Necmettin Erbakan Üniversitesi *Ereğli Eğitim Fakültesi Dergisi* 5/*Özel Sayı (2023), 271-305 | Derleme Makalesi*

Zihinsel Modeller İnşa Etmek: Kavramsal Öğrenmede Matematiksel İletişim ve Söylemler

Mahir BİBER [1]

[1] İstanbul Üniversitesi-Cerrahpaşa, Matematik Eğitimi, İstanbul, Türkiye, mahir.biber@iuc.edu.tr, 0000-0003-4044-6966

Öz

Matematik gibi algılanması güç bir alan için iletişim temelli öğrenme ortamlarının önemi büyüktür. Öğrenme ortamlarında öğretmenlerin öğrencilerinin anlık tepkilerinden matematiksel anlayışlarına dair fikir sahibi olmaları zordur. Öğrencilerin matematiksel fikirlerinin ortaya çıkarılması, karşılıklı diyaloglar, tartışmalar ve iletişimin önemli bir parçası olan söylemlerle mümkün olmaktadır. Diğer yandan, matematik öğretimi kavramsal ve işlemsel öğrenme olmak üzere iki farklı öğrenme türüne dayanmaktadır. Bireylerin matematiksel kavramları ve kavramlar arasındaki ilişkileri zihinlerinde yapılandırmalarını sağlayan ve anlamlı öğrenmenin anahtarı olan öğrenme türü kavramsal öğrenmedir. Matematiksel iletişim temelli bir öğrenme ortamının ayrılmaz bileşenleri olan öğrencilerin diyaloglarının ve etkileşimli alışverişlerinin geliştirilmesinin, onların kavramsal öğrenmelerini derinden geliştirdiğine inanılmaktadır. Bu çalışma, eğitimde iletişim ve söylemlerin merkezi rolünün altını çizmeye çalışmaktadır. Bu bağlamda araştırma, kavramsal öğrenmeyi beslemek ve gerçekleştirmek için iletişim merkezli öğrenme ortamlarının kullandığı derin etkiyi incelemeyi amaçlamaktadır. Araştırma, matematiksel iletişim odaklı öğrenme ortamlarının, kavramsal anlayışın derinden gerçekleştirilmesi için nasıl katalizör görevi gördüğüne dair ilgili literatür ışığında kapsamlı bir değerlendirmeyi içermektedir.

Anahtar Kelimeler

Kavramsal öğrenme, Söylemler, Matematiksel iletişim

Atıf Bilgisi

Biber, M. (2023). Zihinsel Modeller İnşa Etmek: Kavramsal Öğrenmede Matematiksel İletişim ve Söylemler. *Necmettin Erbakan Üniversitesi Ereğli Eğitim Fakültesi Dergisi*, *5*(Özel Sayı), 271-305.
https://doi.org/10.51119/ereegf.2023.45

Geliş Tarihi	11.08.2023
Kabul Tarihi	19.09.2023
Yayım Tarihi	29.10.2023
Etik Beyan	Bu çalışmanın hazırlanma sürecinde bilimsel ve etik ilkelere uyulduğu ve yararlanılan tüm çalışmaların kaynakçada belirtildiği beyan olunur.
Bilgilendirme	Yok.
Benzerlik Taraması	Yapıldı – Turnitin
Etik Bildirim	mahir.biber@iuc.edu.tr
Çıkar Çatışması	Çıkar çatışması beyan edilmemiştir.
Finansman	Bu araştırmayı desteklemek için dış fon kullanılmamıştır.
Telif Hakkı & Lisans	Yazarlar dergide yayınlanan çalışmalarının telif hakkına sahiptirler ve çalışmaları CC BY-NC 4.0 lisansı altında yayımlanmaktadır.

Constructing Mental Models: Mathematical Communication and Discourses in Conceptual Understanding

Mahir BİBER [1]

[1] İstanbul University-Cerrahpaşa, Department of Mathematics Education, İstanbul, Türkiye, mahir.biber@iuc.edu.tr, 0000-0003-4044-6966

Abstract

In the realm of challenging subjects like mathematics, communication-driven learning environments assume paramount significance. Within these environments, gauging students' grasp of mathematical concepts solely through immediate responses can prove to be a daunting task for educators. The unveiling of students' intricate mathematical insights necessitates a platform of reciprocal dialogues, thought-provoking discussions, and engaging discourses. Mathematics pedagogy revolves around two distinct forms of learning: conceptual and procedural. The former, conceptual learning, empowers learners to construct intricate mathematical frameworks within their mental landscapes, interlinking various concepts harmoniously. It is within this context that the cultivation of students' dialogues and interactive exchanges, integral components of a mathematically communicative learning environment, is believed to profoundly enhance their grasp of conceptual dimensions. This study endeavors to underscore the pivotal role of communication and discourses in education. It delves into the profound influence wielded by communication-centric learning settings in nurturing and actualizing conceptual learning. The investigation draws insights from pertinent literature, culminating in a comprehensive exploration of how mathematical communication-oriented learning environments serve as catalysts for the profound realization of conceptual understanding.

Keywords

Conceptual Learning, Discourses, Mathematical communication

Citation

Biber, M. (2023). Constructing mental models: Mathematical Communication and Discourses in Conceptual Understanding. *Journal of Necmettin Erbakan University Ereğli Faculty of Education*. *5*(Special Issue), 271-305 https://doi.org/10.51119/ereegf.2023.45

Date of Submission	11.08.2023
Date of Acceptance	19.09.2023
Date of Publication	29.10.2023
Ethical Statement	It is declared that scientific and ethical principles have been followed while carrying out and writing this study and that all the sources used have been properly cited.
Acknowledgements	No.
Plagiarism Checks	Yes - Turnitin
Conflicts of Interest	The author(s) has no conflict of interest to declare.
Complaints	mahir.biber@iuc.edu.tr
Grant Support	The author(s) acknowledge that they received no external funding in support of this research.
Copyright & License	Authors publishing with the journal retain the copyright to their work licensed under the **CC BY-NC 4.0**.

Introduction

In contemporary information-driven societies, advancements in science and technology have heightened the demands on education. Modern individuals are expected to possess advanced cognitive abilities, including adept problem-solving, critical analysis, and creative ideation. Additionally, qualities like innovation and collaborative prowess are essential for navigating daily life and fostering professional growth. Within this landscape, mathematics emerges as a paramount discipline. Hyde and Jain (2020) underscore that mathematics, and by extension, mathematics education, significantly fosters the development of vital skills. These encompass conceptual comprehension, strategic acumen, logical reasoning, analytical thinking, productivity, and fluency. Thus, the centrality of mathematics and its education becomes indisputably clear, warranting a position of paramount importance.

Mathematics stands as the cornerstone for shaping cognitive frameworks that empower individuals to unravel the complexities, patterns, and interrelationships of the world, catalyzing progress in science and technology (Aufa et al., 2016). Its crucial role extends across disciplines such as astronomy, economics, computer science, physics, and statistics, intricately woven into daily life (Andamon & Tan, 2018). Hence, the urgent call for nurturing mathematical literacy becomes evident. However, certain studies underscore a shortfall in essential mathematical skills and literacy among students (Kushwaha, 2014; Yeh et al., 2019), a challenge influenced by a blend of personal and environmental factors (Savaş et al., 2010). The abstract nature of mathematics compounds the difficulty of grasping its concepts and connections (Andamon & Tan, 2018), often accompanied by feelings of tension and anxiety. Addressing this entails a multifaceted approach, nurturing both knowledge and skills while fostering positive attitudes, interests, and motivations. This holistic strategy holds the promise of bridging gaps and cultivating confidence in the realm of mathematics.

Bandura (1969) asserted that socio-cultural development springs from social interactions, shaping knowledge, skills, and tendencies. Cesar (1998) emphasized the crucial role of social interactions in comprehension, skill acquisition, and socio-cognitive growth. In learning settings, students leverage social interactions to articulate thoughts, guiding their learning journey. Tu and McIsaac (2002) recognize social interaction as a key element of an individual's social presence in learning. Scholarly discourse echoes the enhancing impact of student interactions on understanding (Harianja et al., 2020; Lomibao et al., 2016). Similarly, numerous studies explore social interactions in mathematical pedagogy (Apriliyanto et al., 2018; Boaler, 2000; Cesar, 1998). Exploring social interactions in mathematics education brings forth the concept of mathematical communication, highlighting its central significance.

The mathematical communication process, rooted in individuals' grasp of mathematical concepts and their expression of mathematical thoughts, is denoted as the mathematical communication process (Uygur Kabael & Ata Baran, 2016). Importantly, students' mathematical progress is frequently evaluated based on their proficiency in mathematical communication (Farokhah et al., 2017). Extensive research explores various dimensions of mathematical communication skills (Ansori et al., 2019; Ata Baran & Kabael, 2023; Baxter et al., 2005; Brendefur & Frykholm, 2000; Cai, 2017; Cobb et al., 1997; Kabael & Ata Baran, 2017; Kaya & Aydın, 2016; Lisnani, 2018; Lomibao et al., 2016; Maspe et al., 2021; Mosckhovich, 2007; Mujiasih et al., 2021; Nathan & Knuth, 2003; Ryve, 2006; Sari & Yuberta, 2022; Septiana et al., 2018; Stein, 2007; Susilawati et al., 2018; Tong et al., 2021; Walshaw & Anthony, 2008; Yulian, 2018; Zeybek & Açıl, 2018). These studies uniformly underscore the pivotal role of mathematical communication in effective mathematics education, aligning with educational goals. Recent research emphasizes the positive impact of peer interactions on reducing students' mathematical anxiety and enhancing their performance and academic achievement (Apriliyanto et al., 2018; Garba et al., 2019; Kaymak et al., 2020; Thurston et al., 2020). As Cobb et al. (1997) noted, the classroom's social context substantiates its contribution to students' mathematical understanding. Boaler's (2000) study further amplifies the importance of social interactions in mathematics lessons. Boaler's interviews with 76 students highlight the paramount role of peer communication in learners' eyes, often surpassing seeking guidance from instructors. These findings underscore the potent effect of active student participation and peer interaction in nurturing a profound grasp of mathematics.

This study delves into the crucial connection between meaningful mathematical knowledge and conceptual understanding. Conceptual learning forms the bedrock of mathematics, entailing logical construction and interrelation exploration of mathematical concepts. Notably, a direct link between conceptual learning and mathematical communication emerges, bearing significant implications for math education. Existing research extensively probes various facets of conceptual learning (Baki et al., 2009; Bergsten et al., 2017; Fernández-Millán & Molina, 2017; Forrester & Chinnappan, 2010; Gürbüz & Gülburnu, 2013; İşleyen & Işık, 2003; Jazuli et al., 2017; Kanz, 2020; Konyalioğlu et al., 2011; Malatjie & Machaba, 2019; Mohd Yatim et al., 2022; Nahdi & Jatisunda, 2020; Rillero, 2016; Soylu & Aydın, 2006). However, the intriguing nexus between mathematical communication and conceptual learning remains unexplored. This study aims to fill this gap, investigating how mathematical communication bolsters students' conceptual learning in effective math education. It emphasizes the significance of conceptual learning, underscores the role of mathematical communication in student growth, and advocates communication-centric learning environments. The upcoming sections will delve into mathematical communication and conceptual learning, culminating in an exploration of their interplay and impact.

1. Exploring Mathematical Communication Proficiency and its Significance

Communication skills serve as the bridge through which individuals cultivate self-awareness, navigate their environment, and establish meaningful connections within their social sphere (Yüksel, 2012). Language forms the cornerstone of interpersonal communication. The unique, universal language and terminology inherent in mathematics (MoNE, 2013) underscore the paramount role of communication in mathematics education. Of crucial importance for students is the capacity to articulate their mathematical reasoning, comprehend peers' mathematical insights, and thereby construct a genuine conceptual grasp of mathematics (Martin, 2001). Conversational exchanges that explore diverse perspectives on mathematical ideas foster cognitive clarity and interrelations (Martin, 2001). Paridjo and Waluya (2017) emphasize how such discussions not only unveil mathematical insights but also contribute to the development of an individual's mathematical mindset. Enter mathematical communication skills, which empower individuals to interpret mathematical scenarios, devise effective problem-solving strategies, and convey mathematical concepts coherently to others (Zengin, 2017). This significance particularly resonates in enhancing students' comprehension of mathematical concepts (Baxter et al., 2001). Mathematical communication acts as a conduit for students to transpose their ideas into their surroundings, engaging in activities that amplify their mathematical prowess and knowledge. Summarizing students' endeavors when conveying mathematical ideas to peers or educators, Sammons (2018) outlines these interactions, depicted in Figure 1. **Figure 1**

Activities Performed by Students While Conveying Their Mathematical Ideas

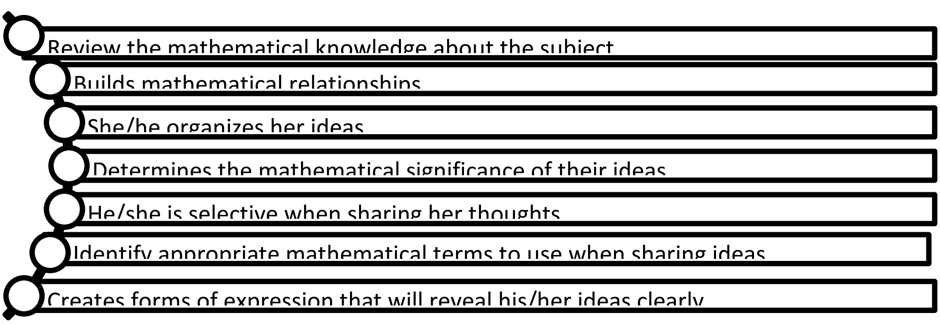
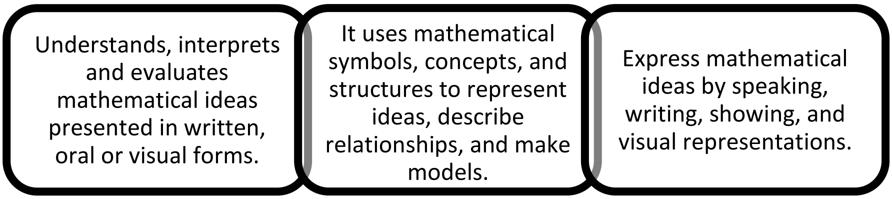


Figure 1 depicts the intricate facets of the mathematical communication process, wherein students deploy a myriad of advanced skills. This process distinctly fortifies the growth of mathematical knowledge and competencies. Within mathematics classrooms, mathematical communication emerges as a cornerstone (Ansori et al., 2019). Baroody (1993) attributes the significance of mathematical communication in learning mathematics to two pivotal reasons. Firstly, mathematics possesses a distinct language, enabling universal communication. Secondly, mathematical communication is intrinsically social within the context of learning mathematics. Amidst broader objectives of mathematics teaching, a paramount aim is nurturing students' social competence, empowering them to effectively convey their mathematical ideas and reasoning to others (Pourdavood et al., 2005). Importantly, mathematical communication skills profoundly facilitate this achievement. This avenue empowers students to engage in discussions, reflection, refinement, and rectification of their ideas (Martin, 2001). Lindquist and Elliott (1996) deem communication pivotal to students' learning and problem-solving endeavors. Thus, the concept of mathematical communication holds a pivotal stance within mathematics education, meriting comprehensive exploration. A rich tapestry of studies underscores the importance and cultivation of mathematical communication skills (Afrianti & Qohar, 2020; Baxter et al., 2005; Brendefur & Frykholm, 2000; Brenner, 1998; Kabael & Ata Baran, 2017; Lee, 2015; Lisnani, 2018; Pujiastuti et al., 2021; Setiyani et al., 2020; Susilawati et al., 2018; Uyen et al., 2021; Yulian, 2018). Furthermore, investigations into the reciprocal influences between mathematical communication skills and variables like students' genders (Ansori et al., 2019), affective traits (Ata Baran & Kabael, 2021; Roble et al., 2018; Septiana et al., 2018; Tahir, 2021), higher-order thinking abilities (Cai et al., 1996), and academic achievements (Baxter et al., 2005; Lomibao et al., 2016) are evident. Studies examining diverse pedagogical approaches impacting mathematical communication skills (Kusumah et al., 2020; Sumaji et al., 2020; Triana et al., 2019), alongside evaluations of how mathematical communication skills influence misconceptions and mathematical literacy (Pugalee, 2001), further enrich the discourse.

When the results obtained in these studies are evaluated in general, it is seen that mathematical communication skills reduce students' anxiety about mathematics and accordingly increase their success levels, while strong mathematical communication established in the classroom significantly affects students' thinking skills, problem-solving, and reasoning skills. All these research results show that mathematical communication skills can be considered as an important variable that reveals the mathematical potential of individuals. There are some clues or indicators that show that an individual has mathematical communication skills. NCTM (1989) lists the indicators of mathematical communication skills as in Figure 2.

Figure 2

Indicators of Mathematical Communication Skills



These indicators reveal that mathematical communication takes place in various dimensions. Ng (2016) also stated that mathematical communication is provided with the help of written and spoken words and algebraic symbols, and it is transformed into a visual form with gestures. Cai et al. (1996) stated that students should be encouraged to benefit from various representations for effective mathematical communication. Thompson and Chappell (2007), on the other hand, stated that for students to make sense of mathematical concepts, symbols, and words, they should perform mathematical communication in four dimensions: speaking, listening, reading, and writing. This situation reveals the importance of knowing the language of mathematics well and using it correctly for healthy mathematical communication.

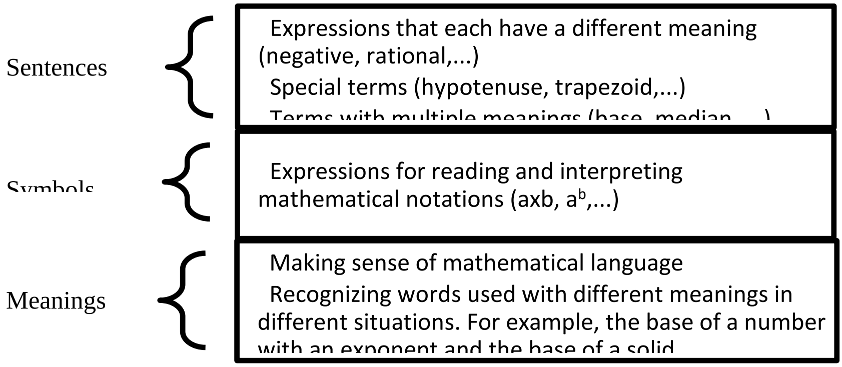
2. The Role of Mathematical Language in Facilitating Effective Communication

Mathematical communication is rooted in conveying mathematical thoughts through speech or writing and comprehending others' ideas. This highlights language and social interaction's pivotal role. Language serves as a universal symbolic system for communication (Akkuş, 2015). Işıkdoğan Uğurlu and Kayhan (2018) define language as a tool for expressing knowledge, emotions, thoughts, learning, cultural heritage, and inquiry. Vygotsky (2012) emphasizes language's role in cognitive development. Mathematics has its language, terminology, and expression (Aydın & Yeşilyurt, 2007). Usiskin (2012) emphasizes mathematical language for meaningful math engagement. Paridjo and Waluya (2017) highlight students' improved math comprehension through language skills. Verbal language empowers students to communicate, expand concepts, and understand others' ideas (Ansori et al., 2019). Written language clarifies mathematical rationale. Language is the foundation of mathematical communication, with utmost significance.

In mathematics teaching, the use of language in interpersonal interactions comes to the fore. Bruner (1974) stated that mathematical communication plays a central role in the development of cognitive structures, and language can be considered a tool not only for representing experiences but also for transforming ideas. Students need to become fluent in mathematical language so that they can perform mathematical communication effectively. This fluency requires knowing the mathematical language with all its elements shown in Figure 3 (Kersaint, 2015).

Figure 3

Elements of Mathematical Language



In Figure 3, it's evident that mathematical language encompasses unique concepts and symbols, with their derived meanings constituting a crucial language component. The significance of language in mathematical communication lies in the internalization of these meanings by individuals. Such internalization transpires through in-class interactions. Notably, classroom interactions encompass two distinct linguistic dimensions: primitive language and mathematical language. Mercer and Sams (2006) outlined two interaction modes linking colloquial language to mathematics education. The first mode involves teacher-guided student interaction, fostering knowledge and understanding development under the teacher's direction. The second mode pertains to peer group interaction, where communication tends to surpass teacher-led instances. In peer interactions, students engage in freer expression of mathematical ideas and contribute more effectively to mathematical discussions.

In recent times, scholars have increasingly focused on the pivotal role of language and social communication in mathematics education (Barwell et al., 2005; Sfard & Kieran, 2001). Given that language forms the bedrock of interpersonal communication, mastering the language of mathematics is paramount for effective mathematical communication. Nonetheless, precise language use alone doesn't guarantee effectiveness, as students often employ common expressions from teachers or textbooks without a comprehensive understanding (Kosko & Wilkins, 2010), hindering meaningful communication. Indeed, students frequently resort to these expressions even when their comprehension is limited, posing a communication challenge. Effective mathematical communication hinges on how teachers and students articulate their mathematical ideas, and crucially, how others interpret this exchange. This bidirectional interaction underscores the notion of "mathematical discourse," emphasizing the indispensability of reciprocal comprehension.

3. Mathematical Discourse: A Crucial Element in Fostering Mathematics Understanding

Altun (2008) suggests that when imparting mathematical concepts, particularly to young learners, it's prudent to minimize mathematical jargon and opt for more relatable language. This calls for teachers to employ everyday language that resonates with students, rather than relying solely on scientific definitions and symbols. Bali Çalıkoğlu (2002) emphasizes that nurturing students' mathematical comprehension involves fostering reciprocal discussions within classroom dialogues, a departure from conventional teaching methods. These dialogues, alongside teachers' use of everyday language to elucidate mathematical concepts, typically manifest as discourses.

Discourses form an integral facet of language. While language serves as the vehicle for expressing thoughts, discourses lend logical cohesion, persuasiveness, and meaning to speech. In essence, discourses are linguistic tools that engender semantic unity and facilitate the attainment of logical conclusions (Çelik & Ekşi, 2013). Sfard (2008) characterizes discourse as a form of communication individuals engage in, either with themselves or others, synchronously or otherwise, through verbal or symbolic language. Discourses are designed to enhance student comprehension through open-ended queries, fostering active contributions to the learning journey. In the realm of mathematics, which possesses its universal language, discourses assume heightened prominence within learning environments, thereby giving rise to the concept of mathematical discourse. Wells (2003) defines mathematical discourse as both spoken and written language used by mathematics students and mathematicians to communicate mathematically.

Sfard (2012) stated that thinking can be defined as a communication activity that an individual carries out with himself and that this situation allows the individual to produce his own story for learning mathematics. In this context, he claimed that mathematical thinking, or mathematics in short, can be thought of as a discourse. Indeed, in the thinking process, the individual synthesizes the ideas he observes from the people he sees as role models or from his peers and creates logical verbal expressions in his mind. These verbal expressions form the basis of the individual's way of speaking and expressing in mathematical communications, that is, mathematical discourses.

Paridjo and Waluya (2017) stated that students can improve their understanding of mathematics by using the language of mathematics correctly while writing their mathematical thoughts, expressing their ideas through symbols, shapes, and verbal expressions, and having mathematical discussions. Discourses allow students to develop a common understanding of mathematical ideas and teachers to evaluate their students' mathematical thoughts (Walshaw & Anthony, 2008).

Soter et al. (2008) highlight that discourses play a pivotal role in identifying students' partial comprehension and misconceptions. Mathematics, inherently abstract, is founded on intricate concept interconnections. Students' challenges often stem from struggling to mentally visualize abstract notions or comprehend inter-concept relationships. The profound links between mathematical concepts amplify the significance of misconceptions in the learning journey. Addressing the needs of mathematically challenged students and fostering their self-assurance requires an initial grasp of their misconceptions. For instance, in the context of defining a square, when a student says the discourse "a quadrilateral with four sides of equal length," it becomes apparent that they might struggle to discern the nuanced differences between a square and a rhombus. Consequently, the role of discourse becomes particularly significant in unveiling and addressing such misconceptions held by the student. So, mathematical discourses are believed to hold substantial potential in enhancing the mathematics learning process, notably aiding in the identification and rectification of students' misconceptions.

Discourses play a crucial role in enhancing students' mathematical knowledge access and fostering their mathematical reasoning. Lomibao et al. (2016) emphasize that conventional math teaching often prioritizes task execution and standardized procedures. Unfortunately, procedure execution can be misleading, misrepresenting true comprehension. This misconception leads to challenges in math education. Sfard (2008) advocates discourse integration in math instruction and assessment to rectify this. Evaluating students' discourse during problem-solving reveals genuine understanding. For instance, the correct procedural application may not signify mastery, as it could result from chance. This misconception intensifies in multiple-choice exams. Opting for discourse-based evaluations, showcasing comprehension, grasp, misunderstandings, and misconceptions, offers more meaningful insights, particularly in our educational context.

When the relevant literature is examined, some studies deal with various aspects of mathematical discourses, evaluate the mathematical discourses used by the example situations, and examine the mathematical discourses of teachers (Bondurant, 2020; Cobb et al., 1997; Kabael & Ata Baran, 2017; Makgakga, 2023; Mosckhovich, 2007; Nathan & Knuth, 2003; Pape et al., 2003; Pourdavood et al., 2005; Pourdavood & Wachira, 2015; Ryve, 2006; Stein, 2007; Toscano et al., 2019; Walshaw & Anthony, 2008; Yimam & Kelkay, 2022). Especially in recent studies, it is striking that teachers' mathematical discourses are focused on. In addition, it is seen that there is a consensus in the context of constructivist learning theory and socio-cultural theories that mathematical discourses positively affect students' conceptual development (Sfard, 2008; Walshaw & Anthony, 2008). In addition, it is thought that mathematical discourses have serious contributions to the communication process.

According to Cobb et al. (1997), teacher-centered dialogues in learning environments are usually one-dimensional, mostly providing real information and rarely rich. When students are involved in these dialogues with their inquiries, teachers tend to give more detailed explanations and decorate their explanations with different examples and representations. This shows that student discourse is an important factor that affects teachers' discourses and thus enriches the communication process. Nakamura (2008) also supported this situation and stated that students should be active participants in mathematical discourses to be able to perform mathematical communication effectively.

Lomibao et al. (2016) underline that students' engagement in mathematical discourses enhances their verbal communication skills. Accordingly, educators should cultivate learning environments conducive to such discourses. Classroom interactions, as discussed earlier and illuminated by Mercer and Sams (2006), prominently feature mathematical discourses. Through interactions led by expert teachers, students are influenced by their discourse and can internalize field-specific mathematical discourses. This dynamic enriches mathematical communication in classrooms, significantly enhancing students' mathematical learning. Moreover, within peer interactions, discourses rooted in everyday life tend to prevail over strict mathematical language. These everyday discourses foster comprehension of mathematics' conceptual structure and bolster emotional aspects by imbuing student-driven mathematical discussions with greater significance.

One of the most important obstacles in the development of the mathematical communication process is students' misconceptions and mislearning. Misconceptions that cause conflicts and disagreements while students are discussing their mathematical ideas with other individuals can seriously damage a healthy mathematical communication process. It is thought that discourses positively affect both the development of the mathematical communication process and the mathematics learning process, with their contribution to identifying misconceptions.

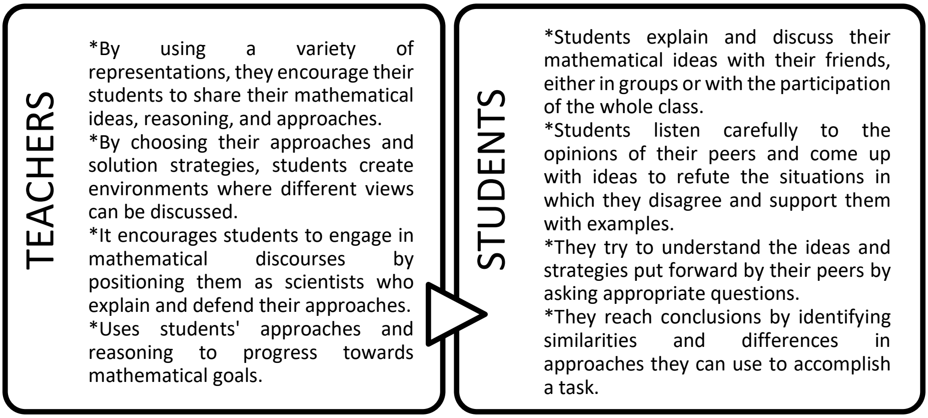
The use of discourses in learning environments is also effective in creating classroom environments suitable for contemporary learning approaches. Kersaint (2015) summarized the communicative processes that take place in a classroom environment rich in mathematical discourse as follows.

- To help students develop a better understanding, both students and teachers acknowledge and discuss their mistakes and the reasons behind them.
- Students question each other using mathematical knowledge to determine the accuracy of their solutions.
- Students arrive at and justify conclusions based on their mathematical knowledge, without seeing teachers as the absolute authority.
- Students are encouraged to use a variety of approaches, such as oral presentations, written explanations, physical, graphical, pictorial, and symbolic representations, to convey their knowledge and solution strategies.

Learning environments rich in discourse provide opportunities for communication processes in line with contemporary approaches such as constructivist learning, which is based on the student's active effort to construct knowledge in his mind. Kersaint (2015) also summarizes the roles of teacher and student in learning environments rich in mathematical discourse as in Figure 4.

Figure 4

Teacher and Student Roles in Mathematical Discourse-Based Rich Learning Environments



As can be seen from the teacher and student roles in Figure 4, mathematical discourses make a significant contribution to the formation of mathematical knowledge by maximizing communication in learning environments. In general, when we consider mathematical communication with language and discourse, many clues appear that it plays an important role in the development of students' mental processes and the perception of the conceptual structure of mathematics. To reveal this situation more clearly, let us consider in detail conceptual learning in mathematics teaching.

4. Exploring the Significance and Role of Conceptual Learning in the Pedagogy of Mathematics

One of the most important purposes of mathematics teaching is to enable students to learn mathematics by understanding it (Ata Baran & Yenilmez, 2014). Van de Walle (1989) attributes students' learning by understanding mathematics to the realization of three conditions: knowledge of mathematical operations, knowledge of mathematical concepts, and understanding of the relations between concepts and operations.

These conditions reveal two types of learning that mathematicians call conceptual and procedural learning. Conceptual learning, which is one of these learning types, especially allows students to construct and make sense of mathematics in their minds. The term 'concept' signifies a generalized abstract notion distilled from specific instances (Merriam Webster Dictionary, 2022). In all scientific fields, foundational concepts drive growth. Mathematics, deeply entrenched in concepts, necessitates understanding individual elements and their relationships. Nurjanah et al. (2021) stress grasping concepts for effective mathematical learning, is echoed by NCTM (2000). This emphasizes conceptual knowledge and learning. Birgin and Gürbüz (2009) define conceptual knowledge as linking, symbolizing, and expressing math concepts diversely, focusing on meaning (Canobi, 2009). Conceptual learning involves establishing logical foundations for concepts and exploring interrelations, as described by

Kilpatrick et al. (2001). Nahdi and Jatisunda (2020) stated that conceptual learning does not only mean knowing the information but also interpreting the information by making it meaningful.

Procedural knowledge, on the other hand, enables us to know and apply situations such as relations, operations, etc. to be used in problem-solving by using the symbols and language of mathematics (Soylu & Aydın, 2006). In short, procedural knowledge includes the practical part of mathematics. Balka et al. (2014) emphasized that mathematics teaching given in schools for many years is based on procedural learning. Especially in countries like Turkey that adopt an exam-based education system, an understanding of mathematics education that is based on procedural learning rather than conceptual learning comes to the fore. On the other hand, Zakaria et al. (2010) stated that knowing mathematics is especially based on knowledge of concepts, in addition to knowing operations and procedures. Mason et al. (2009) supported this view and emphasized the importance of conceptual learning by stating that for a deep understanding of mathematics, it is necessary to make sense of the theoretical structures and relationships on which complex mathematical concepts are based. This situation reveals the necessity of developing students' conceptual learning levels to achieve the goals of mathematics teaching.

A student needs to know how to perform a mathematical operation. But if this student does not know the logical reason for his action, his work is just memorization. For example, a middle school student learns that when adding and subtracting rational numbers, the denominators of numbers must be equalized. Students who learn this information can solve many of the operational questions in this way. Most of these students do not know why they are equating the denominator and unfortunately, the teachers do not explain the logical basis of this operation. Unlike addition and subtraction operations, there is no need to equate the denominator in multiplication and division operations, and equating the denominator causes unnecessary redundancy. Students who forget memorized information make mistakes in applications involving four operations on rational numbers or deal with unnecessary operations. This situation is thought to be the most important reason why mathematical literacy levels do not develop in societies that adopt rote-based education. To overcome this problem, it is seen as a necessity to give importance to conceptual learning in mathematics teaching.

Köğce et al. (2019) stress the importance of accurate concept comprehension in effective teaching, highlighting conceptual learning's role in understanding interconnections between concepts. Scholars also affirm that conceptual learning lays the groundwork for procedural knowledge (Geary, 1994; Halford, 1993). Gelman and Williams (1998) note that mathematical knowledge originates from conceptual understanding, while procedural knowledge stems from practice, counteracting rote memorization. Chi et al. (1989) assert that conceptually adept students provide insightful procedure explanations, especially in problem-solving. However, Aydın et al. (2016) warn against outcomes via formulaic approaches, misleading educators. In societies like Turkey, overemphasizing procedure over concept learning compounds educational challenges. Zakaria et al. (2010) emphasize deep conceptual comprehension for math proficiency. Mason et al. (2009) stress conceptual learning's role in understanding complex math, urging enhanced student conceptual learning for educational goals.

A review of relevant literature reveals a plethora of studies investigating diverse aspects of conceptual learning in mathematics education (Abrahamson & Abdu, 2020; Ata Baran & Yenilmez, 2014; Baki et al., 2009; Becerra Labra et al., 2012; Gürbüz & Gülburnu, 2013; Hussein & Csíkos, 2023; İşleyen & Işık, 2005; İşleyen & Işık, 2003; Lobato et al., 2012; Mareschal, 2016; Rillero, 2016; Schwarz et al., 2018; Simon, 2013; Simon et al., 2018; Soylu & Aydın, 2006; Tzur, 2011, 2011; Tzur & Simon, 2004; Walid, 2023). In a comprehensive review of the academic landscape, it becomes evident that studies focusing on the development of conceptual learning skills and their impact on academic achievement, as well as affective and cognitive traits, prominently occupy the forefront of educational research. While international studies extensively explore various facets of conceptual learning, limited research is evident in Turkey. Exploring the impact of pedagogical methods and techniques on enhancing conceptual learning within mathematics education is crucial for instructional enhancement. Notably, communication-centered learning environments aligned with constructivist principles emerge as significant. Yet, a substantial gap remains, both domestically and globally, concerning the interplay between conceptual learning and mathematical communication. This study aims to bridge this gap, contributing to a deeper understanding of this pivotal intersection.

5. Nurturing Conceptual Understanding: The Role of Mathematical Communication Environments

Many scholars advocate an educational paradigm focused on comprehension-driven mathematics learning (Pirie & Kieren, 1994; Sierpinska, 1994). Research consistently reveals that traditional education impedes conceptual learning, steering students toward procedural knowledge (Mack, 1995; Moseley, 2005). Addressing this, there is a call for a mathematics education framework that remedies traditional education's limitations and nurtures conceptual development. Nurjanah et al. (2021) propose student-centered methodologies that promote active engagement and distance from rote learning, a criticism of traditional approaches. The Everyday Mathematics program, by the University of Chicago's school mathematics project, advocates introducing abstract concepts only after establishing a strong foundation in conceptual knowledge. This program emphasizes real-life examples, concrete objects, and visual aids to enhance students' conceptual development through collaborative discourse (Everyday Mathematics, 2022). In this context, modern pedagogies fostering conceptual growth, often centered on communication-based learning environments, assume vital importance.

Kandemir (2004) emphasizes the significance of students' grasp of the underlying pre-concepts that constitute a given concept for robust conceptual understanding. This comprehension hinges on classroom dialogues, idea-sharing, and participatory educational settings. Sari and Darhim (2020) concur, stating that in participatory classrooms, students demonstrate their grasp of mathematical concepts through effective engagement in discussions, problem-solving, and explanations. Examination of pertinent literature unveils research outcomes showcasing that participatory classroom environments indeed enhance students' conceptual comprehension (Çayci et al., 2007; Kanselaar et al., 2000; Marinopoulos & Stavridou, 2002; Sukmawati et al., 2019). In one such study, Ross and Willson (2012) underscored that participatory classroom contexts, where diverse interpretations of mathematical concepts are shared among educators and learners, foster the establishment of connections between concepts and the acquisition of conceptual knowledge. In sum, these findings collectively underscore the paramount importance of learning environments rooted in research, discussion, collaboration, and inquiry for nurturing students' conceptual development.

Now, let's examine a few examples of situations that can demonstrate that mathematical communication-based learning environments support students' conceptual learning. First, let's take the discussion of a teacher trying to teach his students the concept of angle:

Teacher: *Yes, friends, today we will learn the concept of angle. First, let's talk about the concept of angle. Can anyone define the concept of angle?*

Student A (asks for a voice): *An angle is a number that expresses the size of the aperture formed by the two lines.*

Teacher: *Is an angle a number?*

Student A: *Yes, teacher, for example, 600 is an angle.*

Student B (requests to have a say): *But my teacher, the definition of angle in the sources is expressed as "The union of two rays with the same starting point is called an angle.". In this case, the angle seems to represent a shape rather than a numerical value.*

Teacher: *Well, let's go over the angle definition you made then. What does the union of two rays with the same starting point signify?*

Student C: *Teacher, since the ray is a set of points, the union of the two sets again denotes a set.*

Teacher: *Then can we say that an angle denotes a set of points?*

Student A: *Yes, my teacher, but we always use expressions such as 300, 600, etc. in practice. That's why I thought the angle was a numerical value.*

Teacher: *Yes, you are right, such a misconception can occur. So, what do you think about expressions like 300, and 600?*

Student D: *My teacher, I read in a source that these numerical expressions express the size of the aperture formed by two rays with the same starting point, and they are called the measure of an angle.*

Teacher: *Then, as a result, we will call the shape formed by the union of two rays, namely the set of points, the angle, and the measure of the aperture formed by the rays, the measure of angle.*

Dialogues between the teacher and his students reveal that initially, the students could not make sense of the concept of angle, and then the students reached the truth through a mutual exchange of ideas. For healthy conceptual development, it is thought that for teachers instead of giving the truth to the students' ready-made source of information, the students' access to the right information with their logic filters with this kind of mutual exchange of ideas will provide more permanent learning.

As a second example, consider the concepts of fractions and rational numbers. Let's examine the following dialogue between a teacher who wants to understand the definition of the concept of rational numbers at the secondary school level and his/her students.

Teacher: *Let's define the concept of rational numbers now. Firstly, I want to ask: Do you think fractions and rational numbers are the same things?*

Student A: *Yes, the same thing, I think. When we add the negative fractions, rational numbers appear. (Many students in the classroom support this view)*

Teacher: *How about the negative fraction? Shall we make sense of this situation by remembering the definition of a fraction?*

Student B: *Teacher, the fraction was each of the equal parts of a whole. How can it be a negative fraction since a fraction denotes parts of the whole?*

Student A: *A negative fraction is formed because of a minus sign in front of the fraction, but I cannot say whether it indicates a fraction as it stands.*

Teacher: *I guess it makes no sense to show the parts of the whole with a minus sign. Then, if the concept of fraction expresses the parts of the whole, what do the numbers we write as 1/2, 2/3, … mean?*

Student C: *I researched this, they are called fraction numbers, and they show the amount of the fraction in the whole. So, I agree with the idea that fractions cannot be negative.*

Teacher: *I also agree with this idea. So, let's consider the rational number and fraction relationship again.*

Student B: *Since fractions represent parts of a whole, we will associate fractions with rational numbers. Since fraction numbers cannot be negative, there will be a subset of rational numbers in this case. So, rational numbers are a more general set.*

Student C: *Yes, I agree with this. I think rational numbers are a larger set that includes all the fraction numbers and their minuses.*

Teacher: *Well then, I want to ask a question. Consider the numbers 1/2 and 2/4. We know that both numbers are fractions. But are they both rational numbers?*

Student D: *If we conclude that all fraction numbers are rational numbers, of course, they are rational numbers. But these are equivalent fractions, that is, they represent the same amounts of the whole. Since both show the same amount, wouldn't it be wrong to write both in the same set? Frankly, I'm a bit confused...*

Teacher: *So, the fact that the number 1/2 is in the set does not mean that all the numbers that are equivalent to this number are also present?*

Student A: *Then the set of rational numbers must be a set containing the simplest form of all fraction numbers.*

Teacher: *Well, according to the information we have reached, would it be appropriate if we write a set for the rational number as follows?*

Q = {a/b: a, b∈Z; a and b are relatively prime}

Student C: *Yes, in this case, all the fraction numbers are included in the set, since a and b are integers, negative numbers are also included in the set. Also, since you say that a and b are relatively prime, fraction numbers expressing the same quantities will not be written in the set separately. I think this set is the right teacher…*

Student B: *Only one thing stuck with me. We talked about the fact that 0 cannot be a divisor in a division. Since the number 0 is also an integer, it can be written instead of b. What will it be like then?*

Student D: *I agree too, we should add an expression like b shouldn’t be zero.*

Teacher: *Congratulations, you both saw an important point. Then if we write it like this, I guess there will be no problem.*

Q = {a/b: a, b∈Z; b≠0, a and b are relatively prime}

When we examine the dialogue between the teacher and the students, it is seen that the teacher does not give the relationship between concepts as ready and directs the students to the discussion. With this approach, the teacher encourages his students to learn this way, by showing that he is open to a communication-based learning environment. As it was seen at the beginning, there was a dominant opinion among the students that the concepts of fractions and rational numbers meant the same thing. With the mutual exchange of ideas under the guidance of the teacher, the students were able to reach the right information by using the clues they uncovered. In this example, it is important to note how important students' prior knowledge is in determining the relationships between concepts. Knowing the definition of fractions and fraction numbers, the properties of sets, and the role of 0 in the division process played a key role in reaching a new concept.

In a study conducted by Kabael and Baran (2016) with pre-service secondary school mathematics teachers, pre-service teachers were deliberately given incomplete or erroneous discourses, and they were asked to analyze the discourses. One of the discourses given in the study is "There is one of the numerics 0, 2, 4, 6, and 8 at the end of the numbers divisible by 2, right?". This example is very important in terms of revealing the importance of classroom discourses in conceptual learning. Such a statement means that a number cannot be divided by 2 unless one of these digits is present in the one's digit. However, every number is divisible by 2. But, if the division is desired to be without a remainder, any of these numerics must be in the units' digit of the dividing number. For students to make sense of the difference between division and divisibility, teachers need to make correct statements, and this reflects healthy mathematical communication.

A striking example in terms of revealing the effect of communication-based learning environments on conceptual learning is presented in the study by Chapin et al. (2003). In the study, a situation where a primary school teacher gave a series of numbers while teaching odd and even numbers to his students and aimed to determine whether the numbers were odd or even because of class discussions was exemplified. As a result of a student's statement that 24 is an odd number because it is divisible by 3 in the discussions, the teacher tried to bring the students to the right result by exchanging ideas. Here, the importance of class discussions in terms of revealing students who have conceptual misconceptions and guiding them to correct information is emphasized.

Conclusion and Discussion

In the modern era, education has evolved beyond mere knowledge acquisition to the development of skills aligned with current demands. Educators now prioritize fostering critical thinking, collaborative abilities, information retrieval, analysis, and effective problem-solving. Mathematics plays a crucial role in nurturing these skills, with communication abilities being pivotal. Gauging students' mathematical understanding solely from instant reactions in math classes is challenging. Mutual dialogues and discussions offer a means to unveil students' mathematical ideas. Contemporary learning theories underscore active student engagement and self-guided learning, guided by teachers. Communication-based learning environments are an integral facet of today's education, enabling students to thrive in these contexts.

The essence of understanding mathematics, with its abstract and conceptual nature, lies in comprehending mathematical concepts and their interrelationships. Many students mistakenly believe that memorizing definitions and formulas suffices for success, leading to deeper struggles and a sense of helplessness. Contemporary learning approaches reject rote memorization, advocating for student-centered environments driven by mathematical communication. Such approaches aim to enhance students' conceptual grasp through verbal and written interactions. Chapin et al. (2003) emphasize the significance of asking "Why do you think that?" to encourage explanations based on understanding rather than memorization. In these environments, students engage in dialogues, refining their mathematical ideas. Active participation fosters deeper understanding and the development of unique mathematical discourses, which reflect an individual's comprehension and mental growth. These discourses, as noted by Akkuş (2015), signify a distinct pattern, and contribute to a culture of meaningful discussion. Unlike casual exchanges, discourses convey purposeful speeches aimed at influencing others, serving as indicators of one's mathematical understanding and cognitive advancement.

This study delves into the pivotal role of communication and discourses within contemporary learning approaches, aiming to elucidate their contributions to meaningful conceptual learning in mathematics education. The research underscores that mere regurgitation of information, whether heard or memorized, falls short of genuine conceptual learning. Instead, cultivating a robust conceptual framework based on logical foundations is imperative. Mathematical communication and discourses emerge as powerful tools, revealing the presence of this structure within individuals and enhancing the conceptual learning process. The study emphasizes the significance of fostering a discourse culture in mathematics classrooms to enhance lesson effectiveness, particularly in challenging topics. Furthermore, advocating for communication-driven learning environments and nurturing a culture of mathematical discourse can counter the prevailing perception of mathematics as difficult and counter negative biases even before students embark on their educational journey.

References

Abrahamson, D., & Abdu, R. (2020). Towards an ecological-dynamics design framework for embodied-interaction conceptual learning: The case of dynamic mathematics environments. *Educational Technology Research and Development*, *69*. https://doi.org/10.1007/s11423-020-09805-1

Afrianti, R. E. N., & Qohar, A. (2020). Improving student's mathematical communication ability through think talk write learning model on geometry materials. *JIPM (Jurnal Ilmiah Pendidikan Matematika)*, *8*(2), Article 2. https://doi.org/10.25273/jipm.v8i2.6184

Akkuş, R. (2015). Language and discourse in mathematics. *Elementary Education Online*, *14*(1), Article 1.

Altun, M. (2008). *Teaching mathematics 6-7-8. classes* (6th ed.). Alpha Publications.

Andamon, J., & Tan, D. (2018). Conceptual understanding, attitude and performance in mathematics of grade 7 students. *International Journal of Scientific & Technology Research*, *07*(8), 96–105.

Ansori, H., Budayasa, I. K., & Suwarsono, t. (2019). Mathematical communication profile of female student who is mathematics teacher candidate in implementing teaching practice program. *Math Didactic: Jurnal Pendidikan Matematika*, *5*, 145–153. https://doi.org/10.33654/math.v5i2.605

Apriliyanto, B., Saputro, D., & Riyadi, R. (2018). Student's social interaction in mathematics learning. *Journal of Physics: Conference Series*, *983*(1). https://doi.org/10.1088/1742-6596/983/1/012130

Ata Baran, A., & Kabael, T. (2021). An investigation of eighth grade students' mathematical communication competency and affective characteristics. *The Journal of Educational Research*, *114*(4), 367–380. https://doi.org/10.1080/00220671.2021.1948382

Ata Baran, A., & Kabael, T. (2023). An investigation of eighth-graders' mathematical communication competency and mathematical literacy performance. *The Journal of Educational Research*. https://doi.org/10.1080/00220671.2023.2250752

Ata Baran, A., & Yenilmez, K. (2014). Investigation of prospective teachers' conceptual and procedural knowledge of types of events. *The Journal of Academic Social Science Studies*, *28*, 465–480. http://dx.doi.org/10.9761/JASSS2525

Aufa, M., Saragih, S., & Minarni, A. (2016). Development of learning devices through problem based learning model based on the context of ACEH cultural to improve mathematical communication skills and social skills of SMPN 1 Muara Batu students. *Journal of Education and Practice*, *7*(24), 232–248.

Aydın, S., Ural Keleş, P., Haşıloğlu, M. A., & Aydın, L. (2016). Academicians' views on conceptual and procedural learning in science education. *Participatory Educational Research*, *16*(2), 121–129. https://doi.org/10.17275/per.16.spi.2.13

Aydın, S., & Yeşilyurt, M. (2007). Student's view about the language used in mathematics teaching. *Elektronic Journal of Social Sciences*, *6*(22), Article 22.

Baki, A., Gürbüz, R., Ünal, S., & Atasoy, E. (2009). The effects of activities based on the multiple intelligence theory on students' conceptual learning: A case of basic operations in the integer numbers. *The Journal of Turkish Educational Sciences*, *7*(2), Article 2.

Bali Çalıkoğlu, G. (2002). Language scale in mathematics teaching. *Hacettepe University Journal of Education Faculty*, *2002*(23), Article 23.

Balka, D. S., Hull, T. H., & Miles, R. H. (2014). *What is conceptual understanding?* Retrieved October 24, 2023, from https://studylib.net/doc/7941770/what-is-conceptual-understanding?

Bandura, A. (1969). *Principles of behavior modification*. Holt, Rinehart and Winston.

Baroody, A. J. (1993). *Problem solving, reasoning and communicating, K-8: Helping children think mathematically*. Macmillan.

Barwell, R., Leung, C., & Morgan, C. (2005). Applied linguistics and mathematics education: more than words and numbers. *Language and Education*, *19*, 141–146. https://doi.org/10.1080/09500780508668670

Baxter, J. A., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal*, *101*(5), 529–547.

Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students, *Learning Disabilities Research and Practice*, *20*(2), 119–135. https://doi.org/10.1111/j.1540-5826.2005.00127.x

Becerra Labra, C., Gras Marti, A., & Torregrosa, J. M. (2012). Effects of a problem-based structure of physics contents on conceptual learning and the ability to solve problems. *International Journal of Science Education*, *34*(8), 1235–1253. https://doi.org/10.1080/09500693.2011.619210

Bergsten, C., Engelbrecht, J., & Kagesten, O. (2017). Conceptual and procedural approaches to mathematics in the engineering curriculum – comparing views of junior and senior engineering students in two countries. *EURASIA Journal of Mathematics, Science and Technology Education*, *13*. https://doi.org/10.12973/eurasia.2017.00631a

Birgin, O., & Gürbüz, R. (2009). İlköğretim II. Kademe öğrencilerinin rasyonel sayılar konusundaki işlemsel ve kavramsal bilgi düzeylerinin incelenmesi [Investigation of operational and conceptual knowledge levels of secondary school students on rational numbers]. *Journal of Uludag University Faculty of Education*, *22*(2), Article 2.

Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *The Journal of Mathematical Behavior*, *18*, 379–397. https://doi.org/10.1016/S0732-3123(00)00026-2

Bondurant, L. (2020). Equitable mathematics classroom discourse. *Journal of Practitioner Research*, *5*(2). https://doi.org/10.5038/2379-9951.5.2.1146</p>

Brendefur, J. L., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education*, *3*(2), 125–153. https://doi.org/10.1023/A:1009947032694

Brenner, M. E. (1998). Development of mathematical communication in problem solving groups by language minority students. *Bilingual Research Journal*, *22*(2), 149–174. https://doi.org/10.1080/15235882.1998.10162720

Bruner, J. (1974). *Toward a theory of instruction:* Belknap Press.

Cai, F. (2017). Challenges and strategies of Chinese higher education under the globalization background. *Advances in Computer Science Research*, *76*, 659–662.

Cai, J., Jakabcsin, M. S., & Lane, S. (1996). Assessing students' mathematical communication. *School Science and Mathematics*, *96*(5), 238–246. https://doi.org/10.1111/j.1949-8594.1996.tb10235.x

Canobi, K. H. (2009). Concept–procedure interactions in children's addition and subtraction. *Journal of Experimental Child Psychology*, *102*(2), 131–149. https://doi.org/10.1016/j.jecp.2008.07.008

Çaycı, B., Demir, M. K., Başaran, M., & Demir, M. (2007). Concept teaching with cooperative learning in social studies lesson. *Kastamonu Journal of Education*, *15*(2), Article 2.

Çelik, H., & Ekşi, H. (2013). Discourse analysis. *Marmara University Atatürk Education Faculty Journal of Educational Sciences*, *27*(27), 99 - 117.

Cesar, M. (1998). *Social interactions and mathematics learning* (SE065789; p. 15). https://eric.ed.gov/?id=ED463156

Chapin, S. H., Anderson, N. C., & O'Connor, C. (2003). *Classroom discussions: Using math talk to help students learn: Grades 1-6* (1st ed.). Blindpig Books.

Chi, M. T. H., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science*, *13*(2), 145–182. https://doi.org/10.1016/0364-0213(89)90002-5

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, *28*, 258–277. https://doi.org/10.2307/749781

Everyday Mathematics. (2022). The University of Chicago School Mathematic Project Resource and Information Center. Retrieved October 24, 2023, from https://everydaymath.uchicago.edu/

Farokhah, L., Arisetyawan, A., & Jupri, A. (2017). The effect of ethnomathematics-based SAVI (somatic, auditory, visualization, intellectually) approach on mathematical communication skill on geometry in elementary school. *IJAEDU- International E-Journal of Advances in Education*, *3*(9), Article 9. https://doi.org/10.18768/ijaedu.370417

Fernández-Millán, E., & Molina, M. (2017). Secondary students' implicit conceptual knowledge of algebraic symbolism. An exploratory study through problem posing. *International Electronic Journal of Mathematics Education*, *12*(3), 799–826. https://doi.org/10.29333/iejme/649

Forrester, P., & Chinnappan, M. (2010). The predominance of procedural knowledge in fractions. *Faculty of Education - Papers (Archive)*, 185–192.

Garba, A., Ismail, N., Osman, S., & Mohd Rameli, M. R. (2019). Exploring peer effect on mathematics anxiety among secondary school students of Sokoto State, Nigeria through photovoice approach. *Eurasia Journal of Mathematics, Science and Technology Education*, *16*. https://doi.org/10.29333/ejmste/112622

Geary, D. (1994). *Children's mathematical development: Research and practical applications*. American Psychological Association. https://doi.org/10.1037/10163-000

Gelman, R., & Williams, E. M. (1998). Enabling constraints for cognitive development and learning: Domain specificity and epigenesis. In D. Kuhn & R. S. Siegler (Eds.), *Handbook of child psychology: Cognition, perception, and language* (5th ed, Vol. 2). J. Wiley.

Gürbüz, R., & Gülburnu, M. (2013). *Effect of Teaching Geometry with Use Cabri 3D in Eighth Grade on Conceptual Learning*. *Turkish Journal of Computer and Mathematics Education*, *4*. https://doi.org/10.16949/turcomat.60325

Halford, G. S. (1993). *Children's understanding: The development of mental models*. Psychology Press. https://doi.org/10.4324/9781315801803

Harianja, J. K., Hernadi, S. L., & Indah, I. (2020). Students' mathematical conceptual understanding and its relation to the mathematical communication skills. *Jurnal Penelitian Pendidikan Dan Pengajaran Matematika*, *6*(1), 1–12.

Hussein, Y. F., & Csíkos, C. (2023). The effect of teaching conceptual knowledge on students' achievement, anxiety about, and attitude toward mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(2), Article em2226. https://doi.org/10.29333/ejmste/12938

Hyde, R., & Jain, P. (2020). *Myths and legends of mastery in the mathematics curriculum* (R. Hyde & P. Jain, Eds.). SAGE, Learning Matters.

Işıkdoğan Uğurlu, N., & Kayhan, N. (2018). Examination to attitudes of classroom teachers towards educational evaluation process of students with special needs. *Journal of Adıyaman University Social Sciences Institute*, *29*, Article 29. https://doi.org/10.14520/adyusbd.398385

İşleyen, T., & Işik, A. (2005). On conceptual learning of the concept of sub-vector space. *Journal of Kazım Karabekir Education Faculty*, *11*, Article 11.

İşleyen, T., & Işık, A. (2003). Conceptual and procedural learning in mathematics. *Journal of Korea Society of Mathematical Education Series D: Research in Mathematical Education*, *7*(2), 91–99.

Jazuli, A., Setyosari, P., Sulthon, & Kuswandi, D. (2017). Improving conceptual understanding and problem-solving in mathematics through a contextual learning strategy. *Global Journal of Engineering Education*, *19*, 49–53.

Kabael, T., & Ata Baran, A. (2017). Prospective middle school mathematics teachers' mathematical communication, mathematical and pedagogical knowledge. *Kastamonu Eğitim Dergisi*, *25*(5), Article 5.

Uygur Kabael, T., & Ata Baran, A. (2016). Matematik öğretmenlerinin matematik dili becerilerinin gelişimine yönelik farkındalıklarının incelenmesi. *İlköğretim Online*, *15*(3), 868-881,. https://doi.org/10.17051/io.2016.78518

Kandemir, M. (2004). The permanent concepts in mathematics. *Kastamonu Educational Journal*, *12*(2), Article 2.

Kanselaar, G., van Boxtel, C., & Linden, J. L. (2000). Collaborative Learning tasks and the elaboration of conceptual knowledge. *Learning and Instruction*, *10*. https://doi.org/10.1016/S0959-4752(00)00002-5

Kanz, M. (2020). *Assessing conceptual learning in mathematics classrooms,* [Master Thesis, Massey University].

Kaya, D., & Aydın, H. (2016). Elementary mathematics teachers' perceptions and lived experiences on mathematical communication. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(6), 1619–1629. https://doi.org/10.12973/eurasia.2014.1203a

Kaymak, S., Balta, N., Almas, A., Kazmagambet, B., & Mbala, O. (2020). The impact of peer instruction on students' achievement in mathematics analysis. *Journal of Psychology Research*, *10*(12). https://doi.org/10.17265/2159-5542/2020.12.003

Kersaint, G. (n.d.). *Orchestrating mathematical discourse whitepaper*. I-Ready classroom mathematics. Retrieved October 24, 2023, from https://www.curriculumassociates.com/programs/i-ready-learning/classroom-math/orchestrating-mathematical-discourse-whitepaper

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Köğce, D., Yıldız, C., & Aydın, M. (2019). An investigation of preservice mathematics teachers' opinions on determining and eliminating of mathematical misconceptions and concept teaching. *Cumhuriyet International Journal of Education*, *8*(2), 453–478.

Konyalioğlu, A. C., Tortumlu, N., Kaplan, A., Işık, A., & Hizarcı, S. (2011). On pre-service mathematics teachers' conceptual understanding of the integral concept. *Journal of Bayburt Educational Faculty*, *6*(1), Article 1.

Kosko, K. W., & Wilkins, J. L. M. (2010). Mathematical communication and its relation to the frequency of manipulative use. *International Electronic Journal of Mathematics Education*, *5*(2), 79–90. https://doi.org/10.29333/iejme/251

Kushwaha, S. S. (2014). Trend in researches on mathematics achievement. *IOSR Journal of Research & Method in Education*, *4*, 53–62. https://doi.org/10.9790/7388-04625362

Kusumah, Y., Kustiawati, D., & Herman, T. (2020). The effect of GeoGebra in three-dimensional geometry learning on students' mathematical communication ability. *International Journal of Instruction*, *13*, 895–908. https://doi.org/10.29333/iji.2020.13260a

Lee, J. (2015). "Oh, I just had it in my head": Promoting mathematical communications in early childhood. *Contemporary Issues in Early Childhood*, *16*(3), 284–287. https://doi.org/10.1177/1463949115600054

Lindquist, M. M., & Elliott, P. C. (1996). Communication-an imperative for change: A conversation with Mary Lindquist. In *Communication in mathematics, K-12 and beyond* (pp. 1–10). NCTM.

Lisnani, L. (2018). Design research on plane figure learning by using picture story and pairing game to improve mathematical communication skills of second grade of primary school students. *Journal of Physics: Conference Series*, *1040*, 012027. https://doi.org/10.1088/1742-6596/1040/1/012027

Lobato, J., Hohensee, C., Rhodehamel, B., & Diamond, J. (2012). Using student reasoning to inform the development of conceptual learning goals: The case of quadratic functions. *Mathematical Thinking and Learning*, *14*, 85–119. https://doi.org/10.1080/10986065.2012.656362

Lomibao, L. S., Luna, C. A., & Namoco, R. A. (2016). The influence of mathematical communication on students' mathematics performance and anxiety. *American Journal of Educational Research*, *4*(5), 378–382. https://doi.org/10.12691/education-4-5-3

Mack, N. K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education*, *26*(5), 422–441. https://doi.org/10.2307/749431

Makgakga, T. (2023). Twinned teachers' mathematical discourse using problem-solving. *Journal of Culture and Values in Education*, *6*(2), 138–157. https://doi.org/10.46303/jcve.2023.13

Malatjie, F., & Machaba, F. (2019). Exploring mathematics learners' conceptual understanding of coordinates and transformation geometry through concept mapping. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12), Article em1818. https://doi.org/10.29333/ejmste/110784

Mareschal, D. (2016). The neuroscience of conceptual learning in science and mathematics. *Current Opinion in Behavioral Sciences*, *10*, 114–118. https://doi.org/10.1016/j.cobeha.2016.06.001

Marinopoulos, D., & Stavridou, H. (2002). The influence of a collaborative learning environment on primary students' conceptions about acid rain. *Journal of Biological Education*, *37*. https://doi.org/10.1080/00219266.2002.9655841

Martin, D. (2001). Mathematics success and failure among african-american youth: the roles of sociohistorical context, community forces, school influence, and individual agency. *Contemporary Sociology*, *30*. https://doi.org/10.2307/3089793

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, *21*, 10–32. https://doi.org/10.1007/BF03217543

Maspe, P. W., Awaluddin, Dassa, A., & Rosidah. (2021). *The effect of self-efficacy and self-esteem on students' mathematical communication skills*. 409–415. https://doi.org/10.2991/assehr.k.211211.069

Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education*, *20*(6), 507–528. https://doi.org/10.2167/le678.0

Merriam Webster Dictionary. (2022). Definition of concept. In *Merriam webster dictionary*. https://www.merriam-webster.com/dictionary/concept

Mohd Yatim, S. S. K., Saleh, S., Zulnaidi, H., Yew, W., & Yatim, S. A. M. (2022).

Effects of brain-based teaching approach integrated with GeoGebra (b-geo module) on students' conceptual understanding. *International Journal of Instruction*, *15*, 327–346. https://doi.org/10.29333/iji.2022.15119a

MoNE, (Ministry of National Education). (2013). *Middle school mathematics course (5, 6, 7, and 8th Grades) Curriculum*. Ministry of National Education.

Mosckhovich, J. (2007). Using two languages when learning mathematics. *Educational Studies in Mathematics*, *64*, 121–144. https://doi.org/10.1007/s10649-005-9005-1

Moseley, B. (2005). Students' early mathematical representation knowledge: the effects of emphasizing single or multiple perspectives of the rational number domain in problem solving. *Educational Studies in Mathematics*, *60*, 37–69.

Mujiasih, M., Waluyo, B., Kartono, K., & Mariani, S. (2021). The students' mathematics communication skill performance after GeoGebra-assisted epic-r learning implementation. *International journal of learning, Teaching and Educational Research*, *20*(10), Article 10. https://doi.org/10.26803/ijlter.20.10.14

Nahdi, D. S., & Jatisunda, M. G. (2020). Conceptual understanding and procedural knowledge: A case study on learning mathematics of fractional material in elementary school—iopscience. *Journal of Physics: Conference Series*, *1477*, 1–5. https://doi.org/10.1088/1742-6596/1477/4/042037

Nakamura, K. (2008). Language and affect: Japanese children's use of evaluative expressions in narratives. In *Crosslinguistic approaches to the psychology of language: Research in the traditions of Dan Slobin*. Lawrence Erlbaum Associates.

Nathan, M., & Knuth, E. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction*, *21*, 175–207. https://doi.org/10.1207/S1532690XCI2102_03

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. National Council of Teachers of Mathematics.

NCTM. (2000). *Principles and standards for school mathematics.* National Council of Teachers of Mathematics.

Ng, O. L. (2016). The interplay between language, gestures, dragging and diagrams in bilingual learners' mathematical communications. *Educational Studies in Mathematics*, *91*(3), 307–326. https://doi.org/10.1007/s10649-015-9652-9

Nurjanah, N., Dahlan, J., & Wibisono, Y. (2021). The effect of hands-on and computer-based learning activities on conceptual understanding and mathematical reasoning. *International Journal of Instruction*, *14*, 143–

160. https://doi.org/10.29333/iji.2021.1419a

Pape, S. J., Bell, C. V., & Yetkin, İ. E. (2003). Developing mathematical thinking and self-regulated learning: A teaching experiment in a seventh-grade mathematics classroom. *Educational Studies in Mathematics*, *53*(3), 179–202. https://doi.org/10.1023/A:1026062121857

Paridjo, P., & Waluya, S. (2017). Analysis mathematical communication skills students in the matter algebra based NCTM. *IOSR Journal of Mathematics*, *13*, 60–66. https://doi.org/10.9790/5728-1301056066

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, *26*(2), 165–190. https://doi.org/10.1007/BF01273662

Pourdavood, R. G., & Wachira, P. (2015). Importance of mathematical communication and discourse in secondary classrooms. *Global Journal of Science Frontier Research: F Mathematics and Decision Sciences*, *15*(10), 9–20.

Pourdavood, R., McCarthy, K., & McCafferty, T. (2005). The impact of mental computation on children’s mathematical communication, problem solving, reasoning and algebraic thinking. *Athens Journal of Education*, *7*(3), 241–254. https://doi.org/10.30958/aje.7-3-1

Pugalee, D. K. (2001). Spotlight on the standards: Using communication to develop students’ mathematical literacy. *Mathematics Teaching in the Middle School*, *6*(5), 296–299. https://doi.org/10.5951/MTMS.6.5.0296

Pujiastuti, H., Haryadi, R., & Rosyana, R. F. (2021). The use of set magnetic teaching aids to improve mathematical communication ability. *Unnes Journal of Mathematics Education*, *10*(1), Article 1. https://doi.org/10.15294/ujme.v10i1.43588

Rillero, P. (2016). Deep conceptual learning in science and mathematics: Perspectives of teachers and administrators. *The Electronic Journal of Science Education*, *20*(2), 14–31.

Roble, D. B., Lomibao, L. S., & Luna, C. A. (2018). Promoting students’ achievement and mathematical flexibility through pre-within-post problem posing tasks. *International Journal of Science and Research (IJSR)*, *7*(1), 1830–1834.

Ross, A., & Willson, V. (2012). The effects of representations, constructivist approaches, and engagement on middle school students’ algebraic procedure and conceptual understanding. *School Science and Mathematics*, *112*. https://doi.org/10.1111/j.1949-8594.2011.00125.x

Ryve, A. (2006). Making explicit the analysis of students’ mathematical discourses – revisiting a newly developed methodological framework. *Educational Studies in Mathematics*, *62*(2), 191–209. https://doi.org/10.1007/s10649-006-4834-0

Sammons, L. (2018). *Teaching students to communicate mathematically*. ASCD.

Sari, D. P., & Darhim, D. (2020). Implementation of react strategy to develop mathematical representation, reasoning, and disposition ability. *Journal on Mathematics Education*, *11*(1), Article 1. https://doi.org/10.22342/jme.11.1.7806.145-156

Sari, S. I., & Yuberta, K. R. (2022). Analysis of mathematical communication ability in terms of student's numerical ability. *Rangkiang Mathematics Journal*, *1*(2), Article 2. https://doi.org/10.24036/rmj.v1i2.7

Savaş, E., Taş, S., & Duru, A. (2010). Factors affecting students' achievement in mathematics. *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, *11*(1), Article 1.

Schwarz, B., Prusak, N., Swidan, O., Livny, A., Gal, K., & Segal, A. (2018). Orchestrating the emergence of conceptual learning: A case study in a geometry class. *International Journal of Computer-Supported Collaborative Learning*, *13*. https://doi.org/10.1007/s11412-018-9276-z

Septiana, A., Kusmayati, T., & Fitriana, L. (2018). Mathematical communication skill of senior high school students based on their personality types. *Journal of Physics: Conference Series*, *1108*, 012027. https://doi.org/10.1088/1742-6596/1108/1/012027

Setiyani, S., Putri, D., Ferdianto, F., & Fauji, S. (2020). Designing a digital teaching module based on mathematical communication in relation and function. *Journal on Mathematics Education*, *11*, 223–236. https://doi.org/10.22342/jme.11.2.7320.223-236

Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press.

Sfard, A. (2012). Introduction: Developing mathematical discourse—Some insights from communicational research. *International Journal of Educational Research*, *51–52*, 1–9. https://doi.org/10.1016/j.ijer.2011.12.013

Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, *8*(1), 42–76. https://doi.org/10.1207/S15327884MCA0801_04

Sierpinska, A. (1994). *Understanding in mathematics* (1st ed.). Routledge.

Simon, M. (2013). The need for theories of conceptual learning and teaching of mathematics. In *Vital directions for mathematics education research* (pp. 95–118). https://doi.org/10.1007/978-1-4614-6977-3_5

Simon, M. A., Kara, M., Placa, N., & Avitzur, A. (2018). Towards an integrated theory of mathematics conceptual learning and instructional design: The Learning through activity theoretical framework. *Journal of Mathematical Behavior*, *52*, 95–112. https://doi.org/10.1016/j.jmathb.2018.04.002

Soter, A. O., Wilkinson, I. A., Murphy, P. K., Rudge, L., Reninger, K., & Edwards,

M. (2008). What the discourse tells us: Talk and indicators of high-level comprehension. *International Journal of Educational Research*, *47*(6), 372–391. https://doi.org/10.1016/j.ijer.2009.01.001

Soylu, Y., & Aydın, S. (2006). A study on importance of the conceptual and operational knowledge are balanced in mathematics lessons. *Erzincan University Journal of Education Faculty*, *8*(2), Article 2.

Stein, C. A. (2007). Let's talk: Promoting mathematical discourse in the classroom. *The Mathematics Teacher*, *101*(4), 285–289. https://doi.org/10.5951/MT.101.4.0285

Sukmawati, F., Setyosari, P., Sulton, S., & Purnomo, P. (2019). The effect of project-based collaborative learning strategy and social skill towards conceptual understanding and the application of biology concept. *Journal for the Education of Gifted Young Scientists*, *7*(4), 1325–1344. https://doi.org/10.17478/jegys.630693

Sumaji, S., Sa'dijah, C., Aydın, S., & Sisworo, S. (2020). Mathematical communication process of junior high school students in solving problems based on apos theory. *Journal for the Education of Gifted Young Scientists*, *8*(1), Article 1. https://doi.org/10.17478/jegys.652055

Susilawati, W., Maryono, I., & Maimunah, S. (2018). The development of Adobe Flash-based interactive multimedia to enhance students' mathematical communication skills. *IOP Conference Series: Materials Science and Engineering*, *434*, 012011. https://doi.org/10.1088/1757-899X/434/1/012011

Tahir, T. (2021). The effect of learning styles on students' mathematical communication ability. *Journal Of Medives: Journal of Mathematics Education IKIP Veteran Semarang*, *5*(1), 13–21. https://doi.org/10.31331/medivesveteran.v5i1.1378

Thompson, D., & Chappell, M. (2007). Communication and representation as elements in mathematical literacy. *Reading & Writing Quarterly*, *23*, 179–196. https://doi.org/10.1080/10573560601158495

Thurston, A., Roseth, C., Chiang, T. H., Burns, V., & Topping, K. (2020). The influence of social relationships on outcomes in mathematics when using peer tutoring in elementary school. *International Journal of Educational Research Open*, *1*(100004). https://doi.org/10.1016/j.ijedro.2020.100004

Tong, D. H., Uyen, B. P., & Van Anh Quoc, N. (2021). The improvement of 10th students' mathematical communication skills through learning ellipse topics. *Heliyon*, *7*(11), Article e08282. https://doi.org/10.1016/j.heliyon.2021.e08282

Toscano, R., Gavilán-Izquierdo, J. M., & Sánchez, V. (2019). A study of pre-service primary teachers' discourse when solving didactic-mathematical tasks. *Eurasia Journal of Mathematics, Science and*

Technology Education, *15*(11), em1762. https://doi.org/10.29333/ejmste/108631

Triana, M., Zubainur, C., & Bahrun, B. (2019). Students' mathematical communication ability through the brain-based learning approach using autograph. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *1*, 1. https://doi.org/10.23917/jramathedu.v1i1.6972

Tu, C. H., & McIsaac, M. (2002). The relationship of social presence and interaction in online classes. *American Journal of Distance Education*, *16*, 131–150. https://doi.org/10.1207/S15389286AJDE1603_2

Tzur, R. (2011). Can dual processing theories of thinking inform conceptual learning in mathematics? *The Mathematics Enthusiast*, *8*(3), 597–636. https://doi.org/10.54870/1551-3440.1230

Tzur, R., & Simon, M. (2004). Distinguishing two stages of mathematics conceptual learning. *International Journal of Science and Mathematics Education*, *2*, 287–304. https://doi.org/10.1007/s10763-004-7479-4

Usiskin, Z. (2015). What does it mean to understand some mathematics? In S. J. Cho (Ed.), *selected regular lectures from the 12th international congress on mathematical education* (pp. 821–841). Springer International Publishing. https://doi.org/10.1007/978-3-319-17187-6_46

Uyen, B. P., Tong, D. H., & Tram, N. T. B. (2021). Developing mathematical communication skills for students in grade 8 in teaching congruent triangle topics. *European Journal of Educational Research*, *10*(3), 1287–1302.

Uygur Kabael, T., & Ata Baran, A. (2016). Matematik öğretmenlerinin matematiksel iletişim becerilerinin gelişimine yönelik farkındalıklarının incelenmesi [Investigation of mathematics teachers' awareness of developing mathematical communication skills]. *İlköğretim Online*, *15*, 868–881.

van de Walle, J. A. (1989). *Elementary school mathematics*. Longman.

Vygotsky, L. S. (2012). *Thought and language*. MIT Press.

Walid, A. (2023). Improving conceptual understanding of mathematics through blended learning using google classroom. *International Journal of Education and Teaching Zone*, *2*(1), 130–138. https://doi.org/10.57092/ijetz.v2i1.122

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, *78*, 516–551. https://doi.org/10.3102/0034654308320292

Wells, C. (2003). *A handbook of mathematical discourse*. https://abstractmath.org/Handbook/handbook.pdf

Yeh, C., Cheng, H., Chen, Z. H., Liao, C. Y., & Chan, T. W. (2019). Enhancing achievement and interest in mathematics learning through Math-Island. *Research and Practice in Technology Enhanced Learning*, *14*. https://doi.org/10.1186/s41039-019-0100-9

Yimam, M., & Kelkay, A. D. (2022). Evaluation of the effects of discourse-based mathematics instruction on eleventh grade students' conceptual and procedural understanding of probability and statistics. *Cogent Education*, *9*(1). https://doi.org/10.1080/2331186X.2021.2007742

Yüksel, A. H. (2012). The concept of persuasion. In *Persuasive communication* (p. 18). Anadolu University Open Education Faculty Publications.

Yulian, N. (2018). Developing teaching materials using comic media to enhance students' mathematical communication. *IOP Conference Series: Materials Science and Engineering*, *335*, Article 012110. https://doi.org/10.1088/1757-899X/335/1/012110

Zakaria, E., Yaakob, M., Mistima, S., & Adnan, M. (2010). Conceptual knowledge and mathematics achievement of matriculation students. *Procedia - Social and Behavioral Sciences*, *9*, 1020–1024. https://doi.org/10.1016/j.sbspro.2010.12.279

Zengin, Y. (2017). The potential of GeoGebra software for providing mathematical communication in the light of pre-service teachers' views. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *11*, 101–127.

Zeybek, Z., & Açıl, E. (2018). Investigating seventh grade students' mathematical communication skills: Student journals. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *9*(3), Article 3. https://doi.org/10.16949/turkbilmat.367513