MATHEMATICS TEACHERS' VIEWS ABOUT TEACHING GENERALIZATION OF NUMBER PATTERNS

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ABSTRACT: The present study reports on middle school mathematics teachers' views about teaching generalization of number patterns. Teachers' approaches to teach generalizing and their point of views on using strategies were also examined. Interviews were conducted with sixteen middle school mathematics teachers. Data qualitatively analyzed in terms of Radford's (2008) architecture of algebraic pattern generalization theoretical framework. Analysis of the data indicated that teachers' uses of representations were not effective and teachers' subject matter knowledge on generalization of number patterns is weak. It was found that teachers' commonly used strategy is trial-error. Findings also suggest that although teachers' mostly use strategies and examples in teaching process, they did not use them in order to develop students' understanding, mathematical thinking and reasoning.

Key words: Generalizing number patterns, task design, task implementation

INTRODUCTION

Attention to patterns is acknowledged in its importance as an introduction to algebra (Zazkis and Liljedahl, 2002), hence constructing a conceptual understanding in students' minds is crucial. Because teaching the generalization of number patterns requires both to have specialized subject matter knowledge and purposefully constructed teaching approaches, it could be useful to use tasks within a pedagogy including the meaningful use of representations (Yesildere-Imre and Akkoc, 2012). Here one crucial aspect is the 'didactical aim' and 'pedagogical means' of the teaching process. According to Berg (2009) didactical aim is "the choice of a particular area or knowledge target within a subject-matter", and pedagogical means refers to a task "to use in order to address the chosen didactical aim" (p.100). For example, a didactical aim could be "to generalize a number pattern" and it is possible to find many different tasks used as pedagogical means to achieve this didactical aim (Berg, 2009).

Present study focuses on these both aspects of teaching generalizing number patterns process: subject matter knowledge and teaching approaches. Investigation conducted in terms of (a) teachers' subject matter knowledge about number pattern generalization and (b) teachers' pedagogical content knowledge of teaching generalization of number patterns. This will be done by relying on the theoretical framework of Radford (2008) which will be discussed in the next section.

Theoretical Framework: Algebraic Pattern Generalizations

Radford's (2008) theoretical approach to algebraic pattern generalization arose in the course of a longitudinal classroom-based research which is conducted in the 1990s. He suggested that generalizing a pattern algebraically rests on the capability of grasping a commonality noticed on some particulars (say $p_1, p_2, p_3, \ldots, p_k$); extending or generalizing this commonality to all subsequent terms ($p_{k+1}, p_{k+2}, p_{k+3}, \ldots$), and being able to use the commonality to provide a direct expression of any term of the sequence (Radford, 2006). Figure 1 summarizes the architecture of an algebraic generalization of patterns.

Particulars $p_1, p_2, p_3, ..., p_k$	Noticing a commonality **C**	Making **C** a hypothesis	Producing the expression of p_n

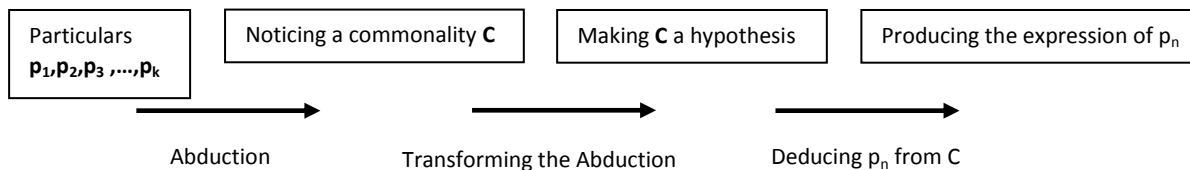Abduction　　　　Transforming the Abduction　　　Deducing p_n from C

Figure 1. The architecture of algebraic pattern generalizations (Radford, 2008)

The architecture of algebraic pattern generalization framework helps us to distinguish it from other strategies that students often use to deal with patterns; In particular, it is possible to distinguish between algebraic and arithmetic generalizations of patterns (Radford, 2008). Radford underlines two other strategies that are use to generalize number patterns; arithmetic generalization and naive induction. He explains these two strategies as follows: "Arithmetic generalization comes into play when students focus on the increase in consecutive numbers or expressing the relation between the terms of the pattern with numbers. If the abductions did not lead to a rule produced by a generalization, but a rule obtained by induction, i.e. a procedure based on probable (or likely) reasoning and whose conclusion goes beyond what is contained in its premises, this type of induction is called naive induction." In line with this framework, middle school mathematics teachers' approaches in order to teach generalizing number patterns were examined through three strategies; algebraic generalization, arithmetic generalization and naive induction.

METHODS

Qualitative research method was employed for this research. Interviews were the main technique. Primary mathematics teachers participated face to face interviews. Interviews were designed to explore teachers' subject matter knowledge about number pattern generalization and pedagogical content knowledge of teaching generalization of number patterns. Each interview was conducted at participant teacher's school. All interviews were audio-taped and transcribed.

Participants

Sixteen primary mathematics teachers were agreed to be interviewed. The teachers had diverse backgrounds in terms of their experience as teachers. Their professional experiences were shown in Table 1.

Table 1. Classifying the participants according to experience as teachers

Less Than 15 Years	More Than 15 Years
K_5 K_6 K_7 K_8 K_9 K_{10} K_{13} K_{14}	K_1 K_2 K_3 K_4 K_{11} K_{12} K_{15} K_{16}

Data Analyses

Data analyses consisted of coding the interview transcripts. Teachers' views were indentified and categories were proposed. The categories for number pattern generalization included teaching approaches, knowledge of students' misconceptions, the importance given to number patterns, knowledge of teaching and subject matter knowledge. In the next section findings about teachers' views and teaching approaches will be discussed around the consisted categories.

RESULTS AND FINDINGS

Mathematics teachers' views are presented in terms of three categories: (i) teaching approaches (ii) knowledge of teaching (iii) subject matter knowledge. The categories and sub-categories are shown in table 2.

Table 2. The Views About Number Patterns and Teaching Generalization of Number Patterns

Categories and Sub-categories		Participants	f
Teaching Approaches	Teaching with Different Strategies	$K_1\ K_2\ K_3\ K_4\ K_5\ K_6\ K_8\ K_{10}\ K_{11}$ $K_{12}\ K_{13}\ K_{14}\ K_{15}\ K_{16}$	14
	Teaching with Examples	$K_2\ K_3\ K_5\ K_6\ K_7\ K_8\ K_9\ K_{11}\ K_{15}$	9
	Teaching with Multiple Representations	$K_1\ K_2\ K_3\ K_4\ K_6\ K_8$ $K_9\ K_{12}\ K_{13}\ K_{14}$	10
	Giving the Rule	$K_1\ K_9$	2
Knowledge of Teaching	Effective Use of Multiple Representations	K_{13}	1
	Ineffective Use of Multiple Representations	$K_2\ K_3\ K_4\ K_6\ K_8\ K_9\ K_{12}\ K_{14}$	8
Subject Matter Knowledge	Sufficient	$K_6\ K_{10}\ K_{13}$	3
	Insufficient	$K_3\ K_5\ K_{13}\ K_{16}$	4

Teachers have a variety of views about how instruction should take place and how generalization of a number pattern should be taught. These opinions are examined in detail below.

Teaching Approaches

As seen in table 2, participants' teaching approaches gathered around four sub-categories: (i) teaching with different strategies, (ii) teaching with examples (iii) teaching with multiple representations (iv)giving the rule. Teachers rarely preferred to give the rule in teaching process, rather they mostly preferred to teach with strategies and representations. They also tend to teach with examples in which also naive induction strategy is utilized.

As stated before, middle school mathematics teachers' approaches in order to teach generalizing number patterns were examined through three strategies in line with the theoretical framework; algebraic generalization, arithmetic generalization and naive induction. It was seen that teachers frequently used arithmetic generalization and naive induction rather than algebraic generalization. Only one of the participants (K_{13}) used algebraic generalization as a strategy. Most of the participants (for example K_5) claimed that naive induction could be the only way to generalize a number pattern:

"... students' computation ability must be very fast and practical to generalize a number pattern. He or she must examine the numbers and understand the relationship between numbers immediately by practicing a lot".

Some teachers (for example K_8) claimed that students need to examine the relationship between consecutive numbers in a pattern with trial and error strategy. This approach leads to use arithmetic generalization and naive induction strategies:

"... students must be encouraged to investigate the relationship between consecutive numbers. I tell my students to add or multiply or subtract or divide numbers to find the number in the pattern."

Most of the teachers stated that it is useful to use multiple representations in teaching process. They usually used table and pictorial representations of the number patterns.

In brief, teachers' one of the approaches for teaching generalization of number patterns were arithmetic and this kind of generalization was not of an algebraic in nature (Radford, 2008). The other approach is naive induction, in which abductions were mere guesses. In other words abductions did not lead to a rule by a generalization, but a rule obtained by induction (Radford, 2008).

Knowledge of Teaching

The use of multiple representations was the main strategy pointed both in primary mathematics curriculum and textbooks. Therefore teachers' knowledge of teaching was determined in terms of the use multiple

representations. Teachers were expected to use any representation in the way of supporting students' algebraic thinking. See the pictorial representation shown in figure 2.
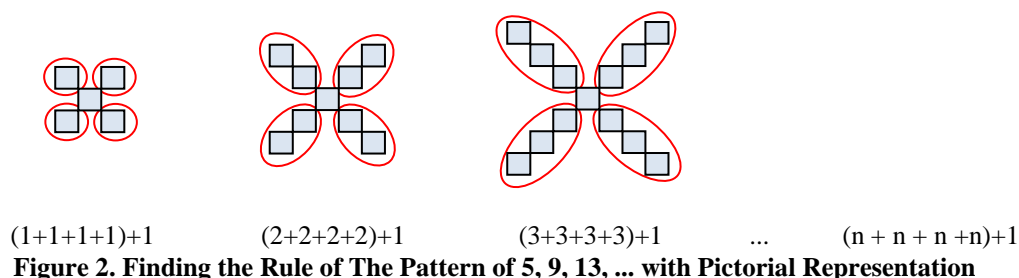

$$(1+1+1+1)+1 \qquad (2+2+2+2)+1 \qquad (3+3+3+3)+1 \qquad ... \qquad (n + n + n +n)+1$$

Figure 2. Finding the Rule of The Pattern of 5, 9, 13, ... with Pictorial Representation

We could find the rule considering the representation and Radford's framework Here a commonality C was inferred from a few particular cases. Then, this commonality was generalized to the rest of the terms of the sequence. Next, the abduction C became an hypothesis and then the rule $(n + n + n + n) + 1$ was calculated.

It was found that seven of sixteen teachers did not use either table or pictorial representations. It was also seen that teachers who used pictorial representations in their lessons did not aware of the potential of the representations. They usually preferred to use them just for counting and determining the number pattern or visualization. One of the teachers (K_9) said:

"*...students complete the pattern and have fun. So the lesson becomes more visual and joyful.*"

Only one teacher (K_{13}) effectively used the pictorial representation at all:

"*Students could easily transfer from pictorial representations to algebraic rule. I encourage students to examine pictorial representation and find a common point. According to me it is important to use pictorial representations which helps students to see the relationship between models and transfer this relationship into algebraic rule.*"

Subject Matter Knowledge

Although we did not ask any question regarding generalizing a number pattern directly, we could decide some teachers' (seven of sixteen) subject matter knowledge from their words during interviews. Four of seven teachers were insufficient subject matter knowledge. One of the misconception teachers have is the idea of there is only one way to represent a number pattern algebraically. Apparently we could also find another rule shown in figure 3.
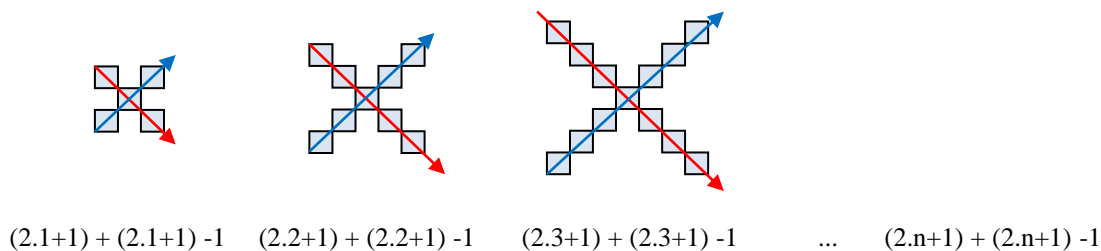

$$(2.1+1) + (2.1+1) -1 \qquad (2.2+1) + (2.2+1) -1 \qquad (2.3+1) + (2.3+1) -1 \qquad ... \qquad (2.n+1) + (2.n+1) -1$$

Figure 3. Different Rule of the Pattern of 5, 9, 13, ...

Although both expressions are algebraically equal, emphasizing the variety of rules helps students to develop algebraic thinking. Teachers also stated that the only strategy to find the algebraic rule of a number pattern is 'just keep trying' which leads either arithmetic generalization or naive induction.

CONCLUSION

As was described before, this study reports on middle school mathematics teachers' views about teaching generalization of number patterns. We concluded that teachers rarely preferred to give the rule in teaching

process, rather they mostly preferred to teach with strategies, examples and representations. However they did not use pattern-specific strategies and encouraged students to make arithmetic generalization or naive induction. This approach may emanate from their lack of subject matter knowledge because it was also seen that they also use these strategies when it comes to find the rule of a pattern by themselves. Seven of the teachers who participated in the study used pictorial representations in their lessons did not aware of the potential of the representations. They usually preferred to use them just for counting and determining the number pattern or visualization.

RECOMMENDATIONS

In our study we found that teachers appeared to teach with notions, however algebraic pattern generalizations are not characterized by the use of notations (Radford, 2008, p.93). Because of lack of subject matter knowledge, teachers usually employ trial-and-error and finite differences as strategies which were also common misconceptions of students reported in literature (Rossi, Becker & Rivera, 2006). In terms of teaching generalization of number patterns, teachers need to deepen their understanding and appreciations of the use of representations.

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