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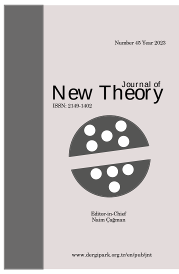

Approximate Solutions of the Fractional Clannish Random Walker's Parabolic Equation with the Residual Power Series Method

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Abstract − One of the prominent nonlinear partial differential equations in mathematical physics is the Clannish Random Walker's Parabolic (CRWP) equation. This study uses Residual Power Series Method (RPSM) to solve the time fractional CRWP equation. In this equation, the fractional derivatives are considered in Caputo's sense. The effectiveness of RPSM is illustrated with graphical results. The series solutions are utilized to represent the approximate solutions. Besides, the approximate solutions found by the suggested method ensure good accuracy when compared with the exact solution. Moreover, RPSM efficiently analyzes complex problems that emerge in the related mathematical and scientific fields.

Keywords *Fractional partial differential equation, Caputo derivative, Clannish Random Walker's Parabolic equation, residual power series method, approximate solution*

Mathematics Subject Classification (2020) 26A33, 35C10

1. Introduction

The Clannish Random Walker's Parabolic (CRWP) equation in the form

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial x}+2u\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial x^2}=0$$

is a mathematical model of physical problems appearing in various scientific fields such as mathematical biology and physics. This equation describes the behavior of two types that carry out a concurrent one-dimensional random walk defined by the condensation of the clannishness of members as the density of another increases. In the literature, various methods, such as the improved tanh function method [1], homotopy perturbation method [2], Jacobi elliptic function method [3], unified rational expansion method [3], and a direct rational exponential scheme [4], have been used to solve the CRWP equation.

Fractional calculus is a quickly developing branch of mathematics with various applications in numerous chemistry, physics, biology, and engineering fields such as thermodynamics, viscoelasticity, electricity, aerodynamics, fluid dynamics, control theory, turbulence, signal processing, and others [5-10]. Thus, finding exact and approximate solutions to fractional differential equations is important in scientific studies. An important one of these fractional differential equations is the time fractional CRWP equation.

Recently, many methods, such as the adapted (G'/G)-expansion scheme [11,12], the $(G'/G\,,1/G)$-expansion method [12,13], the Kudryashov method [14], the improved $\tan(Q(\xi)/2)$-expansion method [15], the generalized homotopy analysis method [16], the modified Kudryashov method [17], the extended $\exp(-\varphi(\xi)/2)$- expansion method [18], the modified extended auxiliary mapping method [19], the modified

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F-expansion method [19, 20], the modified (G'/G^2)-expansion method [20], the power series method [21], the natural decomposition method [22], the energy inequality method [23], and the modified trial equation method [24], have been used to find solutions to the fractional CRWP equation. Residual Power Series Method (RPSM) has not yet been investigated to solve the time fractional CRWP equation in the literature. Thus, the main focus of this paper is to utilize RPSM to calculate the approximate solutions of the time fractional CRWP equation

$$D_t^{\mu}u(x,t) - u_x(x,t) + 2u(x,t)u_x(x,t) + u_{xx}(x,t) = 0, \quad 0 < \mu \leq 1 \tag{1}$$

where D_t^{μ} is the fractional derivative operator in the Caputo sense. Abu Arqub [25] suggested RPSM as a useful method for obtaining coefficients of the power series solution in 2013. RPSM has numerous benefits for solving partial differential equations compared to other methods [26]. RPSM provides an easy and effective power series solution for various equations without linearization, discretization, or perturbation. This method does not need a recursion relationship and does not require comparing the coefficients of the corresponding terms. The suggested method yields the solutions as a convergence series. With this method, infinite series solutions can be gained by iterated operations. Besides, RPSM is unaffected by rounding errors in computation and does not require a lot of computer memory and time. Moreover, there is no need for any transformation with this method. Furthermore, RPSM can be implemented directly into the present equation by choosing an initial guess approximation. In literature, RPSM has been used to find power series solutions for different problems, such as those provided in [27-44].

The organization of the study is as follows: Section 2 provides some definitions and theorems for the Caputo derivative and the fractional power series. Section 3 presents RPSM for the approximate solutions of nonlinear fractional differential equations. Section 4 applies the proposed method for the fractional CRWP equation solutions and exhibits the suggested method's effectiveness with table and graphics. Finally, the last section contains the concluding remarks.

2. Preliminaries

Many fractional derivative definitions, such as Riemann-Liouville, Caputo, Grunwald-Letnikov, Marchaud, Weyl, and Hadamard fractional derivatives, have been used in scientific studies. In this section, the Caputo derivative is considered because the initial conditions of the fractional partial differential equations with the Caputo derivative have the common form of the integer order partial differential equations, and the derivative of the constant is zero.

Definition 2.1. [45] The time-fractional derivative in Caputo sense is described as

$$D_t^{\mu}u(x,t) = \begin{cases} \dfrac{1}{\Gamma(m-\mu)}\displaystyle\int_0^t (t-\tau)^{m-1-\mu}\frac{\partial^m u(x,\tau)}{\partial \tau^m}d\tau, & m-1 < \mu < m \\ \dfrac{\partial^m u(x,t)}{\partial t^m}, & m = \mu \in \mathbb{N} \end{cases}$$

Definition 2.2. [46] The fractional power series about t_0 is defined as

$$\sum_{m=0}^{\infty} c_m(t-t_0)^{m\mu} = c_0 + c_1(t-t_0)^{\mu} + c_2(t-t_0)^{2\mu} + \cdots, \quad 0 \leq m-1 < \mu \leq m \quad \text{and} \quad t \geq t_0$$

Here, c_m are constants, and t is a variable.

Theorem 2.1. [46] Suppose that h is a fractional power series representation about t_0 of the manner

$$h(t) = \sum_{m=0}^{\infty} c_m (t - t_0)^{m\mu}, \quad 0 \leq m - 1 < \mu \leq m \quad \text{and} \quad t_0 \leq t < t_0 + R$$

When $D^{m\mu}h(t)$ are continuous on $(t_0, t_0 + R)$, then coefficients c_m are given as

$$c_m = \frac{D^{m\mu}h(t_0)}{\Gamma(1 + m\mu)}, \quad m \in \{0,1,2,\cdots\}$$

where R is the radius of convergence and $D^{m\mu} = \underbrace{D^{\mu}D^{\mu}\cdots D^{\mu}}_{m\ times}$.

Theorem 2.2. [46] Suppose that $u(x,t)$ has a multivariate fractional power series representation at t_0 of the form

$$u(x,t) = \sum_{m=0}^{\infty} h_m(x)(t - t_0)^{m\mu}, \quad x \in I, \quad 0 \leq m - 1 < \mu \leq m, \quad \text{and} \quad t_0 \leq t < t_0 + R$$

If $D_t^{m\mu}u(x,t)$ are continuous on $I \times (t_0, t_0 + R)$, then $h_m(x)$ are given as

$$h_m(x) = \frac{D_t^{m\mu}u(x,t_0)}{\Gamma(1 + m\mu)}, \quad m \in \{0,1,2,\cdots\}$$

Here, $D_t^{m\mu} = \frac{\partial^{m\mu}}{\partial t^{m\mu}} = \frac{\partial^{\mu}}{\partial t^{\mu}} \frac{\partial^{\mu}}{\partial t^{\mu}} \cdots \frac{\partial^{\mu}}{\partial t^{\mu}}$ and $R = \min_{c \in I} R_c$ that R_c is the radius of convergence of the fractional power series

$$\sum_{m=0}^{\infty} h_m(c)(t - t_0)^{m\mu}$$

3. General Structure of RPSM

In this section, to find the approximate solutions of nonlinear fractional differential equations with the suggested method, we investigate the following general nonlinear fractional differential equation with the initial condition

$$D_t^{\mu}u(x,t) = R(u) + N(u), \quad 0 < \mu \leq 1, \quad t > 0, \quad \text{and} \quad u(x,0) = h(x) \tag{2}$$

where $R(u)$ is the linear term and $N(u)$ is the nonlinear term. Here, D_t^{μ} is the fractional derivative operator in the Caputo sense. The proposed method suggests the solution for Equation 2 as a fractional power series,

$$u(x,t) = \sum_{m=0}^{\infty} h_m(x) \frac{t^{m\mu}}{\Gamma(1 + m\mu)}, \quad x \in I, \quad 0 < \mu \leq 1, \quad \text{and} \quad 0 \leq t < R$$

Then, the $u_k(x,t)$ is given as

$$u_k(x,t) = \sum_{m=0}^{k} h_m(x) \frac{t^{m\mu}}{\Gamma(1 + m\mu)}, \quad x \in I, \quad 0 < \mu \leq 1, \quad \text{and} \quad 0 \leq t < R \tag{3}$$

The 0-th RPSM approximate solution of $u(x,t)$ is expressed as

$$u_0 = h_0(x) = u(x,0) = h(x)$$

Equation 3 can be given as

$$u_k(x,t) = h(x) + \sum_{m=1}^{k} h_m(x)\frac{t^{m\mu}}{\Gamma(1+m\mu)}, \quad x \in I, \quad 0 < \mu \le 1, \quad 0 \le t < R, \quad \text{and} \quad k \in \{1,2,\cdots\} \tag{4}$$

The residual function for Equation 2 is stated by

$$\mathrm{Res}_u(x,t) = D_t^{\mu} u(x,t) - R(u) - N(u)$$

Hence, $\mathrm{Res}_{u,k}$ is expressed as

$$\mathrm{Res}_{u,k}(x,t) = D_t^{\mu} u_k(x,t) - R(u_k) - N(u_k) \tag{5}$$

As can be seen in [25,26, 47-49], it is obvious that $\mathrm{Res}_u(x,t) = 0$ and $\lim_{k\to\infty} \mathrm{Res}_{u,k}(x,t) = \mathrm{Res}_u(x,t)$, for $t \ge 0$ and $x \in I$. Since the fractional derivative of a constant function is zero in the Caputo sense, we express $D_t^{m\mu}\mathrm{Res}_u(x,t) = 0$. Besides, the fractional derivatives of $\mathrm{Res}_u(x,t)$ and $\mathrm{Res}_{u,k}(x,t)$ are matching at $t = 0$ for $m \in \{0,1,\cdots,k\}$; that is $D_t^{m\mu}\mathrm{Res}_u(x,0) = D_t^{m\mu}\mathrm{Res}_{u,k}(x,0) = 0$, $m \in \{0,1,\cdots,k\}$.

To gain the coefficients $h_m(x)$ with $m \in \{1,2,\cdots,k\}$ in Equation 4, we substitute the $u_k(x,t)$ in Equation 5 and calculate the $D_t^{(k-1)\mu}$ of $\mathrm{Res}_{u,k}(x,t)$ for $k \in \{1,2,\cdots\}$ at $t = 0$. Then, we solve the following algebraic equation

$$D_t^{(k-1)\mu}\mathrm{Res}_{u,k}(x,0) = 0, \quad 0 < \mu \le 1, \quad 0 \le t < R, \quad t = 0, \quad \text{and} \quad k \in \{1,2,\cdots\} \tag{6}$$

4. Implementation of RPSM for the Solution of the Fractional CRWP Equation

In this section, the suggested method is used to determine the RPSM solutions for Equation 1 subject to the initial condition

$$u(x,0) = \frac{1}{2} + \frac{1}{1+\cosh x - \sinh x} \tag{7}$$

Here, $u(x,t) = \frac{1}{2} + \frac{1}{1+\cosh(x-t)-\sinh(x-t)}$ is the exact solution of Equation 1 for $\mu = 1$ [14]. We express the residual function of Equation 1 as

$$\mathrm{Res}_u(x,t) = D_t^{\mu} u(x,t) - \frac{\partial}{\partial x}u(x,t) + 2u(x,t)\frac{\partial}{\partial x}u(x,t) + \frac{\partial^2}{\partial x^2}u(x,t)$$

Hence, $\mathrm{Res}_{u,k}(x,t)$ is given as

$$\mathrm{Res}_{u,k}(x,t) = D_t^{\mu} u_k(x,t) - \frac{\partial}{\partial x}u_k(x,t) + 2u_k(x,t)\frac{\partial}{\partial x}u_k(x,t) + \frac{\partial^2}{\partial x^2}u_k(x,t) \tag{8}$$

We investigate $k = 1$ in this equation to determine the $h_1(x)$ and gain

$$\mathrm{Res}_{u,1}(x,t) = D_t^{\mu} u_1(x,t) - \frac{\partial}{\partial x}u_1(x,t) + 2u_1(x,t)\frac{\partial}{\partial x}u_1(x,t) + \frac{\partial^2}{\partial x^2}u_1(x,t)$$

From Equation 4 at $k = 1$,

$$u_1(x,t) = h(x) + h_1(x)\frac{t^{\mu}}{\Gamma(1+\mu)}$$

Therefore,

$$\mathrm{Res}_{u,1}(x,t) = h_1(x) - \left(h'(x) + h_1'(x)\frac{t^{\mu}}{\Gamma(1+\mu)}\right) + 2\left(h(x) + h_1(x)\frac{t^{\mu}}{\Gamma(1+\mu)}\right)\left(h'(x) + h_1'(x)\frac{t^{\mu}}{\Gamma(1+\mu)}\right)$$
$$+h''(x) + h_1''(x)\frac{t^{\mu}}{\Gamma(1+\mu)}$$

We gain $\mathrm{Res}_{u,1}(x,0) = 0$ from Equation 6. Hence,

$$h_1(x) = \frac{1}{-2(1+\cosh x)}$$

Therefore,

$$u_1(x,t) = \frac{1}{2} + \frac{1}{1+\cosh x - \sinh x} - \frac{1}{2(1+\cosh x)}\frac{t^{\mu}}{\Gamma(1+\mu)}$$

To determine $h_2(x)$, we investigate $k = 2$ in Equation 8 and gain

$$\mathrm{Res}_{u,2}(x,t) = D_t^{\mu}u_2(x,t) - \frac{\partial}{\partial x}u_2(x,t) + 2u_2(x,t)\frac{\partial}{\partial x}u_2(x,t) + \frac{\partial^2}{\partial x^2}u_2(x,t)$$

From Equation 4 at $k = 2$,

$$u_2(x,t) = h(x) + h_1(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)}$$

Thus,

$$\mathrm{Res}_{u,2}(x,t) = h_1(x) + h_2(x)\frac{t^{\mu}}{\Gamma(1+\mu)} - \left(h'(x) + h_1'(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2'(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)}\right)$$
$$+2\left(h(x) + h_1(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)}\right)\left(h'(x) + h_1'(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2'(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)}\right)$$
$$+h''(x) + h_1''(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2''(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)}$$

We gain $D_t^{\mu}\mathrm{Res}_{u,2}(x,0) = 0$ from Equation 6. Thus,

$$h_2(x) = -2\mathrm{csch}^3 x \sinh^4\left(\frac{x}{2}\right)$$

Hence,

$$u_2(x,t) = \frac{1}{2} + \frac{1}{1+\cosh x - \sinh x} - \frac{1}{2(1+\cosh x)}\frac{t^{\mu}}{\Gamma(1+\mu)} - 2\mathrm{csch}^3 x \sinh^4\left(\frac{x}{2}\right)\frac{t^{2\mu}}{\Gamma(1+2\mu)}$$

To find $h_3(x)$, we investigate $k = 3$ in Equation 8 and gain

$$\mathrm{Res}_{u,3}(x,t) = D_t^{\mu}u_3(x,t) - \frac{\partial}{\partial x}u_3(x,t) + 2u_3(x,t)\frac{\partial}{\partial x}u_3(x,t) + \frac{\partial^2}{\partial x^2}u_3(x,t)$$

From Equation 4 at $k = 3$,

$$u_3(x,t) = h(x) + h_1(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)} + h_3(x)\frac{t^{3\mu}}{\Gamma(1+3\mu)}$$

Hence,

$$\mathrm{Res}_{u,3}(x,t) = h_1(x) + h_2(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_3(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)} - \left(h'(x) + h_1'(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2'(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)} + h_3'(x)\frac{t^{3\mu}}{\Gamma(1+3\mu)}\right)$$

$$+2\left(h(x) + h_1(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)} + h_3(x)\frac{t^{3\mu}}{\Gamma(1+3\mu)}\right)\left(h'(x) + h_1'(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2'(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)} + h_3'(x)\frac{t^{3\mu}}{\Gamma(1+3\mu)}\right)$$

$$+h''(x) + h_1''(x)\frac{t^{\mu}}{\Gamma(1+\mu)} + h_2''(x)\frac{t^{2\mu}}{\Gamma(1+2\mu)} + h_3''(x)\frac{t^{3\mu}}{\Gamma(1+3\mu)}$$

We gain $D_t^{2\mu}\mathrm{Res}_{u,3}(x,0) = 0$ from Equation 6. Thus,

$$h_3(x) = -\frac{1}{8}(-2 + \cosh x)\mathrm{sech}^4\left(\frac{x}{2}\right)$$

Therefore,

$$u_3(x,t) = \frac{1}{2} + \frac{1}{1 + \cosh x - \sinh x} - \frac{1}{2(1 + \cosh x)}\frac{t^{\mu}}{\Gamma(1+\mu)} - 2\mathrm{csch}^3 x \sinh^4\left(\frac{x}{2}\right)\frac{t^{2\mu}}{\Gamma(1+2\mu)}$$

$$-\frac{1}{8}(\cosh x - 2)\mathrm{sech}^4\left(\frac{x}{2}\right)\frac{t^{3\mu}}{\Gamma(1+3\mu)}$$

Utilizing the same operation for $k = 4$,

$$h_4(x) = -\frac{1}{16}\mathrm{sech}^5\left(\frac{x}{2}\right)\left(-11\sinh\left(\frac{x}{2}\right) + \sinh\left(\frac{3x}{2}\right)\right)$$

and

$$u_4(x,t) = \frac{1}{2} + \frac{1}{1 + \cosh x - \sinh x} - \frac{1}{2(1 + \cosh x)}\frac{t^{\mu}}{\Gamma(1+\mu)} - 2\mathrm{csch}^3 x \sinh^4\left(\frac{x}{2}\right)\frac{t^{2\mu}}{\Gamma(1+2\mu)}$$

$$-\frac{1}{8}(\cosh x - 2x)\mathrm{sech}^4\left(\frac{x}{2}\right)\frac{t^{3\mu}}{\Gamma(1+3\mu)} - \frac{1}{16}\mathrm{sech}^5\left(\frac{x}{2}\right)\left(-11\sinh\left(\frac{x}{2}\right) + \sinh\left(\frac{3x}{2}\right)\right)\frac{t^{4\mu}}{\Gamma(1+4\mu)}$$

The solution $u_4(x,t)$ is obtained for $\mu = 0.25$, $\mu = 0.50$, and $\mu = 1$ with the different values of x and t in Table 1. Besides, $u_4(x,t)$ is compared numerically with the exact solution for $\mu = 1$ in this table. Table 1 indicates that the absolute error increases as the value t increases. When compared with the generalized homotopy analysis method [16] and the natural decomposition method [22], it is seen that more numerical results are presented with the proposed method for the different values of x and t in this table. The comparison of the approximate solution and the exact solution is illustrated for $0 \leq x \leq 1$ and $t = 0.1$ by the natural decomposition method. However, this comparison is illustrated for $-20 \leq x \leq 20$ and $0 \leq t \leq 1$ by the suggested method. Moreover, the comparison of the approximate and exact solutions is demonstrated only with the help of figures by the generalized homotopy analysis method.

Table 1. Comparing the $u_4(x,t)$ solution with the exact solution

x	t	$\mu = 0.25$	$\mu = 0.50$	$\mu = 1$		
		$u_4(x,t)$	$u_4(x,t)$	$u_4(x,t)$	Exact solution	Absolute error
-20	0	0.500000002061	0.500000002061	0.500000002061	0.500000002061	0
	0.2	0.500000001322	0.500000001336	0.500000001688	0.500000001688	5.21805×10^{-15}
	0.4	0.50000000142	0.500000001187	0.500000001382	0.500000001382	1.64757×10^{-13}
	0.6	0.500000001569	0.500000001147	0.500000001132	0.500000001131	1.21259×10^{-12}
	0.8	0.500000001743	0.50000000118	0.500000000931	0.500000000926	4.9557×10^{-12}
	1	0.500000001931	0.500000001277	0.500000000773	0.500000000758	1.46766×10^{-11}

Table 1. (Continued) Comparing the $u_4(x,t)$ solution with the exact solution

x	t	$\mu = 0.25$	$\mu = 0.50$	$\mu = 1$		
		$u_4(x,t)$	$u_4(x,t)$	$u_4(x,t)$	Exact solution	Absolute error
-5	0	0.506692850924	0.506692850924	0.506692850924	0.506692850924	0
	0.2	0.504227945443	0.504341132518	0.50548631283	0.505486298899	1.39308×10^{-8}
	0.4	0.504461951935	0.503840255916	0.504496707853	0.504496273161	4.34692×10^{-7}
	0.6	0.504857371886	0.503672525135	0.503687458916	0.503684239899	3.21902×10^{-6}
	0.8	0.505329920677	0.503726292195	0.503031646234	0.503018416325	1.32299×10^{-5}
	1	0.50585023446	0.50396675685	0.502512007312	0.502472623157	3.93842×10^{-5}
5	0	1.49330714908	1.49330714908	1.49330714908	1.49330714908	0
	0.2	1.48180816187	1.48809037128	1.4918374435	1.49183742885	1.46499×10^{-8}
	0.4	1.47688585161	1.48424157072	1.49004867877	1.49004819813	4.8064×10^{-7}
	0.6	1.47276294279	1.48024305141	1.48787530595	1.48787156502	3.74094×10^{-6}
	0.8	1.46904740969	1.47598316537	1.4852421188	1.48522596831	1.61505×10^{-5}
	1	1.46559068753	1.47142711235	1.48206425377	1.48201379004	5.04637×10^{-5}
20	0	1.5	1.5	1.5	1.5	0
	0.2	1.49999999636	1.49999999837	1.49999999954	1.5	4.56339×10^{-10}
	0.4	1.49999999477	1.49999999715	1.49999999899	1.5	1.01354×10^{-9}
	0.6	1.49999999343	1.49999999587	1.49999999831	1.5	1.69303×10^{-9}
	0.8	1.49999999222	1.4999999945	1.49999999748	1.5	2.51955×10^{-9}
	1	1.4999999911	1.49999999303	1.49999999648	1.4999999851	1.138×10^{-8}

In Figure 1, the comparison between the exact solution and the $u_4(x,t)$ is demonstrated for $-20 \leq x \leq 20$ and $0 \leq t \leq 1$ at $\mu = 1$. When equal parameters are chosen, it is clear that the $u_4(x,t)$ solution has almost the same shape as the exact solution in Figure 1.

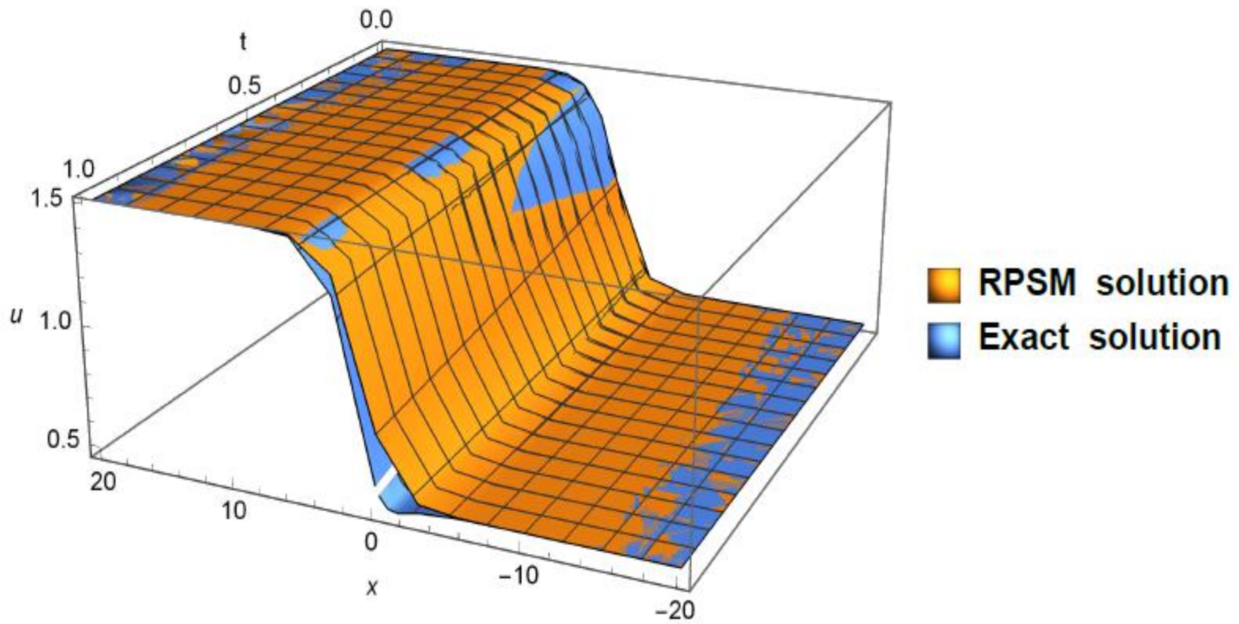

Figure 1. The graphic of the exact solution and $u_4(x,t)$

In Figure 2, the $u_4(x,t)$ is demonstrated for $-10 \le x \le 10$ and $0 \le t \le 5$ when $\mu = 0.1$, $\mu = 0.4$, $\mu = 0.7$, $\mu = 1$. In Figure 3, the same solution is illustrated for $-10 \le x \le 10$ and $t = 4$ with the different values of μ. The solution at $\mu = 0.1$ is demonstrated with the blue line, the solution at $\mu = 0.4$ is demonstrated with the orange line, the solution at $\mu = 0.7$ is demonstrated with the green line, and the solution at $\mu = 1$ is demonstrated with the red line in Figure 3. Cleary observed from Figure 3 that a solitary wave occurs as the values of α increase. All graphics are demonstrated with the aid of Mathematica.

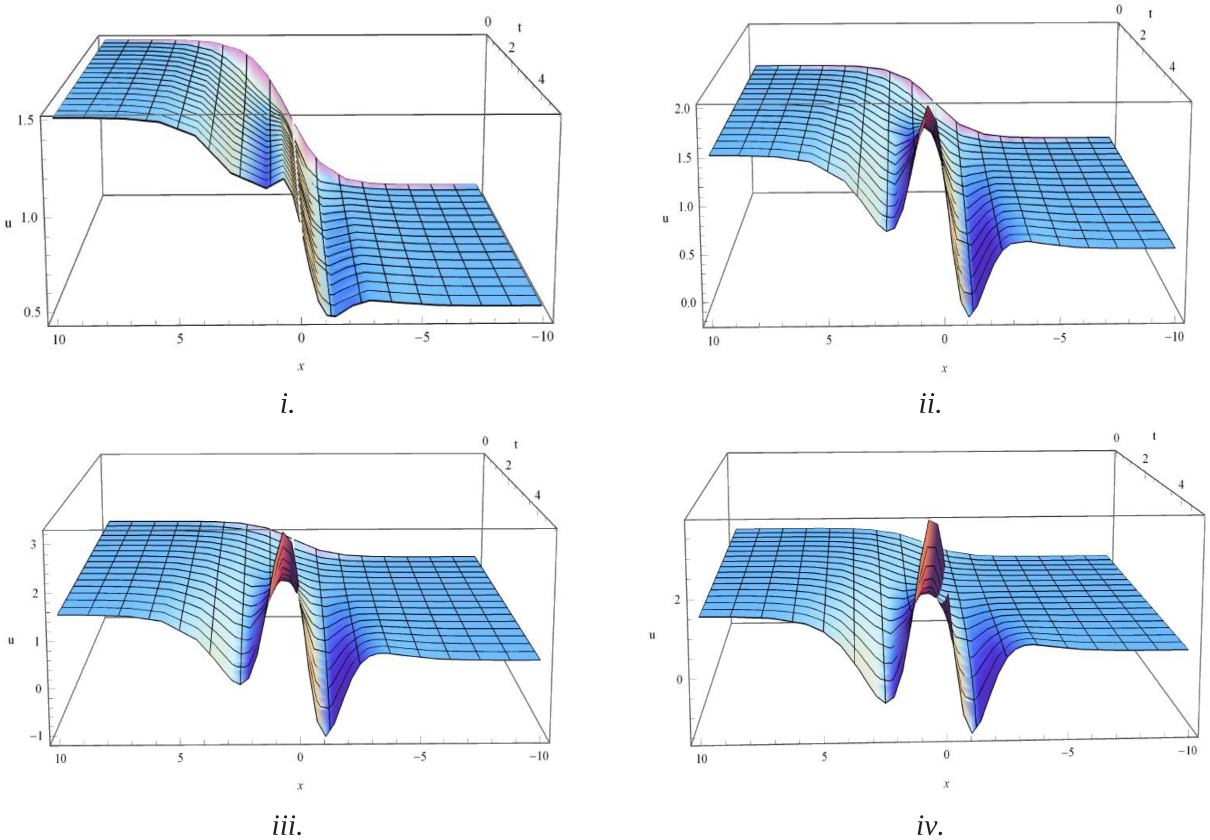

Figure 2. 3D graphics of the $u_4(x,t)$: (*i*) for $\mu = 0.1$, (*ii*) for $\mu = 0.4$, (*iii*) for $\mu = 0.7$, and (*iv*) for $\mu = 1$

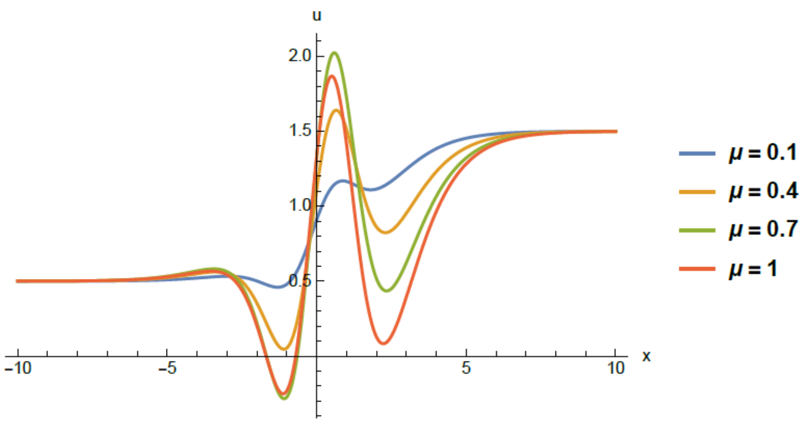

Figure 3. 2D graphic of the $u_4(x,4)$ for the different values of μ

5. Conclusion

In this paper, RPSM is utilized to obtain the approximate solutions of Equation 1. Numerical results are introduced with the different values of μ, x, and t. The proposed method reaches a higher level of accuracy when these results are investigated. It is seen that the approximate solutions are found to have nearly the same shape as the exact solution when equal parameters are chosen. These solutions are also illustrated in 2D and 3D graphics as proof of visualization. The suggested method does not require a lot of calculation work and time. This method can obtain infinite series solutions using only a few iterations. Moreover, RPSM is highly efficient for the fractional CRWP equation. Furthermore, there is no need for perturbation, linearization, discretization, or transformation when utilizing the proposed method. For future studies, RPSM can be used as an alternative to gain the approximate solutions of different types of partial and fractional differential equations encountered in physics, mathematics, and engineering.

Author Contributions

The author read and approved the final version of the paper.

Conflict of Interest

The author declares no conflict of interest.

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