Journal name	International e-Journal of Educational Studies
Abbreviation	IEJES
e-ISSN	2602-4241
Founded	2017
Article link	http://doi.org/10.31458/iejes.1345567
Article type	Research Article
Received date	18.08.2023
Accepted date	09.10.2023
Publication date	21.10.2023
Volume	7
Issue	15
pp-pp	752-768
Section Editor	Prof.Dr. Kürşat YENİLMEZ
Chief-in-Editor	Prof.Dr. Tamer KUTLUCA
Abstracting & Indexing	Education Source Ultimate Database Coverage List EBSCO Education Full Text Database Coverage List H.W. Wilson Index Copernicus DRJI Harvard Library WorldCat SOBIAD
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Conceptualization, literature review, methodology, implementation, data analysis, translation, and writing

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To cite this article:

Arabacı, D., & Kanbolat, O. (2023). Pre-service elementary mathematics teachers' evaluations of meetings with expert and peer participation based on van Hiele geometric thinking levels. *International e-Journal of Educational Studies, 7* (15), 752-768. https://doi.org/10.31458/iejes.1345567

Research Article

Pre-Service Elementary Mathematics Teachers' Evaluations of Meetings with Expert and Peer Participation Based on van Hiele Geometric Thinking Levels*

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Abstract

In this study, it was aimed to present the pre-service elementary mathematics teachers' (PEMT) evaluations of meetings with expert and peer participation based on van Hiele geometric thinking levels (vHGTL). The case study method was used and the study group of the research consists of three PEMT who are studying in the third year of a state university. Data were collected with meeting records, reflection reports of the PEMTs, researcher's field notes and interviews and were analyzed simultaneously by two researchers to ensure coding reliability with content analysis method. As a result of the research, it was seen that there was a positive change in PEMTs' criteria for determining vHGTL. It was determined that they had difficulties in clearly distinguishing the differences between the levels at the beginning, and that in the last meetings, they insisted on the criteria related to the levels they determined and made more accurate determinations about the levels. As a matter of fact, the teacher candidates also stated that meetings enabled them to determine the achievements', solved and unsolved problems' vHGTL better. In addition, they stated that allowing them to freely express their ideas in meetings contributed to their professional development in geometry teaching knowledge. Suggestions regarding experts and peers participatory meetings were presented.

Keywords: : Preservice elementary mathematics teachers, van Hiele theory, meetings, expert, peer

1. INTRODUCTION

The rapid and versatile development of society, science and technology has led to the changes in the individual's needs, interests, sources of motivation, values and characteristics. In this regard, it is crucial for teachers, who are responsible for raising individuals, to keep up with this rapid change and to design and implement learning environments. Indeed, this is inherent in the teaching profession, which is dynamic and open to life-long change and development. Taking the first step into the profession with undergraduate education, teachers have been in an attempt to maintain their professional development through activities such as in-service seminars, courses, projects or collaborative professional practices they have attended since they started to work (Özgenel, 2019). Researchers pointed out that promoting the collaborative work of the pre-service teachers/teachers is regarded as a significant component for professional development practices (Guskey, 2007). The primary objective of these practices is to ensure and durability between pre-service teachers/teachers. Working in cooperative groups has become a tradition of the teaching profession in some countries

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(e.g. Japan) (Lewis, 2002). In our country, pre-service teachers or teachers are involved in collaborative work either through undergraduate courses or projects with the participation of peers or colleagues along with experts such as administrators, families or academicians (Kanbolat, 2015; Watanable, 2005). It is most probable to provide pre-service teachers/teachers with their professional knowledge, skills, attitudes and values thanks to the groups created for professional development in which different knowledge and experiences related to learning and teaching are shared. Because participants and experts have the opportunity to examine and discuss the subject they are working on and teaching of the subject in such environments (Kanbolat, 2015; Kriewaldt, 2012). Cooperation with experts, peers or colleagues is also paramount in these environments (Kanbolat, 2015). The reason for engaging experts in collaborative working teacher groups may be due to either benefiting from in-depth theoretical knowledge, resources or materials related to the subject or providing collaborative work between group members (Kanbolat & Arslan, 2022).

Collaborative teacher practices focused on learning and teaching mathematics in particular maintain the exchange and development of pre-service teachers'/teachers' knowledge of mathematics instruction (Kanbolat, 2015). Teachers are considered as the designers and implementers of mathematics learning environments; moreover, their knowledge of teaching mathematics and how effectively and competently they use this knowledge are fundamental in the teaching and learning process of mathematics (Baki, 2012). Because the knowledge and learning experiences of the teachers, who are one of the most significant components of the learning environment, are reflected in the classroom environments. The components of mathematics instruction knowledge were defined in different ways by various researchers (Ball et al., 2008; Shulman, 1986). Knowledge of teaching mathematics was firstly explained by Shulman (1986) with the components of subject matter content knowledge, pedagogical content knowledge and curriculum knowledge. Afterwards, the researchers added many variables related to the learning process such as knowledge of student recognition, knowledge of teaching methods and strategies, context knowledge, measurement and evaluation knowledge (Baki, 2012). As regards the teacher competencies published by the General Directorate of Teacher Training and Development (GDTTD) under the Ministry of National Education (MoNE) in Turkey, a mathematics teacher's knowledge of teaching mathematics can be classified as mathematics content knowledge, skills, attitudes and values (GDTTD, 2017). Knowledge of mathematics teaching may refer to information about learning and teaching, or it can be classified under categories such as Number Teaching Knowledge, Geometry Teaching Knowledge, and Data Teaching Knowledge according to learning domains (Aydın, 2018). Especially since the beginning of the 20th century, many studies have been conducted to enrich the learning and teaching processes of geometry (Atasoy, 2019). Therefore, teachers equipped with the knowledge of teaching geometry need to possess a comprehensive and deep knowledge of geometry and be receptive about the relationship between geometry subjects and concepts (Altaylı et al., 2014). Teachers' mastery about the conceptual knowledge on geometry is vital for overcoming the problems faced by the students in the classroom (Gürbüz & Durmuş, 2009). Mathematics teachers' ability to design and implement ideal geometry learning environments in which they reflect their current knowledge is as significant as their content knowledge about geometry (Toluk et al., 2002). Hence, a course called teaching geometry and measurement was incorporated in the primary school mathematics teaching undergraduate curriculum, which was implemented in Turkey as of the 2018-2019 academic year. Pre-service teachers are familiar with the concepts and theories that are important in teaching geometry within the scope of this course, and that they are expected to perform tasks by considering these concepts and theories. The van Hiele Theory is considered as one of the most significant theories available in this course.

The van Hiele theory was developed in 1957 by two mathematics educators, Pier M. van Hiele and his wife Dina Van Hiele-Gelfod during their Ph.D. studies in Ultrehct University (Crowley, 1987; Güven, 2006), to explain why some students have difficulties with higher-order cognitive processes,

especially in proof, required for success in geometry (Usiskin, 1982). This theory involves five geometric thinking levels, which indicates that geometric thinking goes through certain stages in a hierarchical manner. In that theory, each level indicates how individuals think about these geometric concepts and these different types of thought processes (Atasoy, 2019, p. 162). Table 1 depicts the names and descriptions of these levels.

Table 1. Names and descriptions of van Hiele geometric thinking levels

Levels	Characteristics	Example
Level 1 (Visualisation)	The student is interested only in the image of the given figure so s/he can learn the names of figures and recognizes the shape as a whole, but cannot recognize the geometric features of the shape. The students can also identify, name and compare the figures only in terms of their appearance.	Squares and rectangles seem different.
Level 2 (Analysis)	The student can identify the features of geometric figures. However, the student cannot associate its features with each other since s/he perceives the features of a geometric shape independently of each other. On the other hand, students can discover the features and rules of a figure by making use of activities such as folding and measuring, and can prove these via experimental methods.	Rectangles have four right angles. The student can list the properties of square and rectangle separately. But (s)he cannot say that every square is also a rectangle.
Level 3 (Informal deduction)	Students at this level can enumerate geometric figures and see the relationships between features of these, but use informal reasoning to establish these relationships. Namely, they can follow a proof, but cannot make their own proofs.	Every square is also a rectangle.
Level 4 (Deduction)	The student understands the importance of definitions, inferences, postulates and theorems in proof and can use them in geometric proofs and determines the necessary and sufficient conditions for a class of shapes. The student can prove other theorems deductively by making use of the already proven theorems and axioms.	The student can prove that the sum of the measures of the interior angles of a triangle is 180^0 through using the postulate of parallelism.
Level 5 (Rigor)	The student at this level can use the axioms, theorems, definitions of Euclidean geometry in non-Euclidean geometries and comprehend the relationships and differences between different axiomatic systems.	The student can work on non-Euclidean geometries.

**Table 1 was developed under favor of Atasoy (2019), Güven (2008) and Usiskin (1982).*

The van Hiele theory offers that students who have trouble are being taught at a higher van Hiele level than they are at or ready for (Usiskin, 1982). The theory offers a remedy for identifying the geometry thinking levels of the students in terms of planning the teaching and helping the students to reach a higher level. Baki (2008) noted that the van Hiele theory holds two different proposals: determining the students' geometric thinking levels, planning the lessons according to the students' levels, and developing geometric thinking through concrete objects. Pre-service teachers need a strong understanding of content knowledge in geometry, awareness of students' misconceptions, learning theories that explain the source of these misconceptions, and teaching strategies to address these misconceptions (Pusey, 2003). This will be possible if teacher education programs enable pre-service teachers to engage in self-reflection and provide plenty of guidance and input for teacher educators to help trainees develop their reflection (Bischoff, Hatch, & Watford, 1999). In fact, studies reported that learning opportunities are effective on students' success, teacher training outcomes and learning (Çelik et al., 2020).

On analyzing the relevant literature, the studies were mostly grounded on determining the pre-service teachers' van Hiele geometric thinking levels (Armah et al., 2017; Atasoy, 2019; Salifu et al., 2018), the effect of dynamic geometry software and/or physical manipulatives on the pre-service teachers' van Hiele geometric thinking levels (Karakuş & Peker, 2015) and the impact of van Hiele theory-based learning environments on pre-service teachers' knowledge of geometry or geometric thinking (Alex & Mammen, 2016; Armah et al., 2018; Erdoğan & Durmuş, 2009; Kaleli-Yılmaz & Koparan, 2016; Yi et al., 2020). These studies analyzed the development of students' existing van

Hiele geometric thinking levels and counted in activities that would improve their geometry thinking. However, there is no such a study specifically published on depicting the reflections from implementation-containing and expert-aided meetings that would help pre-service teachers understand van Hiele Geometric Thinking Levels (vHGTL). Only Yi et al. (2020) conducted a study to examine the impact of van Hiele theory-based instructional activities on pre-service elementary school teachers' knowledge of geometry content, students' geometric thinking levels and geometry activities as well as their relationships. The present study differs from the related study and others in the literature in various ways. This study was conducted with pre-service Elementary Mathematics Teachers (PEMTs); moreover, the study also provides them the opportunity to share and discuss the results of these examinations in expert-aided environments through examining the learning objectives, activities and problems in the textbooks in terms of vHGTL. Some studies suggested that pre-service teachers had low vHGTL (Erdoğan & Durmuş, 2009; Kaleli-Yılmaz & Koparan, 2016), van Hiele theory-based instruction was identified to have a positive effect on high school students' vHGTL (Alex & Mammen, 2016; Kutluca & Gömlekçi, 2022) and undergraduate students' vHGTL (Armah et al., 2018; Erdoğan & Durmuş, 2009; Yi et al., 2020). Moreover, the pre-service teachers were determined to have insufficient geometry content understanding based on van Hiele levels, and pedagogical content knowledge, including students' understanding of geometric thinking levels or geometry instructional activities, may be improved through participation in well-designed instruction tasks (Yi et al., 2020). For this reason, it is critical for pre-service teachers, who are the teachers of the future, to have knowledge and experience about these levels, to be able to determine the levels of students, and to design learning environments in accordance with the levels of students. It is also of great importance to provide guidance and opportunities for teacher educators to help their students in the transitions between these levels and to have first-hand experience about van Hiele levels during their undergraduate education through active participation in subjects such as presenting appropriate activities/problems. It is a matter of curiosity what kind of results will be generated by the meetings created with the participation of expert academicians through concrete examples. In the literature, it is stated that collaborative sharing environments formed by bringing together experts and pre-service teachers positively affect the professional development of pre-service teachers (Ruohotie-Lyhty & Moate, 2016). In sharing environments with the participation of pre-service teachers and experts, experts support pre-service teachers to access theoretical knowledge (Kanbolat, 2015); make self-evaluation (Suh & Parker, 2010); deepen their mathematical knowledge (Bieda et al., 2013); focus on students'mathematical thinking (Bieda et al., 2013; Potari, 2011); and develop their understanding of teaching and learning mathematics (Potari, 2011). Inspired by this fact, the study focused on the creation of sharing environments where pre-service teachers and experts take place together and the interaction experiences of pre-service teachers with their peers and experts with vHGTL content. Thus, it is believed that the sharing environments will contribute to pre-service teachers' professional development as well as their content knowledge about vHGTL. In this regard, this study sought for an answer to the problem "How do the Pre-Service Elementary Mathematics Teachers evaluate the meetings with expert and peers participation based on vHGTL?". In service of this research problem, answers to the following sub-problems were sought:

1. What are the gains of PEMTs about vHGTL through meetings with experts and peers based on vHGTL?
2. How do PEMTs evaluate the presence of experts in the meetings with expert and peers participatory based on vHGTL?
3. How do PEMTs evaluate the presence of their peers in meetings with expert and peers participatory based on vHGTL?
4. How do PEMTs evaluate their professional development through meetings with expert and peers participatory based on vHGTL?

2. METHOD

This section covers information regarding the research design, participants, data collection tools and process, data analysis and ethical statement.

2.1. Research Design

Having a qualitative research method, this study employed the case study design. Creswell (2007) defines the case research as a qualitative research approach in which the researcher explores one bounded case or more bounded cases over time through multiple sources of information (observations, interviews, documents, reports, etc.), and reports a case description and case-based themes. This study is a sample of a case study since it presents PEMTs evaluations of meetings with expert and peer participation, and describes the process with a variety of data collection tools such as meeting records, reflection reports of the PEMTs, researcher field notes and interviews.

2.2. Participants

This study was conducted with three 3rd grade pre-service teachers who study at the department of the elementary mathematics teaching in a state university and two academicians who are experts in the field of mathematics education.The study used convenience sampling method, which is one of the purposive sampling methods. This method refers to the collection of data from a sample that the researcher can easily reach (Büyüköztürk et al., 2015).

The pre-service teachers, who voluntarily participated in the study, have successfully received Teaching Geometry and Measurement course, in which one of the experts was an educator in the previous term, and they have prior knowledge regarding the van Hiele theory. Experts in sharing environments have doctoral degrees in mathematics education and conducting courses related to mathematics teaching at undergraduate and graduate levels. Due to the confidentiality of the participants, the real names of the pre-service teachers were not used; instead, they were represented by the code names as Damla, Ecmel and Zümra.

2.3. Data Collection Tools and Data Collection Process

This study deployed meeting logs, reflection reports of the pre-service teachers, researcher field notes and interviews. Figure 1 summarizes the data collection process.

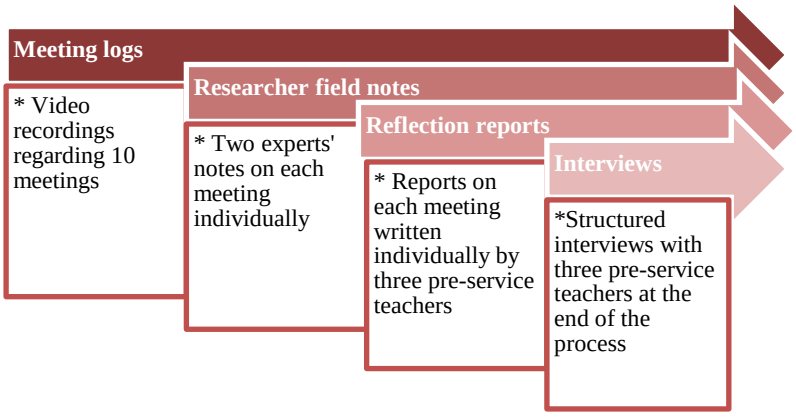

Figure 1. Data collection process

Meeting logs: A total of 7 interviews were conducted with the PEMTs to determine vHGTL. The interviews were conducted online and lasted approximately 45-90 minutes. The interviews were conducted and recorded via the Zoom application. All learning outcomes, solved and unsolved problems in the "Lines and Angles" sub-learning area within the "Geometry and Measurement" learning area of the 5th, 6th and 7th grade mathematics textbooks were examined according to vHGTL in the video-recorded meetings. The textbooks examined during the research process are 5th, 6th and 7th grade secondary school mathematics textbooks belonging to MoNE publications. The reason for the choice of textbooks from this publishing house is that MoNE is the ministry responsible for

national education services in Turkey. The reason why textbooks from this publisher are preferred is that MoNE publishing belongs to the ministry responsible for carrying out national education services in Turkey.

The pre-service teachers identified the relevant parts at each grade level according to the vHGTL through discussion before the meetings. Discussions were carried out on the levels determined by the PEMTs in the meetings with the participation of expert academicians. The discussions were carried out on topics such as how solved/unsolved problems were classified according to vHGTL, why they thought it was at the relevant level, and what distinguished the problem from other levels. In this regard, PEMTs' views were taken into account, and then necessary explanations were made by expert researchers on the accuracy/incorrectness of the levels. The data on the meeting records were used to support the findings.

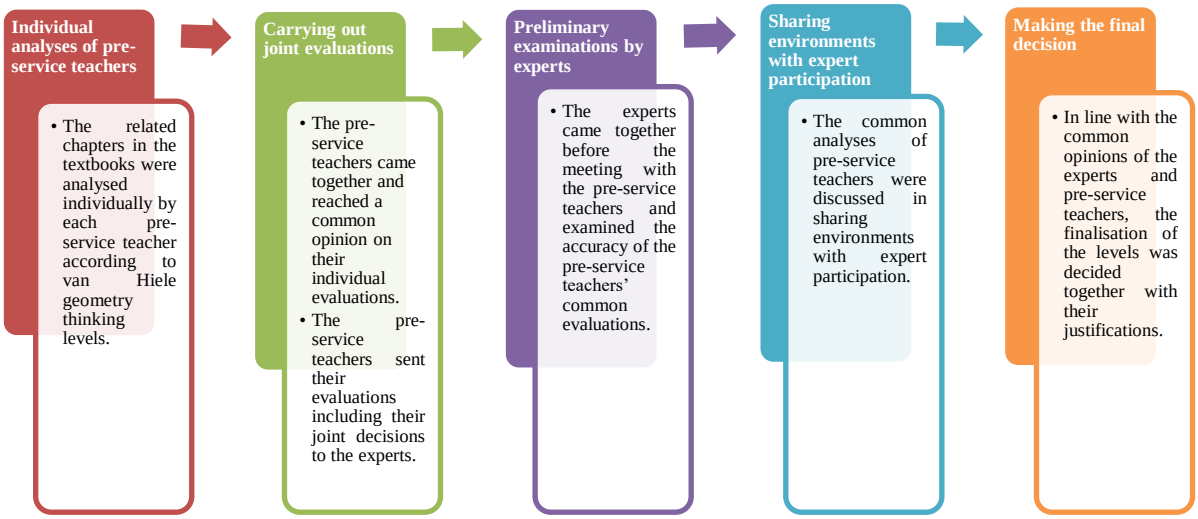

Figure 2. Process of meeting logs

Reflection reports: Unstructured reflection reports, in which pre-service teachers individually reflected on their experiences and professional development after each meeting, were used to support the research data.

Researcher field notes: The unstructured field notes of the experts on meetings and the development of pre-service teachers after each meeting were used to support the findings.

Interviews: Semi-structured interviews were held with PEMTs to evaluate the whole process. The interviews were conducted individually with the PEMTs, they were audio-recorded and lasted an average of 5-15 minutes. The following problems were posed to the PEMTs during the semi-structured interviews:

1. How did this whole process contribute to you? What aspect (variable) of the environment contributed to you? (previous meetings with experts and peers, etc.)
2. What are your views on the presence of experts in the meetings?
3. What are your views on the presence of peers in the meetings?

2.4. Data Analysis

The interview data were primarily analyzed in parallel to the sub-problems. Content analysis was used during data analysis (Patton, 2002). In inductive content analysis, codes emerge from the participants' expressions and form meaning clusters. Similar data are brought together and handled, and documents related to the data are systematically examined and analyzed (Yıldırım & Şimşek, 2013). For this purpose, firstly, the audio recordings of the interviews were transcribed. Then, each participant's transcript was coded and themes were generated by bringing together similar codes. A similar process was carried out for reflection reports. Related codes and themes were presented in tables and direct quotations of the codes are presented. As stated above, the main data source of the

research is interviews. The findings were supported with excerpts from meeting logs, reflection reports and researchers field notes. While the findings of the study are presented, the categories emerging are described with the relevant data in detail (Patton, 2002).

2.5. Validity and Reliability of the Research

Validity in qualitative research includes the strategies researchers use to ensure the reliability of their studies (Creswell & Miller, 2000). Reliability in qualitative research is ensured by consistency, which means that another researcher can reach similar results; and confirmability, which means taking into account the entire process, from the applied method to long-term interaction with the participants, from the researcher's position to the researcher's experience and perspective (Noble & Smith, 2015). In this context, for ensuring a long-term interaction 7 meetings lasting between 45-90 minutes were held with the participants of the research; in addition, the first researcher conducted different courses of the participants for 7 semesters. Researchers attended meetings with participants in the role of experts. All meetings and interviews with participants were recorded to prevent data loss. Both of the researchers are experts in the field of mathematics education and have experience in qualitative studies. One of the researchers teaches "Qualitative Research with NVIVO" as a post graduate course. Hence, the researchers have experience in qualitative research. On the other hand, one of the researchers instructed the "Geometry and Measurement" course for two semesters. Therefore, it can be stated that the researcher has knowledge and experience about van Hiele geometric thinking levels. For ensuring coding reliability the data were analyzed simultaneously by two researchers. Hence, audio recordings of the interviews with PEMTs were transcribed and coded simultaneously. Similar coding processes were also carried out for the pre-service teachers' reflection reports. Afterwards, the researchers came together and analyzed each code and the codes got their final version. Therefore, reliability was ensured by the expert review method suggested by Yıldırım and Şimşek (2013) to provide credibility in qualitative research.

3. FINDINGS

This section presents findings under four main headings in line with the sub-problems of the study.

3.1. Findings Regarding the Pre-service Teachers' Gains about vHGTL through Meetings

Table 2 depicts the pre-service teachers' evaluations on their gains about vHGTL through the meetings.

Table 2. The pre-service teachers' evaluations on their gains about vHGTL through the meetings

Views	f
Recognize their misinformation about vHGTL	D, E, Z
Enhance their knowledge of vHGTL	E, Z
Distinguish vHGTL more clearly	D, E
Opportunity to study more deeply about vHGTL	D

As in Table 2, the pre-service teachers noted that meetings based on vHGTL helped them realize the wrong information they had about vHGTL, contributed to their knowledge about vHGTL, allowed them to distinguish vHGTL more clearly and had the opportunity to study vHGTL in more detail.

All of the pre-service teachers underlined that meetings made them realize the wrong information they had about vHGTL. These statements were frequently encountered especially in the pre-service teachers' reflection reports.

To exemplify, the reflection report that Zümra wrote after the 2nd meeting indicates a change in her knowledge about vHGTL "I missed the detail that these problems could be solved as an activity in

the classroom under the heading "Let's do it together". I evaluated the problems only with student thinking, not with their solution.".

Similarly, Zümra's views changed after the speeches in another meeting related to the problem "Are we ready?" in Figure 2 in the 5th grade mathematics textbook. Zümra's expressions referring that there has been a change in the knowledge regarding this problem are displayed as follows.

Figure 3. The problem "Are we ready?" in the 5th grade secondary school mathematics textbook (MoNE, 2019)

English version of the problem presented in Figure 3.

1) Match the following statements with the appropriate models.

a) Line b) Line segment c) Closed half-line

"This is a problem that we thought as a Level I but agreed that it was Level 2 during the meeting. The problem involved matching the shapes of lines, line segments and rays with daily life contexts such as the light coming out of the lantern and the violin. We believed that this problem was at Level 1 by analyzing the images, yet we decided that the images could not be understood clearly and exceeded the visual level along with the evaluations of our teachers. My perspective towards Level 1 has changed with this problem."

[Excerpt from Zümra's 3rd meeting reflection report.]

Here are the statements of Zümra, who expressed that meetings contributed to PEMTs' knowledge about vHGTL.

"My knowledge of levels was insufficient before I started working. So yes, we learned it in the lesson, you also taught it, but that's it. When I saw the problems, I did not have enough knowledge to categorize them. First of all, I had to do some research on the levels before I started working. I searched on studies first, so I did it myself. Then, I got information about the levels and made a note for myself, I made a note on which level we call this level, whatever, second level, whatever, third level, and I used that note. We also examined the textbooks ourselves, it was very useful for them to examine the books related to our field, to deal with the problems there and to put them into certain categories." [Excerpt from Zümra's interview statements.]

Considering the PEMTs' reflection reports after the interviews, they were identified to take notes on distinguishing across the levels more clearly. For instance, the following statements in the reflection report of the 7th meeting show that Ecmel can distinguish more clearly between the 2nd and 3rd levels.

"[...] we have reached another conclusion that we can set a roadmap for ourselves. This was the 2nd level if there was a direct use of information, and the 3rd level if the student was expected to make inferences."

Besides, Ecmel commented on distinguishing across the levels better in the interview with the following statements.

"I think we had a lesson for levels in the second grade. We learned the van Hiele levels. [...] But it wasn't like that, of course, as we studied it in class, I began to understand more. I started to be able to distinguish more and more which problem belongs to which level. In that sense, the lesson contributed a lot."

One of the PEMTs, Damla evaluated the ability to distinguish the levels more clearly by having the opportunity to study in more detail about vHGTL during the process. Some views of the participant are as follows.

"For instance, we learned the van Hiele Levels in the lesson, teacher, we learned them last year, but we have studied them in more detail. We've learned it in more detail this year. [...] Now, when I analyze a problem, I can understand more clearly what level it is." [Excerpt from Damla's interview statements]

3.2. Findings Regarding the Pre-service Teachers' Evaluations on the Presence of Experts in the Meetings Based on vHGTL

Table 3 displays the pre-service teachers' evaluations regarding the presence of experts in the meetings.

Table 3. The pre-service teachers' views on the presence of experts in the environment

Views	P.T.*
Helping to find the accurate information	D, E, Z
Gaining different perspectives	E, Z
Eliminating ambiguity	Z
Instant feedback	D
Ensuring permanent learning	D
Encouraging systematic work	E
Boosting the use of academic language	E

**P.T.: Pre-service Teacher*

As is seen in Table 3, the pre-service teachers' views on the presence of experts in the meetings were categorized as helping to reach the accurate information; gaining different perspectives; eliminating ambiguity; instant feedback; ensuring permanent learning; encouraging systematic work and boosting the use of academic language.

All the pre-service teachers stated that the experts helped them to have the right information. Damla's statement is depicted as an example.

"[…] (experts) *directed us to find the truth instead of telling directly. The presence of experts in the environment allowed me to learn more permanently and contributed to me academically."* [Excerpt from Damla's interview statements]

The field note of the researcher 2 regarding the following dialogue between PEMTs and experts related to the vHGTL activity at the 5th grade level in the 6th meeting supports the view that the experts encouraged the pre-service teachers to reach the correct answer.

"The pre-service teachers stated that the activity is the third level. Some reasons were presented and the pre-service teachers were requested to discuss the possibility of being considered as a 2nd level to encourage them to think and discuss. While Damla and Zümra argued that their views were correct, Ecmel stated that our problems and the examples we gave confused them. Ecmel was also unsure of their previous decision. Researcher 1 opened an activity that they previously considered as level 3 and asked them to compare the two activities. Zümra shared her opinion with her justifications and stated that she evaluated it as 3 levels. Researcher 2 drew attention to the explanation related to the solution of the activity and asked them to think about it. All the pre-service teachers stated that they were sure of their decisions after reading the explanation." [Excerpt from the field note of the 6th meeting.]

Ecmel and Zümra reported that experts helped them gain different perspectives. Zümra's statements in the reflection report after the first meeting are presented as follows.

"I had a hard time evaluating the learning outcomes before the first meeting. There were points where I hesitated and made me think. I did not think that we would evaluate the level determinations we made during the meeting in this way. Although I determined reasons for each of them, it was very useful to talk to my teachers and evaluate them. I was able to evaluate it from different perspectives."

Zümra concluded that they were hesitant to determine the levels of some problems before the meeting and that they got rid of these uncertainties with the comments of the experts in the reflection report written after the 5th meeting.

"The meeting we held today was mostly grounded on the problems that we talked about with my friends and were undecided about. Frankly, I'm happy about this because we are on the right track in the evaluations we made before the meeting. Discussing and clarifying the problems that we have hesitated during the meeting, clears the problem marks in my mind."

Damla mentioned that they can receive immediate feedback from the experts when they need it, and that the experts contribute to the permanent learning.

"The presence of experts in the environment allowed me to get immediate feedback on the problems I asked. When I did not understand, I was able to consult immediately, I could contact quickly. Besides, they provided us with an environment for discussion. They guided us to find the truth instead of telling directly in this discussion environment. [...]"

Unlike other pre-service teachers, Ecmel pinpointed that experts encouraged PEMTs to work systematically and to use academic language. Ecmel's statements related to the experts' encouragement to work systematically are as follows.

"Not doing the review randomly may be the biggest contribution in the academic sense. In other words, we normally examine the textbook with the assignments in our lessons, but there is a general randomness in accordance with the homework instructions, but in a certain order. Because we have progressed under your [...] management, we have progressed in a certain order. We consciously researched the things we needed to examine, not randomly like this. So this has been very useful for me."

Ecmel's statements regarding the experts' encouragement of the PEMTs to use academic language are summarized as such.

"[...] we tried to use more academic language, which was significant to me. Namely, we tried to talk in a more descriptive way, not in a more friendly environment. [...] Of course, we also need to pay attention to the work we do since you are experts. How can I say, since you can be aware of an information we say without really researching, we have to talk about such things that we need to pay extra attention to, any explanation we would say, either by paying extra attention to a speech or a sentence, and really filling it up."

3.3. Findings Regarding the Pre-service Teachers' Evaluations on the Presence of Peers in the Meetings

Table 4 depicts the pre-service teachers' evaluations on the presence of their peers in the meetings.

Table 4. The pre-service teachers' evaluations regarding the presence of peers in the meetings

Views	P.T.
Gaining different perspectives	D, E, Z
Ensuring they are confident in their decisions	D, Z
Increasing communication skills	E, D
Complementing each other's drawbacks	Z
Feeling comfortable	E

**P.T.: Pre-service Teacher*

Table 4 suggests the PEMTs' views on the presence of their peers in the meetings as gaining different perspectives; being sure of their decisions; increasing communication skills; completing each other's drawbacks and feeling comfortable.

All pre-service teachers confirmed that the presence of their peers in the meetings had them gain different perspectives. In this regard, some views of Ecmel during the interview are suggested as follows.

"My thoughts changed in some problems. Let me give an example. If I believe that a problem is at the second level, but you doubt it, we definitely discuss this problem together. When we discussed, sometimes we left it on the same level, sometimes we took it to a different level as we had different opinions. I gained a different perspective. Before we had a discussion with you, I got some information from my friends. Therefore, we all have different ideas because we think differently. This has also provided benefit for our home meetings."

Damla indicated that she was more confident about the decisions she made thanks to her peers during the interviews, and that her communication skills also improved thanks to her peers.

"They supported me when I was unsure. They helped me have an idea about which direction I should go. In other words, my communication skills developed in this way."

Zümra mentioned in the interviews that working with her peers provided different perspectives and that they complemented each other's shortcomings.

"Since they are my friends, we proceeded comfortably through the process. So everyone knew their own responsibility and what to do. We have a feeling for this. That's why, I think it's a process by which we contribute

to each other. We had an effective process with our friends, we gained different perspectives and made up for each other's deficiencies."

Ecmel also stated that she felt more comfortable in the environment thanks to her peers during the interview.

"My teacher assured me, actually. I think having a friend or a peer reassures you. So I'm not alone. Even if I had an extra task to do, I felt more comfortable and better because we did it together."

3.4. Findings on the Pre-service Teachers' Evaluations Regarding Their Professional Development through Meetings

Table 5 presents the findings on the PEMTs' evaluations regarding their professional development through meetings.

Table 5. The pre-service teachers' evaluations regarding their professional development through meetings

Views	*PT
Reviewing the mathematics curriculum and textbook	D, E
Examine and relate concepts related to geometry	D, E
Update knowledge of geometry related terms	D, E
Solving problems about geometry	D, E
Ability to self-assess	D, E
Gaining different perspectives	E, Z
Ability to work systematically	E
Ability to express oneself	E
Ability to use academic language	E

**P.T.: Pre-service Teacher*

As observed in Table 5, the pre-service teachers' views are categorized as reviewing the mathematics curriculum and textbook, examining and associating concepts related to geometry, updating knowledge of geometry-related terms, solving problems about geometry, ability to self-assess, gaining different perspectives, ability to work systematically and expressing oneself and using academic language.

The statements of Damla and Ecmel regarding that they had the opportunity to examine secondary school mathematics books and curriculum are displayed as follows.

"We had the opportunity to review the textbooks. We revised the variety of problems. We examined the levels of those problems. In addition, we examined the curriculum, the secondary education curriculum." [Excerpt from Damla's interview statements]

"I have examined the curriculum related to my field and the problems in the textbook in more detail. Namely, I have gained experience in my field. This is how I examined the syllabus in detail as a pre-service teacher. I have examined more consciously what kind of problems are included at which grade level."
[Excerpt from Ecmel's interview statements.]

Damla and Ecmel pointed out that they had the opportunity to examine and associate concepts related to geometry from time to time in their meeting. In this regard, it is noteworthy to take the expressions by Ecmel during the interview into consideration.

"I have some drawbacks like this. For instance, sometimes I cannot remember very simple things in geometry and other mathematical sense, but I forget. As we examine everything in detail, whether it is very simple or not, and to what level it belongs, I also learned them in terms of geometry. For example, what exactly a ray is or what kind of examples we can give. Or there were different solutions in the solutions of the problems about angles, I did not know all the solutions, but when we talked about those solutions in detail, I learned more than one way, which contributed a lot."

Similarly, it is remarkable what Damla wrote in the reflection report after the 2nd meeting on examining and associating the concepts related to geometry.

"*One of the problems that we focused on the most at this meeting was the explanation of the parallelism by associating it with the triangle. We did not see the association part in this problem while we were discussing the problems, but I observed that it was obvious to associate with a triangle there, and I began to examine more carefully whether any association is available in such direction and unit problems during the meeting."*

In addition, Damla emphasized that she sometimes had the opportunity to solve problems about geometry and work on mathematical terms during the process.

"You know, I had to do the solutions in order to understand the problems, the level of geometric concepts, etc.. Of course, it contributed in this sense. So, teacher, I forgot some terms. I had the opportunity to remember. I had the opportunity to understand their differences on terms."

The field note received by Researcher 2 for the 3rd meeting may be an example of the work process on the geometry problems as Damla expressed *"We talked to the pre-service teachers about this problem (the one presented in Figure 3) for about 10 minutes. We discussed at length each item of the problem and how the students would solve these items and which skills they would solve. In the meantime, we almost solved the problem together. In particular, we discussed the item 'ç' in detail. We have discussed in detail the cases where two lines are parallel to each other, intersect and even intersect perpendicularly."*

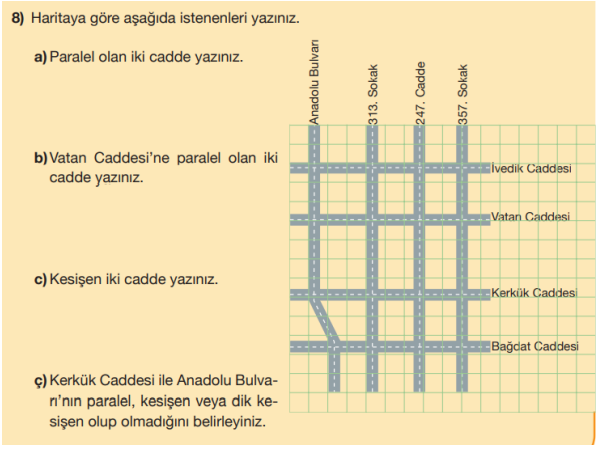

Figure 4. The problem "Your turn" in the 5th grade secondary school mathematics textbook (MoNE, 2019)

English version of the problem presented in Figure 2.

8) Write the following asked according to the map.

a) Write two parallel streets.

b) Write two streets parallel to Vatan street.

c) Write two intersecting streets.

ç) Determine whether Kerkuk Street and Anadolu Boulevard are parallel, intersecting or perpendicular.

Ecmel and Zümra reported that their self-evaluation skills improved in the process with the examples related to the meetings. Ecmel's statements regarding this code are as follows.

"Because, teacher, I have some drawbacks, I am aware of this and I have not fully developed it yet. I have shortcomings like this. For instance, sometimes I cannot remember very simple things in geometry and other mathematical sense, but I forget. As we examine everything in detail, whether it is very simple or not, and to what level it belongs, I also learned them in terms of geometry. For example, what exactly a ray is or what kind of examples we can give. Or there were different solutions in the solutions of the problems about angles, I did not know all the solutions, but when we talked about those solutions in detail, I learned more than one way, which contributed a lot."

Elif and Zümra affirmed that they made progress towards gaining different perspectives. In this sense, the statements of Zümra written in the diary after the 2nd meeting are noteworthy.

"I had the opportunity to examine and query the problems from different perspectives with the explanations of both my friends and instructors during the meeting."

Ecmel evaluated the process as a good experience in terms of systematic working culture.

"We consciously investigated the things we needed to study, not randomly like this. Therefore, this has been very useful for me. In other words, it was good for me to analyze in detail about these van Hiele levels rather than open the textbook and categorize it as an unsolved problem."

Ecmel's statements written in her diary after the 1st meeting was related to finding the opportunity to express herself in a discussion-based environment.

"I would like to state that I like the environment for discussion and that we are defending our ideas, namely, the creation of such an environment. :)"

Moreover, another issue that Ecmel expressed within the context of professional development was about using academic language.

"We tried to use more academic language, which was significant to me. In other words, we tried to speak more descriptively by trying to use more words rather than in a more friendly environment. It has had such an effect."

4. DISCUSSION and CONCLUSION

It is concluded that pre-service teachers had wrong information about vHGTL thanks to the process they experienced on van Hiele geometric thinking levels with the participation of experts; the process contributed to their knowledge about VHGTL; it offered the opportunity to study more about vHGTL, and that they were able to distinguish vHGTL more clearly. On analyzing the relevant literature, the van Hiele theory-based instructional practices (Alex & Mammen, 2016; Armah et al., 2018; Erdoğan & Durmuş, 2009; Kaleli-Yılmaz & Koparan, 2015; Yi et al., 2020), well-structured geometry lessons (Guven, 2006; Toluk et al., 2002; Tutak & Birgin, 2008), dynamic geometry software (DGY), concrete materials and drawing activities (Karakuş & Peker, 2015) were determined to be effective on the pre-service teachers' van Hiele geometric thinking levels. It is recommended to design appropriate experiences so that pre-service teachers are familiar with the van Hiele theory (Alex & Mammen, 2016). Therefore, it is most likely that the meetings on van Hiele geometric thinking levels with expert participation designed for pre-service teachers helped them to better understand their van Hiele geometric thinking levels and to distinguish the levels more clearly. When the pre-service teachers talked about the learning outcomes or activity during the interviews, the experts frequently asked why the objectives did not belong to a lower level or to a higher level. Such inquiries and explanations are thought to raise the pre-service teachers' awareness towards van Hiele geometric thinking levels.

Besides, the pre-service teachers indicated that they had knowledge about vHGTL, and that they had the opportunity to deepen their knowledge and distinguish the differences across levels more clearly through examining many samples of learning outcomes and activities during the process. Thus, it may be wise to mention that the pre-service teachers' knowledge of vHGTL and the classification practices as well as the examinations related to these levels are significant in terms of their knowledge of van Hiele geometric thinking levels. In this regard, it is recommended to provide environments in which they examine resources such as learning outcomes, activities, examples, and problems related to geometry, rather than sharing theoretical content knowledge in the relevant courses to support the knowledge of pre-service teachers regarding van Hiele geometric thinking levels.

Pre-service teachers also suggested that the presence of experts in the environment helped to reach accurate information, gained different perspectives, helped remove uncertainty, had the opportunity to receive instant feedback, provided permanent learning, encouraged systematic study and academic language use. The pre-service teachers examined the learning outcomes and the solved/unsolved problems according to the vHGTL before the meetings. The experts organized the process by means of a calendar to inform the pre-service teachers at the first meeting about which date, which grade level and which section would be discussed. The pre-service teachers stated that this provided them with systematic work. They were also found to have the opportunity to discuss the learning outcomes they examined, solved/unsolved problems with experts and defend their ideas in expert participatory meetings designed on vHGTL. Thus, many ideas were put forward when analyzing a problem, different ideas were discussed and the most appropriate level was determined

through justifications. This may be effective in gaining different perspectives, reaching correct information and eliminating ambiguity. Likewise, Baş and Işık (2014) concluded that face-to-face communication environments were created with the participation of teachers and academicians. It is vital that the category of gaining different perspectives is jointly expressed for both profiles (expert and pre-service teacher) in the meetings. In fact, individuals have the opportunity to gain different perspectives thanks to the sharing of different information, opinions or experiences in various applications within this social environment, which consists of the participants from different profiles (Baki, 2012; Baş & Işık, 2014; Kanbolat, 2015). Watanable (2005) pinpointed that experts may contribute to meetings with their domain knowledge. The pre-service teachers had the opportunity to work one-on-one with the experts in the meetings, to pose problems to the experts whenever they wanted, and to get instant feedback. Besides, the pre-service teachers discussed their ideas with experts, leading them to use academic language during discussions. Kanbolat (2015) confirmed that the presence of experts in meetings to ensure the pre-service teachers' professional development contributes to their knowledge in teaching mathematics. Since the presence of experts in vHGTL content meetings is considered to be more effective and efficient, it is recommended to engage experts in such environments.

The pre-service teachers' views on the presence of their peers in the process were categorized as gaining different perspectives; ensuring assuredness in their decisions; increasing communication skills; complementing each other's drawbacks and feeling comfortable. Some studies demonstrated that the professional development practices that pre-service teachers carried out together by focusing on a certain common purpose contributed to their professional and personal development (Baki, 2012; Lewis, 2000). Hence, the competency of working collaboratively with colleagues is included in the general competencies of the teaching profession (ÖYGM, 2017). It is suggested to create meetings that enable pre-service teachers to interact with their peers and to urge them to work collaboratively.

Given that the pre-service teachers evaluated the process they experienced in terms of their professional development; their views were categorized as examining the mathematics curriculum and the book, examining and associating concepts related to geometry, updating knowledge of geometry-related terms, solving problems about geometry, ability to self-assess, gaining different perspectives, able to work systematically, ability to express themselves and using academic language. In this vein, the environment created in the study may feed the pre-service teachers' knowledge of the curriculum materials defined by Shulman (1986). Furthermore, teachers' sharing on the concepts related to geometry and the associations between these concepts while examining geometry problems may support their knowledge of geometry since they updated their knowledge of geometry-related terms. Likewise, the related literature affirmed that van Hiele-phased education improves the pre-service teachers' knowledge of geometry content (Armah et al., 2018; Yi et al., 2020). The study conducted by Yi et al. (2020) with the pre-service classroom teachers showed that van Hiele theory-based instructional activities were effective on pre-service teachers' understanding of geometry content knowledge, students' knowledge of van Hiele levels, and their geometry teaching activities. Thus, it is vital that the pre-service teachers be provided with opportunities to work actively with their peers through primary sources before starting the profession as well as expert support for their professional development. The relevant literature uncovers that the pre-service teachers should be provided guidance on geometry-related experiences and improving their geometric thinking levels (Erdoğan & Durmuş, 2009), and that higher levels of geometric thinking can be achieved with the implementation of a student-centered and educator-guided applied curriculum (Alex & Mammen, 2016). It is of great significance to design the geometry teaching courses in the undergraduate program by taking this result into account.

Acknowledgment

The researchers confirmed that the data in this study were collected before the year 2020.

Part of this study was presented as an oral presentation at the 5th International Symposium of Turkish Computer and Mathematics Education (TURCOMAT-2021).

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