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DEVELOPING GEOMETRICAL THINKING THROUGH MATHEMATIZATION

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ABSTRACT: This paper summarizes a research done at a graduate course. The researcher investigates the mathematization experience of three graduate students while solving a geometry problem. The participants were expected to go through a vertical mathematization derived from a real life example. However, only one of the participants was successful on exploration whereas the other two failed going further. Data exploring the possible reasons puts forward that their learning habits hindered them from exploring the problem although they agreed on their current knowledge was sufficient to solve the problem.

Key words: Geometrical thinking, mathematization, qualitative research

INTRODUCTION

Realistic Mathematics Education (RME) is one of the leading theories getting more appreciation in recent years. The Realistic Mathematics Education (Freduenthal, 1991) framework suggests taking realistic examples from daily life and connecting it to mathematics. That is, mathematics in daily life should be observed and explored through horizontal mathematization and be extended up to higher levels through vertical mathematization (Treffers, 1987).
For examples, the following picture taken from an ordinary carpet illustrates a square like figure having some extensions at the corner (Figure 1). One can take this specific example by assuming the figure is a regular square with side a. Then, it would be great to extend each side by a certain amount such that he could construct an octagon through the end points of these extensions. Now the question would be how much should one extend sides? Maybe $a/\sqrt{2}$, maybe not!

Figure 1: A Carpet Illustrating A Square Like Figure

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By assuming the figure on the square is a perfect square, one transforming this example on paper-and-pencil or any Dynamic and Interactive Mathematics Learning Environment (DIMLE) (Martinovic and Karadag, 2012) completes a horizontal mathematization process. However, elaborating the example little more and looking for answer for the question, “what about constructing a polygon with 16-sided from an octagon through a similar method?” and even going furthermore and asking a generalization demonstrates a vertical mathematization.

Vertical mathematization

In order to initiate a vertical mathematization process, one should start with a square with a side a in length and construct an octagon. Now the best question at this moment would how much should we extend the sides to obtain an octagon. After a period of exploration at the paper-and-pencil environment, one may find the answer, $^{a}/_{\sqrt{2}}$, and then, in order to verify the answer start drawing a circle with radius, $= {}^{a}/_{\sqrt{2}}$, located its center at the one of the corners, say at B, of the square (Figure 2).

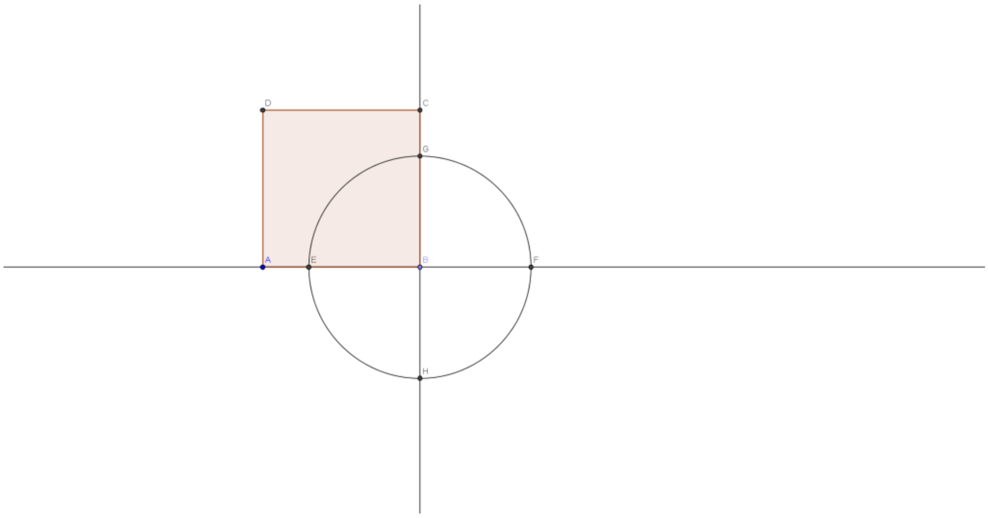

Figure 2: A Circle Created At The Corner B

Since the intersection points of the lines extending the sides and circle located its center B would be the two cornes of the octagon we have been looking for, the rest of the construction would be easier. GeoGebra allows using a tool, helping to construct a polygon if two consecutive corners and the number of sides are known (Figure 3).

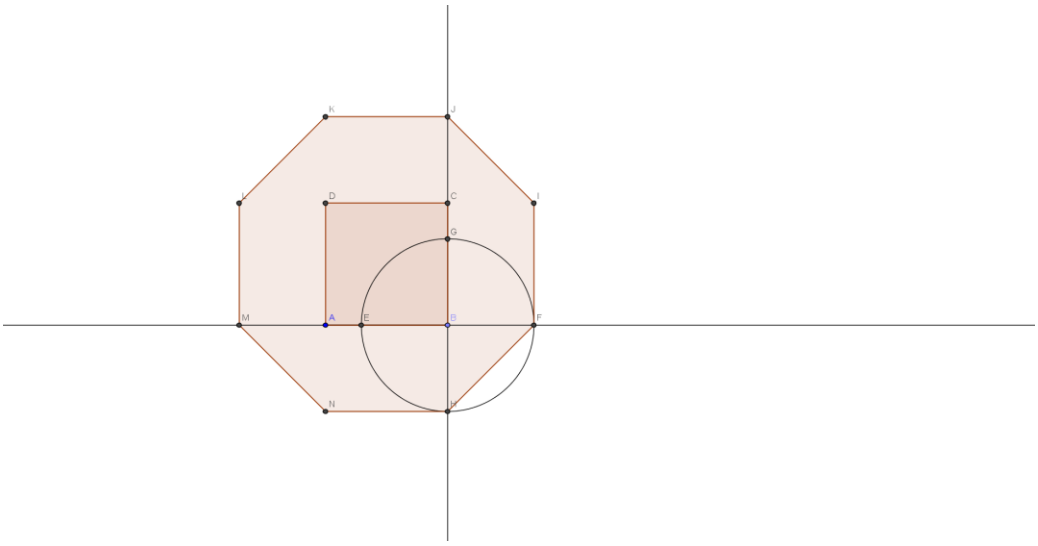

Figure 3: An Octagon Created Around The Square

One could follow similar procedures to construct polygons with 16-sided by drawing a circle at one of the corner with a radius of $R = a / \sqrt{2-\sqrt{2}}$, 32-sided etc. However, a mathematician point of view seeks a generalization to accomplish this task if it is possible. Even it is impossible, mathematician needs to convince herself that the generalization problem has no solution. Fortunately, we have a solution for generalization.

Generalization

One having some experience with geometry in general, polygons in particular may assume that we need a formula consisting some angles, and therefore, may attempt to transform the relationships she developed into more trigonometric forms. For example, the following table summarizes these relationships in more tabular form (Table 1).

Table 1: A Summary Of Relationships Between The Sides And The Radii Of Circles

To construct	Start with	Draw a circle with radius
A octagon	a	$R = \frac{a}{\sqrt{2\left(1-\cos\frac{\pi}{2}\right)}}$
A polygon with 16-sided	a	$R = \frac{a}{\sqrt{2\left(1-\cos\frac{\pi}{4}\right)}}$
A polygon with 32-sided	a	$R = \frac{a}{\sqrt{2\left(1-\cos\frac{\pi}{8}\right)}}$
A polygon with 2n-sided	a	$R = \frac{a}{\sqrt{2\left(1-\cos\frac{2\pi}{n}\right)}}$

This table summarizes the findings of free exploration of the aforementioned vertical mathematization process. The following sections describe how participants could perform in following this anticipated mathematization path.

THE STUDY

This qualitative study explores the graduate students' experience while going through mathematization. The resarch problem for the study has been described as, "how could graduate students experience a non-routine geometry problem through vertical mathematization?"

In order to have them better understand mathematization processes, they were instructed on mathematization, including horizontal and vertical mathematization dimensions. Following the instruction, the carpet picture was shown, and how an octagon could be drawn starting from a square was demonstrated. Then, they were asked to elaborate the problem and to construct (1) a polygon with 16-sided by starting from a octagon and (2) a polygon with 32-sided by starting a polygon with 16-sided. Finally, they were asked if it would be possible to generalize the problem and to generate a formula holds true for all cases.

Participants were three graduate students studying mathematics education and taking a course named, "Teaching Geometry: Yesterday, Today, and the Future." Their teaching experience were at Grades 5-8 at various experience levels. Following the problem was exposed, they were given a week to work on the problem. After exploration time was completed, their results were asked to discuss the mathematization they needed to solve the problem and the challenge they had experienced.

Data included their work on paper-and-pencil and GeoGebra environments and informal interviews including their own reflections. In order to understand their experience the data was analyzed qualitatively. As Charmaz (2006) suggests seeking the truth in qualitative data analysis rather than looking for validity and reliability the data was triangulated with each other to make sure that the research results illustrates the truth.

FINDINGS

Interestingly, one out of three graduate was really succesful in this mathematization process whereas one of them had never done anything. One had declared an unsuccesful attempt. The one who had performed succesfully moved even further and developed a formula for another context.

She declared that once she completed the task she had asked herself “what if I start with triangle rather than a square.” Many mathematicians and mathematics educators suggest encouraging themselves to ask a similar question led them to pose new problems and deepen their understandings.

In contrast, asking the other two what holded them to go further or why they could not solve the problem revealed that their learning habit of mathematics throughout their schooling years was the barrier for them to think further. They put forward that their learning habits of learning geometry led them absorbing some geometrical facts rather than exploring geometry by themselves.

RESULTS and DISCUSSION

This qualitative study reflected on the graduate students’ experience while going through a mathematization process. The data suggests that schooling they had to go through developed a rote learning habit rather than a explorative way of learning. This claim was put forward by one of the participants, and the others also confirmed the claim. They argued that they were supposed to find some certain answers for the test questions on the standardized tests because of the education system they had to follow.

Despite the fact that they appreciated the mathematization they went through they still were unsure about the ways of implementing more problem solving and mathematization sections in their current courses. They claimed that it should be done through Ministry of Education.

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