.

FINDINGS

Findings of the study; (i) Opinions about ML, (ii) Opinions about the contribution of the seminar to preparing the lesson plan, (iii) Opinions about the effectiveness of the teaching and the COM, (iv) Suggestions for the structuring of the education given (v) Reasons for (not) recommending this training to colleagues presented under the headings.

Opinions about ML

At the end of the seminar, questions were asked to the teachers about what ML is and how it can be improved. The findings obtained from the interview data formed three themes: conceptual information about ML, ML questions, and handling ML in the teaching process, which is presented in Table 1.

Table 1. Teachers' Opinions on ML at the end of the Seminar

Theme	Category	Code	f
Conceptual information about ML	Specializing in ML	Gaining awareness of ML	20
		Ability to integrate mathematics into daily life	17
		Domination over ML	13
		Clarification of confusion	4
	Skills included in ML	Deciding	2
		Reasoning	2
		Reading comprehension	1
	ML's contribution to attitudes	Love mathematics more	5
		Students gain confidence in solving ML questions	2
		Changing students' perspective on mathematics	2
	The role of the teacher in the development of ML	The importance of teacher's knowledge for ML	3
		Math literate teacher	3
ML problems	Information on ML problems	Detecting the ML problems	8
		Noticing topics that can/can't be ML problems	3
ML in the Teaching Process	Changes in teacher	Seeing mistakes and shortcomings	7
		Change in teaching style	3
		The tasks that are not difficult for the teacher in the process	2
		Being open to innovation	1

Changes in teaching practices	Make time for events	5
	Creating an environment for discussion	4
	Less room for exercises	4
	Early initiation of ML studies	3
	Working on improving the ML	2

Teachers stated that they specialized in ML with this training (Table 1), gained awareness of ML (20), and were no longer intimidated by ML questions. They dominated ML (13), could integrate mathematics into daily life (17), and were able to think about ML. (4) stated that their confusion became clear. Some have mentioned that the concept of ML includes skills such as decision-making (2), reasoning (2), and reading comprehension (1).

They stated that after this training, they could make students love mathematics more with ML (5), they could gain confidence in solving ML questions (2), and they gained the potential to change the students' perspective on mathematics (2). Finally, they stated that the teacher's knowledge is important in the development of ML (3) and they realized that the teacher should be mathematically literate (3).

The second theme (Table 1) regarding the change in perspectives is related to ML questions, and they stated that not every question is an ML question, and they can now easily identify ML questions (8). Regarding this situation, two teachers have shared their views;

T2: *"I had a different thought when I set out on the road. I thought questions with lots of words and paragraphs were ML questions. The main thing is to be realistic from daily life, interpret it, and make you think."*

T7: "I thought I was doing an ML question by adding a picture. But I realized I was on the wrong track.".

They stated that they understood that ML questions changed their perspectives on mathematics, that not all of the questions used in a course had to be ML questions, and that some subjects (percentage, interest, etc.) were more suitable for ML (3).

The third theme (Table 1) concerns how they perceive ML in the teaching process. Teachers stated that they had the chance to see the mistakes and shortcomings they made during the teaching process (7), they changed their teaching style (3), they realized that they had tasks that were not difficult (2) and that they needed to be open to new things (1). Regarding the change in teaching practices, they stated that while they did not dare to do activities for fear of losing control, they understood that it was necessary to devote time to activities (5). They realized it would be sufficient to create a discussion environment (4) and give less space to exercise; spacey stated that teacher education should start working in the early period (5th grade) to develop ML (3), and after that, they should worry about developing ML and work on it (2).

Opinions on the Contribution of the Seminar to Preparing the Lesson Plan

The teachers participating in this study were asked to what extent they felt ready to prepare a lesson plan suitable for ML, and the findings are presented in Table 2.

Table 2. Contribution of the Seminar to Prepare Lesson Plans for ML Integrated Courses

Theme	Category	Sub-Category	Code	f
Prepare a lesson plan in which ML is integrated	Yes	Absolutely		17
		But…	Practice more	4
			Difficulty preparing events	1
			Difficulty finding an original activity	1
			Unsure of accuracy/Need for confirmation	1
	Not sure	Because…	Don't require much effort	2
			Quantitative adequacy of samples	2
			The difficulty of the first focus	2
			Anxiety about not gaining concept knowledge	1
	No	Because…	Absenteeism in education	3

As seen in Table 2, most teachers (17) stated that they could *prepare*. Some of them said that they *could prepare*. Still, they needed more practice in preparing a plan (4), and they needed help in designing an activity (1) and finding a unique activity (1). They were still determining the accuracy of all plan stages and needed approval (1). Some of the teachers stated that the examples they encountered in the seminar were not quantitatively sufficient (2), that it required much work (2), and that it was challenging to find activities suitable for gaining concept knowledge (2). Three teachers said they could not prepare a lesson plan because they could not fully participate in the training. In addition, they stated that they had the opportunity to see how to attract students' attention through teaching with activities, which is an important dimension of COM (2). Regarding teaching with activity, T2 stated that *it is very important because it will attract the attention of students who have never been interested in mathematics, make them love mathematics, and develop a positive perspective*. Teachers also stated that they realized that there are points where they need to be more sensitive in planning lessons. For example, a teacher expressed, *"Now I choose the questions that I carry into my lesson plans with more subtle thoughts. I want to make sure they are questions based on the construction of knowledge."*.

Opinions on the Effectiveness of Teaching and The Dual-Focus Teaching Model

The teachers participating in this study were asked for their opinions about the effectiveness of the education they received, the usefulness of the COM, and the negative situations that may arise in using the model. The findings are presented in Table 3. The teachers' opinions were gathered under the themes of the operability of the teaching process and the sustainability of the teaching (Table 3).

Table 3. Contribution of the Seminar to Prepare Lesson Plans for ML Integrated Courses

Theme	Category	Sub-category	Code	f
Teaching Process Operability	Functional states	Teaching Qualification	Relating mathematics to life	10
			Providing permanent learning	9
			Fun learning environment	8
		Planning	Lesson planning and method	4
		Content of the Instruction	Teaching with activities	4
			Giving room for discussion	4
			Supporting the subject with applications Developing the ML	4
		Sense of Value	Changing perspective on mathematics	7
			Developing a sense of valuing mathematics	4
		ML problems	Developing question writing	3
			Looking critically at questions	2
Sustainability of Teaching	Blocking situations	Systemic	Having time problems in raising the subjects	10
			The effect of multiple-choice questions in the central examination system	4
			An excess number of students in classes	2
		Subject-based	Difficulty in finding ML questions suitable for the topic	4
			Inability to devote enough time to the second focus	3

As can be seen from the high-frequency values in Table 3, most teachers stated that the COM is a well-functioning teaching model in the mathematics teaching process. They explained the reason for this with expressions such as enabling the student to associate knowledge with life (10), realizing permanent learning (9), and making the learning process enjoyable (9).

While in-service teacher training was continuing, one of the teachers applied his lesson plan, which he had prepared by the COM, in his class and described this process as *"In my classes in 6th and 8th grades, I taught my lesson by applying the plans I had prepared. I started with the question and activity that would interest them in the first focus. At the end of the activity, I enabled the students to conclude. They liked that the students could find*

the formulas they used by the heart. The activities carried out increased the participation of the students. In the second focus, I included reinforcement work. I got good feedback. Thus, I saw the effect and usefulness of the model.".

The teachers (4) stated that the way of teaching the lesson makes the thoughts of students at all levels feel essential and gives the students a personality, makes them realize the necessity of mathematics in real life and its role in life, and encourages them to participate in the lesson. They emphasize the issues of which applications and how to support them to r to provide in-depth learning about mathematical concepts are quite effective for them. T2 expressed his opinion regarding this situation: *"I contributed to conveying to the students where we encounter mathematics in daily life."* Another group of teachers (3) got to know ML for the first time with this training. T4: "*I liked that we will get out of the routine and encourage students to be more literate."* He described the most compelling aspect of education for himself. Another teacher T12 Expressed his opinion: *"I think that if the teacher cares about literacy, the teacher will automatically care about the child. It was the most effective part because it changed my perspective from that point of view."*

In another dimension, teachers reported that there was a change in their perspectives on mathematics (7) and that they realized the importance of valuing mathematics with this education (4). Regarding the sense of value, T8 said, *"Attitude development was not my priority in my classes. But during this training, I saw that this should be the priority."* He stated that their education changed some of their views about students and that only some students could be successful in every subject thanks to this training. In parallel with what they have stated here, during their teacher education, they realized where and how the logic and basis of the things taught by rote for years came from, thanks to this training. They gained experience in how they can transfer the taught mathematics to the students' lives and how the abstraction of the taught information will take place in the minds of the students.

Teachers stated that they learned how to write ML questions thanks to this training (3) and even experienced this process for the first time. They indicated that they realized that the features of ML questions should not include unnecessary information in the context of the questions, be short, contain explanatory statements, be related to daily life, and that mathematical knowledge should be addressed. T3 explained what is meant by not ignoring mathematical knowledge: *"I realized that I ignored mathematics in the things I wrote to be a new generation question, ... I saw where I made a mistake. I saw what I needed to pay attention to".* Regarding the difficulty level of the ML question, the teachers stated that with this training, they realized that not every ML question had to be complicated. At the same time, they expressed that they needed to raise awareness in students that such questions are not complex.

Teachers, who stated that they focused on solving many questions and teaching knowledge rather than ML before the training, even in the same style, said that they realized that they should focus more on ML in their learning processes. One of the teachers expressed his opinion as;

T1: *"I experienced not solving too many questions, but a good activity enables effective learning."*

T6: *Two teachers contributed to my critical approach to all questions regarding their suitability for ML*. Finally, one teacher found it effective to be equipped to write ML questions and said, *"I am not afraid of ML questions anymore. Because I'm writing a question. I am prepared."*.

The teachers were asked for their opinions about the situations that could hinder the implementation of the COM, and their answers were collected in two categories. The first category is the barriers originating from the education system, and due to the intensity of the exams and curriculum in the academic calendar of the schools, time problems in the training of the subjects, multiple choice questions in the central examination system, and the high number of students in the classes are possible obstacles. The second category, on the other hand, is the situation specific to the structure of COM and the difficulty in finding the appropriate ML question for the second focus because it takes too much time to explore in the first focus. The difficulty in finding the appropriate ML question is expressed as situations that may hinder the application of the model (Table 3).

Suggestions for Structuring the Training Provided

Teachers were asked questions to get their suggestions for the studies to be done in case the education will be re-run or the current education will continue, and the findings are presented in Table 4.

Table 4. Teacher Recommendations for the Structuring of the Education

Theme	Category	Code	f
Studies to increase the quality of education	Lesson plan design	Increasing lesson plan discussions	8
		Increasing the lesson plan preparation part with ÇOM	1
		Include lesson plan examples rather than the theory part	1
	Lesson plan applications	Evaluating classroom practices	3
		Changing and implementing lesson plans	2
		Sharing negative aspects of implementations	1
	Write/solve ML problems	Increasing question-writing efforts	5
		Creating a question pool	2
		Sharing more detailed feedback on question writing	1
		See progress in question writing	1
		Not including the solution to known ML questions	1
	Activities	Creating an activity pool	2
		Continuing the activities and writing ML questions together	1
		Carrying the applications of the activities in the classrooms to education	1
		Sharing more activities	1
		Preparing activity papers	1
Studies carried out before, during, and after the training	Implementation of training	Face-to-face (interactive) training	9
		Enrichment of the online environment	3
		Including the use of technology	1
		Preparing a clipboard	1
	Assignment	Giving regular (periodic) assignments	2
		Increasing homework frequency	1
		Giving group assignments	1
		Increasing in-class interaction instead of homework	1
	Studies for post-training use	Material design	2
		Preparing a guidebook	1
		Sharing all work in education	1
		Sharing questions produced through mobile applications with students	1
No Suggestions			1

For the studies that will increase the quality of teaching, it has been requested that more discussions be made about lesson plans (8), that lesson plan preparation studies should be given more place (1), and that application examples rather than theoretical knowledge regarding lesson plan preparation (1) should be included (Table 4). Regarding this, T4 said: *"A little narrowing of the ML part and more space can be given on the plan. There can be more room for writing a lesson plan and asking teachers for it."*. It was asked to follow and evaluate the classroom practices for the implementation of the lesson plans studied (3), to revise and apply the existing plans (2), and to carry them into the process in case of negative situations experienced during these practices (1). In addition, another application that is requested to be increased is about writing ML questions and increasing the number of question writing activities (5), creating a pool of questions for the questions written and to be written (2), getting more detailed feedback from the academicians who provide training on the questions written (1), and sharing the first and last versions of the questions in the writing study (1) were mentioned. While the teachers demanded an increase in activities such as writing activities and preparing lesson plans, they expressed an increase in such studies by giving less space to some existing studies (such as reducing theoretical knowledge) rather than extending the education period. For example, T6 said: "*We saw ML questions a lot through examples, I agree, but especially the theoretical parts were challenging. This part may be reduced and perhaps better by writing one or two questions in consultation, as one makes an idea on writing a question during the lesson, and the other adds to it.".*

Finally, under this theme, teachers had some suggestions for activities. The need to create a pool (2) for the activities that have been written and will be written has been expressed. Teacher T8 expressed his expectation: "*I would like to have an activity pool. For example, it would be nice if we could make a topic. For example, seeing the coordinate system, equation graphics, and what kind of activities there are can give us an idea.".* The fact that the activities related to the activity were towards the end of the training was criticized, and it was suggested that the activities and ML question writing studies to be carried out together (1) and that more activities should be included in quantitative terms (1). About the importance of carrying the practices of the activities in the classroom to the teacher education environment (1), S12 explained: *"For example, I would like the teachers to carry out the activities that you do. It's one thing to see it by watching it; it's another to apply it ourselves. Because if we are going to make that child do it, we must try these kinds of activities ourselves first.".* During the implementation of this activity, they expressed their demands in the form of preparation of activity papers for convenience (1).

Another theme related to the suggestions is associated with the form of teaching. First of all, the situation that is especially emphasized here is related to the conduct of teacher training online due to pandemic conditions. Many teachers (9) preferred a face-to-face education with more intense interaction. T25, one of the teachers, explained this situation: "*our most important shortcoming was that we could not do face-to-face education and that limited education"*. In parallel with this, some teachers (3) wanted the online teaching environment to be enriched, and teacher T10 said: *"For the more active participation of teachers, pdfs can be projected via zoom as a presentation method and an online environment where anyone can write notes and discuss it."*. Similarly, it was stated that studies such as giving more space to the use of technology for the online environment (1) and preparing a board (1) to keep essential/critical information in the foreground in the process could be done.

An important application in the education process is the homework given to the teachers. The teachers generally welcomed the assignments, and T7 gave this situation: *"Assignments were very appropriate. Yes, I was tired while doing my homework, but I put much effort into it. Homework should be increased."* a further increase. In parallel with this, regular (periodic) assignments (2) and group assignments of 3-4 teachers (1) were requested. One teacher stated that in-class interaction should be increased instead of homework.

Teachers also suggested some resources and worked for use after completing the training. It is recommended to design materials that will support the teaching of the subject (2) and to share all the studies in education for later use (1). Preparing a guidebook to provide teachers with ease of application; *"Previously, there was a special guidebook for teachers. They are preparing a good guidebook that my colleagues and I can use. Here, unit by unit, topic by topic. It's like a daily plan. There will be next generation question activity exercise in it. Frankly, I think that doing something like this will be much more efficient and effective in the future."* (T20) with a statement. Finally, it has been suggested (1) to use mobile applications to provide easy access to the questions produced by the students. Regarding this, T11 expressed his opinion: "We give students questions, *but there is nothing that the student can access from a different platform. For example, many mobile applications can now be embedded within the mobile application."*

Reasons for (not) Recommending This Training to Colleagues

The teachers were asked whether they would recommend other teachers on duty to take this training, and if so, the reasons they would put forward for this, and the findings are presented in Table 5.

Table 5. Reasons for Recommending Participants Take This Teacher Training to Other Teachers

Theme	Category	Code	f
Yes, …	Contribution to teachers	Providing professional development in ML	8
		Gaining awareness of ML	4
		Obtaining valuable information	4
	Contribution to enriching the learning-teaching process	Relating mathematics to life	7
		Prepare an ML question	5
		Teaching with activities	4
	Contribution to students through trained teachers	Expanding perspective on math	4
		Contribution to breaking the fear of mathematics in students	2
		Enabling original thinking	2

		Understanding that math is valuable	2
Indecisive	Useful though!	Volunteering should be essential	2

All but two teachers who received training stated that they recommended other teachers to take this training (Table 5). The first reason for their suggestions is related to the individual contribution that teachers will provide to them. These contributions were discussed in three different dimensions, and the teachers stated that they would provide professional development through this training (8) to gain awareness about ML (4) and to acquire valuable information (4).

Another dimension in suggesting that they should take the training is related to the contribution towards enriching the learning-teaching processes. They reported that they would have the opportunity to see how mathematics can be associated with life (7), to prepare ML questions (5), and to teach with activities (4).

T20 said, *"I recommend it. I think there will be an 80% increase in students learning about the activity because it increases retention. I think it will increase its permanence with an event like this instead of memorizing the formula like a theater stage."*. T22, who had the opportunity to personally experience the contribution of teaching to the lesson with COM, said, *"I can recommend it in terms of lecture. I recommend explaining with a dual-focus education model how it contributes to mathematics."*.

The last category is about the contribution to be made to the students through the teachers who are trained. First of all, it was stated that the teacher education received provides the students with the opportunity to expand their perspective on mathematics as the first reflection (4), and one of the teachers T11: *"I definitely recommend it. I have found it very useful and I will apply it. I believe that the student once changed his point of view on mathematics, that is, my students also changed it…"* he shared his opinion on this situation. In addition, the teachers suggested that this training should be taken as it provides opportunity and contribution in terms of thinking originally (2), breaking the fear of mathematics (2), and understanding that mathematics is valuable (2).

Two teachers, who were hesitant to suggest other teachers receive this training, were still determining the benefit it would provide but stated that such training should be taken voluntarily. One of these teachers explained the reason for indecision" T17 said, *"I would like to, but is this actually possible! In fact, the old system is our comfort zone; these new systems are causing us to leave our comfort zone. Otherwise, it would be beneficial for them to receive this training."*

DISCUSSION

This study discusses the way of training teachers who will enable students to become mathematically literate through a program in which ML is integrated into teaching. In this section, the extent to which the seminar's aim was achieved was evaluated according to the order in which the findings were presented. Then the planning and implementation stages of the instruction were discussed. It should be noted that although this study references the COM, it first discusses a "teacher education model". In this respect, it can be considered a teacher education model independent of the COM.

As to Whether the Purpose of The Seminar Was Achieved or Not

Whether the seminar's goal has been achieved can be understood from the participants' thoughts about ML, their opinions about the seminar, and the reasons for recommending the seminar to other teachers. Although the participants had very limited knowledge about ML before the seminar, the high frequency (20, 17, 13) themes of having knowledge about ML, associating mathematics with life, and mastering ML at the end of the seminar (Table 1) suggest that the seminar achieved its purpose.

Other indicators supporting the achievement of the seminar's purpose (Table 1) are participants' emphasis on reasoning and decision-making as skills developed; It can be shown that they adopt the appropriate environment for developing competencies by including activities and discussion as the characteristics of the learning environment. These results indicate that Blomhøj and Jensen (2007) stated that "competencies should be included in teaching for ML; Niss et al. (2016) cared about teachers' knowledge about ML for the development of ML (Table 1); Niss et al. (2016)'s conclusion that the most potent factor in ML teaching is the teacher factor.

Regarding Lesson Planning and Implementation

The majority of the teachers (17) found it easy to plan a lesson by the COM (Table 2), and some of the teachers stated that they had difficulty in finding activities suitable for the content of the first-focus teaching and were unsure whether the activities they found served the purpose (Table 2). suggest the need for more work on

lesson plans. Some concerns were also expressed that including ML questions and practices in the teaching, process might need to be clarified in the classroom. As Burkhard (2008) states, it can be expected that such confusion may occur in innovation initiatives and that it will disappear with time and experience.

The fact that teachers know about the practices facilitates the teaching practices and that they want these practices to be done during the seminar can be considered a sign of their internalization of teaching. However, additional time may be needed in this case, or it can be overcome by including in-class practices within homework studies. Again, teachers' concerns about involving students in the lesson (Table 2, 3) are in line with the result of Staples and Newton (2016) that who "have difficulty in involving students in the discussion." In addition to this, the demands of ML to demand more activities, to experience what has been learned through in-class practices, to increase question writing activities, to create a question and activity pool, to include lesson planning studies rather than theory, to increase discussions, to include the use of technology in the lesson (Table 4) Emphasizing applied studies for the development of both et al. (2013) and shows that there is a need for a change in this direction in the organization of seminar content.

While expressing the possible obstacles to be encountered in implementing the COM (Table 3), teachers generally refer to the concern of educating the curriculum, as stated by Blomhøj and Jense (2007) and Demir et al. As (2017) stated, they emphasized the difficulties required by the examination system and the class crowd. However, this is not related to the Bifocal Teaching Model but applies to almost any innovation initiative. Moreover, no increase in the content will cause teachers to worry about not being able to complete the curriculum in teaching with the COM. Similarly, nowadays, where competencies have gained such importance (Niss et al., 2016; 2017), while it is clear that competencies should take their place in the curriculum, concerns such as completing the curriculum seem unnecessary. It should not be overlooked that it will be a complex and slow process for teachers to try a teaching approach suitable for a new model in the classroom, even in a supportive environment (Schoenfeld, 2010).

When the affective effects of teaching are examined, the thought that these approaches lead to finding mathematics valuable by arousing curiosity, valuing students' thoughts, and revealing the role of mathematics in life (Marks, 2000; Newmann et al., 1992) suggests that the lessons in which ML is integrated will attract the attention of students. It is compatible with the idea that it will increase mathematics achievement. In parallel to this, there are also studies (Bulat, 2023; Karaduman, 2023) reporting positive attitudes of students towards mathematics in the lessons where COM is applied. In these studies, it is stated that students reported positive opinions about enjoying mathematics and valuing mathematics after the training.

"Do you recommend that other mathematics teachers take this training as well? If your answer is yes, explain why." The question, which was asked to the teachers, has two purposes. The first is to test the reliability of the interview content, one of the data collection tools, and the other is to understand the real thoughts of the teachers by obtaining information about the model indirectly. When the opinions expressed by the teachers in this context are examined, it is seen that the findings in Table 5 and the findings in Tables 1 and 3 are consistent. For example, the highest frequency item in Table 3 is "associating mathematics with life" (f=10); It shows parallelism with the expression "associating mathematics with life" (f=7) in Table 5. In the literature, the results related to the benefits of the model in terms of relating mathematics to real life have been shared (Bulat, 2023; Karaduman, 2023). They also evaluated the issues that they emphasized functionally in Table 3 as issues that will contribute to the equipment of the teachers who will retake the seminar. The information in Table 5 and the highlights in Table 3 (learning to patch up questions, teaching with activities, changing perspectives, providing professional development on ML, etc.) overlap to a large extent. This consistency suggests that teachers express their thoughts sincerely and that these statements can be trusted.

Conclusion and Recommendations

This study discusses an in-service teacher training seminar to train teachers who can integrate ML into the teaching process. The seminar's content was created according to the COM. The results showed that the seminar's content and methodology followed in the seminar were suitable for ML teaching. Planned and implemented teacher training seminar teachers; It has produced positive and successful results in planning a lesson by the COM and preparing the modules for the lesson and the contents that make up the modules. Teachers reflected on their statements that they had more detailed information about how they could improve ML. In summary, the participating teachers found the seminar content appropriate, sufficient, applicable, and sustainable in ML teaching. As can be understood from the expressions in Table 3, practice-oriented studies were carried out

throughout the seminar, and teacher initiatives were supported in these applications. They were ensured to act freely in planning the reflections in the focus areas.

As a result, in the practices carried out within the seminar's scope, the "classroom" was considered a living space, and the principle that mathematics is a "matter of thinking and discussion" was always considered. Despite this, for teachers to be well trained in ML;

(i) It is necessary to enrich the teaching with this aspect, taking into account the demands that the applied studies to be carried out in the secondary school mathematics courses are carried out precisely and one-to-one in the seminar environment,

(ii) Providing such resources, taking into account the need for resources such as ML question and activity pool,

(iii) It will become more qualified if the teaching is enriched regarding technological support.

These three items, on which teacher suggestions are concentrated, are not related to a change in the essence of the proposed model but are in the nature of increasing some of the activities in the model, and considering that they can be easily eliminated, it has been concluded that they are valid, applicable and sustainable for mathematics literacy teaching.

Statements of Publication Ethics

This research study complies with research and publishing ethics. The studies involving human participants are reviewed and approved by the … University Ethics Committee (… dated - Number: …).

Researchers' Contribution Rate

Author 1 (%35), Author 2 (%15), Author 3 (%15), Author 4 (15), Author 5 (%5), Author 6 (%5)

Conflict of Interest

The authors declare no conflict of interest.

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