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SOLUTION OF THE (2+1) DIMENSIONAL BREAKING SOLITON EQUATION BY USING TWO DIFFERENT METHODS

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Abstract
In this study, the direct integration and homotopy perturbation method are used for the non-linear partial differential (2+1) dimensional breaking soliton equation. By assigning some special values to the constants in the solutions of the (2+1) dimensional breaking soliton equation, The direct integration was used for obtaining the known solution in the literature in practical and shortest way. By using the homotopy perturbation method with one iteration, it was obtained same type solution to (2+1) dimensional breaking soliton equation. Similarly, same type solutions could be done in different methods such as (G'/G)-expansion method.

Keywords: Breaking soliton equation, homotopy perturbation method, direct integration .

1. Introduction

Breaking soliton equations can be implemented to the model (2+1) dimensional interaction of Riemann-wave and long-wave propagations in engineering and sciences. (2+1) dimensional breaking soliton equation is a kind of different (2+1) dimensional KdV extensions in the following form [1]:

$$u_{xt} - u_{xxxy} - 2u_{xx}u_y - 4u_x u_{xy} = 0 \qquad (1)$$

where $u = u(\mathrm{x}, \mathrm{y}, \mathrm{t})$. This equation was first revealed in [2-3]. In [4] paper it was investigated (2+1)-dimensional breaking soliton equation (1) via the generalized tanh method. In [5] paper using computerised symbolic computation were obtained for (2+1) dimensional breaking soliton equation. This equation that is given in Eq (1) have been investigated by many authors

[6-18]. In this study, we use the direct integration to exact solution of the (2+1)-dimensional breaking soliton equation
In addition, we investigate same type solution for (2+1) dimensional breaking soliton equation by using the homotopy perturbation method with one iteration.

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2. Solution of the breaking soliton equation by using direct integration

In this section, we will present to exact solution of the (2+1) dimensional breaking soliton equation [1] given in Eq (1) by using the direct integration. Seek solution of Eq (1) by taking $u(\mathrm{x}, \mathrm{y}, \mathrm{t}) = \mathrm{U}(\eta)$, $\eta = k_1 x + k_2 y + vt$ and transforming Eq (1) to the ordinary differential equation

$$k_1^2 k_2 U^{(4)} - vU^{''} + 6k_1 k_2 U^{''} U^{'} = 0 \qquad (2)$$

where prime denotes the derivative with respect to η. Integrate Eq (2) term by term one time

$$k_1^2 k_2 U^{(3)} - vU^{'} + 3k_1 k_2 (U^{'})^2 + c = 0$$

where c is an arbitrary integration constant. For simplicity, the integration constant can be set to zero. Defining the transformation $U^{'} = w$ we get

$$k_1^2 k_2 w^{''} - vw + 3k_1 k_2 w^2 = 0 \qquad (3)$$

Integrate Eq (3) term by term one time

$$(w^{'})^2 - \frac{v}{k_1^2 k_2} w^2 + \frac{2}{k_1} w^3 + d = 0$$

where d is an arbitrary integration constant. For simplicity, the integration constant can be set to zero. Thus, we have

$$w^{'} = \mp \sqrt{\frac{v}{k_1^2 k_2} w^2 - \frac{2}{k_1} w^3} \qquad (4)$$

Eq. (4) can be written as follows for plus signature (similar calculation can be done for negative signature):

$$\int \frac{dw}{\sqrt{\frac{v}{k_1^2 k_2} w^2 - \frac{2}{k_1} w^3}} = \int d\eta \qquad (5)$$

By the integrating right side of this equation (5), we get

$$\int \frac{dw}{\sqrt{\frac{v}{k_1^2 k_2} w^2 - \frac{2}{k_1} w^3}} = \eta + e \qquad (6)$$

we choose integration constant $e = 0$ where e is a new arbitrary integration constant. In this case, the solution of the Eq. (6)

$$w(\eta) = \frac{v}{2k_1 k_2}(1 - \tanh^2[\frac{\sqrt{v}\eta}{2k_1\sqrt{k_2}}])$$

Finally, $w(\eta)$ is integrated. The exact solution of the (2+1) Dimensional Breaking Soliton Equation given in Eq. (1) is

$$U = \sqrt{\frac{v}{k_2}} \tanh[\frac{1}{2k_1}\sqrt{\frac{v}{k_2}}\eta] \qquad (7)$$

3. Homotopy perturbation method

For the utility of the reader, we will introduce HPM [19]. The following nonlinear differential equation:

$$A(u) - f(r) = 0, \ r \in \Omega \qquad (8)$$

with boundary conditions

$$B(u) - \partial u / \partial n = 0, \qquad r \in \Gamma$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. The operator A can be divided into two parts L and N,

$$L(u) + N(u) = f(r), \ r \in \Omega$$

where L is linear and N is nonlinear. It was constructed a homotopy $v(r, p) = \Omega \times [0,1] \to R$ in paper [19]. v satisfies

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \qquad (9)$$

where $p \in [0,1]$ is embedding parameter and u_0 is an initial approximation of Eq. expressly we have

$$H(v,0) = L(v) - L(u_0) = 0$$

$$H(v,1) = L(v) + N(v) - f(r) = 0$$

where $L(v) - L(u_0)$ and $A(u) - f(r)$ are called homotopic in topology. According to the homotopy perturbation technique, we can be written the solution Eq (9) as a power series in p small parameter:

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots$$

Setting $p = 1$ results in the approximate solution of Eq. (8)

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots$$

4. Application of homotopy perturbation method for (2+1) dimensional breaking soliton equation

It is well known that the homotopy perturbation method in [18-22], we use for Eq. (10). We compose a homotopy in the form

$$w'' - \frac{v}{k_1^2 k_2} w + \frac{3}{k_1} p w^2 = 0 \qquad (10)$$

where $p \in [0,1]$. In Eq.(4) there are two observations to be considered; the first observation is that for $p = 0$, Eq.(10) becomes a linear equation. The second observation is that for $p = 1$; it becomes the original nonlinear one. Due to the homotopy perturbation method, the solution of Eq.(10) can be expressed in a series of p :

$$w = w_0 + pw_1 \qquad (11)$$

$$\frac{v}{k_1^2 k_2} = h^2 + ph_1 \qquad (12)$$

Substituting Eqs (11) and (12) into Eq. (10) and equating coefficients of like powers of p yields a linear equation:

$$w''_0 - h^2 w_0 = 0 \qquad (13)$$

Solving of Eq. (13)

$$w_0(\eta) = c_1 e^{h\eta} + c_2 e^{-h\eta}$$

This solution is determinant for the $w_0(0)=x,\ \ w_0^{'}(0)=y$ initial conditions, where $y=0$. We get

$x=\frac{1}{2}\sec h^3(\mathrm{h}\,\eta)$. Therefore, we obtain

$$w_0(\eta)=\frac{1}{2}\sec h^2(\mathrm{h}\,\eta)$$

Finally, $w_0(\eta)$ is integrated. Approximate solution of the breaking soliton equation given in Eq. (1) is

$$U=\frac{k_1}{2}\sqrt{\frac{k_2}{v}}\tanh[\frac{1}{k_1}\sqrt{\frac{v}{k_2}}\eta]$$

5. Conclusions

In this study, the direct integration was used to obtain exact solution of the (2+1) dimensional breaking soliton equation. On the other hand, the homotopy perturbation method was used to obtain the perturbative solution of the (2+1) dimensional breaking soliton equation. We compared the results obtained with both methods. Obtained results from direct integration and the homotopy perturbation method are the same with tanh solutions. Furthermore, obtained solution from the direct integration is identical with solution in the article of Arbabi and Najafi (2016) [1].

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