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Hartley-Ross Unbiased Estimator of Population Mean in Ranked Set Sampling Using Two Auxiliary Variables

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ABSTRACT

This paper proposes Hartley-Ross (HR) unbiased estimator of the finite population mean using two auxiliary variables in ranked set sampling (RSS). The variance of the proposed unbiased estimator is obtained to first degree of approximation. Comparisons among the proposed and some existing estimators are made both theoretically and through simulation study. It is shown that the proposed estimator is more efficient as compared to all other competitor estimators under RSS scheme.

1. INTRODUCTION

Ranked set sample scheme is more effective to reduce cost and increase efficiency when the survey variable is costly and time consuming, but it can be ranked easily at no cost or at very little cost. The technique of RSS was first suggested by McIntyre [11] to increase efficiency of the estimator. Takahasi and Wakimoto [19] proved the mathematical theory that the sample mean under RSS is an unbiased estimator of the finite population mean and more precise than the sample mean estimator under simple random sampling (SRS). Stokes [18] did the ranking of elements on basis of the auxiliary variable instead of judgment. Singh et al. [16] proposed an estimator for population mean and ranking of the elements is observed on basis of the auxiliary variable. Kadilar et al. [5] used RSS technique to improve ratio estimator given by Prasad [13].

Hartley and Ross [4] were the first to propose an unbiased ratio estimator for finite population mean in SRS. Later, Pascual [12] proposed an unbiased ratio type estimator in stratified random sampling. Singh et al. [17] and Kadilar and Cekim [6] suggested Hartley-Ross (HR) type unbiased estimators of the finite population mean using auxiliary information of population parameters in SRS. Khan and Shabbir [7,8] suggested a class of HR type unbiased estimators in RSS and stratified ranked set sampling $(SRSS)$. Khan and Shabbir [9] also proposed efficient ratio-type estimators of population mean using two auxiliary variables under RSS scheme. Khan et al. [10] introduced HR type unbiased estimators of population mean using $SRSS$ technique.

Here, we propose an unbiased estimator of the finite population mean using two auxiliary variables under RSS scheme.

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2. RANKED SET SAMPLING AND SOME EXISTING ESTIMATORS

In RSS first m independent random samples each of size m are chosen and the elements in each sample are selected with equal probability and without replacement from a finite population of size N. The elements of each random sample are ranked with respect to the characteristic of the study variable or auxiliary variable. Let Y is the study variable and X and Z are the two auxiliary variables. Then randomly select m^2 trivariate sample elements from the population and allocate them into m sets, each of size m. Each sample is ranked with respect to one of the auxiliary variables X or Z. Here, ranking is done on basis of the auxiliary variable X. An actual measurement from the first sample is then taken on the element with the smallest rank of X, together with variables Y and Z associated with smallest rank of X. From second sample of size m, the variables Y and Z associated with the second smallest rank of X are measured. By this way, this procedure is continued until, the Y and Z values associated with the highest rank of X are measured from the mth sample. This completes one cycle of the sampling. The process is repeated r times to obtain the desired sample of size $n = mr$ elements. Thus in a RSS scheme, a total of $m^2 r$ elements have been drawn from the population and only mr of them are selected for analysis. To estimate population mean $(\bar{Y})$ in RSS using a chain ratio estimator, the procedure can be summarized as follows:

- **Step 1:** Randomly select m^2 trivariate sample units from the population.
- **Step 2:** Allocate these m^2 units into m sets, each of size m.
- **Step 3:** Each set is ranked with respect to the concomitant variable X.
- **Step 4:** Select the ith ranked unit from the ith ($i = 1,2,...,m$) set for actual magnitude.
- **Step 5:** Repeat Steps 1 through 4 for r cycles until the desired sample size $n = mr$, is obtained.

The usual RSS estimator $\bar{y}_{U(rss)}$ and its variance, are given by

$$\bar{y}_{U(rss)} = \frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m} y_{[i:m]j}, \tag{1}$$

$$V(\bar{y}_{U(rss)}) = \bar{Y}^2\left(\gamma C_y^2 - W_y^2\right). \tag{2}$$

where $\gamma = (\frac{1}{mr})$ and $W_y^2 = \frac{1}{m^2 r \bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i:m]}^2$.

Khan and Shabbir [7] suggested the following Hartley-Ross type unbiased estimator in RSS

$$\bar{y}_{HR(rss)} = \bar{r}_{rss}\bar{X} + \frac{n(N-1)}{N(n-1)}(\bar{y}_{rss} - \bar{r}_{rss}\bar{x}_{rss}) \tag{3}$$

and its variance is given by

$$V(\bar{y}_{HR(rss)}) \cong \bar{Y}^2(\gamma C_y^2 - W_y^2) + \bar{X}^2\bar{R}^2(\gamma C_x^2 - W_x^2) - 2\bar{R}\,\overline{YX}(\gamma C_{yx} - W_{yx}). \tag{4}$$

Khan and Shabbir [9] proposed the following ratio-type estimator in RSS

$$\bar{y}_{R(RSS)} = \bar{y}_{[rss]}\left(\frac{\bar{X}}{\bar{x}_{(rss)}}\right)^{\alpha_1}\left(\frac{\bar{Z}}{\bar{z}_{(rss)}}\right)^{\alpha_2} exp\left(\frac{\bar{X} - \bar{x}_{(rss)}}{\bar{X} + \bar{x}_{(rss)}}\right), \tag{5}$$

where α_1 and α_2 are unknown constants. The optimum values of α_1 and α_2 are

$$\alpha^{*}_{1(opt)} = \frac{\left[2C_y(\rho_{yx} - \rho_{xz}\rho_{yz}) - C_x(1-\rho^2_{xz})\right]}{2C_x(1-\rho^2_{xz})}$$

and

$$\alpha^{*}_{2(opt)} = \frac{C_y(\rho_{yz} - \rho_{xz}\rho_{yx})}{C_z(1-\rho^2_{xz})}.$$

The minimum bias and MSE of $\bar{y}_{R(RSS)}$, are given respectively

$$Bias(\bar{y}_{R(RSS)})_{min} \cong \frac{1}{8}\bar{Y}(\gamma C_x^2 - W_{(x)}^2), \tag{6}$$

and

$$MSE(\bar{y}_{R(RSS)})_{min} \cong \bar{Y}^2 V_{200}\left[1 - \frac{C_y^2}{C_x^2(1-\rho^2_{xz})}(\rho_{yx} - \rho_{xz})^2\right]. \tag{7}$$

where $V_{200} = \gamma C_y^2 - W_y^2$.

3. PROPOSED UNBIASED ESTIMATOR IN *RSS*

On the lines of Khan and Shabbir [7], we suggest the following estimator

$$\bar{y}_{P(rss)} = \bar{r}_{rss}\bar{g}_{rss}\bar{Z} \tag{8}$$

where $\bar{r}_{rss} = \frac{\sum_{j=1}^{r}\sum_{i=1}^{m} r_{[i:m]j}}{mr}$, $r_{[i:m]j} = \frac{y_{[i:m]j}}{x_{(i:m)j}}$, $\bar{g}_{rss} = \frac{\sum_{j=1}^{r}\sum_{i=1}^{m} g_{[i:m]j}}{mr}$,

$$g_{[i:m]j} = \frac{x_{[i:m]j}}{z_{(i:m)j}},\ \bar{Z} = \frac{1}{N}\sum_{i=1}^{N} z_i .$$

Now

$$E(\bar{y}_{P(rss)}) = E(\bar{r}_{rss}\bar{g}_{rss})\bar{Z} = \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} + \bar{R}\bar{G}\bar{Z}$$

where $\bar{R} = E(\bar{r}_{rss})$, $\bar{G} = E(\bar{g}_{rss})$ and $S_{rg[i:m]} = \frac{1}{N-1}\sum_{j=1}^{N} g_{[i:m]j}(r_{[i:m]j} - \bar{R})$.

The bias of $\bar{y}_{P(RSS)}$, is given by

$$B(\bar{y}_{P(rss)}) = E(\bar{y}_{P(rss)}) - \bar{Y},$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \left(\bar{Y} - \bar{R}\bar{G}\bar{Z}\right),$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \bar{Y} + \bar{R}\bar{G}\bar{Z} - E(r_{[i:m]}g_{[i:m]})\bar{Z} + E(r_{[i:m]}g_{[i:m]})\bar{Z},$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \bar{Y} + E(r_{[i:m]})E(g_{[i:m]})\bar{Z} - E(r_{[i:m]}g_{[i:m]})\bar{Z} + E(r_{[i:m]}g_{[i:m]})\bar{Z},$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \bar{Y} - \frac{N-1}{N}\bar{Z}S_{rg[i:m]} + E(r_{[i:m]}g_{[i:m]})\bar{Z},$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \frac{N-1}{N}\bar{Z}S_{rg[i:m]} + E(\frac{y_{[i:m]}}{x_{[i:m]}}\frac{x_{[i:m]}}{z_{[i:m]}})E(z_{[i:m]}) - E(\frac{y_{[i:m]}}{z_{[i:m]}}z_{[i:m]}),$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \frac{N-1}{N}\bar{Z}S_{rg[i:m]} + E(h_{[i:m]})E(z_{[i:m]}) - E(h_{[i:m]}z_{[i:m]}),$$

$$= \frac{N-1}{nN}\bar{Z}S_{rg[i:m]} - \frac{N-1}{N}\bar{Z}S_{rg[i:m]} - \frac{N-1}{N}S_{hz[i:m]},$$

or

$$B(\bar{y}_{P(rss)}) = -\frac{N-1}{N}\left(\frac{n-1}{n}\bar{Z}S_{rg[i:m]} + S_{hz[i:m]}\right), \tag{9}$$

where $S_{hz[i:m]} = \frac{1}{N-1}\sum_{j=1}^{N}h_{[i:m]j}(z_{[i:m]j} - \bar{Z})$ and $h_{[i:m]j} = \frac{y_{[i:m]j}}{z_{(i:m)j}}$.

An unbiased estimate of bias given in Eq 9 becomes

$$\hat{B}(\bar{y}_{P(rss)}) = -\frac{N-1}{N}\left(\frac{n-1}{n}\bar{Z}s_{rg[i:m]} + s_{hz[i:m]}\right)$$

where $s_{rg[i:m]} = \frac{1}{n-1}\sum_{j=1}^{n}g_{[i:m]j}(r_{[i:m]j} - \bar{r})$ and $s_{hz[i:m]} = \frac{1}{n-1}\sum_{j=1}^{n}h_{[i:m]j}(z_{[i:m]j} - \bar{z})$.

Thus an unbiased estimator of $\bar{Y}$ is

$$\bar{y}_{P(rss)} = \bar{r}_{rss}\bar{g}_{rss}\bar{Z} + \frac{N-1}{N}\left(\frac{n-1}{n}\bar{Z}s_{rg[i:m]} + s_{hz[i:m]}\right)$$

$$= \bar{r}_{rss}\bar{g}_{rss}\bar{Z} + \frac{n(N-1)}{N(n-1)}\left[\frac{n-1}{n}\left(\bar{h}_{rss} - \bar{r}_{rss}\bar{g}_{rss}\right)\bar{Z} + \left(\bar{y}_{rss} - \bar{z}_{rss}\bar{h}_{rss}\right)\right]. \tag{10}$$

To find the variances of the unbiased estimators, we define the following error terms:

$$\bar{y}_{rss} = \bar{Y}(1+e_0), \bar{x}_{rss} = \bar{X}(1+e_1), \bar{r}_{rss} = \bar{R}(1+e_2),$$

$$\bar{z}_{rss} = \bar{Z}(1+e_3),\ \bar{h}_{rss} = \bar{H}(1+e_4), \bar{g}_{rss} = \bar{G}(1+e_5),$$

such that $E(e_p) = 0,\ (p = 0,1,2,3,4,5)$, and

$$E(e_0^2) = \gamma C_y^2 - W_y^2 = V_{200},\ E(e_1^2) = \gamma C_x^2 - W_x^2 = V_{020},\ E(e_2^2) = \gamma C_r^2 - W_r^2,$$

$$E(e_3^2) = \gamma C_z^2 - W_z^2 = V_{002},\ E(e_4^2) = \gamma C_h^2 - W_h^2,\ E(e_5^2) = \gamma C_g^2 - W_g^2,$$

$$E(e_0e_3) = \gamma C_{yz} - W_{yz},\ E(e_0e_5) = \gamma C_{yg} - W_{yg}, E(e_3e_5) = \gamma C_{zg} - W_{zg},$$

where

$$W_y^2 = \frac{1}{m^2r\bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i:m]}^2, W_x^2 = \frac{1}{m^2r\bar{X}^2}\sum_{i=1}^{m}\tau_{x(i:m)}^2,$$

$$W_r^2 = \frac{1}{m^2 r\bar{R}^2}\sum_{i=1}^{m}\tau_{r[i:m]}^2, W_z^2 = \frac{1}{m^2 r\bar{Z}^2}\sum_{i=1}^{m}\tau_{z(i:m)}^2,$$

$$W_h^2 = \frac{1}{m^2 r\bar{H}^2}\sum_{i=1}^{m}\tau_{h[i:m]}^2, W_g^2 = \frac{1}{m^2 r\bar{G}^2}\sum_{i=1}^{m}\tau_{g[i:m]}^2,$$

$$W_{yz} = \frac{1}{m^2 r\bar{Z}\bar{Y}}\sum_{i=1}^{m}\tau_{yz(i:m)}, W_{yg} = \frac{1}{m^2 r\bar{G}\bar{Y}}\sum_{i=1}^{m}\tau_{yg[i:m]},$$

$$W_{zg} = \frac{1}{m^2 r\bar{Z}\bar{G}}\sum_{i=1}^{m}\tau_{zg(i:m)}, W_{yx} = \frac{1}{m^2 r\bar{Y}\bar{X}}\sum_{i=1}^{m}\tau_{yx(i:m)},$$

and

$$\tau_{y[i:m]} = (\mu_{y([i:m]} - \bar{Y}), \tau_{x(i:m)} = (\mu_{x(i:m)} - \bar{X}), \tau_{r[i:m]} = (\mu_{r([i:m]} - \bar{R}),$$

$$\tau_{z[i:m]} = (\mu_{z([i:m]} - \bar{Z}), \tau_{h[i:m]} = (\mu_{h([i:m]} - \bar{H}), \tau_{g[i:m]} = (\mu_{g([i:m]} - \bar{G}),$$

$$\tau_{yz(i:m)} = (\mu_{y[i:m]} - \bar{Y})(\mu_{z(i:m)} - \bar{Z}), \tau_{yg(i:m)} = (\mu_{y[i:m]} - \bar{Y})(\mu_{g(i:m)} - \bar{G}),$$

$$\tau_{zg(i:m)} = (\mu_{z[i:m]} - \bar{Z})(\mu_{g(i:m)} - \bar{G}), \tau_{yx(i:m)} = (\mu_{y[i:m]} - \bar{Y})(\mu_{x(i:m)} - \bar{X}).$$

Here, $C_{yz} = \rho_{yz}C_yC_z$, $C_{yx} = \rho_{yx}C_yC_x$, $C_{yg} = \rho_{yg}C_yC_g$, $C_{zg} = \rho_{zg}C_zC_g$, where C_y, C_z and C_g are the coefficients of variation of y, z and g respectively. The values of $\mu_{y[i:m]}$, $\mu_{x(i:m)}$, $\mu_{r[i:m]}$, $\mu_{z[i:m]}$, $\mu_{h[i:m]}$, $\mu_{g[i:m]}$ and $\mu_{z(i:m)}$ depend on order statistics from some specific distributions (see Arnold et al. [1]).

In terms of $e's$, we have

$$\bar{y}_{P(rss)} = \bar{R}\bar{G}\bar{Z}(1+e_2)(1+e_5) + \frac{n(N-1)}{N(n-1)}[\frac{n-1}{n}\bar{Z}\bar{H}(1+e_4) - \bar{Z}\bar{R}\bar{G}(1+e_2)(1+e_5)$$

$$+\bar{Y}(1+e_0) - \bar{Z}\bar{H}(1+e_3)(1+e_4)].$$

Under the assumptions $\frac{(N-1)}{N} \cong 1$ and $\frac{(n-1)}{n} \cong 1$, we can write

$$(\bar{y}_{P(rss)} - \bar{Y}) \cong (\bar{Y}e_0 - \bar{Z}\bar{H}e_3).$$

Taking square and then expectation, the variance of $\bar{y}_{P(rss)}$, is given by

$$V(\bar{y}_{P(rss)}) \cong \bar{Y}^2(\gamma C_y^2 - W_y^2) + \bar{Z}^2\bar{H}^2(\gamma C_z^2 - W_z^2) - 2\bar{Y}\bar{Z}\bar{H}(\gamma C_{yz} - W_{yz}). \quad (11)$$

4. EFFICIENCY COMPARISONS

We obtained the conditions under which the proposed unbiased estimator $\bar{y}_{P(rss)}$ is more efficient than the usual RSS mean estimator $\bar{y}_{U(rss)}$, HR type unbiased estimator $\bar{y}_{HR(rss)}$ and ratio-type estimator $\bar{y}_{R(rss)}$.

i. By (2) and (11),

$$V(\bar{y}_{P(rss)}) < V(\bar{y}_{U(rss)}), \quad if \ \frac{\bar{Z}\bar{H}C_z}{2\bar{Y}\rho_{yz}C_y} < 1.$$

ii. By (4) and (11),

$$V(\bar{y}_{P(rss)}) < V(\bar{y}_{HR(rss)}), \quad if \ \frac{\bar{Z}\bar{H}(\bar{Z}\bar{H} - 2\bar{Y}\rho_{yz}\frac{C_y}{C_z})V_{002}}{\bar{R}\bar{X}(\bar{R}\bar{X} - 2\bar{Y}\rho_{yx}\frac{C_y}{C_x})V_{020}} < 1.$$

iii. By (7) and (11),

$$V(\bar{y}_{P(rss)}) < V(\bar{y}_{R(rss)}), \quad if \ \frac{\bar{Z}\bar{H}(\bar{Z}\bar{H} - 2\bar{Y}\rho_{yz}\frac{C_y}{C_z})V_{002}}{\bar{Y}^2 V_{200}\left[\frac{C_y^2}{C_x^2(1-\rho_{xz}^2)}(\rho_{yx} - \rho_{xz})^2\right]} < 1.$$

5. SIMULATION STUDY

To obtain variance of the proposed unbiased estimator, a simulation study is conducted. Ranking is performed on basis of the auxiliary variable X. Trivariate random observations (X, Y, Z), are generated from a trivariate gamma distribution with known population correlation coefficient ρ_{yx}, ρ_{yz} and ρ_{xz}. Using 20,000 simulations, estimates of variance for unbiased ratio estimator are computed under ranked set sampling scheme as described in Section 2. Estimators are compared in terms of variances, percentage relative mean square error ($PRRMSE$) and percentage relative bias (PRB). The values of PRB allow us to analyze the empirical bias of the different estimators, whereas the values of $PRRMSE$ reveal the most efficient estimator from an empirical point of view. Chambers and Dunstan [2], Rao et al. [14], Silva and Skinner [15] and Harms and Duchesne [3] used PRB and $PRRMSE$. The results are presented in Tables 1 and 2. The results showed that with increase in sample size, variances and $PRRMSE$ decrease which is expected results. The expressions of variance, PRB and $PRRMSE$ are defined as follows:

$$V(\bar{y}_{k(rss)}) = \frac{1}{20000}\sum_{i=1}^{20000}(\bar{y}_{k(rss)i} - \bar{Y})^2, \ k = U, HR, R, P.$$

$$PRB(\bar{y}_{k(RSS)}) = \frac{1}{\bar{Y}}\left[\frac{1}{20000}\sum_{i=1}^{20000}\left(\bar{y}_{k(rss)i} - \bar{Y}\right)\right]\times 100,$$

$$PRRMSE_{(k(RSS))} = \frac{1}{\bar{Y}}\left[\frac{1}{20000}\sum_{i=1}^{20000}\left(\bar{y}_{k(rss)i} - \bar{Y}\right)^2\right]^{\frac{1}{2}}\times 100.$$

Table 1. *Variances of different estimators for a simulated trivariate gamma distribution.*

m	r	n	$\bar{y}_{U(rss)}$	$\bar{y}_{HR(rss)}$	$\bar{y}_{R(rss)}$	$\bar{y}_{P(rss)}$
3	3	9	0.159535	0.050708	0.035220	0.005498
	4	12	0.121736	0.038089	0.026410	0.004203
	5	15	0.096354	0.028721	0.022321	0.003237
	10	30	0.047230	0.014200	0.010592	0.001638
4	3	12	0.104722	0.036583	0.027143	0.003980
	5	20	0.059821	0.021148	0.016234	0.002403

	10	40	0.031772	0.010567	0.00791	0.001208
	15	60	0.021210	0.006845	0.005291	0.000772
5	5	25	0.043045	0.016335	0.012887	0.001938
	10	50	0.021152	0.008142	0.006261	0.000974
	15	75	0.014156	0.005491	0.004153	0.000662
	20	100	0.011127	0.004082	0.003110	0.000474

Table 2. *PRRMSE and (PRB) of different estimators for a simulated trivariate gamma distribution.*

m	r	n	$\bar{y}_{U(rss)}$	$\bar{y}_{HR(rss)}$	$\bar{y}_{R(rss)}$	$\bar{y}_{P(rss)}$
3	3	9	20.61 (0.13)	11.62 (-0.22)	9.52 (0.51)	3.82 (-0.02)
	4	12	18.01 (0.28)	10.07 (-0.39)	8.20 (0.18)	3.34 (0.01)
	5	15	16.02 (0.03)	8.74 (-0.13)	7.53 (0.08)	2.93 (0.00)
	10	30	11.22 (-0.17)	6.15 (-0.02)	5.19 (-0.05)	2.08 (0.01)
4	3	12	16.70 (0.23)	9.87 (-0.31)	8.33 (0.20)	3.25 (0.05)
	5	20	12.62 (0.12)	7.50 (0.03)	6.40 (0.20)	2.53 (0.02)
	10	40	9.19 (-0.07)	5.30 (-0.08)	4.53 (0.10)	1.79 (0.07)
	15	60	7.51 (0.01)	4.26 (-0.03)	3.64 (0.05)	1.43 (0.04)
5	5	25	10.71 (-0.07)	6.59 (-0.05)	5.71 (0.14)	2.27 (0.01)
	10	50	7.51 (-0.09)	4.65 (-0.08)	4.01 (-0.02)	1.61 (-0.01)
	15	75	6.14 (-0.02)	3.82 (-0.04)	3.27 (0.03)	1.33 (0.07)
	20	100	5.44 (0.00)	3.29 (0.03)	2.84 (0.00)	1.12 (0.00)

6. CONCLUSION

From Tables 1 and 2, we see that the proposed unbiased estimator $\bar{y}_{P(rss)}$, has less variance and $PRRMSE$ as compared to $\bar{y}_{U(rss)}$, $\bar{y}_{HR(rss)}$ and $\bar{y}_{R(rss)}$. Also, variance and $PRRMSE$ decrease with increase in sample size. So, we conclude that the proposed unbiased estimator is preferable than the usual RSS mean estimator and HR type unbiased estimator and ratio-type estimator $\bar{y}_{R(rss)}$ using two auxiliary variables under RSS scheme. The proposed estimator has reasonable biases, since the values of PRB in Table 2 are all less than 1% in absolute terms.

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CONFLICT OF INTEREST

No conflict of interest was declared by the authors

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