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Sekizinci Sınıf Öğrencilerinin Uzamsal Düşünme Becerisi Gerektiren Gerçek Yaşam Problemlerini Çözme Durumları

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Öz

Bu araştırma 8. Sınıf öğrencilerinin uzamsal düşünme becerisi gerektiren gerçek yaşam problemlerini çözme durumlarını incelemeyi amaçlamaktadır. Araştırma nitel araştırma yöntemlerinden durum çalışması olarak yürütülmüştür. Araştırmanın verileri araştırmacı tarafından hazırlanan bir form aracılığı ile elde edilmiştir. Araştırma çerçevesinde, Türkiye'de bir devlet okulunda öğrenim gören 15 gönüllü sekizinci sınıf öğrencisi ile yarı yapılandırılmış görüşmeler yapılmıştır. Toplanan veriler araştırmacı tarafından her soruya özel olarak hazırlanan bir rubrik ile analiz edilmiştir. Sonuçlar, öğrencilerin problemlerini anlamada genel olarak herhangi bir zorluk yaşamadıklarını, ancak nesneleri veya bileşenlerini görselleştirmede kısmen yetersiz olduklarını göstermiştir. Ayrıca öğrencilerin zihinde canlandırma basamağında başarılı olsalar dahi, canlandırılan şeklin uygulaması olan kâğıda aktarmada beklenen başarıyı yakalayamadıkları tespit edilmiştir. Daha karmaşık ve üç boyutlu şekilleri kâğıda aktarırken zorlandıkları görülen öğrencilerin uzamsal düşünme becerilerini geliştirmelerini desteklemek için okullarda işlenen matematik derslerinin somutlaştırılması uygun olacaktır. Bu doğrultuda derste teknoloji destekli öğretim yöntemlerinin kullanılması ve içeriğin somut materyallerle desteklenmesi önerilebilir. Ayrıca öğrencilere gerçek yaşamda uzamsal düşünme becerisi gerektiren farklı problemlerin fark ettirilmesini sağlayan etkinlikler yaptırılması da faydalı olacaktır.

Anahtar Kelimeler

Uzamsal Düşünme Becerileri, Sekizinci Sınıf Öğrencisi, Gerçek Yaşam Problemleri

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Eighth Grade Students' Solving Real Life Problems Requiring Spatial Thinking Skills

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Abstract

This research aimed to examine how 8th grade students solved real-life problems requiring spatial thinking skills. The research was conducted as a case study, one of the qualitative research methods. The study data were obtained through a form prepared by the researcher. In the framework of the research, semi-structured interviews were conducted with 15 voluntary eighth graders studying at a public school in Turkey. The collected data were analyzed by the researcher with a rubric specifically prepared for each question. The results showed that students generally did not have any difficulty in understanding their problems, but were partially inadequate in visualizing objects or their components. In addition, it was determined that even when the students were successful in the visualization step, they were not successful in the application phase which required the transfer of the visualized shape to paper. It would be appropriate to concretize the mathematics lessons taught in schools in order to support the development of spatial thinking skills of students who have difficulty in transferring more complex and three-dimensional shapes to paper. In this direction, it can be suggested to use technology-supported teaching methods in the course and to support the content with concrete materials. In addition, it will also be helpful to undertake activities that make students aware of different problems that require spatial thinking skills in real life.

Keywords

Spatial Thinking Skills, Eighth Grade Student, Real Life Problems

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Introduction

The general objectives of the mathematics curriculum used in schools include raising individuals who have developed social, verbal and numerical skills in daily life. In this direction, one of the competencies that students will need is determined as "Mathematical competence and basic competencies in science/technology" in the Turkish Qualifications Framework (TQF). Mathematical competence, which includes the ability and willingness to use mathematical modes of thinking (spatial and logical thinking) and presenting (tables, formulas, models, graphs and fictions) at different levels, is defined as developing and applying mathematical thinking to solve real-life problems (Secondary Education Mathematics Curriculum, 2018). According to Charles and Lester (1982), one of the primary goals of education is to educate individuals who can overcome the problems encountered in real life. Daily life problems rather than routine problems provide the suitable setting for students to put their mathematical knowledge into practice (Chacko, 2004).

National Council of Teachers of Mathematics [NCTM], (2000) which defines one of the goals of geometry standards as students' problem solving using spatial thinking, visualization and geometric modeling, emphasizes that spatial thinking and spatial skills should be developed in all students. In this direction, the spatial thinking skill, which has an important place in the mathematics curriculum, is actually included in solving many problems encountered by individuals in their daily lives. Activities such as parking the car, placing the dishes in the dishwasher and relocating items in the house are just a few of examples of using spatial thinking skills in daily life (Tüzün &Yıldız, 2011).

Problem Solving Skills

Problem solving, which marks the twenty-first century, is one of the goals of all disciplines. Problem solving is not about achieving results, but about finding useful and flexible solutions for new cases (Gail, 1996).

Problem solving is a process skill, and so it can be said that the problem-solving steps used to solve the problem are a product of this process (Açıkgöz, 2014). When the research on the problem-solving process is examined, the problem-solving steps discussed by the famous Hungarian mathematician George Polya (1990) are known as the most widely accepted problem-solving steps in the literature. These steps are included in the book of Polya titled “How to Solve it?”, which was published in 1945. According to Altun (2000), the most accepted process for solving routine and non-routine problems is the four-step process given by George Polya. Working in accordance with these four steps in the problem solving process facilitates problem solving. It is seen that Polya (1990) handles the problem-solving process with four different steps: understanding the problem, making a plan, implementing the plan, and checking the solution. In the comprehension step, students are expected to be able to express what is given in their own sentences and to write down what is given and what is required in the problem. The making a plan step involves making a plan suitable for solving the problem and expressing this plan in mathematical sentences. The step of implementing the plan is the application of the methods and strategies to be used in solving the problem. Finally, the step of evaluating the solution involves evaluating whether the methods and strategies used in solving the problem are correct or not (Polya, 1990).

Real Life Problems

In the literature, problems are generally classified as routine (ordinary) problems and non-routine problems (Altun, 2000; Baykul, 2014; Gök & Sılay, 2009). Routine problems can be defined as problems known as four operations problems and are frequently included in textbooks, such as mathematics and physics (Gök & Sılay, 2009). Polya (1990) mentions the importance of utilizing routine (ordinary) problems in mathematics lessons in schools to gain skills related to problem solving. In addition, Polya (1990) defines routine problems as problems that can be solved with certain stereotyped steps or problems that are created by adding or modifying a previously solved problem. Polya (1990) states that students only apply predetermined rules to solve such problems and do nothing that requires logical skills. Routine problems are not the real goal of the problem-solving approach and over time routine problems should be replaced by non-routine real life problems (Altun, 2000). Non-routine problems have important functions in providing students with a way of thinking that will enable them to eliminate the difficulties they face and in the effectiveness of teaching (Azak, 2015).

Spatial Thinking Skill

Spatial thinking is the ability to move or visualize objects consisting of one or more parts and their components in three-dimensional space (Turğut, 2007). Tartre (1990) defined the concept as a combination of mental skills that include interpreting visual relationships, rearranging and changing objects, and expressing their new states. Definitions of spatial thinking skills vary. Likewise, spatial thinking skill has been divided into different sub-components by different researchers. Based on the literature review conducted for this study, the classification of Lohman (1988) including spatial visualization, spatial relationship, and spatial orientation was selected as the theoretical framework in this study.

McGee (1979) defined spatial visualization as the ability to move, rotate or invert a given shape in the mind, while Olkun and Altun (2003) defined it as the ability to visualize new situations that will occur as a result of moving 2D and 3D objects consisting of one or more parts and images of their parts in three-dimensional space. Spatial orientation, on the other hand, involves visualizing the image of an object from other angles, starting from the individual's own position (Clements, 1998). According to McGee (1979), spatial orientation is the task of visualizing the image that occurs as a result of looking at a given shape or object from another angle. Odell (1993) defined spatial relationship as the ability to engage quickly and accurately with the mental rotation process.

Based on the examination of previous studies, it was observed that quantitative studies (Alias, Black ve Gray, 2002; Buckley, 2018; Kök, 2012) in which spatial thinking were investigated from different perspectives were more common while qualitative studies were limited. Hence, it is thought that this study will be important in diversifying the qualitative studies conducted in this line. In addition, although there are studies in the literature examining real life problems from different perspectives, no study has been found on real life problems that require spatial thinking skills. It is thought that the research will contribute to the literature in this respect. Answers to the following questions were sought in the research.

1. How do eighth grade students solve real-life problems involving spatial visualization skills?

2. How do eighth grade students solve real-life problems involving spatial orientation skills?

3. How do eighth grade students solve real-life problems involving spatial relationship skills?

Method

Model of the Research

Qualitative research approach was adopted in this study to examine in-depth how eighth grade students solved real-life problems that required spatial thinking skills. Hancock and Algozzine (2006) defined case studies as deeply grounded studies that try to richly describe events occurring in their natural conditions within a specific time and place, using various data collection tools.

Participants

The participants of the study consisted of 15 volunteer students studying in the eighth grade in a public school in the Mediterranean Region of Turkey. The personal information of the participants, who were determined by adhering to the principle of voluntary participation, was kept confidential by assigning a code name to each participant and the collected data was presented in this manner. The relevant school was selected via the convenience sampling method, one of the non-random sampling methods, to save time and workforce, while the criterion sampling method, a non-random purposeful sampling methods, was used in the selection of the students studying in this school. Criterion sampling is used to study and review all situations that meet some predetermined significance criteria, and the criteria in the sample can be prepared by the researcher or may have been prepared beforehand (Başaran, 2017). In this research, attention was paid to selecting the participants from the voluntary candidates with the following criteria: being in the abstract operational stage (Piaget, 1971) to be able to realize spatial thinking skills and having previously studied subjects that included spatial skills at previous grade levels such as geometric objects and the appearance of objects from different angles.

Data collection tool

A form prepared by the researcher was used as a data collection tool in the study. The form consisted of questions related to the 3 sub-dimensions of spatial thinking skill, namely spatial visualization, spatial relationship and spatial orientation. Various national and international studies were examined in the scope of this study while developing the questions in the form, and a question pool was created. The questions in this pool were evaluated by an expert and a researcher in line with criteria such as suitability for the adopted theoretical framework, suitability for the purpose of the study and suitability for student levels. After the assessment, the questions that were identified to be suitable for the purpose of the study were re-examined and the real-life problem dimension was added to the questions by the researcher to adapt it to the subject of the research. Polya (1990) described a four-step process in problem solving: understanding the problem, making a solution plan, implementing the plan, and reviewing the solution, respectively. These steps were taken into consideration while preparing the questions and getting student answers. Mostly multiple-choice questions were used to determine spatial thinking in the literature but the chance factor in multiple-choice questions may affect the obtained results. In this study, open-ended questions were preferred in the form because the study aimed to measure students' spatial thinking skill quality without affecting the results by the chance factor. The prepared questions were re-evaluated by the researcher and an expert in the field, and were organized based on the 3 sub-dimensions of spatial thinking skill, namely spatial visualization, spatial relationship and spatial orientation, and the first version of the form was created by assigning one question for each sub-dimension. A pilot study was conducted with this version with with 5 voluntary eighth grade students and necessary revisions were made in the questions according to the data obtained from the pilot study. After this phase, expert opinion was sought again from 6 primary school mathematics teachers working in public schools and 3 academicians who are experts in their fields, and the form was given its final form in this direction.

The sub-dimensions and the questions prepared for each sub-dimension are tabulated below.

Table 1.

Prepared Questions and Sub-Dimensions

Sub-Dimensions	Questions
Spatial Visualization	Lego Question (Winter et al., 1989)
Spatial Orientation	Drone Question (Harput, 2019)
Spatial Relationship	Embellishment Question (Karatağ, 2017)

Data Collection

The data were collected by semi-structured interview technique in this study by using the form developed by the researcher. During the interview with the students, questions that are compatible with Polya's (1990) problem solving steps were asked to the students and notes were taken. Afterwards, the problem solving status of the students was tried to be measured according to these steps. During the student evaluation for the problem comprehension step, the students were asked to express the problem in their own words and to explain what was given in the problem by asking what was asked in the problem. For the plan-making step, students were asked to create a solution plan and explain the plan they created. The students were expected to solve the problem for the step of applying the problem. Finally, for the step of evaluating the solution, the students were asked to check the correctness of the solution and explain the reasons for the method used in the solution process.

After receiving the consent of the participants, the interviews were recorded with a voice recorder to use the data during the analysis. The data of the study were collected in settings where the participants could express themselves comfortably and during the time periods when they were available to meet.

Data Analysis

In the study, one question was asked for each of the spatial visualization, spatial orientation and spatial relationship sub-dimensions. The questions were as follows: a lego question for spatial visualization, a drone question for spatial orientation, and a embellishment question for spatial relationship. A specific rubric was developed by the researcher for each question to evaluate the data. The rubrics developed in this direction were revised by taking expert opinion from three academicians who are experts in their field. While developing the rubrics, the mental processes expected to be occur during the solution process were taken into consideration and added to the rubrics item by item. The first items were selected in parallel with the first step in problem solving, which Polya (1990) also considers as the first step in problem solving steps, namely understanding the problem. The steps of “making a solution plan, implementing the plan and reviewing the solution” were integrated into the statements in the rubric, and these statements were supported by face-to-face interviews and what the participant did in these steps was examined. Each item in the rubric was divided into 3 categories as “Adequate”, “Partly Adequate” and “Inadequate”. While the items were evaluated according to these 3 categories, they were analyzed to be *adequate* if the participant completed the item in full, as *partially adequate* if the participant completed a part of the item but not the majority of the item, and as *inadequate* if the participant could not do anything regarding the item. During the data collection process, the participants were asked some questions and participants' mental processes were transferred to the rubrics in line with their explanations and the drawings they made on paper.

Findings

This section provides the student answers to the questions related to spatial visualization, spatial relationship and spatial orientation sub-dimensions.

Findings Regarding First Sub-Problem

The "Lego question" was used to evaluate students’ spatial visualization skills in the first sub-problem of the research. Table 2 shows the student answers to the Lego question.

Table 2.

Student Answers to the Lego Question

Items	Adequate	Partly Adequate	Inadequate
Realizing/understanding how the shape will look after adding a Lego piece.	Kerem Kağan Feyza Furkan Ömer Emre Fatih Yasin Ali Umut Melek İsmet Sevgi	Oğuz Sema	
Visualizing what the shape will look like after the Lego piece is added	Kerem Kağan Furkan Ömer Fatih Yasin Ali Umut Melek İsmet Sevgi	Feyza Oğuz Emre Sema	

The ability to draw the parts on the paper before adding the Lego piece (drawing while looking at the shape)	Kağan Furkan Fatih Yasin Sevgi	Kerem Ömer Oğuz Umut Melek İsmet	Feyza Emre Sema Ali
The ability to draw the new Lego addition in the right place (on the drawing)	Kağan Ömer Fatih Sevgi	Kerem Furkan Oğuz Yasin Umut Melek	Feyza Emre Sema Ali İsmet

Analysis of the data for the Lego question showed that most of the students (86.6%) were adequate/competent in the item "Realizing/understanding how the shape will look after adding a Lego piece". This item in the rubric aimed to measure the skill belonging to *understanding the problem* step, which is the first of Polya's (1990) problem solving steps. In general, the students were evaluated as adequate in this item which shows that they generally did not have problems in understanding the question. Student evaluation on this item included the criteria with the following sub-items: students' ability to express the problem in their own words, to analyze the data in the given problem and to determine what is provided and what is asked. For example, the student coded as İsmet for the purposes of this research expressed what was asked in the question in his own sentences and could explain what was provided and asked in the question. The dialogue between the researcher and İsmet is given below.

Researcher: Do you understand the question?

İsmet: Yes, teacher. What Zeynep is doing... She is going to put two more blocks crossing over on the top of the six horizontal blocks. The questions ask how she will put them in this way.

Examination of the rubric item "visualizing what the shape will look like after the Lego piece is added" showed that 73.3% of the students were evaluated as adequate. This item was prepared to evaluate the students based on plan-making step, which is the second of the Polya's (1990) problem solving steps. Successful implementation of the planning step requires identifying the appropriate strategy for solving the problem. If the appropriate strategy required for the solution is correctly identified during problem solving, the probability of achieving the correct result will increase. In order to evaluate the students' performance in the plan-making step in the Lego question, attention was paid to whether the students made a plan suitable for the solution of the problem and how they expressed this plan in using mathematical concepts in their sentences. During the interview, the students were asked how they created a solution plan to solve this problem and whether they could visualize the shape. Their answers were recorded and recorded in the relevant item in the rubric.

The other items in the rubric, "the ability to draw the parts on the paper before adding the Lego piece (drawing while looking at the shape)" and "the ability to draw the new Lego addition in the right place (on the drawing)" are the items that measure competence in the plan implementation step, the third of Polya's (1990) problem solving steps. The student responses to these items were included in the rubric and the drawings made by the students on the paper and the dialogues between the researcher and the students during the interview were used to assess students' adequacy in these items. The rate of students who were evaluated as adequate in these two items was 33.3% and 26.6%, respectively. Therefore, the students could not display the same achievement rate in implementing the plan as they had in the other steps (understanding the problem and making a plan). There may be more than one reason for the lack of achievement in this steps such as the inability to fully perceive the visual and therefore not being able to implement what is required, inability to apply the strategy correctly even if the identified strategy is correct, and the fact that the implementation step is mostly related to drawing. Although there may be various reasons for the low achievement in these items compared to the first items in the rubric, its relation to drawing skills might be the strongest cause among these. As a matter of fact, even if most of the students were successful in understanding and visualizing the problem, it was noticed during the interview that they had difficulty in transferring it to the drawing which was also expressed by the students. In this context, students expressed the designs they visualized in different ways because they had difficulty in drawing the imagined shape in three dimensions (Figure 1 and Figure 2). In this respect, a dialogue held with the student coded Ali is provided below.

Ali: Move two more unit squares and put a unit cube on the left. Is it okay if I write it down?

Researcher: Sure.

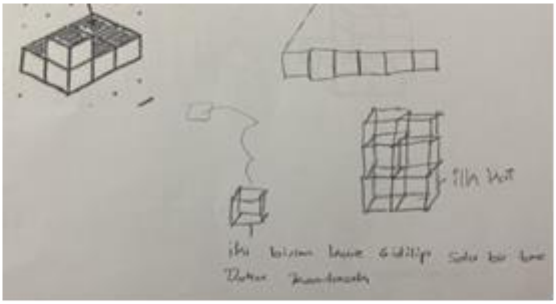

Figure 1.

Ali's response to the Lego question

During the interview, the students who completed the implementation step were asked whether their solution was correct or not in order to evaluate their performance in the evaluation step and they were asked to check their solutions. As a result, students coded Kağan, Ömer, Fatih and Sevgi, who were evaluated as adequate in all items in the rubric, stated that their solutions were correct.

In order to confirm, the student coded Kağan added one unitcube to the desired place on the existing shape and then compared it with the shape he drew and said that it was correct; Sevgi, Fatih and Ömer reached this conclusion by rereading the question and checking their drawings. Some of the other students stated that they understood the question but could not visualize it in their minds, so they could not solve it, and therefore their solutions were not correct. Although it was observed that some students reread the question and tried new drawings afterwards, it was noticed that they said that their solutions were incomplete or incorrect. These results showed that the students were objective while checking their own solutions in the evaluation step of, and that they were able to realize their errors, shortcomings and correct answers.

Findings Regarding Second Sub-Problem

The second sub-problem of the study investigated students' spatial orientation skills based on "Drone question". Table 3 shows the student answers to the Drone question.

Table 3.

Student Responses to the Drone Question

Items	Adequate	Partly Adequate	Inadequate
Realizing that the question is about top view of the toy	Kerem Kağan Feyza Furkan Ömer Oğuz Emre Fatih Sema Yasin Ali Umut Melek İsmet Sevgi		
Visualizing what the top view of the toy would be like	Kerem Kağan Furkan Ömer Oğuz Fatih Yasin Ali Sevgi	Feyza Emre Sema Umut Melek	İsmet
	Kerem		Feyza

Thinking that the top view of the toy will be round	Kağan Furkan Ömer Oğuz Emre Fatih Sema Yasin Ali Sevgi		Umut Melek İsmet
Thinking that there will be a black circle in the middle of the circular shape that will be seen from the top	Kağan Furkan Ömer Oğuz Fatih Yasin Ali Sevgi	Kerem Sema	Feyza Emre Umut Melek İsmet
Visualizing that 2 black and 3 white circles will be formed in addition to the black circle in the middle and that these circles should be arranged as one black and one white.	Kağan Furkan Ömer Oğuz Fatih Yasin Ali Sevgi	Kerem	Feyza Emre Sema Umut Melek İsmet
Thinking that black circles should	Kağan Furkan	Kerem Yasin Ali	Feyza Ömer Oğuz

be thinner than white circles		Melek Sevgi	Emre Fatih Sema Umut İsmet
Drawing the visualization on paper	Kağan Furkan Ömer Fatih Yasin Ali Sevgi	Kerem Oğuz Melek	Feyza Emre Sema Umut İsmet

Examination of the data in the rubric prepared for the drone question showed that all of the students (100%) were evaluated as adequate in the item, "Realizing that the question is about top view of the toy", which belonged to the problem comprehension step. Factors such as students' understanding of what is asked in the problem, determining what is given in the problem and expressing the problem in their own sentences were taken into consideration to make this evaluation. It was observed that the students had no difficulty in expressing the problem in their own sentences, determining what was given and what was required, and thus they were successful in the problem comprehension step. In this direction, for example, the student coded Kerem said, "Teacher, the child's father buys a gift for him as a report card gift, a drone. Well, the child takes a photo from above this object and asks what appears on the object and I will draw it."

Evaluation of the plan-making step, which is another problem solving step for the drone question, showed that the achievement in this step was lower compared to the achievement in the step of understanding the problem. Five of the items in the rubric were directly related to the planning step. During the evaluation of these items, observations were made about how the participants created a solution plan, whether the solution plan they created was suitable for solving the problem, and how they expressed this plan in mathematical concepts. Analyses of items in the rubric showed that the students who were considered adequate in the item "Imagining how the top view of the toy would look like" were 60%. However, 73.3% of the students were considered adequate in the item "Imagining that the top view of the toy will resemble a circle". Based on this result, it is possible to say that although the students experienced uncertainties while imagining how the top view of the toy would look like, they were more successful in thinking that the top view would look like a circle. The other items in the rubric detailed the solution plan of the drawing expected to be made in the implementation step. For these items, it was observed that 53.3% of the students were evaluated as adequate in the items "Thinking that there will be a black circle in the middle of the circular shape that will be seen from the top " and " Visualizing that 2 black and 3 white circles will be formed in addition to the black circle in the middle and that these circles should be arranged as one black and one white.", and the students' assessments of these two items were similar. Of the students, 13.3% was found to be adequate in the item, "Thinking that black circles should be thinner than white circles". The fact that the percentage of students evaluated as sufficient in this item was lower than the other items in the rubric may be related to the fact that the item required more detail in the plan made for the solution. Since the top of the shape given in the question was pointed, it was determined that the students had some confusion about the top view of the shape during the interview. During the interview, it was determined that while planning the solution, some of the students thought that the top view of the toy would be like the one given in the question. This solution plan made by the students was also reflected in the drawing they made in the implementation step. The explanation made by the student coded Feyza supports this: "Since the top will be more prominent, only the top should be visible only in this way. I don't think the bottom part will be very visible."

Although it was observed that some of the students made one-dimensional drawings that resembled a circle in general, it was determined that they said that the shape should be round when describing the shape they imagined in the planning step. Based on this result, it can be argued that some students defined circular models as round in general and that they have some misconceptions in this regard. It was concluded that even if the students were successful in the planning step and in parallel with this, the visualization step; they might have errors in expressing this in mathematical concepts.

The last item in the rubric, "Drawing the visualization on paper", was prepared for the step of implementation. It was observed that 46.6% of the students were evaluated as adequate in this item based on their drawings and explanations on the form. Some students were found to be able to draw the one-dimensional shape with general outlines even if they missed some details of the shape they drew. This result can be considered as a direct reflection of the accuracy or inaccuracy of the plans made by the students in the planning step on the implementation step.

During the interview, it was determined that the student coded İsmet made more than one drawing. When the reason was asked, the student replied, "Teacher, this shape should be in front of us, I can't do it this way." This answer can be considered as an indication that the problem the student had in imagining/visualizing in the planning step was also reflected in the implementation step. It was observed that the student made more than one drawing on the paper. It can be argued that this mistake made in the plan implementation step is due to the student's indecision while making plans for the solution. The drawings made by the student are as shown in the figure (Figure 2).

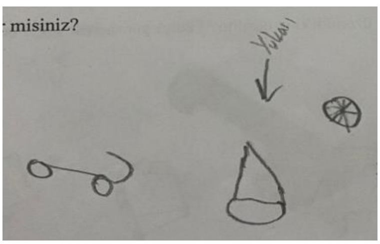

Figure 2.

İsmet's response to the drone question

It was found that some of the students thought that their solutions were correct and some of them said that their solutions could be wrong in the evaluation step, the last step of the problem solving steps. It was determined that while the students whose solutions were correct thought that their solutions were correct, but some of the students who made incorrect solutions also believed that their solutions were correct. In general, when the students' performance in evaluating the solution of the drone question was analyzed, it was observed that the students who were successful in the steps of understanding the problem, making a plan and implementing the plan maintained this success in the step of evaluating as well; some students who had shortcomings in other steps accepted this, while some students evaluated their incorrect solutions as correct.

Findings Regarding Third Sub-Problem

The third sub-problem of the study investigated students' spatial relationship skills via the question on "reflection embellishments". Table 4 shows the student answers.

Table 4.

Student Responses to the Embellishment Question

Items	Adequate	Partly Adequate	Inadequate
Realizing that the question required them to engage in 4 reflection operations	Kerem Kağan Feyza Furkan Ömer Oğuz Emre Fatih Sema Yasin Ali Umut Melek Sevgi	İsmet	
Visualizing the shape for each of these reflection operations	Kerem Kağan Furkan Ömer Oğuz Emre Fatih Sema Yasin Ali Umut	Feyza İsmet	

	Melek Sevgi	
Correctly drawing the visualized shapes on paper	Kerem Kağan Furkan Ömer Oğuz Emre Fatih Sema Yasin Ali Umut Melek Sevgi	Feyza İsmet

Examination of the data in the rubric showed that 93.3% of the students were evaluated as adequate for the item "Realizing that the question required them to engage in 4 reflection operation " in relation to understanding the problem, the first of Polya's (1945) problem solving steps. The obtained results showed that the students did not have much difficulty in understanding the problem. While the students' answers to the embellishment question were evaluated for this item, criteria such as determining what is given and required in the problem, expressing the problem in their own sentences were used and the findings were recorded in the rubric. In this context, some of the explanations made by the students about what was asked in the question during the interview were noteworthy. For example, it was observed that the student coded Melek used expressions such as "We will repeat the shapes five times in a row" and the student coded Umut used expressions such as "The researcher wants us to turn them upside down/reverse them". Although there were partial shortcomings in the students' definitions of the reflection process, it was observed that they did not have any problems in understanding the question. This finding was also supported by the student performances observed in the steps of making a plan and implementing the plan. The student coded İsmet, who was thought to be partly adequate in understanding the problem, said after reading the question "I think the researcher is asking how it will look when we reflect the shape from the right and left". Based on this sentence, it was seen that the student thought that the problem asked how the shape would look as a result of different reflection processes, and based on this, he was evaluated to be partly adequate in the step of understanding the problem.

During the evaluation of the item in the rubric "Visualizing the shape for each of these reflection operations", the planning made by the student and the expressions used while explaining the plan were taken into consideration since this item was related to the planning step. In this direction, it was found that 86.6% of the students were accepted as adequate. It can be argued that because the student coded İsmet was evaluated as partly adequate in understanding the problem, he was partly adequate in the planning step as well. Since the student had problems in understanding the problem, his plan for solving the problem was not sufficient at the expected level. It was also noticed that the student coded Feyza confused the word reflection with the word geometric translation, and therefore was evaluated as partly adequate. Based on this result, it can be argued that shortcomings such as lack of student knowledge about the definitions of the mathematical terms given in the problem, using different terms interchangeably, and having some misconceptions in the problem solving process negatively affected the performance in the planning phase.

The last item in the rubric for the question on embellishment, "Correctly drawing the visualized shapes on paper", was prepared to measure the third step of problem solving, the step of implementing. It was found that 86.6% of the students were evaluated as adequate in this item. The fact that the students achieved similar rates of success in the previous rubric item (planning) was related to the fact students experienced no problems in the implementation of the plan to solve the problem. Lack of problems in implementing the plan may be related to the fact that the shape to be reflected was not three-dimensional. Since the solutions proposed by the students were taken into consideration to evaluate students' activities in the plan implementation step, the drawings made by the students on the form were also effective in evaluation. It was observed that although there were some shortcomings in the participants' definitions of reflection, their knowledge of the reflection process was sufficient and thus they were able to carry out the implementation step correctly.

Based on the examination of the response provided by the student coded Feyza, who was evaluated as partly adequate in the last two items in the rubric (Figure 3), it can be argued that the student's error stemmed from assuming that reflections were translations.

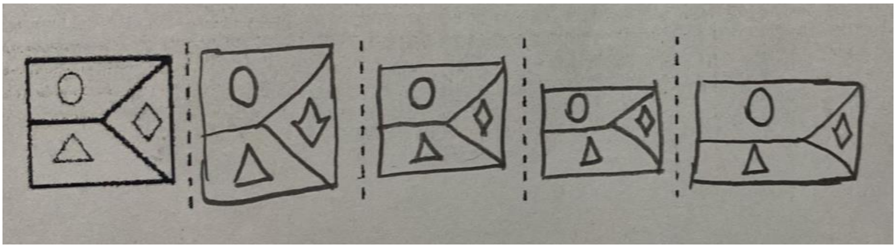

Figure 3.

Feyza's response to the question on embellishment

Student evaluation on implementation step regarding embellishment included asking students, to check the solution they proposed and to evaluate whether the solution was correct or incorrect. As a result, it was observed that some of the students rreead the question and reviewed their drawings, while others checked their drawings without rereading the question. It was observed that most of the students accepted their solutions as correct. It was noticed that some students erased their drawings and drew more carefully, trying to draw the lines and shapes in their drawings more perfectly without affecting the accuracy of the solution. Based on these results, students' performance in evaluating the accuracy of the solution of the question on embellishment can be considered successful in general.

Conclusion and Recommendations

This section presents the findings by supporting them with the findings from the literature and offers some recommendations regarding the study which investigated how 8th graders solved real life problems requiring spatial thinking skills.

Spatial thinking skills were examined under 3 sub-headings: spatial visualization, spatial relationship and spatial orientation, and questions that were asked to students were related to these sub-headings. Students' solutions to the problems were evaluated with a rubric for each question. The first items in the rubrics were chosen in parallel with the part of understanding the problem, which Polya (2014) also considered as the first step in problem solving, and aimed to measure how the questions were understood. When the data in the rubrics were examined, it was seen that the students did not have much difficulty in understanding the question.

The fact that most of the students correctly answered the questions asked by the researcher about understanding the question, and that they explained the question in their own words and summarized the question in a more understandable way shows that the students are generally at an adequate level in understanding the question. However, another result obtained in the study showed that even if the students were successful in visualizing the desired shapes, they could not achieve the same success in making a plan to solve the problem and putting it into practice. It was determined that the students followed the connotations brought by the question, but overlooked the details of the answer at the stage of constructing the solution. Especially when drawing complex structures and three-dimensional shapes, the students' difficulties in some situations and their indecision about whether the view would be three-dimensional or two-dimensional drew attention as a reflection of this situation. In other words, it can be argued that students had difficulty in developing the right perspective. Kepceoğlu and Ercan (2018) stated that preferring open-ended questions instead of multiple-choice questions in determining spatial thinking skills decreases the success and one of the reasons for this is related to the difficulties experienced by students regarding drawing shapes. This finding is in parallel with this study. As a matter of fact, it is thought that students will be more successful in the questions when they combine their existing spatial thinking skills with their drawing skills. Student sentences such as "I can visualize it in my mind, but I cannot draw", making changes on the shape given in the question instead of making three-dimensional drawings, describing the shape they drew by drawing an arrow over the given shape, and using verbal expressions instead of drawing the shape support this view. The reason for this may be related to allocating less time to activities or questions that will activate students' drawing ability in the lessons on geometry subjects and using multiple-choice questions while selecting questions on these subjects. In this regard, the results obtained in the study titled T*he Effect of "Mathematics and Art" Course Activity Applications On Students' Spatial Talents in Izmit Science and Art Center* by Alyeşil-Kabakçı and Demirkapı (2016) are parallel with the findings of the present study. In the aforementioned study, mathematics subjects relevant to spatial skills were taught in the classroom with the help of interdisciplinary connections, without using rote memorization, together with visual arts and applied activities, and students had the opportunity to develop their spatial abilities by establishing relationships between mathematics and art. The results of the study show that mathematics and art activities positively affect the development of students' spatial abilities.

Umay (1992) stated that the importance of the process in problem solving is overlooked because success is measured by multiple-choice questions. However, how the problem is made sense of is at least as important as the solution process. Understanding how the process progresses in problem solving is important in terms of determining at which stages students have difficulties in problem solving. Özsoy (2005) stated that mathematics achievement is related to problem understanding, planning and controlling skills, and is highly related to the ability to implement the plan. It is predicted that success especially in non-routine, real-life problems, and the ability to use mathematical processes correctly will increase mathematics achievement. Indeed, Seligman (2007) found that students learned more about mathematics as a result of solving non-routine mathematical problems.

Interviews with students in this study revealed that students were successful at the problem comprehension stage, but they failed to show the similar success in the following processes. Students experienced various difficulties in the planning and implementation stages, and they mostly ignored the stage of evaluating the result and providing the solution. Gökkurt et al. (2015) also found that students failed in Polya's problem solving stages of understanding the problem, planning, and evaluation. Hence, it is important to include activities for more effective use of Polya's (1945) problem solving steps, especially the evaluation step in the problem solving process. When this study is analyzed in terms of students' spatial skills, it is seen that the difficulties experienced after the problem comprehension stage are due to some deficiencies in spatial skills. The students' inabilities to determine the appropriate strategy for the solution led to difficulties in implementation. There are many studies in the literature on developing spatial thinking skills. These studies revealed that spatial thinking skills can be developed by using virtual environments, differentiated geometry teaching, and concrete objects (Clements and McMillen, 1996; Sünbül and Yurt, 2012; Yolcu and Kurtuluş, 2010; Yurt 2012; Kök 2012). Kurtuluş (2011) stated that painting, drawing, and construction activities have positive effects on spatial skills and mathematics achievement. Olkun (2003) and Kayhan (2005) stated that technical drawing practices support spatial skills.

Within this scope, it can be suggested to emphasize differentiated geometry teaching and using virtual environments and concrete objects in the course to support the development of spatial thinking skills. In addition, it is thought that technology-assisted teaching and using technology-supported software will be beneficial in this regard. Other suggestions include providing cooperation between visual arts and mathematics courses in some subjects, putting emphasis on open-ended as well as multiple-choice items in questions related to spatial thinking skills not to limit students solely with the options in multiple-choice questions and using questions integrated with real-life problems in the lessons.

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