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AN INVESTIGATION INTO EFFECTS OF DYNAMIC GEOMETRY SOFTWARE (DGS) ON STUDENTS' THINKING PREFERENCES: SOLVING GEOMETRY PROBLEMS WITH AND WITHOUT DGS

Didem AKYÜZ
Middle East Technical University

ABSTRACT: Researchers suggest that students have preferences (visual and non-visual) when solving mathematics problems. Many times students have difficulties in solving problems because of one-sided thinking and weakly associating other representations. Reform efforts support connecting visual representations with non-visual representations in order to develop deeper understanding. This study investigates how prospective teachers with different preferences for visual, non-visual, and harmonic thinking solve geometry problems with and without using DGS. The study aims to explore whether students' use of DGS when solving geometry problems is related to their preferences. Suwarsono's mathematical processing instrument (MPI) was administered to determine their preferences for visual and non-visual thinking. Based on MPI instrument's results and their performances of geometry problems solved with and without DGS, three students were selected to be interviewed. Multiple case studies were conducted to conduct a deeper analysis. The reason for selecting three students was to take at least one person from each group based on their thinking preferences so that different cases can be compared and contrasted. The results reveal that regardless of students' preferences preservice teachers preferred to use visual solutions when they are asked to use DGS. When their solutions of DGS and paper-and-pencil were compared, students' solutions with DGS demonstrated more conceptual understanding of the task than paper-and-pencil.

Keywords: Dynamic geometry environment, prospective teachers, geometry.

INTRODUCTION

In the last two decades, the use of technology in mathematics education had a great growth in teaching and learning mathematics. Researchers suggest that technology has an essential role in students' understanding of mathematics with the dragging facilities of dynamic geometry environments (Hollebrands, 2003; Leung, 2008; Hölzl, 2001). There have been many studies that investigated the role of dynamic geometry software (DGS) in understanding geometric constructions, proofs, and measurement (Healy and Hoyles, 2002; Jones, 2000; Mariotti, 2002). These studies showed that dragging is an essential feature of DGS that supports conjecturing, proving, and searching for common properties of geometric shapes.

The effectiveness of using technology depends on the teacher. Teachers have an essential role to increase learning opportunities by using the graphing, visualizing, and computing features of technology. However, many times teachers prefer to use only textbooks and other instructional problems to teach geometry rather than integrating dynamic software into their lessons (Hanna and DeVillers, 2012). One of the reasons of this is that DGS requires teachers to analyze different answers' of students and create different dynamic activities where students can explore mathematical relationships (Healy and Lagrange, 2010).

Studies show that integration of technology into prospective teacher courses enhanced their understanding of mathematical ideas and their instruction (Habre & Grunmeier, 2011). Especially, prospective teachers explored essential advantages of using dynamic geometry in proof problems since they could make conjectures with the help of the software (Christou, 2004; Pandiscio, 2010). Additionally, many prospective teachers stated that using dynamic software helped them gain a broader perspective in solving geometric problems and make geometric conjectures (Pandiscio, 2010). However, the studies also discussed the limitations that might arise from using dynamic software (Habre, 2009). The important point emphasized in these studies is that the properties of the

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*Corresponding author: Didem AKYÜZ- icemstoffice@gmail.com

dynamic software program should match with the aim of the problem and the teacher has an essential role in development of approaches in the solution of the problem by using technology. Thus, the teachers' use of a technological tool is an important component of mathematical learning.

Students' thinking preferences also have an important role in their use of tools. Even though there are some studies that focus on the effect of using technology on students' problem solving strategies (Coskun, 2011; Harskamp, Suhre & Van Streun, 2000; Iranzo-Domenech, 2009; Yerushalmy, 2006), there are not many studies that investigate how students' thinking preferences affect their use of technology.
According to Kruteskii (1976), students thinking preferences can be categorized as analytic (non-visual), geometric (visual), and harmonic (mixed) thinkers. He defined these categories by comparing students' verbal-logical and visual-pictorial components of mathematical abilities. While analytic thinkers generally solve problems using logical reasoning, geometric thinkers utilize mostly visual-pictorial means. Harmonic thinkers, on the other hand, have a relative equilibrium between the two categories. Despite the importance of these thinking preferences, whether they affect the use of dynamic software is an under-explored question.

Koehler and Mishra (2005) define the knowledge the teacher needs to have in order to use technology effectively as the technological pedagogical and content knowledge (TPACK). According to this framework, knowing technology is not enough for teachers to use them effectively. They also need to know how using technology helps students to understand mathematics, how students think about the solutions of problems by using technology, what kind of materials to use in order to integrate technology, and what are the different representations and instructional strategies in learning mathematics with technology. Research shows that prospective teachers as well as secondary mathematics teachers have difficulties changing their practice from direct teaching to guidance where students can explore mathematical relationships using software tool and making deductive reasoning (Christou et al., 2004). They also have difficulties in guiding students to investigate problems without giving answers, reacting "trial and error" methods, and making sense of different unexpected answers (Hahkioniemi and Leppaaho, 2012). Thus, it is important to understand how teachers use technology and provide training to help them overcome their difficulties that might stem from their lack of knowledge (Kokol-Volj, 2007).

In this study, three prospective teachers were taken as cases and their thinking preferences were identified by using Suwarsono (1982)'s mathematical processing instrument (MPI). Then their use of a DGS tool during the solution of several problems were analyzed. In the analysis, the instrumental genesis framework (Trouche, 2004), which defines the interaction between a technological tool and a user, is adopted to understand the nature of their interaction with the tool. The results indicate that regardless of students' preferences, preservice teachers preferred to use visual solutions when they are asked to use DGS. When their solutions of DGS and paper-and-pencil were compared, students' solutions with DGS demonstrated more conceptual understanding of the task than paper-and-pencil.

METHODS

Theoretical Framework

One of the ways to support students' learning of mathematics is the use of tools. While students make use of tools, they facilitate their mathematical activities. Even though these tools are technical, they become internalized and affect the psychology and the learning process of the user (Vygotsky, 1978). In analyzing the interaction between a tool and the user, a commonly used framework is called *instrumental genesis*.
Instrumental genesis is based on the interactions between the tool and the learner. It has two important components: *instrumentation* and *instrumentalization*. While the first one refers to how a tool affects and shapes the thinking of a user, the second one pertains to how the user shapes the tool. According to this framework learners develop conceptual understanding and techniques for using a tool for a specific type of task in time resulting in mutual relationship between the user and the tool (Trouche, 2004). Instrumentation is more related to mental schemes since it is shaped as the user executes the task. On the other side, instrumentalization is a psychological process since organization of the use-schemes is also developed in this process and the user internalizes the use of artifact (Guin & Trouche, 2002).

Another important concept used in the framework is *hotspots* that define the dynamic points in the dynamic software environment. A hotspot is different than a regular point since as the user drags the point the figure is dynamically reconstructed. Thus, a hotspot is the key element between the software environment and the user. As the users move hotspots, they can make conjectures and based on their knowledge they can react the use by moving these points. Thus, hotspots are not owned by users, but they are infrastructural pieces of the environment which give feedback to users. Here it is important to note that when a hotspot is moved, the resulting construction is built by the environment, not by the user, and this provides an important feedback to the user (Hegedus, 2004). In the analysis, the concept of hotspot is also used in interpreting the prospective teachers' interactions with the tool with instrumental genesis.

Data Collection

This study was conducted in a large public university in Ankara, Turkey. The data was collected from the preservice teachers who took an elective course. The aim of this course was teaching students how geometry can be taught using technology and in particular with DGS. Three plane geometry problems were given to 25 prospective teachers who took "exploring geometry with dynamic geometry software (DGS)" course and they were asked to solve the problems with and without DGS. Additionally, Suwarsono's mathematical processing instrument (MPI) was administered to determine their preferences for visual and non-visual thinking. Based on MPI instrument' results and their performances of geometry problems solved with and without DGS, three students were selected to be interviewed. Multiple case studies were conducted to gain a deeper understanding. The reason for selecting three students was to take at least one person from each group based on their thinking preferences (visual, non-visual, harmonic) in order to compare and contrast the cases. The method of selection of these students was purposeful sampling in order to get rich information to answer the research question.

After these three students were selected, the researcher conducted interviews with each of them individually. The interviews lasted about one hour. They were asked to solve six geometry problems during the interview. These problems were selected from Johnston-Wilder and Mason (2005). The selected problems were open-ended with multiple solutions. Students were asked to solve the problems with and without DGS. The choice of which environment to start with was up to the students. In this paper only two questions' data are shared for brevity.

Analysis

Multiple case study was used in this study. In a multiple case study two or more cases are analyzed. Merriam (1998) emphasizes that using more cases in a study results in more compelling interpretation. In this study, the three cases were selected based on their different thinking preferences. The first case was identified as a non-visual thinker, the second case was identified as a visual thinker, and the third one was identified as a harmonic thinker according to the mathematical processing instrument (MPI) developed by Suwarsono (1982).
Suwarsono's MPI test includes two parts with 15 items in each part. The first part contains the questions that the student is required to solve on the paper. The second part contains the possible solution strategies for each question in the first part. Here, the student is expected to choose the solution strategy that he or she followed in the first part. The entire process is repeated for a new set of 15 questions to ensure consistency. MPI included 30 word problems. For each visual solution, a score of 1 was given and for each nonvisual solution a score of 0 was given. Students, whose scores were lower than 15, were considered as nonvisual thinkers. Students, whose scores were higher than 15,were considered as visual thinkers. Students who scored around the cutoff point 15 were considered as harmonic thinkers. S1 scored 10, S2 scored 15 and S3 out of 24 on the MPI. Based on these answers, the thinking preferences of the students were classified as non-visual, harmonic, and visual.

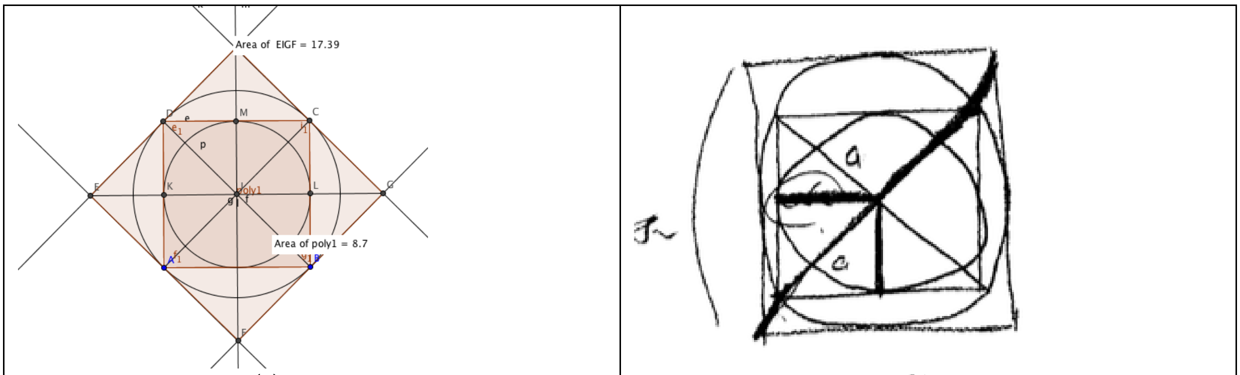
In addition to this instrument, students solutions of problems with and without DGS during the classroom were used to cross-check their thinking preferences. This data was found to be congruent with the MPI results. During the interview the students were asked six questions, two of which are discussed here for brevity. One of the questions was Q1: "Draw a circle with one square inside and one square outside. Calculate the ratio of two squares. Draw a circle inside the small square. Find the ratio of two circles". The other question was Q2: "Suppose you have a chord AB of a circle. Find the triangle inscribed with AB as a side, which has the largest area". Use this result to solve: In a circle with Radius 1, what inscribed triangle has the largest area?"

RESULTS AND FINDINGS

Case One: S1's solution strategies (non-visual)

S1 was found to be the highest scoring non-visual thinker according to Suwarsono's instrument as well as the analysis of problems solved in the classroom. During her solution of Q1, she first tried to construct the file with Geogebra (GGB) without solving it on the paper.

I: How would you solve this problem?
S1: I would use GGB to see the solution. First, I will construct the problem and then I will try to find out the answer (she creates the construction in Figure (1a)).



(a) (b)

Figure 1: S1's Solution Of First Problem With And Without GGB

S1: Without using the area tool of GGB, I can say the answer.
I: How did you see the relationship?
S1: It is half of it.
I: Why?
S1: Because, I look at the big square. It includes four small triangles. And then I look at the inside square. It includes four triangles. If we combine two of the triangles, it makes one small square. So inside square includes two small squares. Outside the square included four small squares (She moves the hotpoint and states that the result does not change).

She checks the result with the area tool of GGB. Next, she was asked to solve this problem on the paper. She did the similar method on the paper. She stated that she would create small triangles and then combine them to make a square. Additionally, she stated that she would use similarity. However, she had difficulty in drawing the problem on the paper. She stated that if she said *a* to half of the diagonal of the inside square, then one side of the square would be $a\sqrt{2}$ and one side of the outside square would be $2a$. The ratio would be 2.
However, even though she assigned a variable to the side of squares, she did this using the GGB file. She stated that she could not see any relationship on the paper since her shape did not look right. Thus, even in her algebraic solution she used the shape on the GGB.
I: What about the second part of the question?
S1: The half of the outside square's side is equal to the radius of outside circle and the half of the inside square's side is equal to the radius of inside square. Since the side's ratio of circles were 2, the ratio of radius's would be 2 and the ratio of areas would be 4 because of the area formula which is $pi*r^2$*. However, I could not tell this by looking at my drawing on the paper. I am lucky that I can use Geogebra. Otherwise it is important to be able to draw the shape beautifully in order to answer these questions.*

Even though, she was asked to solve by using paper-pencil, she stated that from her drawing she could not see the relationship. Thus it would be more useful for her to look at the GGB file while she was trying to explain the relationship. She tried to do the solution on the paper but because of difficulty in drawing, she did not use it in her explanation (see Figure 1(b)).
In the solution of this problem, S1 found the ratio of two squares by using the dynamic features of DGS. She also stated that it was very useful for her to create the activity itself since they need to know what constitutes the center of the inscribed circle as well as the circumscribed circle. During the creation of the activity she had to use bisectors and perpendicular lines. Construction of the shapes helped her to understand mathematical relationship between the circle and the square. Using hotspots was essential to generalize the problem and make conjectures. As the user executed the tasks on DGS, the environment gave feedback to reflect on. This was essential to support instrumental genesis as the co-action emerged and shaped the sustainable bi-directional process that support developing mathematical concepts.
Next she was asked about the second question, Q2:
S1: (She starts with GGB, draws a circle and then a triangle inside of it). I think the area would not change. Would it? But, it changes. Perhaps when the midpoint of AB passes through the center...Mmm..I do not know.
I: Try to think about it.
S1: I think, it would be the greatest area when it is equilateral triangle (she uses hotspots on Figure 2(a)).
I: Why?
S1: When three sides are equal than it looks like the area becomes the biggest (she finds this by using the area tool of GGB, area of BDC in Figure 2(a)). But I do not know why.
I: Do you think the triangle you show is equilateral?
S1: (She checks the length of sides by using GGB). Yes. And then she draws the same Radius circle (5 cm) and a equilateral triangle inside of it by using rotation (Figure 2(b)). I think the area of circle is two times bigger than the area of triangle (she checks with area tool). No it is not like that.

I: Why do you think it is equilateral triangle?
S1: I do not know. But I can try it on other circle. I will create a slider. I can rotate the points by γ angle and then create the triangle. I can find the area of triangle. I can see that when slider is 120 degrees it has the maximum area. But I do not know why it works.

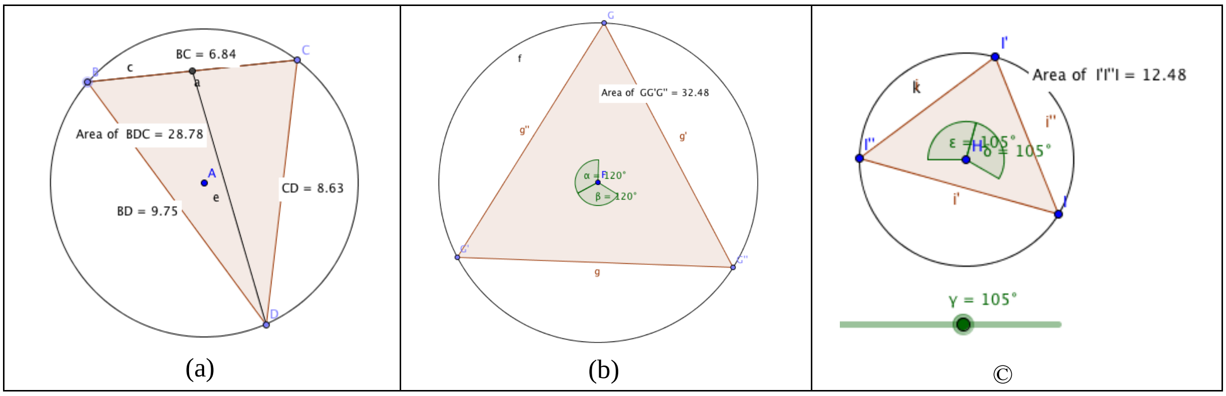


Figure 2: S1's Second Problem's Solution

S1: Angles would be the same. The side of the triangle is a and a is the chord. So we need to find the maximum value of multiplication of side and height. Actually, I first thought when the base is diameter it should be maximum. However, the result did not come out like that when I checked it with GGB. When we make it equilateral the base decreases but height increases. If r is equal to 2a√3 then the height is 4a (she writes on the paper). When it is 90 degrees the side is 2a and the height is 4a.

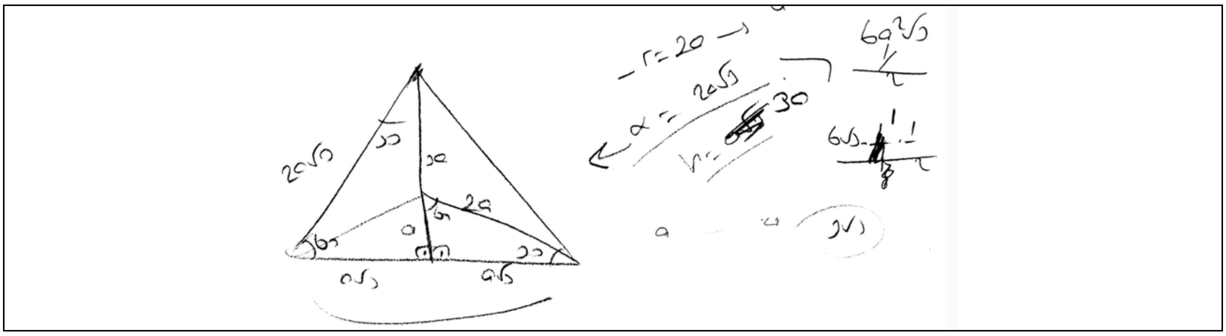


Figure 3. S1' S Solution Of Second Problem On The Paper

Since S1 was very comfortable using the tool, the features of the tool allowed her to find different solutions than the paper-pencil methods. The instrumentation level was very high as the tool had important role shaping her solution strategies. Especially, for the first solution, she used the dynamic feature of the software program and without that she might not have been able to make the conclusion that the ratio of the outer square to the inner one is 2. In the second question, she also found the equilateral triangle by using the hotspot on the software. By using the dynamic feature of the software she calculated both areas and generalized it by dragging the hotpoint. In her second solution, she used different properties of the tool such as making rotation of the given shapes by creating sliders. It can be seen from the solution strategies that instrumentation and instrumentalization was intertwined. For example, in the second solution she could find the strategy of using rotation thanks to the tool since it is easy to make transformations with the DGS. Her mathematical knowledge was also essential to find this solution strategy by using the tool. In her last solution she used the hotspot in order to make a conjecture. She preferred to use sliders instead of dragging from the hotspot; both of which allow dynamic variation. Here it is important to note that even though she moved the sliders to change the figure dynamically, the resulting construction was developed automatically by the environment. As Hegedus (2005) emphasizes, "the artificial realities of the diagram obey the rules of geometry that are preserved in the elements of diagram". This feature of the tool is critical in supporting both instrumentation and instrumentalization.

Case Two: S2's solution strategies (harmonic)

When S2 was asked about the first question, before trying the solution on the paper, she opened Geogebra and first created the problem on it. However she did not use the file created on the GGB in order to find the answer. Instead, she used a paper solution to give the answer.
S2: If I say √2 to half of the diagonal, the diagonal is equal to 2√2. And one side would be 2. Then outside square area would be 4. The ratio would be 2 (Figure 3(b)). We can use GGB to check our result. We can

compare poly1 and poly2 and see that the ratio is 2. I use numbers since it is ratio, it does not matter to use variable.
I: How would you use GGB to solve this problem?
S2: I would use in the same way. I can create a segment between center and midpoint of the outside square's side. I would say r to that segment. I would find out the lengths by using the GGB tool. I can divide the big square's side to the small square's and find out the ratio (1/2). Even though I move the square dynamically, the ratio would not change (Figure 3(a)). I can also see the ratios from the triangle.

After this she was asked to solve the second part of the question.
S2: They would be also 2. One of them would be pi($r\sqrt{2}$)^2 and the other would be pi*r^2 (Figure 3(b)). And then I would use the GGB to check the result.*

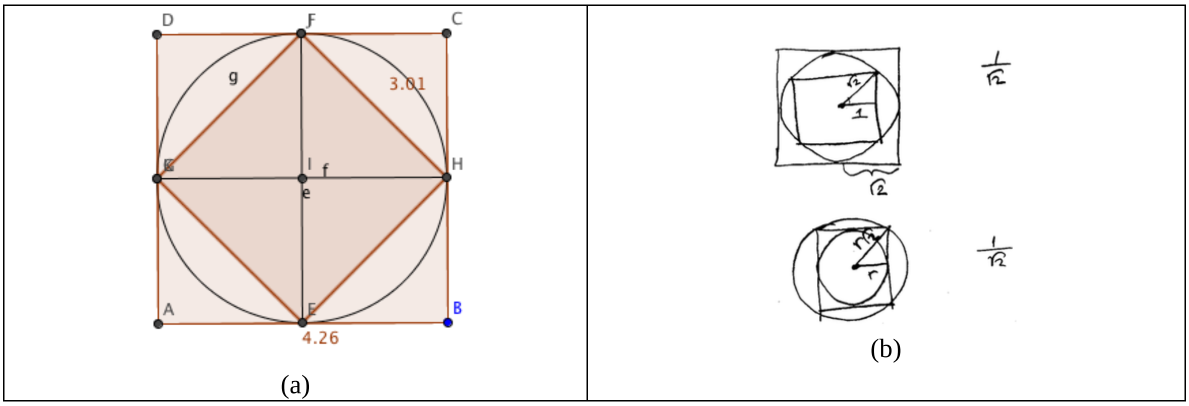


Figure 3. **S2'solution Of First Question With And Without DGS**

S2 used the dynamic software as a checking tool in the first question. First, she created the figure of a square, she found the length of inside and outside squares. Next, she calculated the ratio of inside square's side to the outside square's side. The tool helped her to construct the shape precisely and show the lengths in the question. She checked the area of polygons from the algebra view. Next, she made an explanation based on the triangles inside the squares. She moved the hotpoint and showed the results do not change even though squares size change. Her instrumentalization level was high. She could use the tool effectively. Once she finished solving the first problem, she was asked to solve the second one. In this question she started to solve the problem by using paper and pencil.
S2: AB chord is constant. I am trying to find out the maximum height. I need to draw a perpendicular line that passes through the center. The intersection of the line and circle would be my third point. This triangle would be an isosceles triangle (Figure 4(a)).
I: What about the second part of the problem?
S2: Before I start I thought regular shapes cover more area. But how can I show it on paper? I can use derivative.
S2: If I say 1 to the radius, then this length would be $\sqrt{1-x^2}$. And the height would be 1+x. I want the area to be maximum. So I would find out the derivative of this function and make it equal to zero . There are two numbers that make it zero but since I want the length, I take the positive one (Figure 4(b)). I got the x value as ½. Now, I need to find out what kind of triangle is when x =1/2. I will find the arctan which is $\sqrt{3}$. That angle is 60^0. Since it is isosceles triangle, it would be equilateral triangle.

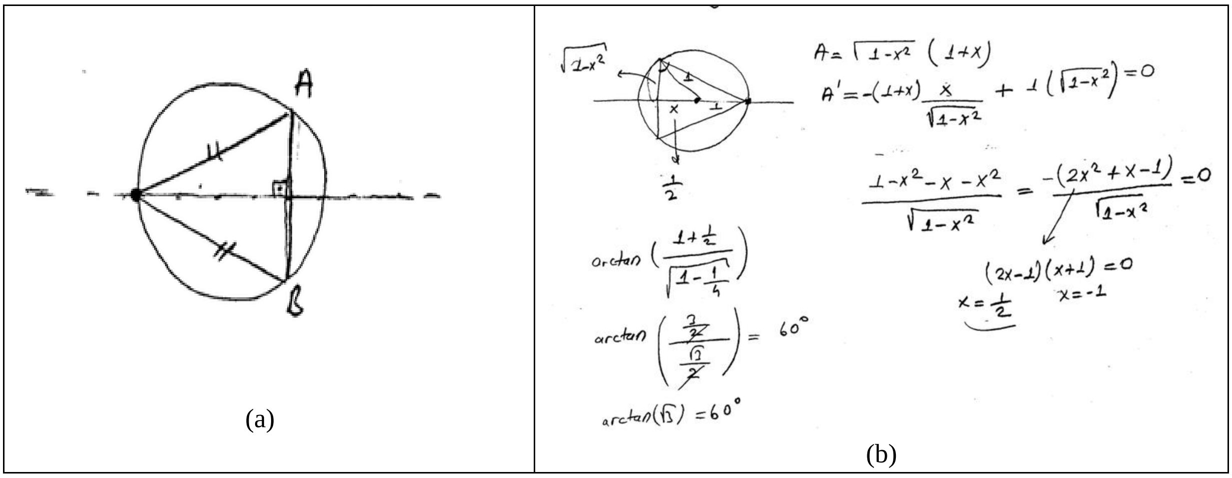


Figure 4. S2's Solution Of Second Problem On The Paper

In this question, she preferred to start from the paper solution similar to the previous one. She could solve the problem mathematically on the paper by using her content knowledge. After this solution, she was asked to solve it on GGB and she did the solution below:

I: How would you solve it with GGB?
S2: I can draw a unit circle on the axis where the center is origin. I take the point C and draw a perpendicular line from the point c. Find out the intersection of it with circle, which is D. I also take the E point, which is the height of the triangle. Next, I can find the area of the triangle, which is poly1. Then I took the point F. The coordinate of F can be (x(C), poly1). This way I can see when the x of point C gives the maximum area (Figure 5(a)). When I turn on trace on tool, I can see that F is maximum when the length are equal.
I: How can you show it on GGB?

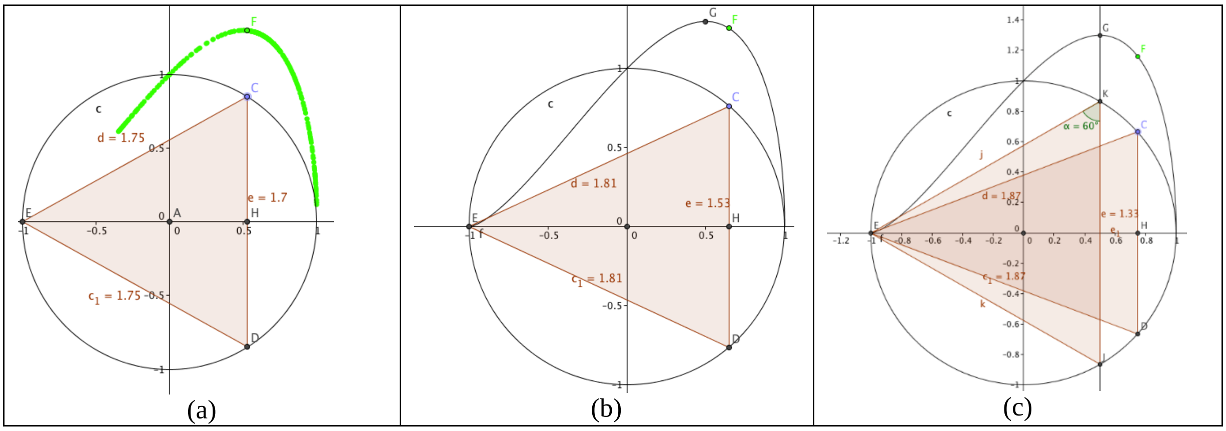


Figure 5. S2's Solution Of Second Problem With DGS

S2: I can draw the function which is $(x+1)\sqrt{1-x^2}$, since we used this function in order to find the critical points (Figure 5(b)). And then I can find the maximum point of f(x) function which is point G. Actually F is also the maximum when we move it on G.
I: How can you show the triangle is equilateral when the point is on G?
S2: I can draw a perpendicular from G to circle and get the intersection points (K and J). If I create the triangle KEJ, that triangle is equilateral. I can show each angle is 60^0 (Figure 5(c)).

From her solution, we can see that she could visualize the solution. In this question, it was more obvious that she could use the tool effectively. Thus, the type of questions were important to understand instrumental genesis. By using GGB, she could prove why the equilateral triangle has the largest area. Her GGB showed that her instrumentalization was very high. She is a harmonic thinker and similar to other students she preferred to use Geogebra to visualize the problem. Her Geogebra solution was more conceptual than her paper solution. Here it is important to say that her content knowledge was also very high. This helped her to use the tool effectively.

Case 3: S3's solution strategies (visual)

When S3 was asked by the first question she did it very similar to S1. She first drew the squares and then she created triangles inside of the square.

S3: There are four squares inside of the outside square and there are two small squares inside of the square. So the ratio is 2.
I: How would you solve it on the paper?
S3: I would give variables. One side of the outside square is 2a and one side of the inside square would be $a\sqrt{2}$. So the ratio would be 2.
I: What about the circles if we compare?
S3. (She draws on GGB and finds the area of circles by using area tool). I think there is the same ratio between the radius' of two circles. We can calculate it on the paper. The outside circle's radius is b and inside circle's radius is $\sqrt{2}b$. So the ratio would be 2.

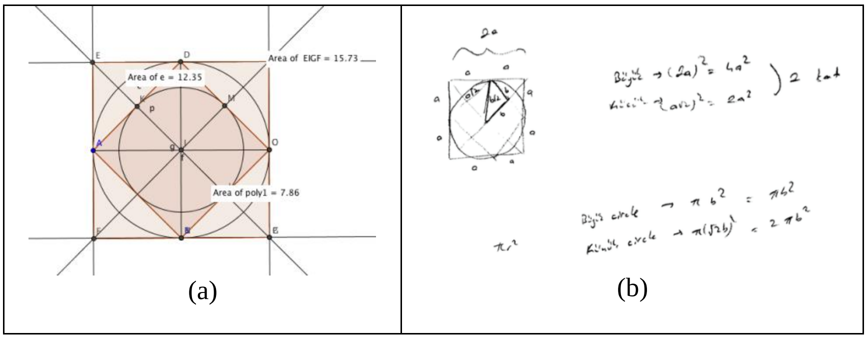


Figure 5. S3's Solution Of First Problem With And Without DGS

As it might be seen from the solutions of S3, she used the dynamic software environment in order to create dynamic constructions. In her solution strategies, by dragging hotspots on the object she had a chance to test several iterations of the geometrical constructions (Figure 5(a)). The figures above show the dynamism of the figures with different values. During her solution, she dragged the blue points which are hotspots in order to enlarge or minimize the figures to illustrate the mathematical ratio is conserved for many points. Dragging hotspots illustrated Euclidean construction of squares could be created in DGS environment and the relationship between interior and outer squares can be observed. Using hotspots were essential to give feedback to users as S3 moved those points they inquired the mathematical reasoning behind the construction that conserve the geometric relationship. Next, she was asked to solve the second question. She drew a circle and then created the chord BC.

S3: In order to have the largest triangle I need the maximum height. I will find the perpendicular line to the chord. Then I will take the intersection of the line and the circle to find the vertex of the triangle. Then I can create the triangle. This triangle has the maximum area. I can check it with GGB (up to this part it was similar to S2's solution). I can take a random point on the circle and create a triangle. Then compare the area of the first and second triangle (she moved point F in order to test (Figure 6(a)).
I: Is this a special triangle?
S3: It is an isosceles triangle since I drew the height from the midpoint of the chord. I can also see that the sides are equal by using the length tool.

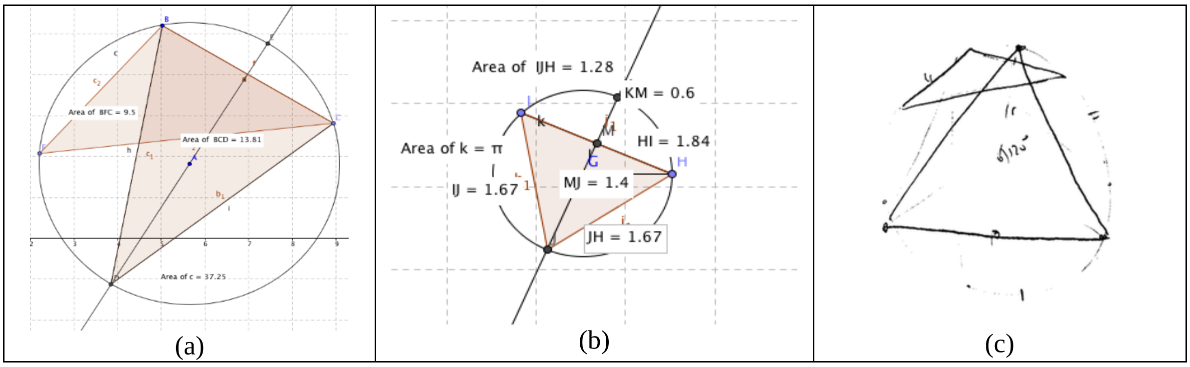


Figure 6. S3's Solution Of Second Problem With And Without DGS

I: What about the second part of the question?
S3: (She draws a circle which has radius 1 and tries to find the triangle that has the maximum area). If I look at the area of circle it is 3,14. This question is totally different from the other one. Here we can change the base. (She moves the triangle dynamically and sees that it has maximum area around the 1.3 value.) There might be a ratio between KM and MJ. I need to think again. I can find out the sides. All sides are equal.
I: Why?
S3: The height and base's values are closest when the triangle is equilateral. But I am not sure why it works. This one is also equilateral, but I cannot relate it with the other question. Because in the other question it asks me to take a chord and do not ask to find a maximum area in a given random triangle.
I: Okay, how can you explain it?
S3: Logically, when I change these points, the side becomes bigger but height gets smaller. But when they are equal we spread the circle equally. I am trying to explain mathematically, but I am not sure how to do this.

She continued by drawing triangles on the paper and stated that the triangle becomes bigger when we increase the length of the side but from one point, it starts to increase since the height becomes smaller (Figure 6(c)). For this problem, the instrumentation level was very high as the tool had an important role shaping her solution strategies. However, here it is important to note that it is difficult to separate instrumentation and instrumentalization since both affect each other continuously. These two concepts are interwoven and interdependent as was stated before. For example, in the second solution, S3 knew that the height of the triangle is passing through the midpoint of the BC chord. Without this knowledge, she would not have been able to create the correct construction given in the problem. So her knowledge allowed her to shape the use of the tool. Once she created the triangle that she thought has the largest area, she created another triangle and compared the area of the triangle with the one she created first by using hotspots.

CONCLUSION AND RECOMMENDATION

Many researchers emphasize the importance of using tools in teaching and learning mathematics (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Lagrange et al., 2003). However, they also state teachers have an essential role in using tools in order to help students understand mathematical concepts (Hoyles and Lagrange, 2008).

This study shows how the use of the same tool can change from one prospective teacher to another. The first and third cases, S1 and S3 (non-visual and visual thinker), used the dynamic software program as a tool to solve the problem. They constructed geometrical solution strategies by using the DGS environment. They used the hotspot effectively in making conjecture and generalization. Using hotspots was critical in supporting instrumentation as they received feedback from the tool while using the DGS. The use of the tool helped them to shape their mathematical knowledge and with the feedback taken from the tool, they developed new mathematical ideas and applied them again by using the tool. The interaction between the instrumentation and instrumentalization was highly effective in their solutions.

The second student (harmonic thinker) also used many dynamic features of the program in the second question; while she preferred to use the program to show the results that she did on paper for the second question. One of the reasons that she didn’t use the program effectively in the first question might be related to the type of the problem. Even though, the problem could be solved more dynamically with DGS, it was possible to find a solution without using the software program. Thus, one of the important points in teaching with technology is the selection of suitable tasks. Another important point that is related to the instrumental genesis was using the technology in different solution strategies of the problem. It is important to facilitate instrumental genesis by making connections between DGS and the paper-pencil environment (Bretscher, 2009). When we compare the solution of the first question, using the triangles to compare the area of inside and outside squares was more conceptual than just using formulas. Especially for the second question, two students (S1 and S3) could not even interpret what question means without DGS. By using DGS they made conjecture about the type of triangle. Even though S2’s solution on the paper was non-visual, she preferred to use visual method with DGS. She used different representations during the solution of the second problem. She wrote the function for which she wanted to find the critical points. Next, she drew this function and took a point on it. Also she created the triangle. Thus, her solution included three types of representation, algebraic, geometric, and numeric. This also revealed that the use of DGS facilitated students’ use of representations (Huntley et al., 2000). Thus, the use of DGS might be important in supporting students’ conceptual understanding.

In other words, students tended to solve the problems algebraically on paper, while they solved the problems geometrically on DGS environment. Regardless of students’ thinking preferences, they used more visual methods when they solved problems by using the DGS environment. This result is also compatible with Harskamp et al. (2000)’s study. According to this study, the tools of Geogebra, which help to create lines and graphs automatically are important factors to have these results. Similarly Yerushalmy (2006) also stated the differences between the solution of the problem based on the environment. DGS environment helped students to make conjectures and check their answers. However, as it was seen in the second question, even though three students guessed the answer of the problem from the tool, they could not explain the answer mathematically. This shows the importance of content knowledge in supporting instrumental genesis. These results are consistent with Drijvers and Gravemeijer (2005), as they also emphasize the importance of interrelation between the tool and students’ conceptual background in instrumental genesis.
Even though there have been some studies that compare the different strategies of students on DGS who have different preferences (Yerushalmy, 2006; Coskun, 2011), the importance of thinking preferences on supporting instrumental genesis has not been investigated up to know.

Since the use of DGS support using different type of representations, effective use of DGS might contribute to an in depth understanding of mathematics (de Jong & van Joolingen, 1998). In order to support instrumental genesis, teachers can determine students’ thinking preferences from how they use the virtual manipulatives and how they use them to solve problems. Once they identify students’ difficulties in their use of visual or non-visual representations, they can focus on the problems where they can connect visual representations with non-visual aspect of the problem. They can use DGS to solve the problem with different representations. This might support instrumental genesis. This way, regardless of students' thinking preferences, students can use DGS effectively.

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