The Eurasia Proceedings of
Educational & Social Sciences (EPESS)
ISSN: 2587-1730

The Eurasia Proceedings of Educational & Social Sciences (EPESS), 2014

Volume 1, Pages 124-134

ICEMST 2014: International Conference on Education in Mathematics, Science & Technology

NON-MATHEMATICS STUDENTS' REASONING IN NON-ROUTINE CALCULUS TASKS

Ljerka Jukić MATIĆ
Department of Mathematics
University of Osijek

ABSTRACT: This paper investigates reasoning of first year non-mathematics students in non-routine calculus tasks. The students in this study were accustomed to imitative reasoning during their schooling in primary and secondary education. In order to move from imitative reasoning toward more creative reasoning, the non-routine tasks were implemented as the part of the calculus course. Using qualitative strategy in a form of interview, we examined reasoning of six students in the middle of the calculus course and at the end of the course. Analyzed data showed that creative reasoning develops slowly even when students are exposed to the non-routine tasks. Also, we have found several negative met-befores and met-afters affecting students' knowledge and interfering with the reasoning process.

Keywords: Calculus, reasoning, non-mathematics students

INTRODUCTION

Society today highlights mathematical literacy as an important educational goal. Mathematical literacy means that one should possess certain mathematical competencies. Niss (2003) distinguishes eight, distinct and clearly recognizable competencies: thinking mathematically, posing and solving mathematical problems, modelling mathematically, reasoning mathematically, representing mathematical entities, handling mathematical symbols and formalisms, communicating in, with and about mathematics and making use of aids and tools. These mathematical competencies should be developed not only through primary and secondary education, but in tertiary education as well. This is surely in line with demands of industries and businesses worldwide; mathematics provides a powerful tool to understand, to investigate and to make predictions in the solution of a wide range of problems, therefore it is important to produce employees who are both mathematically capable and trained (Chinnappan et al., 2009). But, we can ask ourselves what competencies students are developing in tertiary education when they are given the same types of tasks in the exams as they have met in the mathematics courses. When it comes to exam requirements, students actually do expect this kind of situation to happen. Their expectations are anchored in previous accessible exams copies, and this is usually part of didactical contract between students and lectures (Lithner, 2010). The reasoning which students employ in such situations Lithner (2003) calls imitative reasoning. This kind of reasoning is founded on copying task solutions, for example by looking at a textbook example or remembering an algorithm. Examining final exams from the introductory calculus courses at Swedish universities, Bergqvist (2007) found that most of the tasks can be solved using imitative reasoning. Investigating teachers' view on the reasoning requirements in the calculus exams, Bergqvist (2012) found that teachers were concerned with the exam passing rate, therefore majority demanded imitative reasoning. The situation is no different in Croatian universities as well. Even though there are no published studies, browsing exams and course material accessible on the web pages of different Croatian universities, one can reach the similar conclusion, especially when it comes to calculus courses for non-mathematics students.

Lithner (2008) points out that even after many years of research, students still do inefficient rote thinking and rely on imitative reasoning. Guided with the aforementioned concerns, we examined reasoning of several non-mathematics students who have been exposed to non-routine calculus tasks. Our goal was to see how the non-routine tasks affected students' reasoning.

THEORETICAL FRAMEWORK

- Selection and peer-review under responsibility of the Organizing Committee of the conference

*Corresponding author: Ljerka Jukić MATIĆ- ljukic@mathos.hr

In the mathematics education literature, there can be found many definitions describing the term reasoning. Lithner (2008) defined reasoning as the line of thought that is adopted to produce assertions and to reach conclusions when solving tasks. It is not necessarily based on formal logic, nor restricted to proof. It may even be incorrect as long as there are some sensible reasons (to the reasoner) supporting it and can be seen as thinking process, as the product of these processes, or as both. It can have characteristics of high and low quality. In this paper we adopt aforementioned definition of reasoning.

Lithner (2008) distinguishes two types of mathematical reasoning: imitative and creative mathematically founded reasoning. Everything that includes rote learning reasoning is in fact imitative reasoning, and the opposite reasoning is creative reasoning. Creative mathematically founded reasoning fulfills following criteria: novelty, plausibility, flexibility and mathematical foundation. *Novelty* includes new reasoning sequence that is created or recreated if forgotten. *Plausibility* can be described as using arguments to support the strategy choice and/or strategy implementation. *Flexibility* admits different approaches and adaptions to the situation and it does not suffer from fixation that hinders the progress. *Mathematical foundation* means the argumentation is founded on intrinsic mathematical properties of the component involved in the reasoning. Creative reasoning does not have to be a challenge as problem solving but conceptual understanding is deeply anchored in it, unlike in imitative reasoning.

Imitative reasoning consists of several different types of superficial reasoning. *Memorized Reasoning (MR)* is when the strategy choice is founded on recalling an answer and the strategy implementation consists of writing this answer down with no further consideration. *Algorithmic Reasoning (AR)* is implemented when the strategy choice involves recalling a certain algorithm (set of rules) for solving the given problem. The strategy implementation is trivial, straightforward once the rules are recalled. AR has several variants: familiar algorithmic reasoning, delimiting algorithmic reasoning and guided algorithmic reasoning. The strategy choice in *Familiar AR* is founded on recognizing the task as being familiar, which can be solved by a corresponding known algorithm. In *Delimiting AR*, an algorithm is chosen from a set that is delimited through surface relation with the task. Following the algorithm carries out the implementation of the strategy. If this implementation does not produce desired outcome, the algorithm is abandoned and a new one is chosen. In *Guided* AR, the reasoning is mainly guided by two types of sources that are external to the task. In person-guided AR, a teacher pilots the student's solution. In text-guided AR, the strategy choice is founded on identifying, in the task to be solved, similar surface properties to those in a text source (e.g., a textbook).

The main problem with algorithmic reasoning is not that students do not learn creative reasoning, but that they do not develop the conceptual knowledge necessary for learning different aspects of mathematics (Lithner, 2004). Cox (1994) argued that many first-year university students obtain good grades by concentrating on routine topics, instead of aiming at deep understanding of fundamental topics. Schoenfeld (1991) pointed out that while many teachers are trying to reduce complexity of mathematical concepts and processes, students are trying to cope with curriculum goals so they often use quicker short-cut strategies to learning and passing exams. Lithner (2008) claimed that the most frequent type of reduction of complexity is focused on algorithmic procedures that can solve advanced task and not using conceptual understanding or creative reasoning at all. He supported Tall's (1997) conclusion on "vicious circle" of procedural learning and teaching where, for example, calculus lecturers focus on the differentiation and integration on symbolic level and ask similar question in examinations.

Met-befores and met-afters

A met-before is a mental construct that person uses at a given time based on prior experiences (Tall, 2006). Using met-befores can sometimes be an advantage when person is learning a new mathematical concept and sometimes it can be an obstacle which causes severe difficulties. Hence, met-befores affect learning of new concepts, but new mathematical concepts may also affect older knowledge. Such mental constructs are called *met-afters* (Lima & Tall, 2008). Met-afters are those experiences met at the later time that affect the retention of old knowledge. Met-afters can also be both positive and negative, and the negative effect of some met-after shows fragility or inconsistency of the earlier learned knowledge.

New knowledge that builds on previous knowledge is much better remembered, but concepts that do not fit into earlier experience are learned temporarily and easily forgotten or not learned at all. According to McGowen and Tall (2010), this can be observed when student, for instance, is interested only in algorithmic reasoning, relying on well-established procedures or algorithms. If there is no conceptual meaning, this kind of knowledge is stored improperly and is very fragile when person tries to adapt it to new situation. This previous knowledge makes it difficult to understand new subject matter, since the student is trying to distinguish among accessible rules and is trying to imbed new knowledge into his fragmented knowledge structure.

Setting up the scene

The tasks that are given to students within some mathematics course can be categorized as routine and non-routine tasks. The main difference between the non-routine and routine task is that in the former the solver has to, at least partially, construct his/her solution method, while in the routine task, the method is already known by the solver or provided by an external source such as the book or the teacher (Lithner, 2012). To be able to determine if a task is routine or not, it is insufficient to consider properties of the task alone, but the relation between the task and the solver has to be considered (Schoenfeld, 1985).
In order to move from aforementioned "vicious circle", where university calculus courses promote procedural knowledge and imitative reasoning, the non-routine tasks were implemented in a calculus course given to one group of civil engineering students at one university in Croatia. Selden et al. (1998) suggested that the non-routine tasks should be implemented as the explicit part of the curriculum in traditional calculus courses, not at the end, but throughout the course in the exercises sessions or in the homework. This group of civil engineering students obtained two or three non-routine tasks for homework in each exercise session. The homework should have been handed over to the teaching assistant of the course. The homework tasks were solved sometimes in the next exercise session, and sometimes the solution was only commented. The routine calculus tasks, which were given to the students in the course, asked for the application of some procedure that was shown to them either by lecturer or teaching assistant, while the non-routine tasks asked for more conceptual understanding. For instance, the routine tasks, that students solved in exercise session, asked for evaluation of function limit at some point e.g. "Find $\lim_{x\to 1}\frac{1-x^3}{1-x}$", while the corresponding non-routine task given for a homework had the following form "Is there an a such that $\lim_{x\to 3}\frac{x^2+x-ax-a+4}{x^2-2x-3}$ exists? Explain your answer."

Since Hiebert (2003) argues that students learn when they are given an opportunity to learn, we wanted to see if, and how, the non-routine tasks influenced on students' reasoning.

METHODOLOGY

Participants

This study was conducted at one university in Croatia, and participants were the first year students who belonged to the civil engineering study program. The participants were chosen according to the scores obtained in the calculus mid-term exam. We have chosen students who scored between 65% and 70% in the exam, and we have classified them as average students, i.e. students who possessed some knowledge, were far from failing the exam, but also were not close to excellent scores. Their scores represent the most common results in the calculus exam among civil engineering students. In order to pass the calculus course, the students in this study program have to pass both written exam and oral exam. Moreover, the written exam consists of the mid-term exam and the final exam. In the written exam students have to solve various tasks, while the oral exam puts emphasis on the mathematical theory. Participating in the study was voluntary and students had right to withdraw from the study at any time, therefore we believe that students invested a significant effort into solving given tasks. The students in this study will be named only with the capital letters of their names in order to assure their privacy.

Method

The empirical data was collected from six task-based interviews. The students were interviewed in pairs. Arksey & Knight (1999) argue that this method is better for establishing an atmosphere of confidence with two students being interviewed at the same time and because the interviewees may 'fill in gaps' for each other. Also, students' interaction may also be of interest. Schoenfeld (1985) gives support for this kind of interview, stating that two-person protocols often provide better insight and information about students' reasoning and knowledge.

In order to estimate if there were any changes in students' reasoning, students were interviewed in two occasions: in the middle of the course, just after the mid-term exam, and after passing the course. In the interview, the participants were given specific non-routine tasks designed in collaboration with lecturer and teaching assistant of the course. At the start of the interview, participants were given directions how to behave

when solving given tasks. They were asked to talk to each other when solving the task, to state out loud everything they are thinking at the moment, not to plan what to say, and to behave like they are alone in the room working together on their homework or any other assignment. Similar directions were recommended by Ericsson & Simon (1993) in order to initiate the student's thinking out loud.
In both occasions, the interview was separated into two parts. In the first minutes, the author only reminded the participants to keep talking if they were silent for a while. If the students struggled with the given tasks more than several minutes, the author asked direct questions, trying to get the students to explain what they were doing and why they were doing it.

The interviews were video-taped, transcribed and analyzed together with the students' written work. We videotaped students work because we wanted to see their gestures, type of interaction between the students and their tone of voice. We believe this plays an important part in the student's behavior and can say something about the student's mathematical reasoning. However, in this paper we will focus more on the verbal expressions.

Tasks

Before the interviews took place, we examined participants' results and solutions in the mid-term exam and in the final exam where they had to solve several routine tasks from differential calculus. This was necessary to determine whether the students had the requisite calculus knowledge to solve the non-routine tasks.
The routine tasks in the-mid-term asked students to:

- calculate the limit of the function at the point,
- examine whether or not the function is continuous at the point
- determine extreme values for the given function

The routine task in the final exam asked students to investigate all properties (domain, zero points, continuity, asymptotes, extremes, intervals of decrease and increase, the point of inflection, intervals of concavity) of certain function given with algebraic expression and to draw the graph of the function accordingly.
In the interview, students were given the non-routine task with concepts that emerged in the routine tasks in the exercise sessions and in the exams. However, there were no concrete algebraic expression for the functions in the tasks, wherefore those tasks asked for flexibility and creative reasoning.
Following tasks were given to students:

1. It is given function $f: R \rightarrow$R. Let $f'(x_0) = 0$ for only one point x_0. Also let $x_0 > 0$ and $f(x_0) < 0$. If $\lim_{x\to+\infty} f(x) = \lim_{x\to-\infty} f(x) = +\infty$, how many zero points does function *f* have?
2. Sketch the graph of a function *f* which satisfies the following conditions:

a. f is discontinuous at $x = 0$ and $f(0) = 1$;
b. $f''(x) < 0$ for all $x < 0$ and $f''(x) > 0$ for all $x > 0$;
c. $f'(-1) = 0$ and $f'(x) \neq 0$ for $x \neq -1$.

The first task was given to students after the mid-term exam, and the second task was given after passing the course. Students were given sheets of papers for solving tasks. Also they could use their notebooks if they felt they should look something up.

RESULTS

Here we will not provide only students' answer to non-routine tasks, but we also will describe the whole process of solving to be able to observe students' reasoning.

Task 1
Firstly, we will report on Task 1, given in the middle of the course, and afterwards we will report on Task 2, given to students after passing the course.

Students D&P

Task 1 was given to students D&P when they were in the middle of the calculus course. When confronted with the task, students sat in a silence for a while and then student P gave his solution to the task where he identified $f'(x_0) = 0$ as zero point of the function and consequently concluded how this function had only one zero point. Furthermore, he placed this zero point in the origin of coordinate system.

P: It is equal with zero so… When we search for zero point of the function, it is in fact first derivative. Here it says $f'(x_0) = 0$, therefore it means that we have only one zero point. And it's 0. This point is in the origin of coordinate system.
Student D did not interfere while student P was explaining his solution; however, his face mimic implicated that he was not sure that this solution was right. Some time elapsed since students spoke, so interviewer started the second part of the interview. Here the interviewer asked students if they have ever met derivatives and where derivatives were used in the course. Student P interpreted $f'(x_0) = 0$ again as zero point of the function, and even when interviewer explained that in fact x_0 represents critical point of the function, student P resumed with reasoning in wrong way. However, student D claimed that x_0 was point of inflection:

I (interviewer): Have you ever used a first derivative? What for?
P: Yes, we have. It's point in which graph of the function crosses x-axis.
I: Hm…What if I say it's, in fact, a critical point. What would that be?
P: Zero point.
I: Hm… Have you ever used another derivative?
P: Yes, second. When something is maximum or minimum.
D: I think $f'(x_0) = 0$ says x_0 is point of inflection.
[silence]

Even with significant guidance from the interviewer, students D&P could not move in the reasoning sequence. They claimed the task was too difficult pointing out the fact they had never met task phrased this way, and that task was created for mathematics students. Although students had experienced tasks without calculation, they eventually gave up, not examining other given conditions and claiming there are no concrete numbers to work with.

I: Hm…Let's try this way...Where is this point? What else is given?
D: It's in the origin.
I: Do you have some other information?
P: I do not understand. I have never met task like this.
D: It' too difficult. I know to calculate zero point for the function but this way no… We are not mathematics students.
I: But you met tasks without calculation in the course, didn't you?
P: Yes.

Students N&M

Students N&M solved task without any assistance or prompts from the interviewer. They had read the task silently and afterwards they commented the amount of the written text in the tasks. Their whole reasoning process lasted very shortly, but they also experienced some difficulties in the process: the student N identified x_0 as the zero point, and the student M situated the point in the wrong part of the coordinate system, namely the first quadrant:
N: Too much text [laughs].
M: Hm, well…
N: How many zero points does function f have?
M: So for only one x we have $f'(x_0) = 0$ … Aha, it means that function has only one extreme.
N: Zero point.
M: Zero point and extreme are not the same.
N: Oh, yes yes… It's about extreme values.
M: This extreme value is here [shows first quadrant].…
N: No... x_0 is here [point at positive part of the x axis]… and $f(x_0)$ is negative…. so the point is in fourth quadrant.

Students got the idea to separate function limit in two parts and to see what each part represents. Student M said that function values go to the positive infinity for all positive x-values, and students N concluded the same for negative x-values. They made the figure of the given function in their head and this enabled them to give answer to the posed question. Interestingly, when the reasoning went in the wrong direction, the students helped each other; one member of the pair steered the reasoning process on the right track.

Students B&J

The student B had read Task 1 out loud, and both students had the same reaction to the given task, namely they were intimidated with the amount of data in the task. When students began to interpret the given data, student B concluded that $f'(x_0) = 0$ is a derivative of the constant, but student J corrected him.
J: This is weird…Never seen something like this....this information…too much data.
B: Yes. I agree… Much data…. It [tasks] says that derivative of the constant is zero.
J: No, that's critical point.
B: Hm…
[silence]
The interviewer asked students to express out loud what they were thinking that moment, but the students stayed silent for several minutes. In order to stimulate the reasoning process, the interviewer gave students a prompt:
I (interviewer): Go on... What seems to be the problem?
B: Do not know what to do with the data.
I: What about drawing what is given in the task?

Even with this prompt, students were not able to move further in the reasoning sequence, and the task was solved with a significant guidance from the interviewer. The interviewer asked what the given data represented and the students responded accordingly. The students B&J alternated in data interpretation, but none of them was independent in the reasoning process. The point $(x_0, f(x_0))$ was drawn in the correct quadrant, and students concluded that the function values go to infinity:
I: Where does the point $(x_0, f(x_0))$ lie?
B: In the second quadrant [pointing at the fourth quadrant] because x_0 is positive and that is on the right from the origin on x- axis, and $f(x_0)$ is negative and that is down on y-axis.
I: What about next piece of data? What does it says?
B: That everything goes to infinity.

At the end of the task, student B concluded that he can sketch parabola from the given data but he was uncertain how to draw it because there is only one point given in the task. However, student J took initiative and made drawing.
B: Hmm… this is kind of parabola... (laughs). But where to put it? I have only one point at disposition.
J: [takes pencil and draws] You do not need another point. See? If you connect all conditions, you get graph.
B: Oh… and there are two zero points

Task 2

Here we will present detail of students' solution for Task 2. This task was given to students shortly after they passed the calculus course. Here we will also provide detailed descriptions of students' reasoning and excerpts.

Students D&P

Initial behavior of students D&P was similar as in Task 1, but this time student D was involved in solving from the start. Student P stated that function crosses over x-axis when the function has discontinuity in zero, and student D stated that second derivative is usually used for calculating minimum and maximum of some function.
P: [draws coordinate system] This means that function in 0 would cross axis. [draws small circle in the origin of coordinate system]
D: Second condition....We used this when we calculated minimum and maximum.
[silence]

Students stopped after interpretation of two conditions. It seemed that they did not know what to do next, so after several minutes of silence, the interviewer asked them to explain the role of the first and second derivative when examining properties of the function. The students remained silent, so the interviewer tried another approach asking them how they determined intervals of concavity for some function. It was only after this prompt that students connected concavity and second derivative. Later student P interpreted -1 as the zero point of the function in third condition $f'(-1) = 0$, but student D corrected him. However, the students could not incorporate all conditions to sketch the figure.

I (interviewer): How did you determine where the function is concave upward or concave downward?
P: It' s concave upward when second derivative is less than zero.
D: Hm...no... I think is concave downward in that case.
P: [draws graph of function where certain parts are concave upward and concave downward] So let's look at parabola… second derivative… greater than zero here where [it] is concave upward, and concave downward here [pointing to the figure]
[silence]
I: What about next condition? What does -1 represent?
P: It's zero point.
D: Hmm… no it's maximum. See here at the figure [pointing to upper figure]… the derivative is zero in the critical point and then it follows it's maximum form other given conditions
[silence]
I: And now, can you incorporate all condition in your sketch?
P: I cannot make figure in my head.

Students N&M

After several months, when faced with Task 2, students N&M employed the same strategy as they had in Task 1. They read the task silently and then they started to interpret conditions like they are talking to each other. First of all, student N decided to draw the coordinate system. Then student M said that the function f had a hole in 0 and student N sketched that hole in the coordinate system as an empty circle at the point (0,0) and drew a full black circle at the point (0,1). Then they interpreted condition b. Student M concluded that condition b. described when the function was convergent or divergent, and student N concluded that function increased or decreased at given intervals. On the other side of the paper, student M drew his sketch of divergent and convergent functions. In fact, those figures represented curves being concave upward and concave downward. Student N pointed out that discontinuity should be incorporated in the drawing, and it should be where the function was changing its shape from concave downward to concave upward. But, this remark had disturbed the reasoning sequence of student M. He became puzzled with the outcome, but student N took over and drew that part of figure.
M: Second condition... It could be … [silence]
I (interviewer): Where have you used second derivative?
M: When something is convergent and divergent….but I do not remember when you use the first thing and when the second
N: For intervals of decrease or increase.
M: That is the first derivative, the second is used for that when something converges or diverges [draws a figure of curves being concave upward and concave downward]
N: Ok, you mean concave upward and downward…Discontinuity is here where the function changes its shape.
M: Well, no… that would mean… when $f''(x) > 0$, it is concave upward, when $f''(x) < 0$, it is concave downward.
N: See, here where $y = 1$, it fits… it goes like this [corrects figure, draws curve looking like parabola, having minimum]

The students M&N switched to condition c. identifying -1 as the only extreme value of this function. They changed their figure according to a new condition, and again the student M was a bit puzzled if new figure was correct solution of the task. The student N explained that new figure fulfills all conditions, but the student M expressed his doubts once more because that graph did not look alike any other graph he had ever seen:
M: But we have two functions now...
N: We fulfilled all conditions. It's alright.
M: So it's ok that we have two parts?
N: The function has discontinuity in 0.
M: Hm…I tried to imagine something I have seen before, and now I see it doesn't make sense.

Students B&J

In Task 2, the students B&J decided to draw the coordinate system firstly, and then they read the rest of the task. Reading out loud the condition a., the student B noticed there are some points in the text and drew them in the coordinate system. The points, he marked, had following coordinates (1,0) and (0,0). He said that condition b. was about intervals of increase and decrease of that function, and then he stopped looking puzzled. During that

time, student J was silent, examining the task. Since both students did not say a word for longer time, the interviewer got involved asking questions about given condition. Those questions served as an aid in the reasoning sequence. The student B corrected his drawing by marking point (0,1) instead of point (1,0), but he interpreted condition b. again in the same manner. This time student J got involved and corrected him.
I (interviewer): Are you sure you drew it correctly?
B: Well...no [corrects his drawing]
I: What about condition b.?
B: We had this at maximum and minimum. When f'' is less than zero, than we have maximum. When f'' is greater than zero we have minimum.
J: No. This [condition] says when the function is concave upward and when the function is concave downward… [silence]
I: Where the function is concave upward and where the function is concave downward?
J: On the left side of x-axis is concave upward and on the right side is concave downward.
[silence]

Students B&J discussed how shapes of concave upward and concave downward looked like, trying to decide what shape parabola $y = x^2$ has. After discussion, the student J drew the graph of function that satisfied condition b. Students moved to condition c. However, this new information caused confusion when students tried to incorporate it with other data they had. The aid of interviewer was needed again in the form of leading questions. At the end of the task, the students did not adjust their figure so that all conditions were met. Moreover, drawing represented the graph of the function with two extrema; having maximum on the left side of x-axis, and having minimum on the right side of x-axis. In this part of interview, students J had took the lead role.
I: What does the third condition says?
J: That's critical point.
I: Where?
J: On the negative part of the x-axis.
I: What property does function have there?
J: It's concave upward
I: So what kind of point we have there.
J: Hm … It's maximum. [puts the pencil down]
I: Is this solution?
B: Yes.

DISCUSSION

These non-routine tasks do not represent problems in terms of Schoenfeld (1985), but can be characterized as the moderately non-routine tasks. The interviews given in two occasions highlighted some difficulties, which students can encounter when solving such non-routine tasks. Also, we were able to detect if there were changes in the students' reasoning as they progressed through the course.

Students D&P experienced significant difficulties in the process of reasoning in both occasions. The absences of computation in the given tasks prevented students to rely on procedures and to use algorithmic reasoning to which they were more accustomed. Their reasoning in Task 1 had erroneous base from the start what disabled them to move further from the first condition. In the beginning, student P used familiar AR when he concluded that the function in fact has one zero point. Here he relied on superficial property, namely the expression where the function f was equal with the zero. Student D also used imitative reasoning when he concluded that x_0 is the point of inflection. He relied on recalling something he has seen before, not giving any arguments why he reached such conclusion. The students experienced significant frustration because they could not interpret the given condition in the right way and this reflected on their reluctance to examine other conditions. They pointed out that they knew how to calculate derivatives, but that the tasks without any calculations were only for mathematics students. Certain difficulties appeared also in Task 2, where students, only after the intervention, could move on further in the task. The students showed they were quite dependent on the external guidance in their reasoning. But, if we compare students' reasoning examined in two occasions, in the middle of the course (Task 1), and the other after the course (Task 2), we can see some positive shifts in the sequence of reasoning. Reasoning in the second task is in some part local creative since students did use argumentation and considered intrinsic properties of problematic components all together.

We identified some negative met-befores and met-afters in the base of their knowledge structure, which hindered their reasoning sequence. For example, student P interpreted $f'(x_0) = 0$ or $f'(-1) = 0$ as x_0 or -1 were the zero point of the function, and that had a stopping effect in his further reasoning. Calculation of zero points of some function is frequently done in high school mathematics, wherefore student P disregarded a sign for the derivative and identified this expression with commonly seen expression $f(x_0) = 0$ and this triggered familiar AR. On the other hand, in this calculus course, students learn about the concepts of critical points and extrema before the concept of point of inflection. Therefore identifying the expression $f'(x_0) = 0$ in Task 1 as a property which satisfies the point of inflection can be considered as the met-after which, we believe, influenced on student D's reasoning.

Students M&N solved given tasks themselves with almost no assistance from the interviewer in both occasions. They complemented their reasoning in both tasks and together reached conclusions that were anchored in relevant mathematical properties of the concepts they were reasoning about. In Task 1, student positioned given point in the right quadrant and concluded it was a critical point from the relations that were provided. They reached meaningful conclusion about function values after separating the function limit in two parts and they verified their solution at the end of the task. In Task 2, they interpreted all conditions in the right way, and sketched required graph of the function. But, in Task 2, it became more evident that student M aspired toward something he had seen before, showing the desire toward familiar AR. Even though students relied on conceptual understanding, the puzzlement of student M at the end of the task exposed his uncertainty. The obtained graph did not resemble any graph he had seen before and he believed this was not the right solution.

We identified certain negative met-befores in their knowledge structure. In Task 1, those were interpretation of x_0 as zero point in $f'(x_0) = 0$ (student N), and placing the point $(x_0, f(x_0))$ in the first quadrant (student M). The first is similar as in the case of students D&P, and the latter can be connected with presentation of many function graphs in the textbooks and lectures. Usually, when the lecturer (either at university or secondary school) sketches graph of arbitrary function as an example to show some property, it is mainly placed in the first quadrant, or the major part of the graph is sketched there. In Task 2, student N identified the second derivative with properties of increase or decrease of the function. In this calculus course, usage of first derivative for intervals of increase and decrease is taught before the second derivative and its connection with concavity. This met-before indicates that concepts related to the first and second derivatives were not properly understood. But, it seems that this mental construct did not have negative effect on solving the tasks in calculus exams, because student did pass the course. Even though these met-befores were evident in students' knowledge, their reasoning was not suppressed like in the case of students D&P. The members of the pair corrected each other, and consequently, produced a valid solution for the tasks. The question that remains unanswered is whether students alone would be able to solve the tasks, and whether these negative met-befores would stop them to proceed further in the reasoning process.

Reasoning of students B&J was guided with the assistance of the interviewer in both occasions (in the middle of the course and after the course). Moreover, in Task 1, students showed inability to move further beyond their inference about the amount of the text and data in the task. Besides giving a prompt for drawing data in the coordinate system, the interviewer asked what the certain piece of data represents, but did not participate in the interpretation. Therefore components of the local creative reasoning can be found in students' reasoning sequence. Using their conceptual understanding, students obtained the right solution, but the final conclusion could not be reached without interviewer's initial help. In Task 2, students showed more initiative, namely they were more creative, but the interviewer again served as the support and the guidance in the reasoning sequence. The students were reluctant to express their thinking out loud and to continue their reasoning, what can be inferred from silence gaps in the process. However, we would not say that students lacked resources or knowledge for the tasks. Students used arguments that are mathematically founded to provide validity of their conclusions. Their reasoning was anchored in intrinsic properties of the components in the reasoning: the relation between the property of function being concave upward or downward and the shape of parabola to determine what shape given function has, or the relation between critical point and the shape of the function to conclude what extreme value is given.

Some negative met-befores were found in knowledge structure of student B. At the end of the Task 1, student B identified graph of the function as the graph of well-known quadratic function. This interpretation of the obtained graph does not have to be necessarily problematic met-before, but it certainly can have negative impact in other situations. Here we see another met-before as much more problematic and that is a need for "formula" i.e. concrete expression which should help the student to draw "parabola". According to Tall (2006), seeking for a function that person has already met and the need for formula hinders the development of advanced mathematical thinking. Another negative met-before is the interpretation of condition b., namely connecting $f'' < 0$ with the property of increase or decrease of some function, what together with the interpretation of $f'(x_0) = 0$ as the derivative of the constant, indicates that student did not quite understand the topic of first derivative and related concepts of: extrema and intervals of increase and decrease. However, the student did pass the calculus course with the average grade, what indicates that these problematic met-befores were not visible in reasoning that was required in the calculus exams.

CONCLUSION

The students in this study frequently used imitative reasoning during their schooling in primary and secondary education. The aim of the study was to examine if meeting the non-routine tasks in the calculus course had any effect on their reasoning. The results of the study showed that creative reasoning was developing slowly. When solving the non-routine tasks, students showed tendency to be guided in the reasoning sequence even at the end of the course, or more likely, after passing the course. Also, they expressed the need for concrete expressions to work with. On the other hand, what students had met before and after learning some mathematical topic had a wide impact on their reasoning. Negative met-befores and met-afters stopped students in further task solving, or turned them to imitative reasoning. However, our intention is not to classify all met-befores and met-afters that can appear in students' knowledge and that can inhibit creative reasoning, but to caution to their existence. The students in our study passed the calculus course, what indicates that those met-befores and met-afters did not prevent them to successfully solve tasks that asked for imitative reasoning. We based our conclusion also on students' mid-term and final exams that we have examined before each interview session.

When faced with new situations, students tend to look for something familiar, and usually seek for a remedy in the form of imitative reasoning, like searching the textbook for similar solutions or going through their recollection of similar tasks (e.g. Boesen et al., 2010, Haavold, 2012). Selden et al. (1998) pointed out that students lack tentative solution starts, i.e. general ideas for beginning the process of finding a solution, and that, together with mental constructs of met-befores and met-afters, provides significant obstacle for creative reasoning. We argue that mathematics educators and lecturers should take this into consideration when teaching students. On the other hand, creative reasoning is beneficial to investigate students' understanding and to check the quality of their long-term knowledge. In imitative reasoning, students do not consider intrinsic properties of the objects they are reasoning about, and frequently they rely on well-established procedure, mimicking, almost unconsciously, its every step (Lithner, 2012). Even though imitative reasoning provides reduction of complexity in the course requirements, students do not construct appropriate meaning in such process. The remedy is not avoidance of non-routine tasks, but quite opposite, facing students with new situations. The non-routine tasks and creative reasoning can uncover negative met-befores and met-afters which students are oblivious to when they perform imitative reasoning. This uncovering is important for the sequencing courses that build upon previous courses, i.e. where new knowledge is building up previous acquired and mastered concepts.

But is it possible that the non-routine tasks become more visible in the course, not only as the part of the homework, but also as the part of exercise sessions? And to whom does this matter? Giving answers to these questions is not simple. In the calculus course that our participants took, the non-routine tasks were implemented mainly in the homework part. The course syllabus is overloaded, and it is difficult to explicitly deal with non-routine task on regular basis. During the course, students showed the resistance toward non-routine tasks that demanded more of their invested time than usual routine tasks. But we as educators argue that it does matter, because we want to build up working force that can adapt to any requirements business and economy demand today. We believe that flexible thinking and creative reasoning are part of this ability. Looking from the students' position, this question does not have unique answer. Students, not only in this study program, but in many other science and technical study programs, have many requirements in the courses more related to their profession. They usually lack time for deeper engagement in mathematics, but at the same time they want to pass the mathematics course and would like to know how to apply gained mathematical knowledge (Jukić Matić, 2013). There is no simple solution to this problem and these facts put us in very difficult position. We conclude this paper with a note that the non-routine tasks, besides being part of the homework, became the part of the oral exam in this study program.

REFERENCES

Arksey, H., & Knight, P. (1999). *Interviewing for social scientists*. London: Sage.

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *Journal of Mathematical Behavior*, *26*(4) 348-370.

Bergqvist, E. (2012). University mathematics teachers' views on the required reasoning in calculus exams. *The Montana Mathematics Enthusiast*, 9(3), 371-408

Boesen, J., Lithner, J., & Palm T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics*, *75* (1), 89-105

Chinnappan M., Dinham S., Herrington A., & Scott, D. (2007). Year 12 students and Higher Mathematics: Emerging issues, *AARE 2007 International education research conference*

Cox, W. (1994). Strategic learning in a-level mathematics? *Teaching Mathematics and its Applications*, 13, 11–21.

Ericsson, K., & Simon, H. (1993). *Protocol analysis: Verbal reports as data*. Cambridge: MIT Press.

Haavold, P. (2010). What characterizes high achieving students' mathematical reasoning? In *The elements of creativity and giftedness in mathematics*. Sense Publisher

Hiebert, J. (2003). What research says about the NCTM standards. In Kilpatrick, J., Martin, G., & Schifter, D. (eds) *A Research Companion to Principles and Standards for School Mathematics*, pp 5-26. Reston Va., National Council of Teachers of Mathematics.

Jukić Matić, Lj. (2013). Non-mathematics students' knowledge and their beliefs about mathematics (under review)

Lima R.N. de, & Tall D. (2008). Procedural embodiment and magic in linear equations. *Educational Studies in Mathematics*, 67(1), 3-18.

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics*, 52, 29–55.

Lithner, J. (2004). Mathematical reasoning in calculus textbook exercises. *Journal of Mathematical Behavior*, 23, 405–427.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics* 67(3).

Lithner, J. (2012). University mathematics students' learning difficulties, *Education Inquiry*, *2*(2), 289-303

McGowen, M., & Tall, D. (2010). Metaphor or Met-before? The effects of previous experience on the practice and theory of learning mathematics. *Journal of Mathematical Behavior*, *29*, 169--179.

McNeal, B. (1995). Learning not to think in a textbook-based mathematics class. *Journal of Mathematical Behavior* 14 18–32.

Niss, M. A. (2003). Quantitative literacy and mathematical competencies. In B. L. Madison, & L. A. Steen (Eds.), *Quantitative literacy: why numeracy matters for schools and colleges.* (pp. 215-220). Princeton: National Council on Education and the Disciplines.

Schoenfeld, A H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J. Voss, D. Perkins, and J. Segal, editors, *Informal Reasoning and Education*, pages 311–344. Hillsdale, NJ: Lawrence Erlbaum Associates

Schoenfeld, A. (1985). Mathematical problem solving. Orlando, FL: Academic Press.

Selden, J., Mason, A., & Selden, A. (1998). Can Average Calculus Students Solve Nonrutine Problems? *Journal of Mathematical Behavior*, *8*, 45-50.

Tall, D. (1997). Functions and Calculus. In A. J. Bishop et al (Eds.), *International Handbook of Mathematics Education*, 289–325, Dordrecht: Kluwer.

Tall, D. (2006). A Theory of Mathematical Growth through Embodiment, Symbolism and Proof, *Annales de Didactique et de Sciences Cognitives*, IREM de Strasbourg, 11, pp. 195-215.