DOI: 10.34056/aujef.1346962

Semiotic Examination of Algebra Teaching Process in the Context of Gestures[1]

Cebir Öğretim Sürecinin Jestler Bağlamında Göstergebilimsel Olarak İncelenmesi[1]

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Article Type[4]: Research Article

Application Date: 20.08.2023 **Accepted Date:** 18.10.2023

To Cite This Article: Danişman, Ş. & Tanışlı, D. (2023). Semiotic examination of algebra teaching process in the context of gestures. *Anadolu University Journal of Education Faculty (AUJEF), 7*(4), 1065-1092.

ABSTRACT: This research aims to examine the gestures used by teachers and students in the algebra learning-teaching process and the purpose of these signs. The participants of this case study are 25 eigth graders and their mathematics teacher. The data were collected through video recordings and clinical interviews. Deductive analysis was used to analyze the data, while synchronic and diachronic analyses were utilized to analyze the signs in terms of semiotics. Findings revealed that the teacher used a bunch of iconic, metaphoric, deictic, and beat gestures during the algebra teaching process. The meaning of the iconic gestures that resulted in the study can be listed as centering and itemization, illustration of a table, constant term, preceding element, decrease, separation, base and power, rectangle and side, length, and vertical-horizontal. On the other hand, the metaphorical gestures attribute holding, equality, moving the term from one side to the other, numbers or letters, equality, distributing and hiding. The deictic gestures imply pointing out boards with fingers, board markers or hands, and indicating students and notebooks with fingers, while the beat gestures refer to lowering the hands from top to bottom, waving the fist up and down and moving the hand back and forth. It is seen that the teacher aims to strengthen her expression and support students' mathematical understanding through the gestures she uses.

Keywords: Semiotics, gesture, algebra, mathematics instruction

[1] This article is derived from the first author's PhD dissertation under the supervision of the second author.

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[4] This study was conducted with the approval of the Ethics Committee of the Anadolu University, dated 27.12.2017 and numbered 133081.

ÖZ: Bu araştırma, öğretmen ve öğrencilerin cebir öğrenme-öğretme sürecinde kullandıkları jestleri ve bu işaretlerin kullanım amacını incelemeyi amaçlamaktadır. Durum çalışması desenindeki bu çalışmanın katılımcıları, 25 sekizinci sınıf öğrencisi ve onların matematik öğretmenidir. Veriler video kayıtları ve klinik görüşmeler yoluyla toplanmıştır. Verilerin analizinde tümdengelimli analiz, göstergelerin göstergebilimsel açıdan analizinde ise eşzamanlı ve artzamanlı analizlerden yararlanılmıştır. Bulgular, öğretmenin cebir öğretimi sürecinde bir dizi ikonik, metaforik, işaret ve vurgu jestlerini kullandığını ortaya koymuştur. Çalışmada ortaya çıkan ikonik jestlerin anlamı, ortalama ve maddeleştirme, tablo gösterimi, sabit terim, önceki öğe, eksiltme, ayırma, taban ve kuvvet, dikdörtgen ve kenar, uzunluk ve dikey-yatay olarak sıralanabilir. Öte yandan metaforik jestler, tutma, eşitlik, terimi bir taraftan diğer tarafa taşıma, sayılar veya harfler, eşitlik, dağıtma ve gizleme gibi nitelikler taşımaktadır. İşaret jestleri tahtayı parmakla, tahta kalemiyle veya ellerle gösterme, öğrencileri ve defterleri parmakla gösterme anlamı taşırken, vurgu jestleri söylemlere vurgu anlamı katmak için elleri yukarıdan aşağıya indirmeyi, yumruğu yukarı ve aşağı sallamayı ve eli ileri geri hareket ettirmeyi içermektedir. Öğretmenin, kullandığı jestler aracılığıyla anlatımını güçlendirmeyi ve öğrencilerin matematiksel anlayışlarını desteklemeyi amaçladığı görülmektedir.

Anahtar sözcükler: Göstergebilim, jest, cebir, matematik öğretimi

1. INTRODUCTION

A number of symbols such as signs, movements, and sounds are used by living things to communicate with one another (Elden, 2009). Communication, which can be defined as the process of making meanings common between two or more people, gives people, who are social beings, the opportunity to make sense of the environment. Words, non-verbal behaviors, postures, facial expressions, and extra-verbal behaviors (tone of voice, emphasis, and silence) are all ways in which messages are encoded in interpersonal communication (Radford, 2010; Wertsch, 1998).

In the sense that thinking is a form of communication, a conversation between two people can be viewed as a complex process, of which only a small part is visible to the observer. In an attempt to form a sense of integrity, the things heard are the missing parts of what is meant or intended to be said (Sfard, 2001). Any discussion of meaning would be pointless without an activity system in which meaning is created (Levinson, 1983, as cited in Razfar, 2012). Hence, communicating with words requires more than just linguistic elements; it also requires elements that affect communication. According to Cazden and Beck (2003), curriculum reform movements for curriculum development focused on classroom discourse as communication became more important in the education and training process. This is where semiotics, also known as the "science of signs", comes in. Semiotics is concerned with how signs support discourse and support communication. Semiotic research aims to uncover the functioning and meaning systems of all kinds of observational activities and subjects beyond words (Barthes, 2012). Despite its broad scope and wide range of subjects, semiotics is an important research field in terms of analyzing languages, sign systems, and visual, auditory, verbal, and non-verbal communication and cultural materials (Sayın, 2014).

The term sign encompasses any form, object, phenomenon, etc., that represents something outside of itself. In this sense, signs can serve as substitutes for what they represent (Eco, 1984; Rifat, 2013). Various feelings, thoughts, attitudes, intentions, expectations, needs, and desires are mediated through signs. All communication and social interactions are actually signs. Words, symbols, body postures, behaviors, looks, gestures and facial expressions can be considered as signs. This research focuses on the gestures in line with its aim. The formation of thoughts and speech is assisted by gestures. Along with the formation of thought, it has an important effect on speech (McNeill, 1992; Roth & Lawless, 2002). McNeill (1992) proposed the following classification scheme for gestures: iconic, metaphoric, deictic, and beat.

Iconic: A gesture of this kind depicts a concrete entity or action. Through their formal and structural similarity to events or objects, gestures serve as referential symbols. For example, the teacher's movement of his/her hand from left to right, while talking about a number line.

Metaphoric: The act of gesturing can also be used to visualize abstract content, effectively imagining the inexplicable. As part of a metaphoric gesture, abstract meaning is rendered as if it occupied space and/or had form. When metaphoric gestures accompany speech, they generally indicate a meta-level rather than an object-level perspective. For example, the teacher's expression of "let's send this term to the other side" while depicting it with his/her hands.

Deictic: Prototypically, deictic gestures are made by extending the finger or holding an object. Adults often point abstractly at objects or locations when they converse and tell stories rather than pointing at physical objects. For example, the teacher's pointing at the figures on the board with index finger while talking.

Beat: Beats are simply hand movements that identify rhythmically the prosodic peaks of speech by flicking them up and down or back and forth. Since beats are rhythmically related to speech, they seem purely speech-related. Besides signaling the speaker's sense of the importance of something within a larger context, they also serve as discourse functions. The beat can be thought of as a gestural yellow highlighter. For example, the teacher's emphasis on the expressions with his/her hand moves as underlining the 'sentences' or moving up and down.

The mathematical language is said to be a unique language due to its distinctive way of thinking and its unique systematic structure (Yıldırım, 1996). Mathematics, however, is different from other languages in that it internalizes abstraction and has a unique syntax and grammatical structure (Bullock, 1994; Umay, 2002). Communication in mathematics lessons is heavily reliant on the representations used by teachers and students. Gestures, facial expressions, and body movements, along with symbols, figures, drawings, texts, and graphics are frequently used in the classroom to reveal thoughts. Hence, the use of signs in mathematics is of great importance for the development of mathematical understanding. This is because we need signs to understand mathematics. Mathematics involves signs that act as bridges between us and mathematical objects, allowing us to comprehend and understand them. It is possible in many sciences to do experiments and investigations to reach explanations by nature (astronomy, physics, biology, etc.), but not in mathematics (Duval, 2006). From another perspective, Aktaş and Argün (2020) concluded that the simultaneous formation of mathematical language visually and verbally in mathematical communication depends on the simultaneity of tactile movements and discourse for visually impaired individuals. Therefore, drawing attention to the gestures, they suggest that visually impaired individuals should consider the sense of touch (hand movements like gestures and mimics, handwriting or figure analysis, etc.) as a component of mathematical language skills.

A student constantly oscillates between images in his or her mind and external representations when doing math. He/she presents the image in his/her mind as an external representation once he or she reaches a conclusion (Gutierrez, 1996). The study of mathematics is concerned with the relationships between abstract objects. We must embody these abstract objects to a certain extent in order to work with them. Signs, words, symbols, expressions, and drawings can all be used to accomplish this. Thus, semiotics becomes the study of the relationship between mathematical objects and their representations. It is important not to confuse mathematical objects with their semiotic representations at this point (Duval, 2006). Considering the relationship between signifier and signified, that is, representation, as well as the entity characterized by representation, it is likely that because there is more than one semiotic representation of mathematical objects, there are multiple signifiers for the same signified (Santi, 2010). The production and use of signs play an important role in mathematical thinking, and they develop together in close relation to meaning (Rotman, 2006; van Oers, 2000). A number of signs are used by teachers and students in mathematics learning-teaching processes, and these signs affect the quality of the learning-teaching process. There are a number of discourses involved in this interaction process, including diagrams, drawings, notation, symbols, etc. (Sfard, 1998). Signs used in mathematics teaching and learning mostly have a visual nature (Presmeg, 2014). In light of the necessity of taking into account the interaction between individual and environment (Barab & Duffy, 2000), the context in which students learn mathematics cannot be considered separately from the students' mathematical thinking and mathematics learning processes.

According to Radford (2006, 2010), signs are used to identify patterns consisting of culturally determined mathematical activities. Communication in the semiotic reality of people is essentially an exchange of messages depending on: sociocultural contexts, the content of the message, the language

used to convey the message (syntax, grammar, semantics, active-passive terms), human interaction tools (voice-intonation, diagrams and graphics, writing, phrases), and visual tools for their transmission (gesture-mimics, pointing-display, gaze, posture, etc.) (Saenz-Ludlow & Kadunz, 2016). When dealing with mathematical activities, transitions are made between symbols, diagrams, and figures. In mathematics, representations often replace other representations and students' transitions between these representations provide convenience in problem solving and learning (Duval, 2006).

Value attributions of signs only occur within a context (Erkman-Akerson, 2005). During the meaning-making process, cultural tools such as social languages, speech forms, and other semiotic tools are used (Wertsch, 1998). Members of each community have their own meaning-making methods, and it is essential that they attribute the same meaning to the signs that are used in this meaning-making process (Cope & Kalantzis, 1993; Sayın, 2014). For this reason, it is crucial for community members to attribute similar meanings to the same signs in order to establish a healthy communication. As a result, context and meaning are interrelated and affect one another (Halliday, 1996; Hasan, 1995). Therefore, one can only evaluate the signs used in a class during its communication process in terms of the class's culture. Among the functions of semiotics is to analyze and reveal the meaning-making processes in the classroom. Yackel and Cobb (1996) point out that discourses in the classroom may differ in different social structures, communities or cultures. Mathematical values are also important in mathematics education (Dede, 2014; Seah & Bishop, 2002). Mathematics teachers may reflect values in the classrooms whether they are aware of it or not (Aktaş & Argün 2018). Accordingly, body movements that support teachers' discourses can affect students' mathematical values.

Analysis of the signs used by students and teachers in classroom communication can help improve mathematics education by providing insight into students' thinking processes. Because it is believed that the semiotic analysis of these signs will reveal communication barriers in mathematics learning and teaching, the reasons why students understand or do not understand the subject, how students and teachers interpret and think about signs in their environment, and so on. Moreover, although semiotics studies abroad are quite common, they are rare in Turkey, where most of them are carried out in fields other than education. Çalışkan (2011), one of the studies related to mathematics education, revealed that the students mostly preferred verbal semiotic representations in algebra and in verbal geometry, while they mostly used non-verbal semiotic representations in figural geometry. Akıncı (2014) identified the integrity of gesture and speech, the meaning that gestures add to speech and gestures' guidance for speech as the significant findings of his study. Gürefe (2015) investigated deaf students' use of semiotic resources for some geometric concepts and put forth the gestures preferred by the participants. In a similar vein, a study focusing on the use of symbols in algebra by Aktaş and Argün (2020) highlighted the diversity and significance of symbolic representations by connecting the function of braille for visually impaired pupils to the use of students' hand motions.

Akçakoca (2018) found out that participants' deictic gestures were the reflections of cognitive construction in the physical environment, representational gestures (iconic and some metaphorical) were the mental simulations of action and perception, while other metaphorical gestures were the conceptual metaphors based on the body. Çetinbaş (2022) studied the semiotic representations and the modeling routes of pre-service teachers and found out that the semiotic register in which the groups were included differed according to the purposes of their actions in the different parts of the modeling activity. Balcı (2022) examined geometry teaching processes with semiotic mediation theory and revealed a number of different signs, which are the indicators of the transition to mathematical meanings created by students in teaching geometry. Geometry appears to be the main focus of most of the few studies that used

semiotics and were conducted in Turkey. In this respect, it can be thought that this study, which will examine the gestures used in algebra, is important in terms of revealing the use of signs in algebra teaching. Therefore, this research aimed to analyze the gestures used by secondary school students and their teacher in the algebra teaching processes with the semiotic method and to reveal what meanings individuals attribute to these signs.

2. METHOD

2.1. Research Design

In line with the purpose of the research, case study design, one of the qualitative research methods, was used in the research. A case study is a qualitative research approach that examines a phenomenon in its real environment (Yin, 2003) and includes in-depth data collection such as observations, interviews, documents, reports, audio-visual materials (Creswell, 2007). Considering the purpose of the research, the holistic single-case study design was adopted because of the focus on only one classroom (Merriam, 1998; Yin, 2003).

2.2. Participants

This study involved 25 secondary school students at the eighth grade level and their mathematics teacher from a secondary school in the Western Black Sea Region. She holds a non-thesis master's degree in Curriculum and Instruction in Education and has 7 years of experience teaching secondary school mathematics. The participants for the study were selected by criterion sampling, one of the purposive sampling methods. Purposeful sampling allows for the in-depth study of situations that are thought to have rich information, and the discovery and explanation of facts and events (Yin, 2003). The research employs a qualitative research approach, which generally uses the purposive sampling method. Due to the nature of qualitative research, in-depth understanding and knowledge about a subject are developed rather than generalizations. Therefore, the research was conducted at a secondary school that allows for detailed data collection during the research process.

In the research, grade level and the teacher's level of experience were determined as criteria. Since the pilot study was conducted at the seventh grade level, which covered the algebra learning field in a comprehensive manner, it was decided that the participants would be at the eighth grade level. Also, considering that the teacher's experience will affect the signs used, it has been decided that the teacher should have at least 5 years of experience.

2.3. Data Collection

The data within the scope of the research was obtained by using video recordings and clinical interviews.

2.3.1. Video Recordings

Video recordings of mathematics lessons taught in the classroom are the main source of data to be obtained for the research. In the study, video recordings are considered an important source of data to examine instantaneous interactions in the classroom in more detail, as well as to analyze the signs and

the meanings attributed to them in a more accurate and detailed way later. It is possible to review data repeatedly by recording patterns of behavior and complex interactions in the classroom (including both speech and nonverbal communication) (Clement, 2000). In this way, all kinds of signs could be observed, such as classroom interactions, expressions, body movements, etc. The class and the teacher were captured separately with cameras in the classroom to record teaching processes. Video recordings were made for 15 lesson hours for the four acquisitions within Algebraic Expressions and Identities.

2.3.2. Clinical Interviews

Clinical interviews were conducted to investigate in detail the meanings attributed to the signs used by the teacher, taking into account the views of the teacher, and to reveal the mental processes of the participants. Interviews with clinical participants reveal the hidden elements behind the participants' thinking processes and help reveal their mental processes (Clement, 2000). Additional questions may be asked during clinical interviews in addition to those prepared for the interview. Clinical interview forms prepared by the researchers were used to allow additional questions to be asked during the clinical interviews with the student and teacher.

These interviews were conducted so that it would be possible to understand the students' thoughts on algebraic expressions and to examine the signs they used to do so. A weekly clinical interview was conducted with the selected focus student throughout the study period. At the end of the teaching process, a clinical interview was conducted with the teacher. It took an average of 60 minutes to conduct a clinical interview. During the pilot application, the clinical interview questions were tested, and the clinical interview forms from the initial application were reviewed in light of the results of the pilot application as well as the eighth grade level content of the lesson.

2.4. Data Analysis

Rather than focusing on the product or result, qualitative research examines the process and analyzes the data holistically (Bogdan & Biklen, 1998). This context involved transferring all data obtained from algebra learning-teaching sources to the computer environment in order to allow holistic analysis of the data.

The data were analyzed using deductive analysis which is a top-down approach to data analysis. One starts with a set of predetermined codes and applies them to the data (Saldana, 2009). Hence, this study utilized this top-down approach and took gesture classification by McNeill (1992) into consideration as the predetermined codes. McNeill (1992) created a classification scheme for gestures with four categories: iconic, metaphoric, deictic, and beat. Iconic gestures refer to resemblance to concrete objects, while metaphorical gestures refer to abstract objects. Beat gestures include gestures used to support the expression, while deictic gestures include hand and arm movements to point at the target.

Furthermore, a focus is placed on how the signs are used both sequentially and simultaneously when analyzing data obtained from videos and clinical interviews. The synchronic and diachronic analyses by Arzarello (2006) and Arzarello et al. (2009) were performed in this context. As opposed to diachronic analysis, which analyzes signs individually over time, synchronic analysis looks at signs together at a given point in time. Diachronic analysis refers to signs produced consecutively by individuals within a certain period, whereas synchronic analysis refers to signs used simultaneously. The

synchronic and diachronic analyses allow for revealing the roles of different types of signs (such as gesture, mimicry, speech, writing, etc.). This approach was taken during the analysis of the data within the scope of the study, as both synchronic and diachronic determination of the signs are necessary to understand the gestures used in algebra learning and teaching processes as well as the purposes for which they are used.

2.5. Validity and Reliability

Validity and reliability are important for the acceptability of the results obtained within the scope of the research. In qualitative research, it is seen that these concepts have been replaced by the concepts of credibility, the competence of the researcher, and the accuracy of the results (Creswell, 2007). According to Guba and Lincoln (1994), validity and reliability in qualitative research can be considered in four parts: credibility, dependability, confirmability and transferability. Table 1 summarizes the components and the recommended methods for delivering them (Başkale, 2016).

Table 1: *Credibility in Qualitative Research*

Validity and reliability	Methods
Credibility	• Long-term interaction • Reducing researcher biases • Participant confirmation • Triangulation
Transferability	• Purposeful sampling • Inclusion/exclusion criteria • Detailed description of the environment • Detailed description of the participants
Dependability	• Supervision • Literature • Detailed description of research methods • Triangulation • Independent researcher
Confirmability	• Reducing researcher biases • Triangulation

Accordingly, the following methods were followed in order to increase the validity and reliability of the study:

Long-Term Interaction: Interaction was established with students and teachers, who were the data sources, through classroom visits and clinical interviews during the research.

Reducing researcher biases: The role of the researcher is stated in a separate title, and it is stated how the researcher's biases are reduced with the explanations in this title. In addition, the diversification of the data can be considered as a support for reducing the prejudices and for the findings to be based on the data obtained directly.

Confirmation by the participants: During the research process, the opinions of the teacher regarding the results obtained in the interviews were taken. In addition, after the analysis, the teacher was interviewed and her views and comments on the results were obtained.

Triangulation: Instead of using a single method or source in the research, data diversity was made by using video recording and clinical interviews.

Purposeful sampling: For the purpose of the research, a school and classroom were preferred at the eighth grade level, where parents, students, and the teacher had the opportunity to voluntarily participate in the research. The participants and the school administration willingly supported the research in the data collection process. In this context, a school that is open to cooperation was preferred for research.

Inclusion/exclusion criteria: The class in which the research will be conducted was chosen among the other classes upon the recommendation of the teacher. In addition, the student who was interviewed in the focus group was also selected at the suggestion of the teacher.

Detailed description of the environment: The teachers and administrators at the school, which is not in a very central location, provided the necessary support for the healthy conduct of the research process. The research process, carried out with 25 students in a class with a capacity of 30, allows for taking images of the teacher and students and observing the teacher-student/student-student interaction.

Detailed description of the participants: Information about the participants has been explained in detail above.

Supervision: Audio recordings, videos, and materials obtained in the research were recorded and kept by the researcher, and the methods and processes used in the research, data collection tools and how the findings were obtained were explained in detail. In addition, direct quotations and images from the participants were included in the study.

Literature: The literature was used throughout the research process, and the literature data related to the research were presented by citing the source in the research.

Detailed description of research methods: The methods followed in the research, data collection tools, analyses etc. were described in detail in the method section.

Independent researcher (expert review): The data were coded by two independent researchers and the coding was controlled by a third independent researcher. In addition, the third independent researcher gave feedback to the researcher about the research process, data collection, analysis and presentation of the research.

2.6. The role of the Researchers

During the course of the research, two people assume the role of researcher, one who conducts the research as part of his dissertation and the other who guides. As part of the implementation process for this dissertation, the first researcher conducted clinical interviews and analyzed the data together with the second researcher by ensuring that the lectures were recorded with video. In addition to her academic studies in algebra, the second researcher has a graduate course related to semiotics. The second researcher guided and took an active role in determining the scope of the research and planning the process, while the first researcher collected, analyzed and reported the data.

2.7. Research Ethics

An Ethics Committee Report was received from the Anadolu University Ethics Committee before the research began. Based on the positive ethics committee report, the Anadolu University Rectorate and the provincial governorship where the research was conducted granted permission for the research to be conducted in the schools. Having obtained the governorship's permission, the administrators and mathematics teachers of the selected schools were interviewed and informed about the scope of the study. Both students and parents signed the consent form before the research began, providing information about the purpose, scope, content, and participant rights of the study. The confidentiality of the participants was protected in the course of the research, and no information about the schools or names of participants or their own images was included in the report or on any platform. Whenever students are addressed, the phrase “student” is used instead of their names, and the faces of students and teachers are hidden. Research and publication ethics were complied with in the study.

3. FINDINGS

Using gesture classification, the researchers investigated the signs and usage purposes used in algebraic expressions, identity, and factorization. Gestures are frequently used by the teacher to express algebraic expressions. The gestures discovered within the current study are summarized in Table 2.

Table 2: *Gesture Classifications with Implied Meanings*

Gestures	Implied meaning	
Iconic	✓ Centering and itemization	✓ Seperation
	✓ Illustration of a table	✓ Base and power
	✓ Constant term	✓ Rectangle and side
	✓ Preceding element	✓ Length
	✓ Decrease	✓ Vertical-horizontal
Methaporical	✓ Holding	✓ Numbers/letters
	✓ Equality	✓ Equality
	✓ Moving the term	✓ Distributing and hiding
Deictic	✓ Pointing out board with fingers	
	✓ Pointing out board by hands	
	✓ Indicating students and notebooks with fingers	
Beat	✓ Lowering the hands	
	✓ Waving the fist	
	✓ Moving the hand	

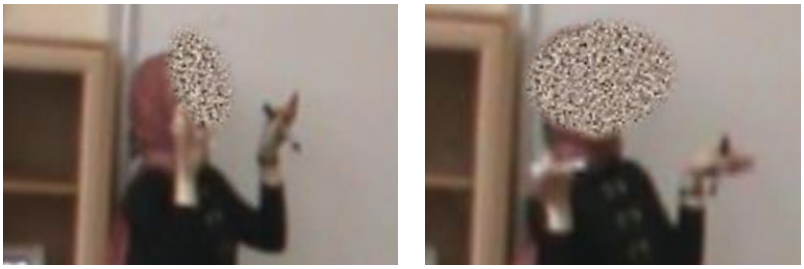
As Table 2 presents, the gestures, which include hand-arm gestures, used by the teacher were divided into four categories: iconic gestures, metaphoric gestures, deictic gestures, and beat gestures. The iconic gesture can be seen in the teacher's use of the "middle" sign at the beginning of the subject when she says, "Let’s write the title in the middle" (Figure 1a). This was a reference to the subject's importance and to inform the students about the beginning of the lesson. While writing on the board, the teacher centers his writing at the top, demonstrating his preference to correspond to this gesture. As well

as this gesture, Figure 1b shows the teacher's iconic gesture of lowering his hand from top to bottom to indicate that the features are itemized.

(a) (b)

Figure 1: *Example for Iconic Gesture: Centering and Itemization*

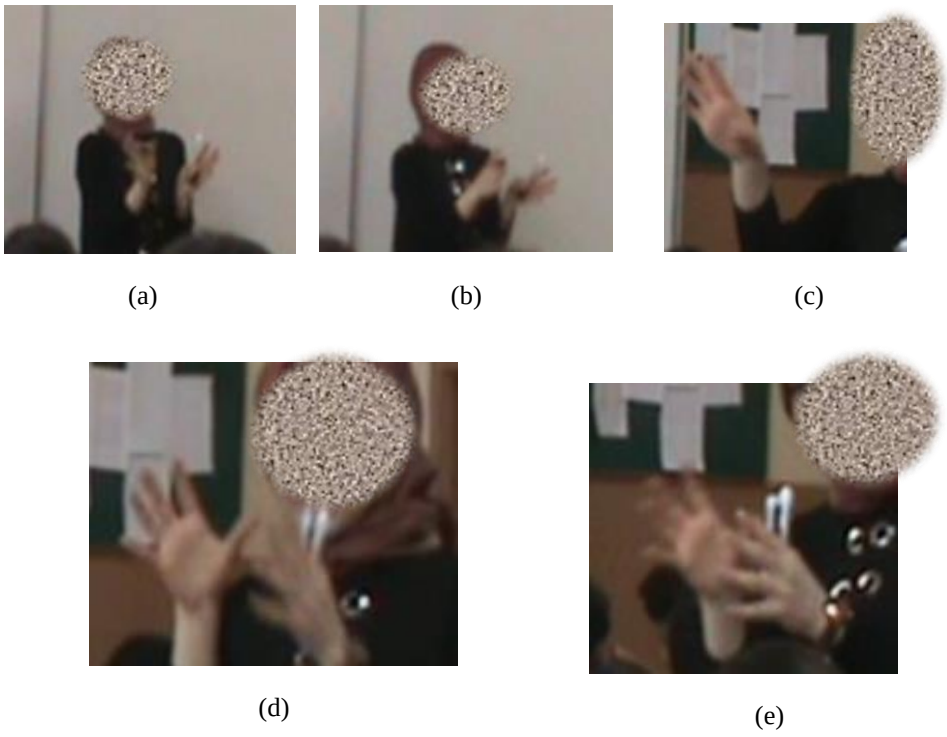
Figures 2a and 2b depict the iconic gesture used by the teacher for the table. Using Figure 2a and Figure 2b, the teacher showed that she would draw the upper edge of a rectangular table and the side edges in the space to distinguish the properties of algebraic expressions using table representation.



(a) (b)

Figure 2: *Example for Iconic Gesture: Drawing a Table*

Using the iconic gesture in Figure 3a, the teacher pretended to hold something stationary with both hands as a reference to the constant term's fixed nature. Her next statement was "think like a constant number" to emphasize the constant feature of "constant number" (Figure 3b). A similar sign is seen in Figure 3c when she says "fixed", meaning "standing on the side, unlike the others". For a student who failed to understand, the teacher used this gesture to support the words "fixed-pebbled part," using a similar gesture as in Figures 3d and 3e.



(a) (b) (c)

(d) (e)

Figure 3: *Example for Iconic Gesture: Constant Term*

While explaining the coefficient, the teacher said "there will always be the number before a" in the example of "6a" and showed that the coefficients should come before the variables, both with words and with the iconic gesture in Figures 4a and 4b. While using this gesture, it is seen that the teacher moves her body from her left to her right side with her hand to refer to "before".

(a) (b)

Figure 4: *Example for Iconic Gesture: Preceding*

Figure 5a shows the teacher's gesture as she explained algebraic operations, lowering her hand gradually while saying "you will decrease -30 by 3" for the operation "-30-3". However, she then proceeded in the opposite direction of the instruction she gave by attributing the growth of the number in absolute value due to the processing of negative numbers, and used the iconic gesture in Figure 5b.

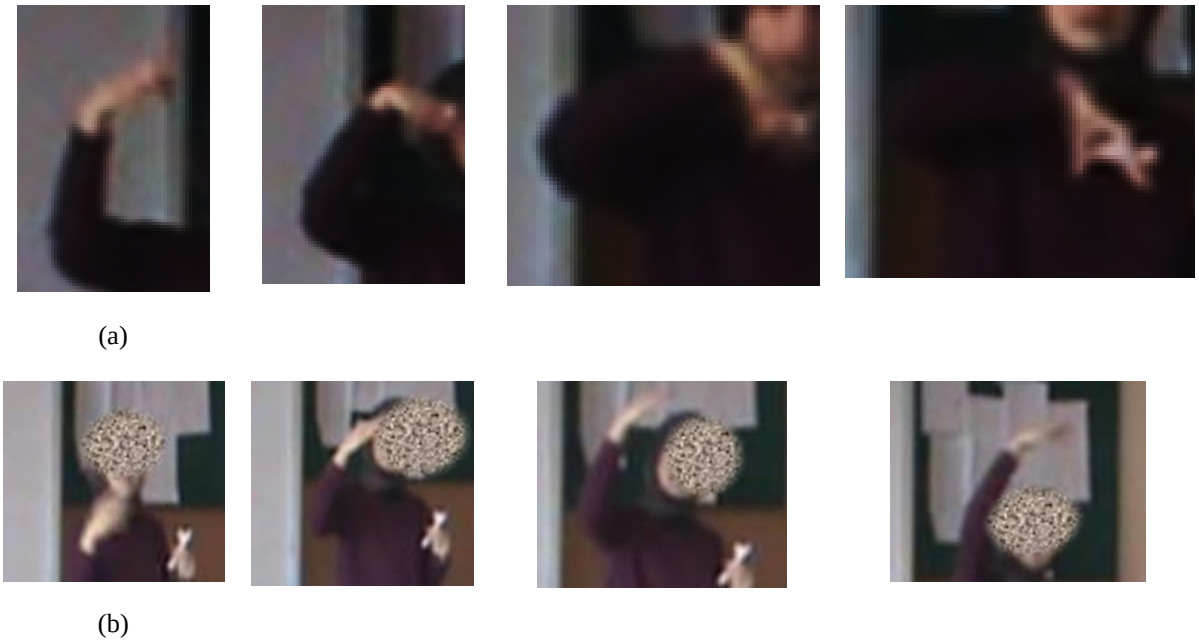
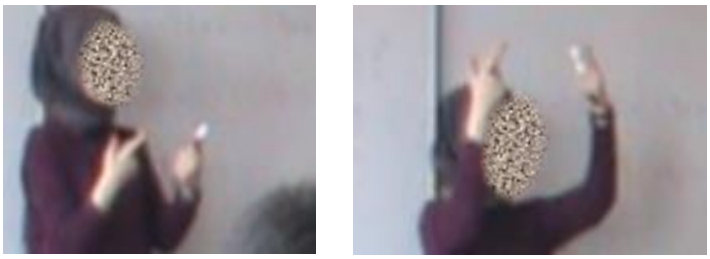


Figure 5: *Example for Iconic Gesture: Decreasing*

In the examples given by the teacher about algebraic operations, when writing a term as different factors (as 12x can be written as 4.3x), the "separation" gesture that he uses when he says "you will only separate this expression" to guide the students can also be considered as a metaphorical gesture. Figure 6 illustrates how the teacher opened her hands from the elbow where she joined them to express what she said with the corresponding action when she said "you will separate".

Figure 6: *Example for Iconic Gesture: Seperating*

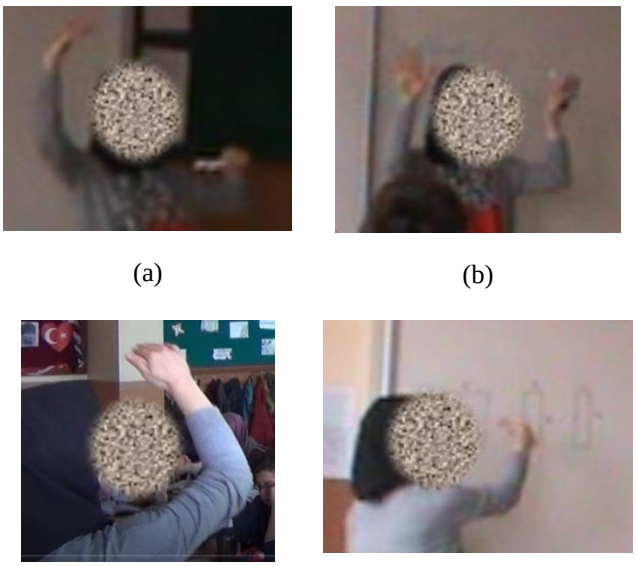
The teacher also used an iconic gesture for exponential numbers when teaching algebra. When using the base-exponent notation in exponential numbers, it can be seen that the teacher used iconic gestures to teach the concepts of base and exponent, while she also said that "when the bases are the same (Figure 7a), the exponents are added (Figure 7b) in multiplication" for the same variables to be multiplied.



(a) (b)

Figure 7: *Example for Iconic Gesture: Base and Power*

Also, the teacher used several gestures to convey the idea of figure drawing. One of these gestures was for the outer edges of the rectangle created while modeling. In addition to the teacher starting from the edges and completing the drawing for the drawing of the rectangle (Figures 8a and 8b), she illustrated the sides of the rectangle through gestures, for example, as illustrated in Figure 8c in which a hand gesture represented the rectangle's upper edge. In a similar manner, the teacher's movement up and down while showing the heights of the rectangles and squares she drew (Figure 8d) is an iconic gesture.



(a) (b)

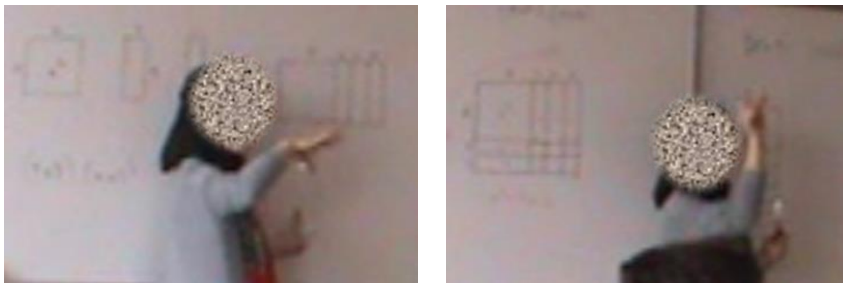
(c) (d)

Figure 8: *Example for Iconic Gesture: Rectangle and Side*

By holding the index fingers of both hands parallel to the length and referring to the distance between them, the teacher uses the sign in Figure 9, which is an iconic gesture.

Figure 9: *Example for Iconic Gesture: Length*

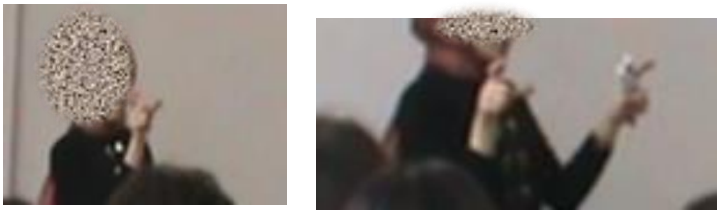
Additionally, the gestures used by the teacher for the horizontal (Figure 10a) and vertical (Figure 10b) positions of the figures during modeling were considered iconic gestures. It is described here that small shapes used in modeling can be placed in either position to achieve the desired product.



(a) (b)

Figure 10: *Example for Iconic Gesture: Vertical-Horizontal*

The teacher frequently used the "holding" gesture, which she made with her thumb and forefinger in a C shape. Throughout the lesson, the teacher utilized this gesture extensively. The teacher's use of this gesture kept the concept she refers to in a sense and created the meaning that she talked about that concept. As such, it can be considered a metaphorical gesture. Although the teacher used the same gesture in itemizing the features, it was used there as a substitute for itemization marks. As shown in Figure 11a, the object of the algebraic expression is to be "held." For the equation, a similar gesture can be seen in Figure 11b.



(a) “For the letter” (b) “For the equation”

Figure 11: *Example for Metaphorical Gesture: Holding*

The metaphorical gesture used by the teacher for the concept of "equality" is to first hit something in the space with her hands (Figure 12a: To the right) and then to the opposite side in the middle (Figure 12b) and to the other side (Figure 12c: To the left) and do the same movement.

(a) (b)“equals...” (c)“something...”

Figure 12: *Example for Metaphorical Gesture: Equality-1*

Another gesture that the teacher attributed a similar meaning to the meaning of "equal" was related to modelling. The teacher, who modeled the multiplication operation in algebraic expressions, asked the students to verify whether the results from the modeling were the same as the results obtained by using the distributive property of the multiplication operation over the addition operation. The metaphorical gesture she used to describe both results (Figure 13) supports this explanation.

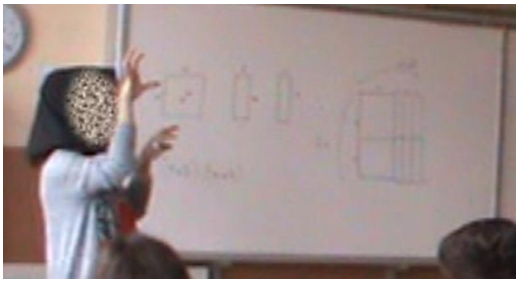


Figure 13: *Example for Metaphorical Gesture: Equality-2*

As seen in Figure 14, the teacher kept both hands parallel and fixed in the air, referring to the equality of the two sides while indicating that numbers are equal when the value is attributed. Hence, the teacher emphasized balance and equality for identity with this metaphorical gesture.

Figure 14: *Example for Metaphorical Gesture: Equality-3*

The sign (Figure 15a) that the teacher used to “cross over” some terms related to algebraic operations refers to the transfer of the term or terms on one side of the equation to the other. Similarly, it is seen that the students express their thoughts by using the same sign used by the teacher (Figure 15b).

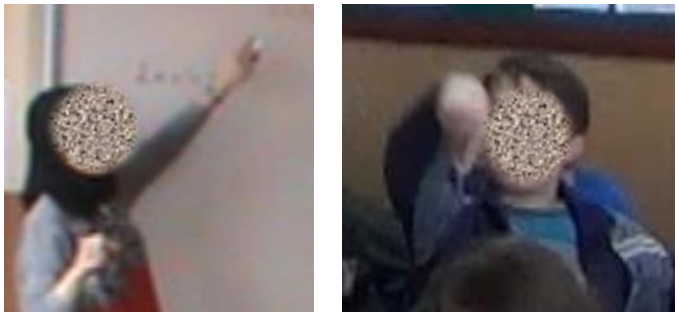
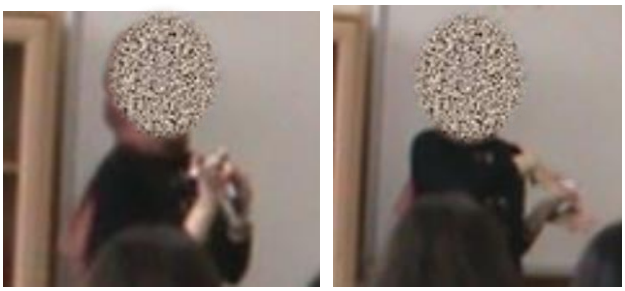


Figure 15: *Example for Metaphorical Gesture: Moving the Term from One Side to the Other*

When the teacher did not get the answer she expected from the students, she gave an example with a metaphor that he thought would help students understand the concept of "term" more easily. Having told the students that the terms can be considered as numbers and letters, the teacher stated, "think like numbers (Figure 16a: pretending to put both hands inside something) and letters (Figure 16b: pretending to put both hands inside something on the other side)". She tries to emphasize that numbers and letters are different from each other by considering them in two separate boxes. Students classified the terms in this way as known and unknown as a result of the expression of the teacher. As a matter of fact, the answers of the students to the questions asked by the teacher support this. Figures 16c and 16d represent how the teacher represents the thinking styles of students who put x and y together as unknowns in the trinomial example. "There may be different types of unknowns," the teacher said, insisting that this is not true.



(a) “numbers” (b) “letters”

(c) “numbers, knowns” (d) “unknowns”

Figure 16: *Example for Metaphorical Gesture: Numbers-Letters*

This part also includes the metaphorical gesture that the teacher and students used to express numbers by raising their fingers. Students and the teacher used metaphorical gestures to refer to numbers, which are abstract mathematical objects. Figure 17a shows the teacher saying "there are two terms", while Figure 17b shows the student's metaphorical gesture when he says "one" in reference to the answer.

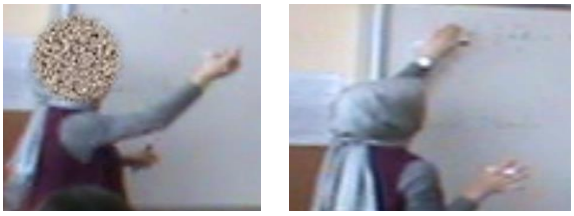
(a) (b)

Figure 17: *Example for Metaphorical Gesture: Number*

As seen in Figure 18, the teacher used the gesture to emphasize the similarity of the terms by saying "I am adding the same expressions" in order to illustrate that the operation can be carried out by pointing out their similarities. By using this gesture, the teacher emphasized that the terms were similar, and that the terms must have this feature in order to add them.

Figure 18: *Example for Metaphorical Gesture: Equality*

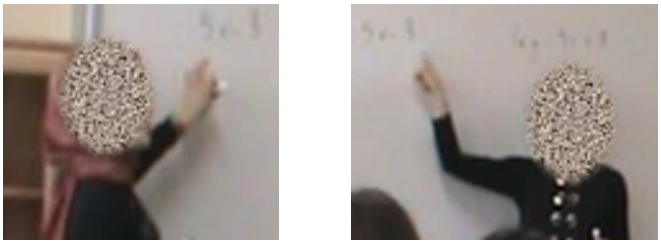
Multiplying algebraic expressions with the distributive property of multiplication over addition is often demonstrated by the teacher by "distributing" with his hand. The gesture used here is a metaphorical gesture, since it is used to express the operation carried out. By drawing an arc four times in the air, the teacher illustrated the relationship between the terms to be multiplied (Figure 19a). As the teacher closed the first term in order to proceed with the second term after performing the operations on the first term (Figure 19b), she meant that that term was no longer relevant. This is another metaphorical gesture for the message to be delivered to the students.



(a) (b)

Figure 19: *Example for Metaphorical Gesture: Distributing and Hiding*

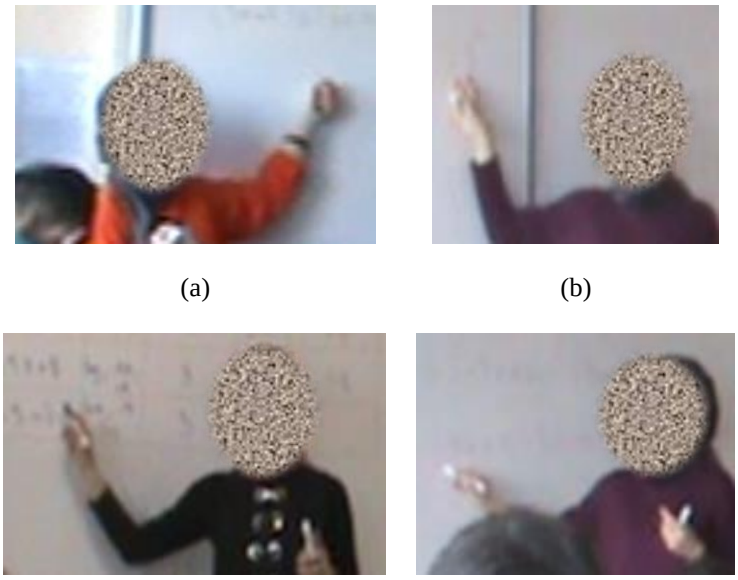
It is noteworthy that the teacher frequently used deictic gestures. These deictic gestures were used to indicate students, as well as point out what was written on the board. The teacher often used this gesture to allow students to talk or to establish a relationship between what she said and those on the board. When the deictic gesture was used, what was pointed out can be the student, a certain part of the board, a certain expression or just a certain object. In Figure 20a, the teacher is making an explanation on the expression “5x-3”, while in Figure 20b, she is talking about the term “3” in the algebraic expression.



(a) (b)

Figure 20: *Example for Deictic Gesture: Boards with Fingers*

Figure 21 shows examples where the teacher used deictic gestures. While finding the number of terms, it is seen that the teacher pointed to every term counted as "1, 2, 3". Not only the index finger, but also the whole hand (Figure 21a), the top of the fingers (Figure 21b), the board marker (Figures 21c and 21d) and the head are used for pointing. In Figure 21d, it is also seen that she pretends to underline with a pencil while showing similar terms. Therefore, the teacher also used the deictic gesture for the point she wanted to draw attention to.



(a) (b)

(c) (d)

Figure 21: *Example for Deictic Gesture: Board by Board Marker or Hands*

One of the most frequently used deictic gestures was for the teacher to give the students the right to speak (Figure 22a). Another example of these gestures was the teacher's use of the student's notebook while explaining how to help the student (Figure 22b).

(a) (b)

Figure 22: *Example for Deictic Gesture: Students and Notebooks with Fingers*

In order to emphasize important points and add strength to her expression, the teacher frequently used beat gestures. Figure 23 shows him using the beat gesture for “algebraic expressions” when she asked the class about algebraic expressions by lowering her hands from top to bottom and abruptly stopping them.

(a) (b)

Figure 23: *Example for Beat Gesture: Lowering the Hands From top to Bottom*

The teacher used a similar beat gesture when saying “what do you remember for this”. After the students did not remember and gave wrong answers for a while, she made them feel that she was waiting for different and possible correct answers by emphasizing them with her hands. Another example of a beat gesture is when the teacher said "this sign is very, very important" while explaining the operations in algebraic expressions, turning to the students to emphasize the importance of her words, making a fist and waving it up and down in the air (Figure 24).

(a) (b)

Figure 24: *Example for Beat Gesture: Waving the Fist Up and Down*

Repetition was one of the important components of the beat gesture, and the teacher's repeating the number 1 three times for the answer, showing the number 1 with her hand each time and moving it back and forth (Figure 25) is an example of this.

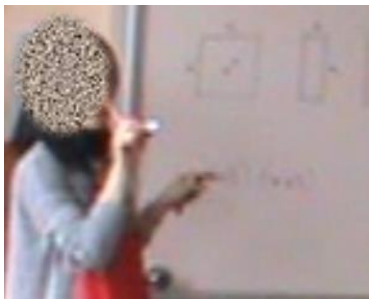


Figure 25: *Example for Beat Gesture: Moving the Hand Back and Forth*

The following thoughts on these body movements used by the teacher in algebra teaching reveal that most of the body movements she used were voluntary and purposeful:

> *If I talk about my teaching in algebra, I try to pay attention to visuality, simulations and differences in this regard.*

It is seen that the teacher made the following statements regarding the body movements she preferred in algebra teaching:

> *In order to visualize the concept I am explaining in the eyes of the students, I pretend to draw what I am going to tell with my hand and arm movements in the air. Sometimes I draw the same concept on the board and go over it. While emphasizing the crucial points of the subject, I make my face look confused and use gestures, as if I have seen something important. With these actions, I expect the students to understand that there is an important detail right in the subject and that my facial expression is not an expression in its normal course, but when they see that it has changed, it is a place that needs attention.*

Therefore, it is seen that the teacher makes use of these signs in order to add vitality to the abstract mathematical terms and concepts and to create a better image in the minds of the students. The teacher also states that she can read the body movements exhibited by the students:

> *On the other hand, I see that students' faces take on an expression like helplessness and difficulty while trying to understand the complex parts of the subject. Apart from that, when I solve the questions on the board, which are actually easy but the student cannot do, on the board, in the face of the ease of the solution, "I couldn't answer this question now?" I can see him waving his arm.*

In the clinical interview, it was observed that although the teacher used various gestures, the student did not know whether the teacher used them purposefully or not. The conversation about this is as follows:

> Student: *In algebraic expressions, for example, she says x and does so (Figure 26: He opens his palm by pushing his hand forward.)*

Figure 26: *The "x" sign of the teacher according to the student*

Researcher: *What is she using that hand sign for?*

Student: *She points to x.*

Researcher: *OK, for demonstration purposes. Another?*

Student: *This is how she uses her hand while talking, for example, while teaching.*

Researcher: *How, how does she use it?*

Student: *How can I say? (He closes his eyes for a moment and thinks.) She uses her hands, but I don't know if it's related to the lesson.*

Researcher: *So, how does she do it?*

Student: *She does it like this (Figure 27) while she is talking.*

Figure 27: *Student's Example of the Teacher's Gesture*

Researcher: *Randomly? So is she using it randomly or to mean something?*

Student: *Yes, it seems so to me.*

Researcher: *Well, does it make any sense for you to gesture like this while showing x, what does it mean when she says x, for example?*

Student: *For example, it shows x, what x is. That is, while she is saying it, she is showing it at the same time. Furthermore, she supports what she says with her hand movements.*

The student's interview reveals that the students cannot understand the teacher's gestures, they believe that these gestures are used randomly during conversation, and deictic gestures are generally remembered by the students. It may be that the messages given by the teacher are not clearly understood or interpreted by the student as desired, or it may be that the gestures used by the teacher during the narration of the students unconsciously affect the understanding of the student and cannot be specifically remembered by the student.

4. DISCUSSION and RESULT

The findings of the current study suggest that the teacher provides students with visuals and guides their thinking by using body movements, especially gestures, thus overcoming difficulties within the teaching of algebra. During communication, gestures are an important component that accompany speech, according to Novack and Goldin-Meadow (2016). A gesture is an important supporter of speech, according to the authors, and it plays a significant role in the meaning conveyed to recipients. A number of studies have shown that different gestures used for various purposes are effective in shaping students' understanding (e.g. Alibali, Nathan, & Fujimori, 2011; Cook, Duffy, & Fenn, 2013). Arzarello and Edwards (2005) found that gestures that support the content of a speech can contribute to and affect students' cognitive and psychological learning experiences in an evaluation study conducted within the scope of the relationship between gestures and learning mathematics. Similarly, Nemirovsky (2003) argues that thinking is not something that occurs behind body movements, but rather occurs as a result of them. He emphasizes, therefore, that body movements themselves are direct thinking. We can conclude from the results of this study that the teacher's gestures and other body movements both affect the students' understanding of algebra-related concepts. Several, if not all, of the signs used by the teacher reflect both the expressions and signs used by the students in the classroom and in the clinical interviews. The teacher used a number of iconic, metaphoric, deictic, and beat gestures during the algebra teaching process. Most of the time, metaphorical and iconic gestures are used to support teaching. The meaning of the iconic gestures that resulted in the study can be listed as centering and itemization, illustration of a table, constant term, preceding element, decrease, separation, base and power, rectangle and side, length, and vertical-horizontal. As seen, the iconic gestures help the teacher support her linguistic communication. On the other hand, the metaphorical gestures attribute holding, equality, moving the term from one side to the other, numbers or letters, equality, distributing and hiding. The metaphorical gestures, as the iconic ones, strengthen the expressions and wordings of the teacher. Deictic and beat gestures, on the other hand, are used to facilitate communication rather than provide direct instruction. The deictic gestures imply pointing out boards with fingers, board markers or hands, and indicating students and notebooks with fingers, while beat gestures refer to lowering the hands from top to bottom, waving the fist up and down and moving the hand back and forth. As seen, the deictic and beat gestures are used to draw the attention of the students to the instructional moments. In studying and teaching mathematics, a variety of studies have demonstrated the importance of metaphor (Lakoff & Nunez, 2000; Nunez, 2000; Presmeg, 1992). According to studies on gestures, a gesture accompanying each word in the communication later on helps the listener remember the message (Hostetter, 2011). Similarly, Cook, Duffy, and Fenn (2013) found that students who use gestures with speech perform better in the classroom than those who use gestures without speech. In this sense, it can be said that the teacher's preference for metaphorical and iconic gestures facilitates algebraic instruction. Several gestures are also used by students to reveal their thoughts. Using these gestures by the students as a means to reveal the students' thinking will contribute to the teaching process. The studies on gestures conclude that the gestures used

by the students in the teaching process, as well as their gestures, will guide the teacher in terms of instruction (Cook & Goldin-Meadow, 2006; Novak & Goldin-Meadow, 2015).

According to the research, the gestures used in algebra learning and teaching support the instruction, express the action, animate the concepts and words, emphasize, give directions, point, count, itemize, and draw attention. According to Edwards' (2009) research on fractions, pre-service teachers use metaphorical gestures the most, followed by iconic, beat and deictic gestures. This sign has been found to be used by preservice teachers as a tool to represent their understanding of concepts and to communicate with other parties about these concepts. A metaphorical gesture represents an abstract idea or concept, whereas an iconic gesture represents a concrete object. In this context, it can be said that the themes of supporting instruction obtained in response to these gestures, enlivening words/concepts, emphasizing and pointing are similar to Edwards's (2009) study. According to Herbert (2012), students use five different types of hand gestures to communicate. The hand gestures contain signs that describe the relationship, formula, graph, axes, and difference for the function, and it is likely that the indicated signs are similar to what is being explored in this study, namely supporting instruction and animating words and concepts. According to Streeck (2008), hand gestures can be classified as shaping, drawing, modeling and expressing action. Streeck's (2008) general classification of hand gestures appears to be more detailed than the hand gestures found in the scope of the study. On the other hand, Akıncı (2014) put forth that the students used quite a variety of gesture types for the concepts of geometry, which serves as a reflection of their thoughts. Throughout the course of this study, it has been established that there are different types of gestures that serve different purposes, just as it has been shown in the literature studies on this topic. Gestures can be interpreted as an important part of the methodology used in the teaching and learning of mathematics, as they can support teachers as well as students in the process of expressing themselves and their thoughts.

In light of the findings of the study, it was found that the teacher uses gestures to support the instruction of mathematics and that students are also affected by the gestures used by the teacher. Furthermore, there are certain body movements that the teacher uses without any apparent purpose and without any awareness of them. In order to strengthen the teacher's instruction in algebra, it might be useful to suggest that the signs within the scope of the teacher's body movements should be used purposefully and consciously. Hence, this will support and enhance the lecture and the teaching process. Particularly, teachers who are just starting their careers should be given training on the importance of body movements as a means of supporting their teaching by raising awareness about the importance of body movements. Therefore, it would be beneficial to make the use of these signs more widespread so that the teaching process could be more effective.

Author Contributions

We declare that the authors contributed to the research jointly.

Statement of Support and Acknowledgment

We would like to express our sincere thanks for their support to Anadolu University Scientific Research Projects Commission.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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