# THE DEVELOPMENT OF STUDENTS' MATHEMATICAL SKILLS IN THE EVALUATION OF NUMERICAL EXPRESSIONS INVOLVING ORDER OF OPERATIONS 

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#### Abstract

This paper examines the students' ability in using their mathematical skills when performing order of operations. In this study, the 'hierarchy-of-operators triangle by Ameis (2011)' was introduced as an alternative BODMAS approach to help students in gaining a better understanding behind the concept of the order of operations. The study of 21 Year 9 students in one of the government secondary schools in Brunei Darussalam used mixed qualitative and quantitative methods. Comparisons of the scores showed positive progress and greater improvement in the students' performance. Interviews from the students were also analysed to gain further insight. Most of the interviewed students responded that it is easier for them to remember the triangle to remember the order of operations.


Key words: Order of operations, numerical expressions, secondary mathematics

## INTRODUCTION

Producing highly skilled people requires students to excel in Mathematics. These students must master fundamental arithmetic operation through the understanding of conceptual knowledge. Bautista (2010) suggested that when evaluating numerical expression, it has to 'operate' in an order.

The acronym BODMAS (Brackets, Order, Division, Multiplication, Addition and Subtraction) has been the common teaching method used to help students (Headlam \& Graham, 2009). Although the acronym helps the students to remember the order of operations, it does not develop their conceptual knowledge (Ameis, 2011). Thus, this issue needs to be addressed.

## The Study

In the local context, when performing operations on the number, students must apply the BODMAS rule. Generally, students are still weak in performing operations. Vanderbeek (2007) suggested that when teaching the order of operations, instead of memorizing the mnemonic device using BODMAS, it should be focusing on the basic fundamental mathematical principle. However, there are some students who understood the mnemonic used and knew which order to perform first, but they still have difficulty in manipulating and solving expressions.

Consequently, this study was based on observations of Year 9 students simplifying mathematical expression containing multiple operations. Hence, the purpose of this study was to investigate how Year 9 students develop

[^0]their mathematical skills. Carpenter and Lehrer (1999) suggested that the importance of learning skills with understanding to avoid rote application.

One of the purposes of this study was to enhance the basic mathematical skills. Acquiring these skills will help students in the algebra topic (Yahya \& Shahrill, 2015). Therefore in this study, another approach was introduced apart from the mnemonic. The focus is on the ability of students performing the order of operations. Students were exposed to the hierarchy-of-operators triangle (Ameis, 2011) focusing on the conceptual knowledge of the order of operations.

This study is guided by the following research questions: Did the Year 9 students develop their mathematical skills when performing the Order of Operations during the evaluation of numerical expression? And how difficult it is for Year 9 students to accept another approach in remembering the Order of Operation?

## LITERATURE REVIEW

A study done by DeLashmutt (2007) investigated the role of mnemonics in learning mathematics. She stated that students who practice mnemonics may remember the math concept and will be "able to retrieve them at a later date" (p. 2). However, Kalder (2012) argued that the use of mnemonic such as PEMDAS might also hinder the students. A notable disadvantage of mnemonic is that it does not connect to the conceptual understanding of the order of operations.

In a study by Lee et al. (2013), the researchers used alternative approach. Participants from the experimental group were exposed to the 'Rearranging Numerical Expression' approach whereas the control group used the ordinary Order of Operations method. They found that the experimental group managed to simplify questions by applying the 'Rearranging Numerical Furthermore, the experimental group showed that the students understood the method and this could be one of the ways to develop their mathematical reasoning in using the mnemonic, PEMDAS.

The triangle shown in Figure 1, taken from Ameis (2011), illustrated with 'powers' on top of the triangle, followed by multiplication and division as having the same priority. The addition and subtraction are placed at the bottom of the triangle, also shared the same priority.


Figure 1. Hierarchy-of-Operators Triangle by Jerry Ameis (2011)
METHODOLOGY

## Research design

An action research approach was used as the research design in this study. Yasmeen (2008) mentioned that in educational system, action research is conducted in classroom not only to gain student response, but also to gain feedback on teaching strategies.

## The Sample

This study was conducted in a government secondary school in the Brunei-Muara district. There were 21 students from Year 9 doing Mathematics at the International General Certificate for Secondary Education level (IGCSE) were involved.

## Procedure

Permissions and consents were sought from the relevant parties before conducting this study. The participants' details are strictly confidentiality in this study. Before the intervention lessons, the students were given the pretest which is the first step to identify the problem. In the first cycle of action research, intervention lessons in three sessions were completed where one session lasted around 50 minutes. Subsequently, a post-test was given
to the students. From the first post-test, we felt that the students had not acquired the necessary skills. By comparing the results in the first post- and pre-test, it was necessary to design another cycle for this study. In the second cycle, the students were placed into groups of two depending on their scores. During the intervention lesson, an activity was set up for the groups. Upon completion, another post-test were given to check the students' development.

## Instruments

## Mathematics Achievement Test

Scores from pre-test and post-test were collected to assess the students' mathematical skills in evaluating numerical expression involving order of operations. The students were given only 30 minutes. During the test, the students were not allowed to use calculator. Table 1 below listed the problems for different types of operations.

Table 1. Problems for Each Item and the Types of Operations Included

| Item Number | Problem structure | Problem | Types of operations included |
| :---: | :---: | :---: | :---: |
| 1 | Multiple choice question | $9 \div 1+3 \times 0$ | Division, addition and multiplication |
| 2 | Multiple choice question | $8+2 \times 2-6-1^{2}$ | Addition, multiplication, subtraction and exponent |
| 3 | Multiple choice question | $\begin{aligned} & 9+3 \times(6-2)-8 \\ & \div 4 \end{aligned}$ | Addition, multiplication, bracket, subtraction and division |
| 4 | Multiple choice question | $3+9 \div 3+1$ | Addition and division |
| 5 | Multiple choice question | $14-3 \times 2-5$ | Subtraction and multiplication |
| 6 | Multiple choice question | $(3+2) \times 2 \div(1+9)$ | Brackets, addition, multiplication and division |
| 7a | Inserting brackets | $3 \times 4+5-15=12$ | Multiplication, addition and subtraction |
| 7 b | Inserting brackets | $4+8-2 \div 2+3=10$ | Addition, subtraction and division |
| 8 | Justification of answer | $9+2 \times 6=66$ | Addition and multiplication |
| 9 | Comparison | $\begin{aligned} & 5 \times 2-2 \times 4 \text { or } \\ & 5 \times(2-2) \times 4 \\ & \hline \end{aligned}$ | Subtraction, multiplication and bracket |

Each item was scored on a 0-2 point scale. It was essential for the students to show their workings and have correct answers with the appropriate use of the correct order of operations to earn 2 points. However, one point was given only when they showed acceptable workings even though the answers were not exact. Otherwise, the students will get zero point for incorrect answers or no attempt in the problem.

## Audio-Recorded Interview

A semi-structured interview was conducted with selected participants from the study sample. The purpose of this interview was to extract more information, ideas from the interviewee and to avoid biasness and for analytical purpose (Gill et al., 2008). The selected students for the interview were the ones who had improved. During the interview, students were essentially asked their understanding of the topics and the difficulty in remembering the methodologies.

## RESULTS and DISCUSSIONS

The results of the pre- and post-tests of the students were analysed quantitatively by using the Statistical Package for the Social Science (SPSS). Meanwhile, the information collected during the interview was combined and analysed for emerging patterns.

The study compared the results between the participants pre- and post-tests. Paired $t$-test was used to determine significance at the 0.05 level. Table 2 shows the students had great improvement during the second post-tests from the pre-test, the mean of the second post-test (13.43) was greater than the mean of the pre-test (6.14).

Table 2. Mean and Standard Deviation of the Achievement Tests

|  | Mean | Standard deviation |
| :--- | ---: | ---: |
| Pre-test | 6.14 | 4.767 |
| Post-test 1 | 8.71 | 7.072 |
| Post-test 2 | 13.43 | 6.408 |

Using paired sample $t$-test the p-value of 0.000 supports the fact that there is a significant difference between the second post-test and the pre-test of the students $(p=0.000<0.05)$. Although the results in Table 2 showed the mean of the first post-test (8.71) was greater than the mean of the pre-test, but the study did not show statistically significant difference as the p-value of $0.127>0.05$.


Figure 2. Graphs of Students' Marks Obtained During Pre-test and the Improvement for the Post-tests
Figure 2 above shows the graphs between pre-test of the students and their improvement $t$. There was greater improvement in the second post-test compared to the first. From analysing the students' pre-test, there was several common type of mistakes, as listed below.

Mistake \#1: Evaluating the numerical expression by performing the operations from left to right.
Mistake \#2: Multiplying any number by zero.
Mistake \#3: Weakness in Exponents
From the interview, it can be concluded that students' conceptual understanding has improved and they prefer to use hierarchy-of-operators triangle.

## CONCLUSIONS

Students with strong foundation in conceptual knowledge of Order of Operations may reduce the misconceptions in the topic. Reducing misconceptions will help students be more confident. For every level, it is important that students be given continuous practice and consistent review of the topic to reinforce the concept. Some limitation for this study is the lack of ability of the participants and their lack of English proficiency (Pungut \& Shahrill, 2014).

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