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THE EXAMINATION OF 7TH GRADE STUDENTS' ACHIEVEMENTS IN MATHEMATICAL PATTERNS

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ABSTRACT: The aim of this study is to determine the 7th grade students' achievements in mathematical patterns presented by figures, tables, number sequences, and word problems. This research is a situation determination study where quantitative methodology is used. The sample of the study consisted of 47 female and 50 male students, totally 97 students from 7th grades in Giresun city on 2015-2016 academic year. Pattern topic oriented 7 questions were used as the data gathering tool. The questions which focused on the attainments of pattern topic were prepared by the researcher. It was determined that students could perform specialization in figural patterns but they couldn't reach a generalization. In other words, it was observed that students could find the required steps according to a given rule and so they could easily reach the situation which involved operational knowledge. Also, it was seen that although students found the number of figures in the next step of the pattern, they couldn't find the general rule that represented the pattern. Another result of the study was that students could recognize patterns of number sequences but they couldn't find the general term of the pattern. Lastly, it was determined that students couldn't understand patterns which were represented as word problems and they failed at these kind of pattern questions but they had success in pattern questions represented by tables. In this context, it can be offered to give much place to representation forms of patterns by figures, tables, number sequences, and word problems while students are given experiences of patterns.

Key words: Elementary school mathematics curriculum, numbers and operations learning domain, algebra learning domain, pattern representation forms, 7th grade students

INTRODUCTION

Mathematics, as the leading discipline aiming at what is learnt in real life, is a field that has an application for sciences like engineering and technology and in the real world (Hacısalihoğlu Karadeniz, Aydın Güç, & Tülek, 2014). It can be said that "mathematical patterns exist in nature (Devlin, 1998)" as there are patterns in sunflowers, cabbages, and the flow of water. Mathematical patterns are an important subject which are often encountered in daily life and allow connections to be made with other disciplines. For this reason the study of patterns has been included in the mathematics curricula of many countries (Australian Education Council, 1994; Ministry of National Education (MoNE), 2005, 2009a, 2009b, 2013b; National Council of Teachers of Mathematics, 2000).

The pattern, which is a regular order of mathematical objects such as numbers, figures, etc., has been defined in various ways by mathematics educators (Tanışlı & Özdaş, 2009). For example, the pattern can be described as a repeating regularly arrayed combination of elements such as geometrical figures, symbols, signs, actions or mathematical objects (Burns, 2000; Fox, 2005; Souviney, 1994; Waters, 2004). According to Guerrero and Rivera (2002) the pattern is the rule functioning between the elements of structured mathematical objects (numbers, figures, etc.). According to Olkun and Toluk-Uçar (2006) the pattern is like a poem which is composed of regularly arrayed repeating objects or figures. Papic and Mulligan (2007) have defined the pattern as spatial or numeric regularity.

The patterns used in the school curricula are divided into four categories: a figure (visual), a table or graphic, a number sequence, and a word problem (Yaman & Umay, 2013). *Patterns given as a figure (visual)* may be either

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an increasing series of points or may be composed of figures made from matchsticks, counting rods, unit squares, unit spheres, blocks, and tiles (Ley, 2005). In *patterns given as a table or graphic*, the students record the outcomes in each line systematically and look for a pattern among the resulting outcomes (Schliemann, Carraher, & Brizuella, 2001). In *patterns given as number sequences*, the students are expected to determine the connections between the given terms of the pattern, to write the unknown one in the sequence (Yaman & Umay, 2013), and to express the pattern rule using the connection that they have found (Ley, 2005). Moreover, the patterns may also be represented as a word problem (Yaman & Umay, 2013).

When the curricula are examined, it can be seen that the topic of patterns may be highlighted from the pre-school period. In the renewed and updated pre-school curricula, the attainment, "*He/she forms a pattern with the objects*", is given with regard to the cognitive domain (MoNE, 2006, 2013a).

In the primary school mathematics curriculum (1^{st}-5^{th} grade), which was renewed in 2005, patterns are given in the learning domains "*geometry*" and "*numbers*". The topic was given in the form, "*He/she determines the pattern of connection. He/she determines the elements which are missing in a pattern*", in the sub-learning domain "*pattern and tessellation*" in the learning domain "*geometry*" for the 1^{st} grade. Patterns were discussed under the heading, "*He/she forms number patterns*", in the sub-learning domain "*natural numbers*" in the learning domain "*numbers*" in the 2^{nd} grade. The topic of patterns was given as, "*He/she determines the connection in a pattern and extends the pattern*", in the sub-learning domain "*natural numbers*" in the learning domain "*numbers*" in the 3^{rd} grade curriculum. Patterns were expressed through the idea that, "*He/she associates a pattern with numbers and completes the missing part*", in the sub-learning domain "*natural numbers*" of the learning domain "*numbers*" in the 4^{th} grade. Finally, an attainment related to patterns given is observed as, *"He/she forms the pattern, using the rule and procedure; he/she determines the number or numbers which are not given in a pattern*", in the sub-learning domain "*natural numbers*" in the learning domain "*numbers*" in the 5^{th} grade (MoNE, 2005, 2009a).

In the elementary school mathematics curriculum (6^{th}-8^{th} grade), which was renewed in 2005, it is seen that the topic of patterns is also given in the learning domain "*geometry*". The topic of patterns was given in two attainments as "*He/she forms patterns using polygons and identical and similar ones of the polygonal areas*" and "*He/she creates tessellations by translation*", in the sub-learning domain "*pattern and tessellations*" of the learning domain "*geometry*" in the 6^{th} grade. Similarly, the topic was introduced as "*He/she makes a tessellation by tessellating an area with polygonal models*", "*He/she determines the codes of the tessellations created with uniform polygonal models*" and "*He/she makes tessellations by reflecting, translational, and rotational motions*" in the sub-learning domain "*pattern and tessellations*" of the learning domain "*geometry*" in the 7^{th} grade. There after patterns were represented in *"He/she builds patterns from linear, polygonal, and circular models and determines the fractal ones from among these patterns*" in the 8^{th} grade (MoNE, 2009b).

The elementary school mathematics curriculum was updated in 2013 and grades 5, 6, 7, and 8 were renamed the "Middle School Mathematics Curriculum". There are two attainments which explicitly contain the topic of patterns in the updated middle school mathematics curriculum. The attainment "*He/she produces the desired sequence number and figural patterns, the rule for which is given*", the first one of these attainments, belongs to the sub-learning domain "*natural numbers*" within the learning domain "*numbers and procedures*". The attainment "*He/she express the rule of the arithmetic sequence using a letter; he/she finds the desired term for the sequence, the rule of which is expressed with a letter*", the second attainment, belongs to the sub-learning domain "*algebraic expression*" in the learning domain "*algebra*" of the 6^{th} grade (MoNE, 2013b).

Grades 1, 2, 3, and 4 of the elementary school mathematic curriculum were renamed the "Primary School Mathematics Curriculum". In this curriculum, the topic of patterns was given with the attainments "*He/she recognizes the single-rule number pattern, he/she finds the pattern rule*" and "*He/she completes the pattern by finding the missing element in a number pattern*" in the learning domain "*introduction to algebra*" in the 1^{st} grade. The attainments for the learning domain "*geometric patterns*" at the same grade were given as "*He/she finds the rule in a pattern consisting of geometric objects or figures and he/she completes the pattern by finding the missing element of the pattern*" and "*He/she creates a single-rule geometric object or figural pattern of maximum three-element*". The attainment in the learning domain "*introduction to algebra*" in the 2^{nd} grade of the pattern curriculum was given as "*He/she extends the number pattern, the rule of which requires one procedure*". The attainments for the learning domain "*geometric patterns*" in the same grade were given as "*He/she determines the missing elements in a repeating geometric pattern and completes the pattern*" and "*He/she creates new patterns having the same connection with different materials using the connection in a geometric pattern*". The attainment for the learning domain "*introduction to algebra*" in the 3^{rd} grade of the curriculum was given as "*He/she creates a number pattern. He/she creates a pattern, with one procedure as the rule*". The attainments of the learning domain "*geometric patterns*" of the same grade were given as "*He/she makes a design using the figure models. He/she draws the pattern on dotted or squared paper*". In the 4^{th} grade of curriculum the learning domain "*introduction to algebra*" is further extended with the attainments "*He/she*

determines the rules of a number pattern containing a maximum of two different procedures and extends the pattern" and "*He/she creates and describes repeating, increasing, and decreasing number patterns*" (MoNE, 2015).

When the curricula in which the new approaches have begun to be implemented in Turkey since 2005 are examined, it can be understood that the topic of pattern has been given a place in every curriculum. According to MoNE (2009a, 2009b) the main objectives for placing patterns to be in mathematics curricula are to reveal the students' situations in their cognitive schemes, to enhance the psychomotor skills of the students, and to explain mathematics to the students, moving from the abstract to the concrete, using their daily activities.

This study is needed because there are few studies related to patterns conducted with primary school and middle school students in Turkey. Thus, this study will likely make a contribution to this gap in the literature. Accordingly, this study aims to reveal 7th grade students' perceptions of mathematical patterns represented as figures, tables, number sequences, and word problems.

METHOD

This section includes information on the research method, study group, data collection tool, and data analysis.

Research Method

This study is a case determination study using quantitative methodology. Considering the study participants and the subject of the study; this research is a case study method. Because case study method pave the way for profoundly examining the subject, investigating the relations of data and explaining the cause and effect relations (Çepni, 2014).

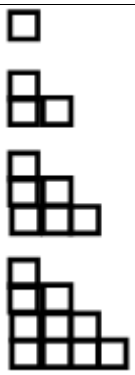
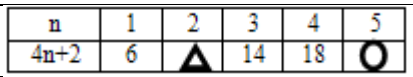
Study Group

The study group comprised of 97 7th grade students consisting of 47 female and 50 male students being educated in a middle school in the center of Giresun during 2015-2016 school year. The names of the students were kept confidential due to ethical reasons and the students were coded as follows: "S1, S2, S3, … , S97".

Data Collection Tool

In this study, totally 7 questions regarding pattern presentation forms were used in order to collect data. The questions were prepared by focusing on two attainments related to patterns in the middle school mathematics curriculum which was updated in 2013 and they were practiced by the students in classroom environment during a single class period individually. The questions used in the study and the attainments on which the questions were focused are provided in Table 1:

Table 1.The Questions Used in the Study and the Attainments on Which these Questions Were Focused

Attainments	Questions
A1. He/she produces the desired sequence in number and figural patterns, the rule of which is given.	**Q1.**Find the sum of the first 7 terms of the pattern, the rule of which is 3a+1.  **Q2.**How many squares are there in the next step according to the figure above? Find the general rule of the pattern?
A2. He/she express the rule of an arithmetic sequence with a letter; he/she	

finds the desired term of the sequence, the rule of which is expressed with a letter.

Q3. What is the sum of  using the pattern, the rule for which is given in the table?

Q4.A model of a pattern which is created by circles is given above. Fill in the table below by determining the pattern used for each step in the model.

Sequence number of the number in the pattern	The number of circles used for the number
1	2
2	4
3	6
4	8
5	…
6	…
…	…
n	…

Q5. Cem, who is a mason, builds a wall by working for 3 hours on the first day and for 4 hours more on the second day than the first day and by repeating the same rule on all subsequent days. What is the general rule and pattern of this activity?

Q6. An entrance fee for a car park is 5 Turkish Liras and is 2 Turkish Liras for every hour thereafter. Find the general rule of this pattern.

Q7. Find the algebraic expression corresponding to the number pattern 0-8-16-24-32-40-…

The 1st and 7th questions are the questions given as a number sequence. The 2nd question is given for determining the next step of a pattern represented as a figure and for finding the general rule of the pattern. The 3rd question includes the determination of the desired sequence of a pattern given in a table. The 4th question which was represented as a figure and a table requires specialization of a figural pattern primarily and than a generalization of it. The 5th and 6th questions are represented as word problems and require finding the general rule of a pattern.

Data Analysis

While analyzing the data, the questions answered by the students were categorized as; correct (answers containing all aspects of the valid answer), partially correct (answers containing one aspect of the valid answer, but not all aspects thereof), incorrect (scientifically incorrect answers), and blank (leaving blank, giving meaningless answers, answers such as "I do not know" or "I did not understand") and the frequencies (f) and percentages (%) were calculated.

FINDINGS and COMMENTS

The frequencies (f) and percentages (%) calculated according to the answers of the students which are correct, partially correct, and incorrect, or left empty are provided in Table 2:

Table 2.The Answers of the Students Given to the Questions

Attainments	Questions	Representation Forms of the Patterns	Answers							
			Correct		Partially Correct		Incorrect		Blank	
			f	%	f	%	f	%	f	%
A1	Q1	Number sequence	30	30.9	2	2.1	43	44.3	22	22.7
	Q2	Figure	3	3.1	36	37.1	19	19.6	39	40.2
A2	Q3	Table	81	83.5	7	7.2	3	3.1	6	6.2
	Q4	Figure and table	45	46.4	43	44.3	8	8.3	1	1.0
	Q5	Word problem	5	5.1	25	25.8	39	40.2	28	28.9
	Q6	Word problem	4	4.1	6	6.2	59	60.8	28	28.9
	Q7	Number sequence	35	36.1	6	6.2	29	29.9	27	27.8

The findings and comments related to the questions are provided below.

Findings and Comments related to Q1

When the findings for the first question are examined it is seen that 44.3% of the students gave incorrect answers, 30.9% of them gave correct answers, 22.7% of them left the question blank, and 2.1% of them gave partially correct answers. Examples of correct, partially correct, and incorrect answers given to the 1st question by the students are provided in Figures 1, 2, and 3:

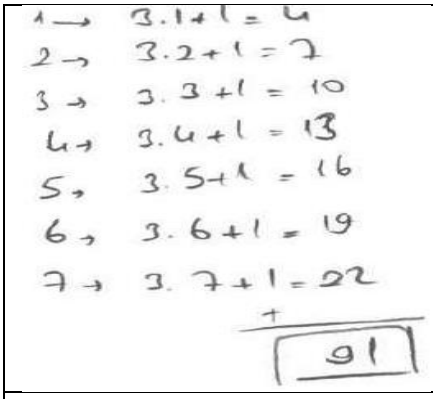


Figure 1. The Correct Answer Given To the 1st Question By S26

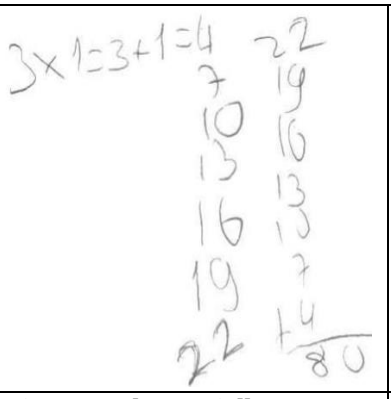


Figure 2. The Partially Correct Answer Given To the 1st Question By S47

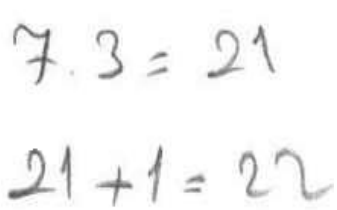


Figure 3. The Incorrect Answer Given To the 1st Question By S34

In this question the students were asked to find the sum of the first 7 terms of a number pattern, the rule of which was given and the expected answer was "4+7+10+13+16+19+22=91". When we examine Figure 2, it is seen that students found all the required terms but they made a mistake in the process of addition. When Figure 3 is examined it was determined that S34 found only the 7th term, not the sum of the first seven terms of the pattern like the majority of the students.

Findings and Comments related to Q2

When the answers given to the 2nd question are examined it is seen that 40.2% of the students left the question blank, 37.1% of them gave partially correct answers, 19.6% of them gave incorrect answers, and 3.1% of them gave correct answers. It was determined that although 37.1% of the students found how many squares would be in the next step of the pattern, they could not obtain the general rule expressing the pattern. Examples of correct, partially correct, and incorrect answers given to 2nd question by the students are provided in Figures 4, 5, and 6:

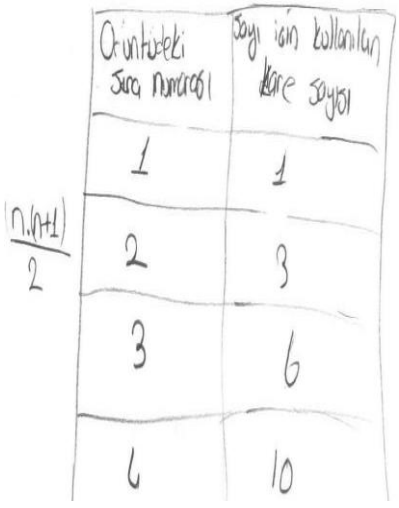


Figure 4. The Correct Answer Given To the 2nd Question By S76

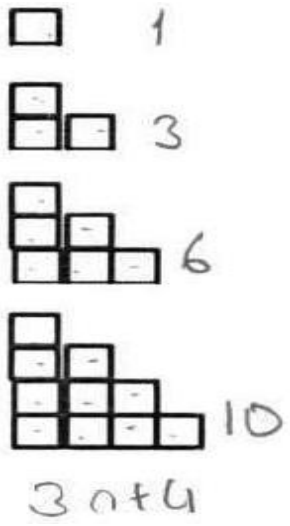


Figure 5. The Partially Correct Answer Given To the 2nd Question By S61

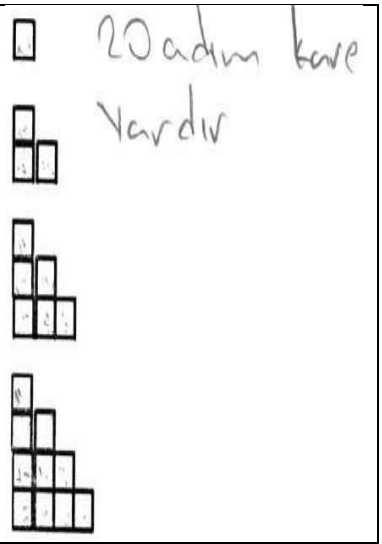


Figure 6. The Incorrect Answer Given To the 2nd Question By S19

What is expected from the students is that they reach the rule "n.(n+1)/2". When Figure 4 is examined it is seen that S76 used the table for solving the 2nd question represented as a figure and solved the problem correctly.

"Making a table, i.e. arranging the data during solving some problems, or knowledge obtained during problem-solving into a table, facilitates seeing the connection between data or the obtained knowledge. Thus, the rule used for obtaining the results is found and the problem is solved" (Altun, 2015, p.76 and 123). When Figure 5 is examined it is seen that although the student correctly found the number of the squares, he/she got the rule of the pattern wrong.

Findings and Comments related to Q3

When the findings for the 3rd question are examined it is seen that 83.5% of the students gave correct answers, 7.2% of them gave partially correct answers, 6.2% of them left the question blank, and 3.1% of them gave incorrect answers. Examples of correct, partially correct, and incorrect answers given to the 3rd question by the students are provided in Figures 7, 8, and 9:

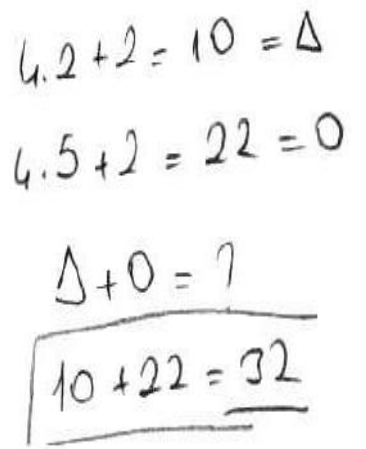


Figure 7. The Correct Answer Given To the 3rd Question By S73

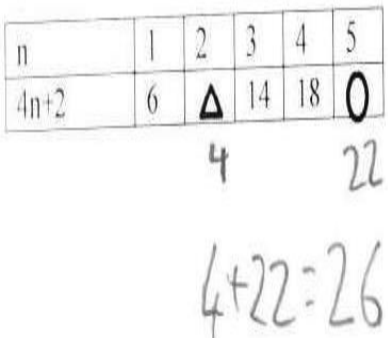


Figure 8. The Partially Correct Answer Given To the 3rd Question By S71

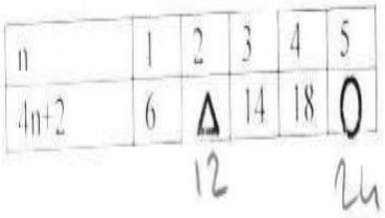


Figure 9. The Incorrect Answer Given To the 3rd Question By S72

In this question, which was represented as a table, what is expected from the students is that they obtain a result which reveals the procedure to be "10+22=32". It is seen that the majority of the students did not have any difficulty answering this question which requires the skill of procedural knowledge. The students who gave a partially correct answer to the question could not get the result as they found a value belonging only to one of the terms, as in Figure 8. "The primary school mathematics curriculum aims to make connections between concepts and procedures and to teach conceptual learning rather than procedural learning. Moreover, the curriculum expects the students to use mathematical terminology correctly and develop their communication skills via concepts, terms, and numbers" (MoNE, 2015, p.4).

Findings and Comments related to Q4

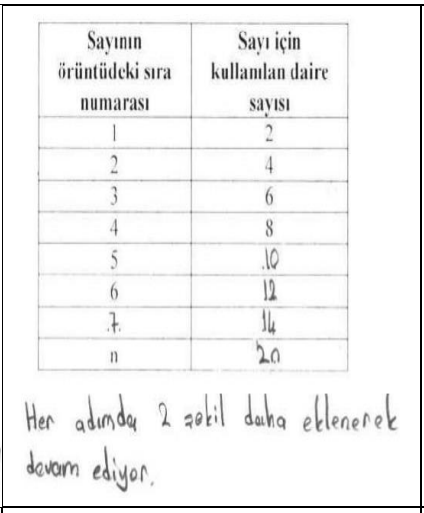
When the findings for the 4th question are examined it is seen that 46.4% of the students gave correct answers, 44.3% of them gave partially correct answers, 8.3% of them gave incorrect answers, and 1.0% of them left the question blank. The 4th question which was represented as a figure and a table, is a question which requires students first to specialize a figural pattern and then to reach a generalization. Specialization means to choose particular or systematic examples in order to understand and give a meaning to a problem, and to examine these examples with the problem (Keskin, Akbaba Dağ, & Altun, 2013). During specialization there are a number of actions such as choosing one or more example, giving examples, identifying, displaying, explaining, and drawing an example (Arslan & Yıldız, 2010). While 46.4% of the students in the study group gave correct answers for this question, 44.3% of them could make a specialization but could not reach a generalization. Examples of correct, partially correct, and incorrect answers given to the 4th question by the students are provided in Figures 10, 11, and 12:

Sayının örüntüdeki sıra numarası	Sayı için kullanılan daire sayısı
1	2
2	4
3	6
4	8
5	10
6	12
...	...
n	2n

2.1 - 2.2 - 2.3 - 2.4 ... 2n

2 - 4 - 6 - 8 ... 2n

Figure 10. The Correct Answer Given To the 4th Question By S36



Sayının örüntüdeki sıra numarası	Sayı için kullanılan daire sayısı
1	2
2	4
3	6
4	8
5	10
6	12
7	14
n	2n

Her adımda 2 şekil daha eklenerek devam ediyor.

Figure 11. The Partially Correct Answer Given To the 4th Question By S61

Sayının örüntüdeki sıra numarası	Sayı için kullanılan daire sayısı
1	2
2	4
3	6
4	8
5	9
6	8
4	6
n	1

Figure 12. The Incorrect Answer Given To the 4th Question By S31

In this question, represented as a figure and table, what is expected from the students is that they obtain the general rule "2.1, 2.2, 2.3,…,2n". With reference to Figure 11 it is understood that the majority of the students who gave a partially correct answer filled in the table by expressing the sequence number of the number which represents the variable "n" in the table as a number rather than writing the general rule.

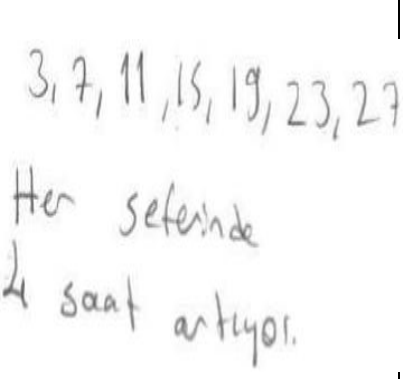
Findings and Comments related to Q5

When the findings for the 5th question are examined it is seen that 40.2% of the students gave incorrect answers, 28.9% of them left the question blank, 25.8% of them gave partially correct answer, and 5.1% of them gave correct answers. Examples of correct, partially correct, and incorrect answers given to the 5th question by the students are provided in Figures 13, 14, and 15:

Gün	Saat
1	3
2	7
3	11

4n-1 olur.

Figure 13. The Correct Answer Given To the 5th Question By S75



3, 7, 11, 15, 19, 23, 27

Her seferinde 4 saat artıyor.

Figure 14. The Partially Correct Answer Given To the 5th Question By S12

her gün 1 saat daha

1.gün 2.gün 3.gün

3 4 5

Figure 15. The Incorrect Answer Given To the 5th Question By S59

In this question, represented as a word problem, what is expected from the students is that they obtain the general rule "4n-1". When Figure 13 is examined it is seen that S75 solved the question by making a table. When Figure 14 is examined it is seen that S12 continued the number pattern correctly in order to find the general rule but got the general rule incorrect. It can be understood from Figure 15 that the student tried to find the correct answer using the figural pattern but failed.

Findings and Comments related to Q6

When the findings for the 6th question are examined it is seen that 60.8% of the students gave incorrect answers, 28.9% of them left the question blank, 6.2% of them gave partially correct answers, and 4.1% of them gave

correct answers. Examples of correct, partially correct, and incorrect answers given by the students to the 6th question are provided in Figures 16, 17, and 18:

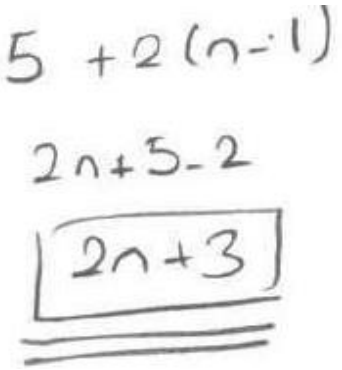


Figure 16. The Correct Answer Given To the 6th Question By S15

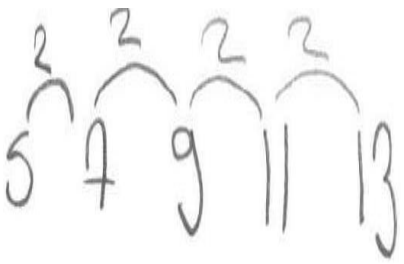


Figure 17. The Partially Correct Answer Given To the 6th Question By S44

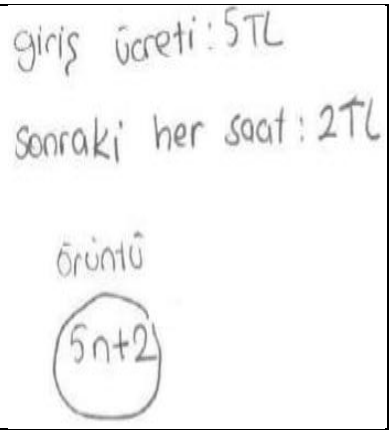


Figure 18. The Incorrect Answer Given To the 6th Question By S37

In this question, represented as a word problem, what is expected from the students is that they reach a generalization as "2n+3". When Figure 17 is examined it is seen that S44 determined the number pattern expressing the pattern but could not obtain the general term in the 6th question represented as a word problem.

Findings and Comments related to Q7

When the findings for the 7th question are examined it is seen that 36.1% of the students gave correct answers, 29.9% of them gave incorrect answers, 27.8% of them left the question blank, and 6.2% gave partially correct answers. Examples of correct, partially correct, and incorrect answers given to the 7th question by the students are provided in Figures 19, 20, and 21:

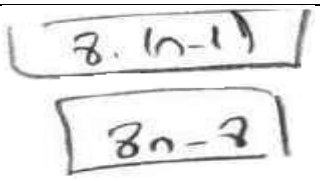


Figure 19. The Correct Answer Given To the 7th Question By S18

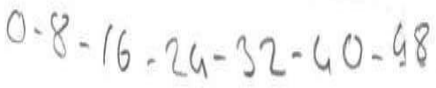


Figure 20. The Partially Correct Answer Given To the 7th Question By S20

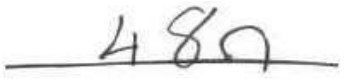


Figure 21. The Incorrect Answer Given To the 7th Question By S25

In this question, given as a word problem, what is expected from the students is that they obtain the general rule "8.(n-1)". When Figure 20 is examined it is seen that S20 found the number pattern but could not make a generalization. Generalization is defined as extending the effect of the results obtained by the students using mathematical thinking and problem-solving, or restating the results using some examples in a more general and broader way (Mullis, Martin, & Foy, 2005; Stacey, Burton, & Mason, 1985). Students have to perform the following steps to make generalizations: determining connections, preparing examples to test hypotheses, finding a number of different examples, classifying the examples systematically, determining which exercises have the same results, carrying out similar tests and making assumptions (Bell, 1976; cited by Pilten, 2008).

CONCLUSIONS and RECOMMENDATIONS

At the end of the study, it was seen that the students could make specializations but could not reach a generalization for the patterns given as figures. It was concluded from the answers given to the questions by the students that although the students could find out how many figures would be present in the next step of the figural pattern, they could not find the general rule expressing the pattern. In addition to this, it was determined that the students were able to find the desired steps according to a given rule, and thus they easily attained procedural knowledge. This situation indicates that the students had memorized the procedures performed on the concept and its definition. Attention must be paid to conceptual learning to overcome this difficulty (Hacısalihoğlu Karadeniz, Bozkuş, Gündüz, & Baran, 2015). In procedural knowledge, while the idea is to know how to use a concept or procedure without necessarily knowing the reason for it, in conceptual knowledge the process of understanding becomes prominent (Baki, 1997). Conceptual knowledge encompasses the core

meanings of the rules, generalizations, and the connection between them and procedures (Bekdemir, Okur, & Gelen, 2010).

In the studies of Soylu and Aydın (2006), it was determined that procedural and conceptual learning could not be balanced in the mathematics teaching process and that subjects could not be learned at a conceptual level as the procedural and conceptual learning could not be balanced. In the studies of Kaya and Keşan (2012), it was determined that the students were more successful in questions which required procedural knowledge and that they better understood these questions. Palabıyık and İspir (2011) specified in their studies that pattern-based algebra teaching increased the success in conceptual algebra of students in the experimental group and did not cause any difference in success in procedural algebra in the experimental and control groups. Similarly, as a result of the study carried by Bekdemir et al. (2010) it was revealed that score average of the students related to procedural knowledge was higher than the score average related to conceptual knowledge. Having difficulties in making connections between the problem and the subject and not correlating and reading what is given in the graphics indicated that the students had inadequate knowledge and could not internalize or learn the subject (Baki & Kartal, 2004). These findings from the study support the findings of this study. Every science field has its own teaching methods in accordance with its objects (Soylu & Aydın, 2006). A teaching process suitable for the structure of mathematics should be planned such that the students are able to understand both conceptual and procedural knowledge in mathematics and to make connections between them (Van De Wella, 2004).

As concepts in mathematics are the connections structured within the cognitions of individuals, these concepts cannot be learned and the next stage of the learning process becomes difficult if mathematical connections are not created in the mind of the student (Soylu & Aydın, 2006). The updated middle school mathematics curriculum (MoNE, 2013b) intends that the use of informal knowledge obtained from concrete experiences, feelings, and the daily life of the students support conceptual learning. When this situation is considered what is expected from the teachers is that they create environments in which the students can make connections between their past knowledge and new knowledge (Hacısalihoğlu Karadeniz et al., 2015).

It was concluded in the study that the students could reproduce a pattern related to the number sequences but could not find the general rule containing the pattern. As another result, it was determined that the students did not understand the questions represented as a word problem, thereby they were not successful in solving these kinds of questions. However it was seen that the students were successful in the pattern questions given as a table. In this context, activities involving the figure, table, number sequence, and word problem forms of patterns should play a greater part in the lessons.

In order to eliminate the difficulties related to the patterns encountered by the students, various examples should be given from environments closely related to these concepts; students' attentions should be attracted by using appropriate materials or models in the class; and the students should be allowed to better understand the subject using different methods and techniques such as games or drama. Similarly, the topic of pattern should be correlated with daily life by using concrete materials, enriched activities, and different models for the process of teaching patterns. Thus, "students will be able to understand the concepts, make connections between concepts and procedures (MoNE, 2013b)" and achieve the *general purposes of mathematics education* by coordinating these with daily life and the other disciplines.

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