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A Computational Analysis of Long Transfer Line Behavior

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Highlights

- This paper focuses on the convergence rate of the decomposition method for long transfer lines.
- The system behavior is examined by applying different parameters for the production rate.
- Additional tests are performed for the production rate as well as the profit value and WIP values.
- Good initial buffer configurations have been generated for the design of long transfer lines.

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Abstract

Meeting customer demands for order-based production and make-to-stock production policies against holding and non-holding costs are fundamental functions for businesses to ensure. For these policies, finite capacity buffers between machines is of great importance. WIP, production rate and profit values, the key performance indicators of the transfer line, affect the sustainable economics of companies. It is important to investigate how the production rate, one of the most important performance indicators, and its CPU time are affected by the reliability parameters of the machines, the convergence rate and the analytical methods applied. In this study, the theoretical computational convergence analysis of the Dallery-David-Xie (DDX) algorithm is conducted on balanced transfer lines consisting 20, 30 and 50-machines with four different reliability parameters, each having finite buffers. The results show that the performance of the DDX algorithm is very sensitive to the convergence rate. The CPU times spent based on the different convergence rates used in the applied DDX algorithm significantly differ from each other at a 95% confidence interval. Additionally, the study investigates uniformly, ascending order and descending order buffer distributions to maximize the profit value and minimize WIP in the transfer line. The initial buffer configuration affects the key performance indicators on balanced transfer lines with different reliability parameters.

1. INTRODUCTION AND LITERATURE REVIEW

The analysis and design of production lines require decisions at the tactical and strategic levels. Line designers and production line engineers can make relatively long-term decisions only by evaluating production line performance indicators with the most appropriate method and achieving the best line configuration [1]. While simulation is an important tool for evaluating complex production lines from the simplest line design, its use is limited by license fees, specialized knowledge, large replication numbers and long run/execution times. However, various analytical methods can be used depending on the production line topology (i.e., transfer, assembly, split/merge, parallel), line size (small, medium, long), line reliability, line balance, part flow pattern and time distribution [2]. While exact analytical methods based on Markov chains have been proposed to calculate performance indicators accurately and quickly for small lines, various approximate analytical methods based on several statistical distribution assumptions have been used for long lines [2]. Most of these analytical methods are iterative and stop when the performance indicators reach a specific convergence rate. The convergence rate, which is usually determined by the line designer or engineer, may vary slightly depending on the line size, failure-repair times, processing times of machines, distribution of these times and size of the buffer areas. Because the analytical methods are deterministic, for a given constant convergence rate, the same production rate is produced each time these methods are run on the same production line. Evaluations

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of the performance of transfer lines of small, medium and long sizes have received much attention in the literature [2]. In particular, the calculation of line performance in a reasonable amount of time employing analytical methods for increasing the number of machines has attracted much attention [2, 3].

A long transfer line is a flow (serial, tandem) line with 20 or more machines lined up consecutively with buffer areas between adjacent machines. Large-volume and complex products are usually produced on long transfer lines, which require various process types. However, the methods developed for performance calculation are applied to lines with 20 machines to prove that they are efficient [2]. Increases in production rates and revenues have been achieved through the analysis of complex and long production lines of large companies such as Hewlett Packard, PSA Peugeot Citroen and General Motors [1]. For this justification, the sensitivity of evaluation methods for analyzing long transfer lines becomes more important.

When one or more machines fail, they cannot process the parts and require repair before they can process them again. A machine can have three states on any production line: busy (operate), failed (under repair), and idle (blocked or starving) [3, 4]. Since a machine can fail only when it is in operation, it is also under repair when it fails. A blocking after-service mechanism is applied for parts waiting in machines. This means that a part that has been processed on one machine is transferred only when the next buffer area is available and the machine starts processing another part, if available [2]. A machine is idle if it is blocked or starving; otherwise, it is under repair or busy. Successive busy states are considered a single busy state (see Figure 1). The machine with the highest average busy time is the highest bottleneck [5]. Buffers are used to reduce the number of idle states (blocked and starving), and consequently the idle time of one or more machines. If the size of the buffer areas to be assigned to a production line is fixed, more buffers are allocated around bottleneck machines [6].

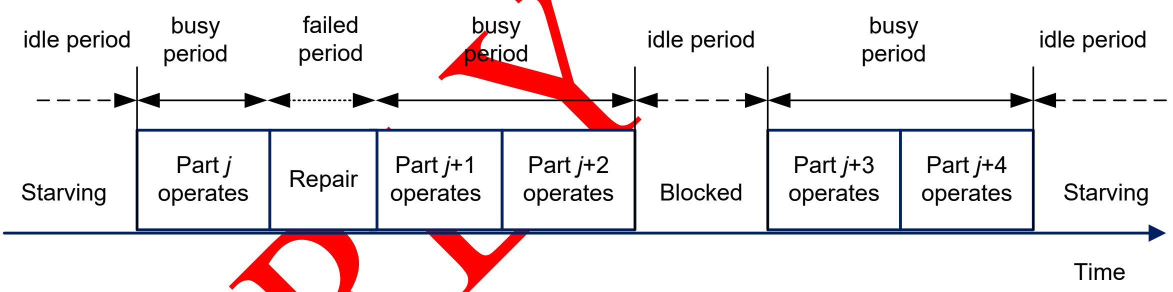


***Figure 1.** All States of a Machine (Adapted from the study of Roser* et al. *[5])*

Due to the highest production volume and the most common line structure, approximate analytical methods have focused more on transfer (tandem queueing) lines, also known as serial production lines. Systems in which workpieces move from one station to another in sequence, such as manufacturing, chemical processes, computer operations, etc., are called transfer systems, while in production, they are called transfer lines [7]. In transfer lines, workpieces arrive at the first machine within a certain arrival time. In production line analyses, it is generally assumed that part of the arrivals to the first machine are intermittent. After the workpieces have been processed on the first machine, if any, they move to the second machine through the buffer area, known as the physical storage area [8]. In this way, parts are processed on all machines and then stored away from the last machine. The time between when a workpiece enters the first machine and when it leaves the last machine is called cycle time [9]. The cycle time is minimized to maximize the number of parts produced per unit time, i.e., the production rate.

In the literature, algorithms for analyzing the performance indicators of transfer lines have been developed approximately half a century ago. Buzacott and Hanifin [10] compared seven different models proposed to estimate the efficiency of transfer lines and presented their formulations. A production system is called unreliable if random failures occur in its subsystems, whereas it is called reliable if no failures occur. Gershwin and Berman [11] proposed an analytical method to calculate the performance of lines with up to two machines, random processing times and limited buffers. Altıok [12] proposed an approximate analytical method to analyze the performance of exponential tandem queueing lines with up to six machines and compared it with the exact analytical method and simulation. Ho and Cassandra [13] analyzed real or near real transfer lines and assembly lines with a discrete flow model based on empirical studies. Using analytical methods consisting of a set of equations based on Markov chains, Gershwin and Schick [7] calculated the exact performance indicators of unreliable, balanced and finite buffer transfer lines with up to three machines. Using the set of equations proposed by [7], a decomposition method was proposed by [14] to calculate the production rate of transfer lines with unreliable, balanced, and finite buffers. The performance of the decomposition method was demonstrated by comparing the obtained results with the simulation ones. Additionally, Gershwin [14] mentioned that the computational complexity of the decomposition method is $O(K^3)$, where K is the number of machines. In the same year, a method called aggregation was proposed by [15]. With this method, the author first considered a line with two machines and then aggregated the rest of the line as a virtual machine to obtain performance indicators for the whole line. The use of the aggregation method and its variations are limited compared to those of the decomposition method [9, 16].

Dallery et al. [17] presented an approximate algorithm, named DDX (Dallery-David-Xie), for the analysis of discrete structure transfer lines with unreliable machines and finite buffers. The same authors (Dallery et al. [18]) proposed an approximate algorithm for the analysis of continuous structure transfer lines with unreliable machines and finite buffers in 1989. In the literature, these two algorithms are known as the discrete flow model and the continuous flow model, in which the machines have the same processing time (homogeneous line). These algorithms rapidly and accurately calculate the production rate considering blocking and starvation phenomena due to machine failures in an unreliable transfer line. Both algorithms are based on an approximate decomposition method proposed by [14] for the analysis of the discrete flow model of large homogeneous transfer lines. A transfer line with K machines is decomposed into K-1 two-machine lines. The failure and repair rates of each machine on the two machine lines downstream and upstream are characterized by the failure and repair rates, respectively. The partial/temporary production rate of each two-machine line is calculated to approximate the production rate of the whole line such that this value for all two-machine lines converges to (below) a certain convergence rate. Gershwin proposed an iterative method called decomposition to solve this equation with 4(K-1) unknown quantities. The DDX algorithm improves the computational efficiency of the method proposed by [14]. The description, formulation, and algorithm of the DDX algorithm are given in section 2.

Lim et al. [19] introduced a performance evaluation method for analyzing paint shops in automobile assembly plants with homogeneous, asymptotically reliable transfer line structures. The aggregation-based asymptotic model developed by the authors represented the production rate as a function of system parameters and demonstrated its efficiency through analytical formulas. Glassey and Hong [3] introduced a method that considers the continuous flow model and analyzed it with the DDX algorithm. Glassey and Hong [3] investigated the relationship between the steady-state behavior of decomposed sublines and the failure and repair rates of sublines, as well as the whole line, showing that the method is computationally efficient as a result of numerical and simulation experiments. Later, Burman [20] extended the DDX algorithm for transfer lines in which machines have different processing times (nonhomogeneous) and proposed an efficient analytical algorithm named *accelerated*-DDX. In his doctoral thesis, Burman [20] successfully used an algorithm that efficiently calculates the production rate of the Hewlett-Packard transfer line and provides financial benefits to a division of Johnson & Johnson. In this way, the production rate of an unreliable, nonhomogeneous transfer line may be estimated very accurately and quickly.

In production lines, machines may fail in either a time-dependent or operation-dependent manner. Hanifin [21] reported that most failures are operation dependent. Le Bihan and Dallery [22] proposed an analytical method to analyze a transfer line, where the processing times of all machines are deterministic and identical, random operation-dependent failures occur, and the failure and repair times are exponentially distributed. The authors (Dallery and Le Bihan [23]) emphasized that the DDX algorithm would not yield accurate results due to reliability parameters such as failure and repair times. Furthermore, in 1999, an improved version of the decomposition method was developed to provide more accurate results for lines with similar characteristics. The approaches proposed to achieve more reliable and accurate results than the DDX algorithm are usually based on the values of the reliability parameters and different strengths of the exponential distribution assumption. Dallery and Le Bihan [23] reported that the proposed method also performs poorly when the buffer size used between machines is too small to cover the repair time.

Li et al. [24] presented eight different two-machine models with assumptions and formulations, classified as synchronous and asynchronous lines with time- and operation-dependent failures. The authors showed that the proposed models exhibit similar performance. Xia et al. [25] extended the analytical methods developed by [23] and [22] for nonhomogeneous lines and proposed a new evaluation algorithm for transfer lines with deterministic processing times and exponential distributions of failure and repair times. Göttlich et al. [26] developed two approaches to evaluate the performance of transfer lines with deterministic processing times and random failures, considering both discrete and continuous flow. The authors showed that the proposed approaches are equivalent in terms of the underlying formulations under the assumption of linearity and can be implemented accurately. Using the method presented by [11], Matta and Simone [27] developed an analytical model for unreliable transfer lines with steady state and multiple failure modes subject to time- and operation-dependent failure. The accuracy of the developed analytical model is compared with that of existing methods. Li et al. [28] proposed two analytical approaches based on the classical decomposition method to estimate the production rate of approximately balanced reliable transfer lines. The authors applied these approaches to lines with identical or optimal buffer values and showed that the experiments yielded accurate production rate values. Decomposition algorithms calculate adequate and reliable production rate values but do not reveal a precise relationship between transfer line performance indicators and system parameters [28].

The characteristics of transfer lines and the applied analysis algorithms are already covered in detail. Especially, the studies in references [3], [14], [18], [19], [20], [22], [23] and [25] are analyzed in the context of the DDX algorithm, which is the focus of this study, and relations/comparisons are performed.

It is necessary to clearly and emphatically state that in this study, no performance comparison is made on any benchmark example taken from the literature; instead, the sensitivity of the DDX algorithm with respect to different input and output parameters is analyzed.

The contributions of this study are fivefold:

i. Considering the limitations of previous papers, this study extensively investigated the sensitivity of the DDX algorithm, which is the most widely used transfer line evaluation algorithm in the literature, in terms of production rate.
ii. It is important to investigate the effect of different buffer sizes and different convergence rates on the production rate obtained with the DDX algorithm [17]. To the best of the authors' knowledge, the performance of the DDX algorithm has not been analyzed in terms of the convergence rate parameter when considering CPU times.
iii. The differences between the production rates (minimum and maximum values) have been analyzed for the first time based on different indicators.
iv. The authors present examples of small and medium-sized transfer lines for which unreliable lines are usually not available. In addition to using existing reliability parameters from the literature, the present study extends them to new benchmark examples involving up to 50-machines.
v. Furthermore, additional experimental tests investigated how to design the initial buffer configuration to reduce the effort of the optimization problems to maximize production rates and profit values and minimize WIP. The production rate, WIP and profit values of the uniform, ascending and descending order buffer configurations are shown in the figures detailed.

The remainder of this study is organized as follows. The details of the DDX algorithm are given in the section 2. The system behavior of long transfer lines is investigated in section 3. Additional experimental tests for performance indicators of transfer lines are provided and discussed in section 3. The study is summarized and several future research directions are presented in section 4.

2. METHOD: ALGORITHM

The decomposition method has logic that can be used adaptively for different objectives in a wide range of fields, from economics to traffic systems and from computer science to management science. In addition, it aims to facilitate the analysis of the whole system by decomposing it into subsystem parts. The decomposition method is known to be an efficient approach for the analytical solution of a large set of dynamical systems without linearization or weak nonlinearity assumptions or other assumptions on stochasticity [29]. The decomposition method was proposed by [14] in the field of management science to estimate the performance parameters of transfer lines, also known as tandem queueing and serial production lines.

The decomposition method for transfer lines comprises three steps [28]:

i. decomposing the original line into K-1 sets of two machine sublines $SL(i)$ for i=1, …, K-1 with some unknown parameters,
ii. deriving 4(K-1) equations to determine the unknown parameters,
iii. developing an iterative algorithmic approach to solve for unknown parameters.

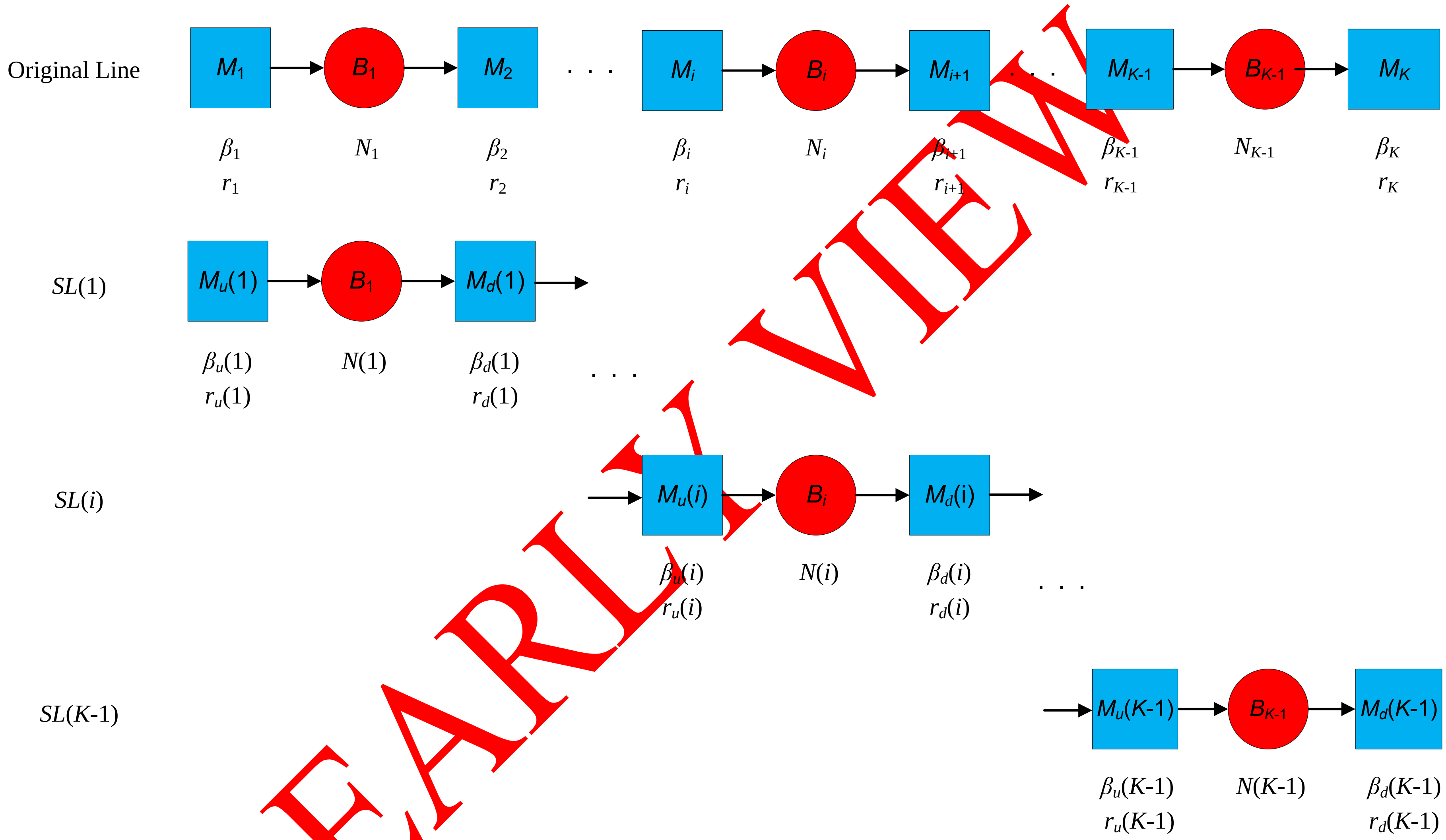


***Figure 2.** Implementation of the Decomposition Method on the Transfer Line [14, 17]*

In Figure 2, the machines are shown with blue squares, while the buffer areas are shown with red circles. Subline $SL(i)$ consists of an upstream machine $M_u(i)$ and a downstream machine $M_d(i)$, which have the same reliability parameters as the original line, and a buffer area $B(i)$, which has the same capacity as buffer B_i in sublines SL and N_i. Thus, the $M_u(i)$ and $M_d(i)$ machines have failure and repair rates $\beta_u(i)$ and $r_u(i)$, and $\beta_d(i)$ and $r_d(i)$, respectively. The method aims to determine these unknown parameters so that the behavior of the discrete material flow in buffer $B(i)$ in subline $SL(i)$ closely overlaps that of the flow in buffer area B_i of subline SL (Figure 2). To determine the unknown parameters of each 2-machine subline, a set of nonlinear equations was developed by [14], and these equations were solved using the *Newton–Raphson* approach as follows:

$$e_i = \frac{r_i}{r_i + \beta_i} \text{ for } i = 1, 2, \dots, K \tag{1}$$

e_i is the isolated efficiency of machine i and is calculated from the failure and repair rate values of machine i (Equation (1)). The value of Equation (1) is also equal to the mean effective service rate on the transfer lines for which the machines' processing times ($\mu_i = 1$ for $i = 1, 2, \dots, K$) are equal, deterministic and one unit.

$$PR(1) = PR(2) = \dots = PR(K-1) \tag{2}$$

Equation (2) is related to the conservation of discrete part flow considering the given convergence rate (Δ). The difference between the production rates of any two-machine sublines is assumed to be less than or equal to Δ. For example, the difference between $PR(1)$, the production rate value of the $SL(1)$ subline in Figure 2, and $PR(K\text{-}1)$, the production rate value of the $SL(K\text{-}1)$ subline, should be less than or equal to Δ while conserving part of the flow.

$$PR(i) = e_i(1 - p_b(i) - p_s(i-1)) \tag{3}$$

Equation (3) calculates the production rate of each subline. In Equation (3), $p_s(i)$ is the probability of starvation of $M_u(i)$ and $p_b(i)$ is the probability of blockage of $M_d(i)$. The DDX algorithm used in this study assumes that there is unlimited supply space in front of the first machine and the first machine never starving. However, there is unlimited storage space after the last machine and the last machine is never blocked.

$$\frac{\beta_d(i-1)}{r_d(i-1)} + \frac{\beta_u(i)}{r_u(i)} = \frac{1}{PR(i-1)} + \frac{1}{e_i} - 2 \text{ for } i = 2, \dots, K-1 \tag{4}$$

In Equation (4), the flow rate-idle time is shown.

$$X = \frac{r_u(i)p_s(i-1)}{\beta_u(i)PR(i-1)} \tag{5}$$

Equation (5) shows that a failure occurring from one or more of the upstream machines (M_{i-1}, M_{i-2}, etc.) causes a starvation of the machine M_i, or the failure of the M_i machine denotes the failure of the upstream machine $M_u(i)$.

$$Y = \frac{r_d(i-1)p_b(i)}{\beta_d(i-1)PR(i)} \tag{6}$$

Similarly, considering the failure of machine, $M_d(i)$ is calculated with Equation (6).

$$I_u(i) = \frac{\beta_u(i)}{r_u(i)} \qquad \text{for } i = 1, 2, \dots, K-1$$
$$I_d(i) = \frac{\beta_d(i)}{r_d(i)} \qquad \text{for } i = 1, 2, \dots, K-1 \tag{7}$$

In Equation (7), the quantities $I_u(i)$ and $I_d(i)$ are calculated by [17] as follows: The equations introduced by [14] can be rewritten as follows using the quantitative values obtained from Equation (7) and the above equations:

$$I_u(i) = \frac{1}{PR(i-1)} + \frac{1}{e_i} - I_d(i-1) - 2 \text{ for } i = 2, \dots, K-1 \tag{8}$$

$$r_u(i) = Xr_u(i-1) + (1-X)r_i \text{ for } i = 2, \dots, K-1 \tag{9}$$

where $\quad X = \dfrac{p_s(i-1)}{I_u(i)PR(i-1)}$

$$I_d(i) = \frac{1}{PR(i+1)} + \frac{1}{e_{i+1}} - I_u(i+1) - 2 \text{ for } i = 1, 2, \dots, K-2 \tag{10}$$

$$r_d(i) = Yr_d(i+1) + (1-Y)r_{i+1} \text{ for } i = 1, \dots, K-2 \tag{11}$$

where $\quad Y = \dfrac{p_b(i+1)}{I_d(i)PR(i+1)}$

The pseudocode of the DDX algorithm by [17] is given as a flow diagram in Figure 3. The DDX algorithm takes as input data the number of machines, buffer configuration, failure rate, repair rate and convergence rate. In Step 1, initialization operations are used to set the reliability parameters of the sublines. Because the difference between the production rate values of any two sublines is greater than the convergence rate, which is initially set as the stopping criterion and usually assigned a value between 10^{-2} and 10^{-6}, the algorithm will proceed to Step 2. In Step 2, the parameters of the upstream machines are calculated using Equations (7), (8) and (9). In Step 3, Equations (7), (10) and (11) are used to calculate the parameters of the downstream machines. This loop of the algorithm is terminated if the calculated convergence rate is reached.

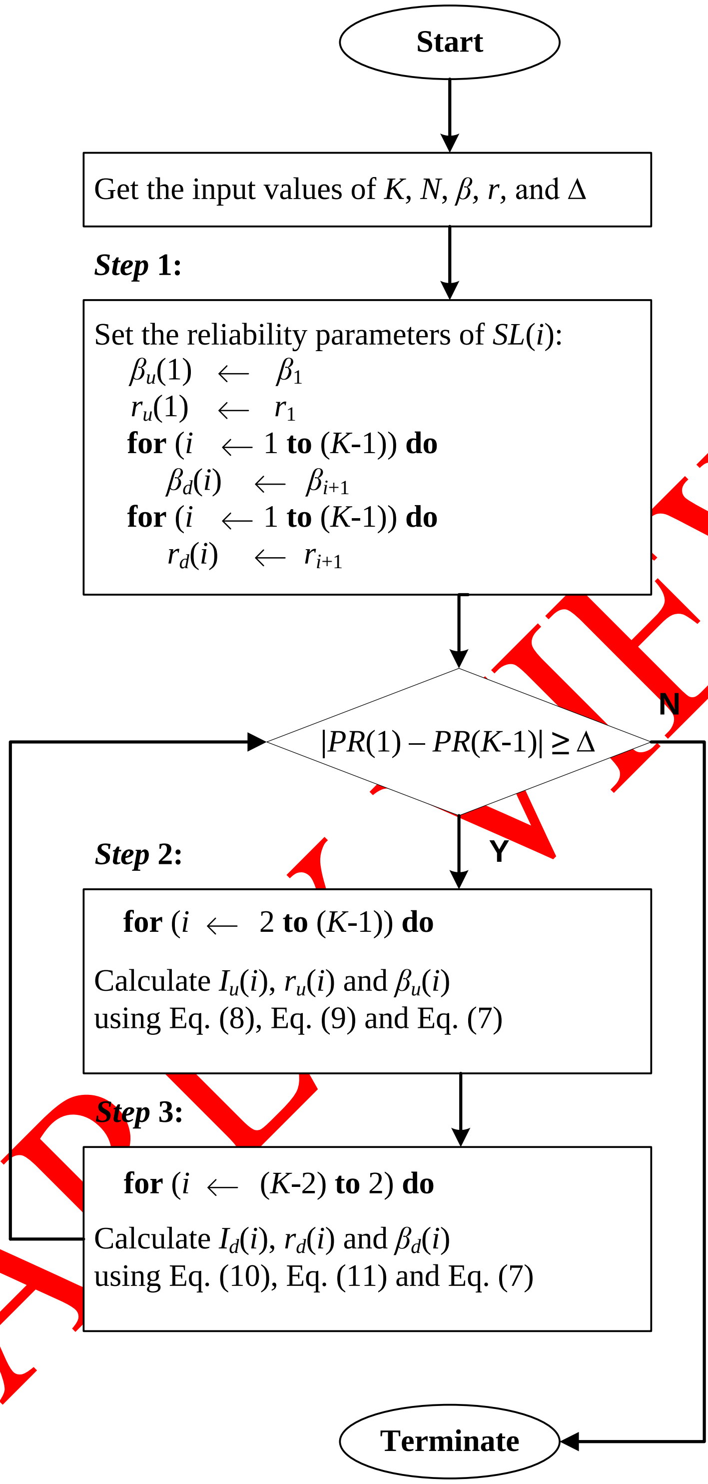


Figure 3. *Flowchart of the DDX algorithm [17]*

The smaller the user-specified convergence rate in Figure 3 is, the more loops the algorithm will perform within itself.

3. SYSTEM BEHAVIOR OF LONG TRANSFER LINES

In this section, we first describe the experimental design and the computer specifications used in the computational study (section 3.1). Then, in section 3.2, computational convergence analyses are performed for various cases. Finally, in section 3.3, the WIP values, production rate and profit value are calculated, and conclusions are drawn from different buffer configurations to provide an initial solution to the optimization problems used in production line design.

3.1. Experiment Design and Computer Specifications

The benchmark instances (extended from references [30] and [31]) in the transfer line performance analysis and buffer allocation problem are used to evaluate the behavior of the DDX algorithm [17]. CPU times, throughput rates and indicators are calculated and analyzed considering different convergence rates for cases with different reliability parameters for balanced transfer lines. The DDX algorithm was implemented in MATLAB. The experiments are conducted on an Intel (R) 2.40 GHz i5 processor with 4 GB of RAM.

3.2. Benchmark Instances and Computational Convergence Analysis

The accurate production rate calculation of the DDX algorithm is tested against reliability parameters retrieved from the literature. In these tests, literature examples of 20, 30 and 50-machines are considered and an accurate production rate is obtained with the relevant buffer configuration. Because transfer lines with more than 20 machines are called long lines, these reliability sets from the literature are extended to transfer lines with 20, 30 and 50-machines. The present study investigates the effect of long transfer lines on performance indicators (production rate) according to the applied evaluation algorithm.

To analyze the system behavior, balanced transfer lines with four different reliability parameter sets (four cases) are analyzed (Table 1). Considering these four different cases [30, 31], the mean effective service rate values of these transfer lines are the same for all machines. For each of these reliability parameters, transfer line experiments with 20, 30 and 50-machines are conducted. Additionally, the Δ value in the DDX algorithm is set to 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5} and 10^{-6} for each case and for each number of machines. Ten replications are executed for each instance.

In the experimental study, the initial buffer configuration for each case starts with all buffers equal to four (20 machines for i=1, 2, ..., 19), and each buffer value is increased by a constant one until the total number of buffers (N) reaches 1000 times the total number of machines ($N \leq K \times 1000$). The reason for choosing four here is that it is the minimum value at which the DDX algorithm converges precisely. For example, in the initial buffer configuration for a 20-machine line, the total number of buffers is 4×19=76. For this incrementation process, the buffer configurations are changed with a simple algorithm (5, 5, ..., 5; 6, 6, ..., 6; ...; 1052, 1052, ..., 1052). The tables presented in this section provide information on the properties of the machines in the line, i.e., the failure rates (β_i) and the repair rates (r_i). The processing times of the machines are equal, constant and one unit ($\mu_i = 1$) for the whole line.

***Table 1.** Reliability parameters of the transfer lines [30, 31]*

Case	Reliability parameters		Mean effective service rate of machines on the lines
	β_i	r_i	
1	0.01	0.1	0.9090
2	0.1	0.2	0.6667
3	0.5	0.5	0.5000
4	0.3	0.03	0.0909

Table 2 presents the CPU time values (seconds) of the experimental study. The CPU time of the DDX algorithm increases as the convergence rate decreases, i.e., as the partial production rate values of the two machine lines approach each other. Table 2 clearly shows that the CPU time increases when the number of machines and buffers increases. The optimization algorithms used in the buffer allocation problem, one of the most important production line design problems that aims to determine the amount of buffer to be kept between machines, typically search for results in an iterative manner.

Table 2. *Numerical results: CPU time (sec.) values according to the convergence rate*

Case	**Number of Machine**	**10^{-2}**	**10^{-3}**	**10^{-4}**	**10^{-5}**	**10^{-6}**
	20	17.8344	18.2813	25.3781	60.1000	104.6686
1	30	20.0906	25.0903	28.6781	101.3344	204.4998
	50	25.4719	25.2031	34.5144	211.5538	498.7310
	20	18.3440	18.5430	24.3053	60.9489	112.5354
2	30	21.8277	22.0183	26.1216	103.3402	213.3042
	50	26.2002	26.7758	32.1457	206.9142	501.8148
	20	18.4063	19.2082	18.4165	35.9136	74.9428
3	30	20.5562	21.7379	22.0497	45.5494	134.7704
	50	25.4587	27.3903	26.8246	51.9340	287.1998
	20	17.7998	18.8304	18.5194	43.7844	84.5841
4	30	20.8682	21.5245	21.4595	62.4450	155.6096
	50	26.2259	27.2097	27.5963	86.2606	344.6574

A performance measurement method like the DDX algorithm evaluates a buffer configuration based on certain reliability parameters of the machines and calculates values such as the throughput rate. If a proposed optimization method for the considered transfer line design needs to be compared with other methods in the literature, a large number of buffer configurations may need to be evaluated with many iterations. Depending on the stopping condition of the optimization algorithm, the overall computational performance is affected by the convergence rate.

The experimental results presented here show that the convergence rates yield different results in terms of CPU times. Since the differences in some convergence rates seemed small, a Friedman test was used to determine whether the differences in CPU time values were statistically significant. The H_0 (null) hypothesis is tested at the 95% confidence level as follows:
H_0: *All convergence rates yield the same CPU time*

Table 3 shows the comparison results. In Table 3, the Friedman test has an asymptotic significance (p) value of 0.000, which indicates that the differences between the convergence rates are significant. The H_0 (null) hypothesis is rejected.

Table 3. *Results of the Friedman test*

Convergence Rate	Mean Rank	Test Statistics[a]	
10^{-2}	1.08	N	12
10^{-3}	2.25	Chi-Square	44.867
10^{-4}	2.67	df	4
10^{-5}	4.00	Asymp. Sig. (p)	0.000
10^{-6}	5.00	**a.** Friedman Test	

This study conducted a comparison according to the convergence rates, which are sensitive for the DDX algorithm. The Wilcoxon signed-rank test is more sensitive than the parametric t test. Since the observed values in Table 2 for CPU times are close to each other and the assumption of a normal distribution is not made, the Wilcoxon signed-rank test is safer [32]. When the five convergence rates were compared closely pairwise with the Wilcoxon signed rank test, significant differences were obtained (Table 4).

***Table 4.** Results of the Wilcoxon signed rank test*

Test Statistics[a]	0,01 – 0,001	0,001 – 0,0001	0,0001 – 0,00001	0,00001 – 0,000001
Z	-2,824[b]	-1,961[b]	-3,059[b]	-3,059[b]
Asymp. Sig. (2-tailed)	0,004742	0,049860	0,002218	0,002218

a. Wilcoxon Signed Ranks Test
b. Based on negative ranks.

It is important for long-term decisions to calculate the production rate accurately and precisely rather than quickly. Although a small convergence rate requires more CPU time, a more precise production rate is obtained with a small convergence rate in the transfer lines.

Table 5 shows the results of computational tests using the case data (reliability parameters) in Table 1. Table 5 presents the minimum production rate (Min PR), maximum production rate (Max PR), root mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) to indicate the lower and upper limits of the production rate values for these parameters. For example, in Case 1, for a 20-machine transfer line, the buffer configuration (4, 4, ..., 4) yields a throughput rate of 0.3989 with a convergence rate of 10^{-2} and a throughput rate of 0.5356 with a convergence rate of 10^{-6}. However, in Case 1 for a 20-machine transfer line, the buffer configuration (1052, 1052, ..., 1052) yields a production rate of 0.9034 with a convergence rate of 10^{-2} and a production rate of 0.9064 with a convergence rate of 10^{-6}.

In addition, varying statistical indicators can be used to measure the forecasting accuracy (forecasting performance) of models, such as the mean squared error, root mean squared error, mean absolute error, and mean absolute percentage error. In a proposed new analytical method (or data), the root mean squared error with respect to the existing (original) method is usually considered. All production rate values from the buffer configuration with minimum values (4, 4, ..., 4) to the maximum buffer configuration are calculated based on the convergence rate. It is important to compare the accuracy of the production rate values obtained with the convergence rates in pairs. In this respect, each convergence rate is compared according to the RMSE, MAE and MAPE values in terms of the throughput rate against 10^{-6}. Table 5 shows the RMSE values (Equation (12)) of the convergence rates according to 10^{-6} for each case and each line size (20, 30 and 50-machines lines). In Equation (12), T is the number of buffer configurations (and hence production rates) evaluated for each line size, while $diffPR_j$ is the difference between the convergence rate considered and the production rate values obtained from j. buffer configuration for a convergence rate of 10^{-6}. Similarly, using Equations (13) and (14), the MAE and MAPE values for transfer lines of 20, 30 and 50-machines are calculated and given in Table 5.

$$RMSE = \sqrt{\frac{\sum_{j=1}^{T}(diffPR_j)^2}{T}} \tag{12}$$

$$MAE = \frac{\sum_{j=1}^{T}\left|diffPR_j\right|}{T} \tag{13}$$

$$MAPE = \frac{\sum_{j=1}^{T}\frac{\left|diffPR_j\right|}{Max\ PR}}{T} \times 100\% \tag{14}$$

In Table 5, as the convergence rate receives smaller, the minimum and the maximum production rate values receive as close to each other as possible. In other words, for the steady-state production rate of the system, the convergence rate should be chosen to be as low as possible within a reasonable number of CPUs. The RMSE, MAE and MAPE values decrease as the convergence rate decreases. The mean effective service rate values given in Table 1 are the maximum production rate values that can be achieved in the transfer line for the relevant case. When considering Table 5 from perspective, the maximum PR values approach the maximum mean effective service rate values. The RMSE, MAE and MAPE increase as the size of the line increases. This is because the difference between the Min PR and Max PR increases as the size of the line increases.

Table 5. *Results of Production Rate Values and Statistical Indicator Values*

	Number of Machine	20					30					50				
Case	Convergence Rate	Min PR	Max PR	RMSE	MAE	MAPE (%)	Min PR	Max PR	RMSE	MAE	MAPE (%)	Min PR	Max PR	RMSE	MAE	MAPE (%)
1	10^{-2}	0.3989	0.9034	0.0300	0.0148	1.7633	0.3207	0.9026	0.0402	0.0193	2.3218	0.2400	0.9015	0.0535	0.0252	3.0450
	10^{-3}	0.5079	0.9034	0.0117	0.0092	1.0514	0.4446	0.9026	0.0180	0.0131	1.5063	0.3556	0.9015	0.0294	0.0191	2.2259
	10^{-4}	0.5299	0.9034	0.0023	0.0023	0.2552	0.5029	0.9026	0.0044	0.0043	0.4805	0.4718	0.9015	0.0079	0.0076	0.8572
	10^{-5}	0.5354	0.9062	0.0002	0.0002	0.0239	0.5104	0.9060	0.0004	0.0004	0.0418	0.4917	0.9057	0.0007	0.0007	0.0761
	10^{-6}	0.5356	0.9064	0.0000	0.0000	0.0000	0.5116	0.9063	0.0000	0.0000	0.0000	0.4943	0.9062	0.0000	0.0000	0.0000
2	10^{-2}	0.2187	0.6617	0.0221	0.0115	1.9006	0.1911	0.6610	0.0271	0.0143	2.3595	0.1654	0.6601	0.0330	0.0177	2.9157
	10^{-3}	0.3232	0.6617	0.0109	0.0083	1.3048	0.3060	0.6610	0.0165	0.0115	1.8229	0.1654	0.6601	0.0287	0.0167	2.7152
	10^{-4}	0.3503	0.6617	0.0024	0.0023	0.3571	0.3429	0.6610	0.0042	0.0041	0.6338	0.3349	0.6601	0.0074	0.0071	1.0860
	10^{-5}	0.3534	0.6641	0.0002	0.0002	0.0312	0.3487	0.6639	0.0004	0.0003	0.0536	0.3451	0.6638	0.0006	0.0006	0.0952
	10^{-6}	0.3537	0.6643	0.0000	0.0000	0.0000	0.3494	0.6642	0.0000	0.0000	0.0000	0.3468	0.6641	0.0000	0.0000	0.0000
3	10^{-2}	0.3149	0.4990	0.0067	0.0027	0.5583	0.3034	0.4989	0.0080	0.0033	0.6863	0.2917	0.4987	0.0096	0.0040	0.8404
	10^{-3}	0.3680	0.4990	0.0049	0.0024	0.4987	0.3034	0.4989	0.0080	0.0033	0.6863	0.2917	0.4987	0.0096	0.0040	0.8404
	10^{-4}	0.3861	0.4990	0.0015	0.0013	0.2598	0.3839	0.4989	0.0023	0.0018	0.3711	0.3803	0.4987	0.0036	0.0026	0.5362
	10^{-5}	0.3883	0.4993	0.0002	0.0002	0.0357	0.3873	0.4991	0.0003	0.0003	0.0672	0.3863	0.4987	0.0008	0.0008	0.1556
	10^{-6}	0.3885	0.4995	0.0000	0.0000	0.0000	0.3878	0.4995	0.0000	0.0000	0.0000	0.3874	0.4994	0.0000	0.0000	0.0000
4	10^{-2}	0.0226	0.0893	0.0042	0.0029	3.5739	0.0208	0.0890	0.0050	0.0035	4.3290	0.0192	0.0888	0.0060	0.0042	5.2154
	10^{-3}	0.0226	0.0893	0.0042	0.0029	3.5739	0.0208	0.0890	0.0050	0.0035	4.3290	0.0192	0.0888	0.0060	0.0042	5.2154
	10^{-4}	0.0343	0.0893	0.0018	0.0017	1.9776	0.0323	0.0890	0.0029	0.0025	2.9405	0.0192	0.0888	0.0055	0.0040	4.9031
	10^{-5}	0.0364	0.0899	0.0002	0.0002	0.2063	0.0358	0.0897	0.0003	0.0003	0.3549	0.0353	0.0888	0.0007	0.0007	0.7586
	10^{-6}	0.0367	0.9001	0.0000	0.0000	0.0000	0.0364	0.0901	0.0000	0.0000	0.0000	0.0361	0.0900	0.0000	0.0000	0.0000

To obtain the production rate accurately, a convergence rate of 10^{-6} is applied in the following additional experimental study section, and several performance indicators are obtained.

3.3. Additional Experimental Tests for Performance Indicators of Long Transfer Lines

Because simulation and Markovian methods are too time consuming, approximate analytical methods have been developed to evaluate the throughput of long transfer lines with finite buffers [28]. The efficiency and behavior of the approximate analytical methods proposed to calculate the performance indicators of long transfer lines and help in designing decision-making are of particular interest for long lines. The line design decision also considers the cost of the initial setup (capital) of the production line because the buffer (storage) is very expensive and can vary along the line [31, 33]. This cost may even limit the desired production rate value of the production line [28]. Thus, the calculation of line performance indicators based on varying reliability parameters with a certain convergence rate using an approximate analytical method can help in the design decisions of production line managers.

To understand the system behavior in detail, additional tests are performed for long transfer lines with different parameters and production rates. Additionally, work-in-process (the WIP is also known as the average buffer level) and profit values are analyzed.

There are different formulations in the literature for calculating the profit value (*Pro*) based on the production rate (*PR*), holding cost (h_i) and coefficient (*C*).

In Equation (15), *Pro* is formulated as in the study of Massim et al. [34]:

$$Pro\ (B) = C \times PR\ (B) - \sum_{i}^{K-1} h_i \bar{n}_i \tag{15}$$

s.t.

$$\sum_{i=1}^{K-1} B_i = N \tag{16}$$
$$B_i \geq 0 \text{ and integer for } i = 1, 2, \ldots, K-1 \tag{17}$$
$$\bar{n}_i \geq 0, \text{for } i = 1, 2, \ldots, K-1 \tag{18}$$

where *C* is the revenue per part produced and h_i is the cost coefficient associated with the average inventory (WIP) for the *i*th buffer area. *B* is the depicted buffer configuration. Additionally, *Pro*(*B*) is the profit of the line obtained with buffer configuration *B*. Equation (16) depicts the total buffer size (*N*) constraint. Equation (17) emphasizes that the buffer value in each buffer area (B_i) is greater than or equal to 0, being also an integer. Similarly, the average inventory ($\bar{n}_i$) for the *i*th buffer area is greater than or equal to 0 (Equation (18)) [14, 33]. The *C* coefficient is set to 50*N*, and the h_i cost coefficient is set to 1 for all additional tests (Massim et al. [34]). For example, for a 20-machine line, *C* is equal to 47500. In Table 6, the average inventory in B_i is denoted by $\bar{n}_i$ (for $i = 1, 2, \ldots, K$-1).

Considering the reliability parameters in Table 1, Table 6 provides the performance indicators for transfer lines of different sizes with 20, 30 and 50-machines and equal buffer sizes between these machines, with total buffer sizes of 950, 2900 and 4900, respectively. The aim here is to investigate the WIP values ($\bar{n}_i$), total WIP, production rate, and profit value of long transfer lines with machines that have different mean effective service rates in Table 1 (0.9090, 0.6667, 0.5000, 0.0909).

As shown in Table 6, the balanced lines with different reliability parameters exhibit almost the same pattern, when considering the WIP values. As the size of the line increases, there are only small differences in the WIP values between the different reliability parameters. In this balanced production line where all machines have the same processing time (one unit), the average effective service rate of the machines affects the production rate. However, the production rate and profit values of the lines with the same buffer configuration are quite different from each other. Since the cost parameters (WIP) are the same for all the lines considered, the main difference in profit values for lines with the same buffer configuration is due to the production rate of the line.

For a transfer line, if the costs of semi-finished products (work in process) and/or holding costs along the line are too high, machine selection (and therefore machines with different reliability parameters) should be performed to minimize the total WIP. If the WIP values and related costs are insignificant compared to the production rate and profit value of the line, the machine selection in this case should be performed by considering the production rate and profit values (Table 6).

Table 6. *Performance indicators for the 20-, 30-, and 50-machine transfer lines*

(K, N)	Reliability parameters [30, 31] β_i	r_i	WIP values ($\bar{n}_i$)	Total WIP ($\sum_i^{K-1} \bar{n}_i$)	Production Rate ($PR(B)$)	Profit ($Pro(B)$)
(20, 950)	0.01	0.1	33.752, 30.735, 29.136, 28.096, 27.336, 26.735, 26.229, 25.783, 25.371, 24.975, 24.579, 24.169, 23.725, 23.223, 22.627, 21.872, 20.837, 19.245, 16.234	474.658	0.8567	40218.59
	0.1	0.2	32.222, 29.600, 28.265, 27.418, 26.810, 26.335, 25.939, 25.592, 25.272, 24.967, 24.662, 24.345, 24.001, 23.610, 23.141, 22.540, 21.701, 20.374, 17.761	474.551	0.621	29022.95
	0.5	0.5	31.253, 28.916, 27.739, 26.996, 26.465, 26.051, 25.706, 25.406, 25.131, 24.870, 24.612, 24.346, 24.059, 23.733, 23.342, 22.838, 22.127, 20.985, 18.682	473.258	0.4902	22811.24
	0.3	0.03	32.500, 29.711, 28.304, 27.416, 26.781, 26.286, 25.875, 25.516, 25.188, 24.875, 24.565, 24.245, 23.898, 23.505, 23.033, 22.424, 21.567, 20.194, 17.439	473.324	0.0778	3222.18
(30, 2900)	0.01	0.1	65.322, 60.051, 57.381, 55.715, 54.552, 53.676, 52.980, 52.403, 51.910, 51.476, 51.084, 50.724, 50.385, 50.061, 49.746, 49.433, 49.116, 48.789, 48.446, 48.075, 47.667, 47.203, 46.660, 46.001, 45.166, 44.045, 42.426, 39.805, 34.584	1444.884	0.8817	126401.62
	0.1	0.2	63.618, 58.772, 56.369, 54.891, 53.866, 53.100, 52.493, 51.992, 51.564, 51.188, 50.851, 50.540, 50.249, 49.972, 49.702, 49.435, 49.166, 48.890, 48.599, 48.286, 47.941, 47.548, 47.087, 46.524, 45.805, 44.832, 43.408, 41.064, 36.277	1444.029	0.6426	91732.97
	0.5	0.5	62.532, 57.935, 55.649, 54.231, 53.238, 52.484, 51.880, 51.375, 50.939, 50.553, 50.204, 49.884, 49.585, 49.302, 49.031, 48.768, 48.509, 48.251, 47.987, 47.711, 47.415, 47.085, 46.703, 46.238, 45.640, 44.817, 43.583, 41.496, 37.109	1430.133	0.495	70344.87
	0.3	0.03	63.927, 58.850, 56.328, 54.769, 53.680, 52.857, 52.200, 51.653, 51.182, 50.766, 50.391, 50.047, 49.726, 49.422, 49.131, 48.847, 48.565, 48.282, 47.991, 47.685, 47.352, 46.979, 46.545, 46.014, 45.331, 44.395, 42.999, 40.654, 35.762	1432.330	0.0835	10675.17
(50, 4900)	0.01	0.1	65.422, 60.209, 57.595, 55.985, 54.879, 54.061, 53.425, 52.911, 52.483, 52.118, 51.800, 51.519, 51.266, 51.037, 50.826, 50.631, 50.449, 50.277, 50.114, 49.959, 49.811, 49.668, 49.529, 49.394, 49.263, 49.133, 49.005, 48.878, 48.751, 48.623, 48.493, 48.360, 48.223, 48.080, 47.928, 47.766, 47.590, 47.395, 47.176, 46.924, 46.630, 46.277, 45.842, 45.289, 44.557, 43.540, 42.024, 39.507, 34.396	2425.017	0.8814	213517.98
	0.1	0.2	63.699, 58.900, 56.545, 55.114, 54.137, 53.419, 52.862, 52.413, 52.039, 51.720, 51.442, 51.196, 50.975, 50.773, 50.588, 50.416, 50.255, 50.104, 49.960, 49.823, 49.692, 49.565, 49.443, 49.323, 49.207, 49.093, 48.981, 48.870, 48.760, 48.649, 48.538, 48.424, 48.308, 48.187, 48.061, 47.926, 47.780, 47.620, 47.440, 47.233, 46.991, 46.698, 46.335, 45.869, 45.246, 44.369, 43.041, 40.794, 36.110	2422.938	0.6424	154965.06

	0.5	0.5	62.621, 58.076, 55.843, 54.477, 53.535, 52.833, 52.281, 51.828, 51.444, 51.110, 50.815, 50.549, 50.305, 50.081, 49.871, 49.673, 49.486, 49.307, 49.136, 48.972, 48.813, 48.659, 48.509, 48.363, 48.220, 48.080, 47.944, 47.809, 47.677, 47.546, 47.417, 47.289, 47.162, 47.035, 46.908, 46.779, 46.647, 46.510, 46.364, 46.206, 46.027, 45.816, 45.556, 45.218, 44.752, 44.067, 42.980, 41.048, 36.829	2384.473	0.4949	118866.03
	0.3	0.03	64.018, 58.995, 56.528, 55.021, 53.986, 53.216, 52.613, 52.120, 51.704, 51.344, 51.026, 50.740, 50.480, 50.241, 50.017, 49.808, 49.610, 49.422, 49.242, 49.069, 48.902, 48.740, 48.584, 48.431, 48.282, 48.136, 47.993, 47.853, 47.714, 47.578, 47.443, 47.308, 47.175, 47.040, 46.905, 46.766, 46.623, 46.472, 46.309, 46.130, 45.924, 45.679, 45.375, 44.978, 44.434, 43.641, 42.396, 40.208, 35.487	2387.704	0.0834	18045.3

When designing a production line, manufacturing systems engineers and plant managers must decide how to distribute the number of buffers between machines along the line. The cases in Table 1 are used to show how different patterns of distributing the total buffer amount along the line have an effect on the line performance indicators (WIP values, production rate values and profit values) in transfer lines with balanced and unreliable machines. The mean effective service rate of Case 1 decreased from 0.9090 to 0.0909 for Case 4 and 30 lines with different reliability parameters were created with values in between (Figure 4). Considering 20 machines, 950 total buffer sizes, and C equal to 47500, the variation in performance indicators for 30 different lines with a uniform (equal), ascending, and descending order distribution of buffer values (buffer configuration) along the line [33, 35] is presented.

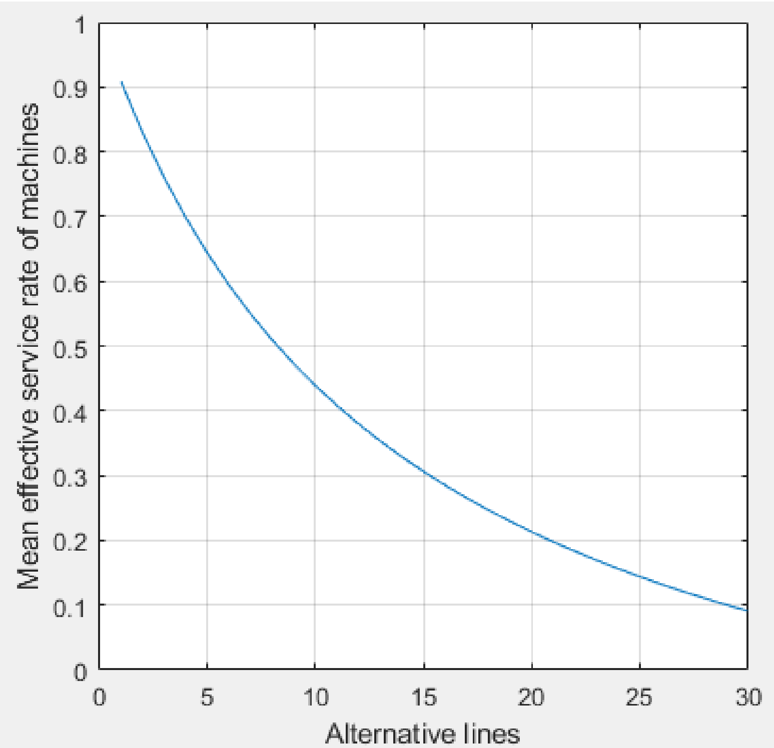
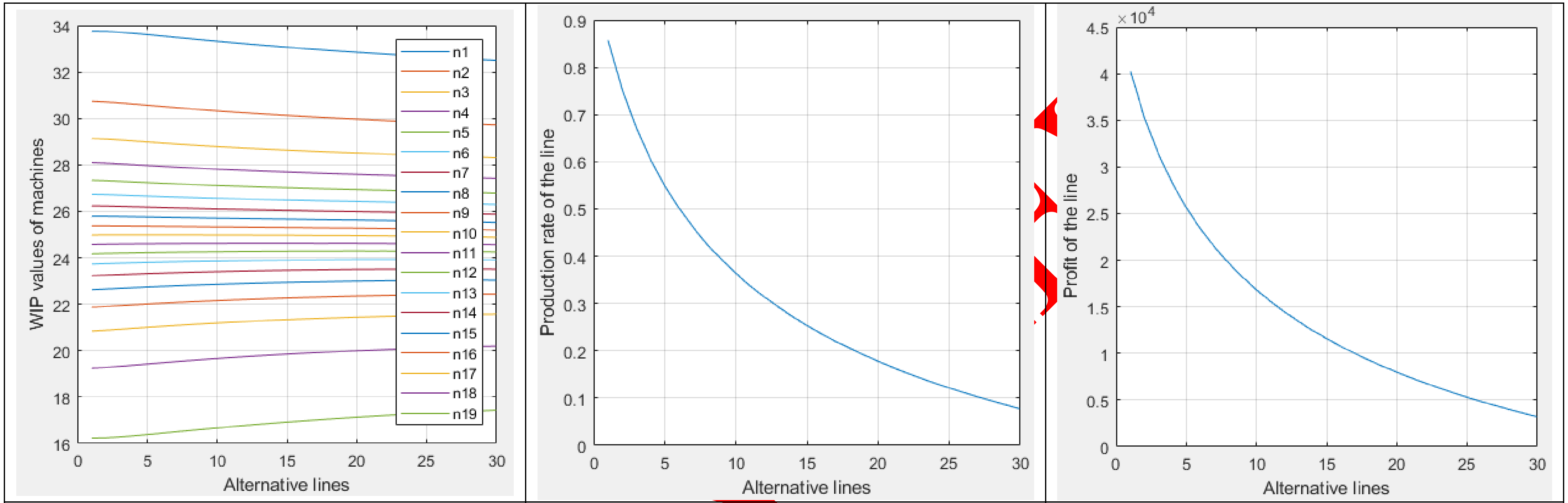


Figure 4. *Mean effective service rate of machines in 30 alternative balanced transfer lines*

To analyze the WIP values in more detail, 20-machine transfer lines with mean effective service rates (Figure 4) ranging from 0.9090 to 0.0909 are considered. The buffer configuration of this line is set to equal (uniformly) [50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50]. In this way, a total of 30 different line alternatives are generated (Figure 5). WIP values obtained from 30 generated alternative lines are shown in Figure 5a. The total WIP values of the lines are in the range of 474.658-473.324. WIP values decrease from the beginning to the middle of the line ($n1$, $n2$, …), while WIP values increase from the end to the middle of the line ($n19$, $n18$, …). Furthermore, the total WIP values of all the considered lines remain almost the same. In general, because the part processed in machines toward the end of the line is closer to the final product stage, its cost is considered to be greater. Therefore, the WIP holding cost at the end of the line is greater. Under these circumstances, in balanced transfer lines, it may be relatively cost advantageous to prefer machines with a high mean effective service rate.

Additionally, the obtained production rate and profit values of the 30 alternative lines are presented in Figure 5b and Figure 5c, respectively. The production rate and profit of the alternative lines are in the range of 0.8567-0.0778 and 40218.59-3222.18, respectively. As shown in Figure 5, as the mean effective service rate of the machines on the line decreases, the production rate and profit of the lines also decrease. The profit value of a production line is influenced by the production rate value when WIP quantities and therefore costs are almost equal. If a higher production rate and a higher profit value are anticipated in a transfer line, machines with a higher mean effective service rate should be preferred.



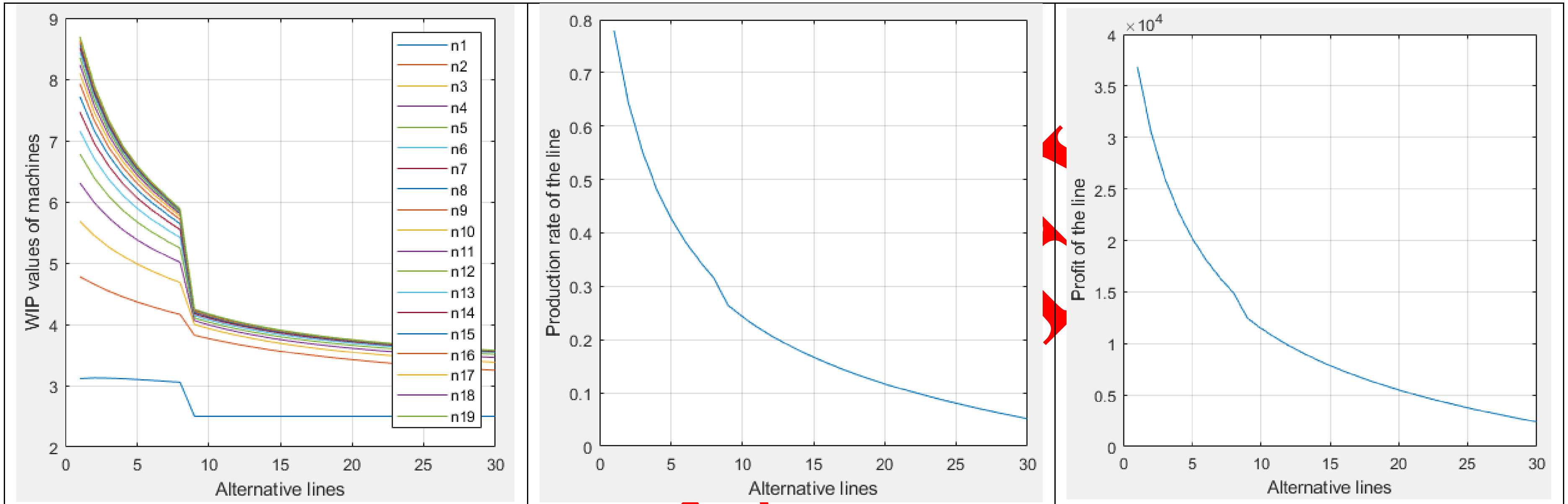
a) WIP values of machines	**b)** Production rate values	**c)** Profit values

Figure 5. *Performance measures of 30 alternative lines with the uniform buffer configuration* [50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50]

After examining the effect of a uniformly distributed buffer configuration on system behavior, the study also examined the effects of different buffer configurations on profit. To do that, two different buffer configurations were generated [33, 35]. The first one is an ascending order buffer configuration and set as [5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95]. The second one is the descending order buffer configuration, which is set to [95, 90, 85, 80, 75, 70, 65, 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5] by taking the inverse symmetry of the ascending-order buffer configuration.

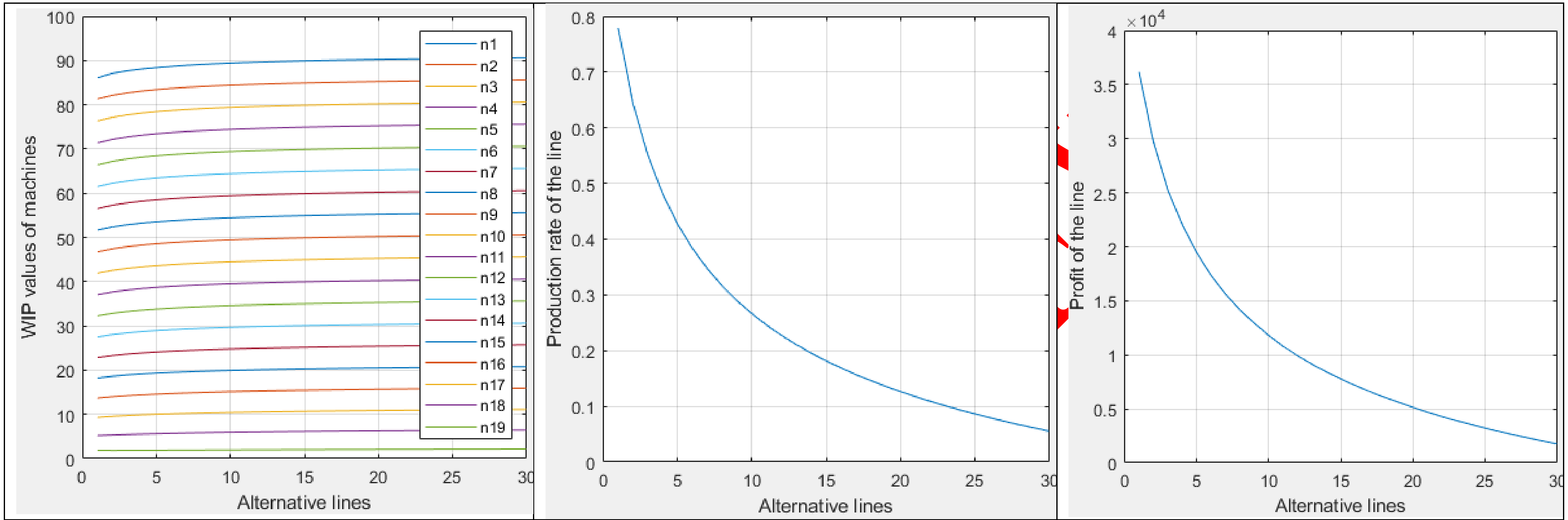
Figures 6 and 7 represent the values of the performance measures for these buffer configurations for the 30 generated alternative lines, as shown in Table 1. These sets of experiments were designed to test the effect of ascending order and descending order distribution against the performance of the uniform buffer configuration. The motivation for these sets of experiments is that increasing the buffers toward the end of the line increases the profit value and provides relatively less WIP accumulation in the buffer areas. The WIP values of 30 alternative lines are given in Figure 6a for the [5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95] buffer configuration. The WIP values of the lines are in the range of 141.780-66.106. The ascending order buffer configuration WIP values decrease as the mean effective service rate of the line decreases. Additionally, the production rates and profits of the 30 alternative lines are given in Figure 6b and Figure 6c, respectively. The production rate and profit values of the lines are in the range of 0.7792-0.0520 and 36868.60-2405.88, respectively.

For the [95, 90, 85, 80, 75, 70, 65, 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5] buffer configuration, the WIP values, production rates and profit values of the 30 alternative lines are in the ranges of 807.808-869.695, 0.7792-0.0554 and 36202.78-1761.79, respectively (Figure 7a, 7b and 7c). The total WIP values increase only slightly from the high mean effective service rate to the low mean service rate at the beginning and then remain almost constant. As evident from the performance indicators of both buffer configurations, the difference in profit between the descending-order buffer configuration and the ascending-order buffer configuration is attributed to the difference in WIP values. Therefore, for low WIP and high profit values of balanced unreliable transfer lines with a finite buffer, the initial buffer configuration should be generated in ascending order rather than descending order.



a) WIP values of machines	**b)** Production rate values	**c)** Profit values

Figure 6. *Performance measures of 30 alternative lines with the ascending order buffer configuration* [5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95]



a) WIP values of machines	**b)** Production rate values	**c)** Profit values

Figure 7. *Performance measures of 30 alternative lines with the descending order of buffer configuration* [95, 90, 85, 80, 75, 70, 65, 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5]

As evidenced by these additional tests, while the production rate and profits are greater for the uniformly distributed buffer configuration, the WIP values are lower for the ascending order buffer configuration. To maximize the profit of the balanced unreliable transfer line, it is necessary to obtain the buffer configuration that can maximize the production rate and minimize the WIP value.

It is more difficult to determine the optimum buffer configuration when the reliability parameters of machines differ from each other. Therefore, the optimization algorithm used to determine the optimum buffer configuration should be combined with a good initial solution to rapidly reach the best solution.

4. RESULTS AND DISCUSSION

The analysis and design of production lines is one of the main tasks for production plant managers and production line engineers. Calculation of the production rate with the highest precision possible is crucial for meeting customer demands completely and on time and managing costs. Long transfer lines are usually complex and serve as production lines in which high-tech products such as automobiles, computers and vehicle engines are manufactured. As the most important performance indicator, the production rate value, which represents the amount of production per unit time, is affected by the reliability of the calculation technique applied in terms of production quantity in long transfer lines where high volumes are produced. The performances of the new analytical methods are tested against exact analytical methods for small lines, while for long lines, they are tested against simulation and other validated methods. On long transfer lines and machines with variable reliability parameters, convergence (and computational time) is a significant problem. For this reason, the convergence and accuracy of these methods should be tested. Although previous studies have shown that analytical methods developed and applied for half a century can calculate production rates slightly differently based on certain parameters, the effectiveness of the DDX algorithm has not been discussed in detail thus far. In the present study, the DDX algorithm is considered for different line sizes (20, 30 and 50-machines), different cases, and different convergence rates, production rate values and CPU times. The results can be summarized as follows:

a) Dallery et al.'s algorithm has several limitations/features related to balanced unreliable long transfer lines:
   i. The production rate of a line is sensitive to the convergence rate.
   ii. The number of cycles executed is sensitive to the convergence rate and hence the CPU time.
   iii. The production rate is sensitive to the reliability parameters and (total) buffer size.

b) The production rate and profit are greater for the uniformly distributed buffer configuration and the WIP is lower for the ascending order buffer configuration. To maximize the profit of the line, it is necessary to maximize the production rate and minimize the WIP value.

It would be more interesting to examine the convergence rate effect on the DDX algorithm with respect to the randomly generated mean effective service rate, which is different for each machine. Furthermore, an alternative analytical approximation method can be studied for a different line topology.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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