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Research Article

Investigation of Middle School Students' Area Measurement Knowledge and Skills

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Abstract – In this study, it is aimed to examine the area measurement knowledge and skills of middle school students. In the study, in which the descriptive method was used, the data were collected in written form with 8 open-ended questions from 92 seventh grade students. Students' answers to open-ended questions were analyzed with a scoring scale. The research shows that the majority of students who correctly estimate the size of the area measurement units are in the majority, but they have difficulty in transforming the standard area measurement units. Students who can calculate the area as the number of unit squares covering a region are in the majority. It was observed that the students who were successful in measuring the area of the rectangle had low success in measuring the area of the parallelogram, triangle and trapezoid, and it was observed that the students tended to multiply the two sides of the polygon while measuring the area. In addition, it was determined that some students confused the area with the environment.

Key words: area measurement knowledge, area measurement skill, geometry teaching, middle student.

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Introduction

Mathematics is an effective tool that helps individuals understand the world and the competency they should have for the beginning of a career (Dursun & Dede, 2004; Stafslien, 2001). Therefore, mathematics is taught as an introductory course at any level today, and mathematics skill is questioned as a fundamental skill in many national assessment and evaluation exams for the selection of students. Success in mathematics lessons is highly related to IQ, a sign of giftedness (Konold & Canivez, 2010). Therefore, students see mathematics as an important lesson and a tool for life (Yaman & Yaman, 2020). Students only sometimes have positive feelings about mathematics, which is an essential course. Lim and Ernest (1999) stated that many students have negative thoughts about mathematics, and even teachers and parents have these negative thoughts. While there are many reasons for this, one of the main reasons is that mathematics is considered a complex subject to achieve. Although there are many reasons for the student to have this perception, both from the teacher, family, and friends, the student's failure in the course as an individual factor is essential (Savaş et al., 2010). In many countries, studies aiming at making students successful in mathematics are carried out to eliminate the perception that mathematics is complex (Şahin, 2013). In our country, the "Mathematics Mobilization" studies (MEB, 2022), which were initiated in cooperation with the Ministry of National Education, TUBITAK, and universities, in order to both facilitate the learning of mathematics by adapting it to daily life skills and to ensure that students like this lesson from a young age, continue actively.

Many factors affect students' success in mathematics. Thomson et al. (2003) discussed the factors affecting students' success under four headings: student attitude, student-related factors, teacher factor, and school factor. The researchers, who considered the affective components such as the student's anxiety towards mathematics, and the perception of self-efficacy under the student attitude, considered the components such as the thought, age, and experience of the individual's teacher with mathematics teaching under the teacher factor. Stating that the success of the student is affected by the technological facilities of the school, its perspective on education, its size, and the student-parent profile, Thomson et al. (2003) finally discussed the components such as the student's gender, family structure, socioeconomic status, and cognitive characteristics under the title of student-related factors. Students' knowledge and skills about a mathematical concept and subject also affect mathematics achievement (Şenol et al., 2015).

Therefore, studies examining students' knowledge, skills, and learning difficulties regarding mathematical subjects and concepts are frequently encountered in the literature (Çavuş Erdem, 2013; 2018; Öztürk & Gürefe, 2021; Tan Şişman & Aksu, 2009). Explaining students' deficiencies and mistakes in mathematical concepts is also crucial in paying attention to these issues in the design of instruction. Based on this idea, this study aimed to examine the knowledge and skills of secondary school seventh-grade students on area measurement.

Although making mistakes and experiencing difficulties in learning mathematics subjects are a natural part of the learning process of students (Hansen, 2014), it negatively affects their learning. Mathematics is a cumulative course with significant prerequisite knowledge. Therefore, student mistakes on a subject create an obstacle in learning other subjects that are fundamental to that subject, making it difficult to fully learn the subject. The subject of area measurement is the basis for volume measurement and land measurements, and it is also used as an effective tool in associating mathematical concepts such as algebraic expressions and identities (Bingölbali & Çoşkun, 2016). Therefore, it is important to identify students' errors and difficulties in measuring area, because detecting the mistakes and misconceptions at an early stage will enable the problem to be solved more easily. In addition, the student mistakes regarding the subject reveal important points about how the subject teaching should be designed and which issues should come to the fore. It is very important to increase teacher awareness on this issue. It is stated that some of the student errors are pedagogical, in other words teaching-related (Bingölbali & Özmantar, 2009). Therefore, conducting studies that determine the level of students' knowledge and skills and which subjects they learn incorrectly makes important contributions from the teacher and student perspective. Considering the studies that emphasize that students' learning deficiencies and obvious errors on the subject are caused by the curriculum (Yorulmaz & Önal; 2017), it is thought that studies examining students' knowledge and skills will also provide important information for program developers. Based on this idea, this research aims to examine the knowledge and skills of seventh grade secondary school students about area measurement and to reveal their learning deficiencies.

Area measurement

One of the learning fields of the mathematics curriculum is measurement, and one of the sub-learning is area measurement (MEB, 2018). *Area measurement* is an important concept that affects the understanding of mathematics subjects in upper grades (Cavanagh, 2008). Because area measurement represents a transition in teaching other measurement types, such as volume (Smith et al., 2016). Therefore, area measurement gains are at every grade level from the third to the seventh grade in the curriculum (MEB, 2018). Area measurement is expressed as determining the amount of a region in terms of a unit (Fauzan, 2002). Two essential concepts emerge in area measurement. These are the concept of area and the concept of measurement. The concept of area refers to the amount that covers a limited space, and measurement determines this amount with a unit (Simon & Blume, 1994). The first stage involves understanding that the area is a planar region, that is, interpreting the area conceptually. In the second step, the amount is determined. Studies indicate that students need to clarify these two concepts and consider the area separate but interpret it as area measurement (Huang & Witz, 2013). It is important to emphasize these two concepts separately while teaching area measurement in order to prevent mistakes in teaching the subject.

In learning area measurement conceptually, it is necessary to pay attention to some issues. Clement and Stephan (2004) stated that in order to avoid rote learning in area measurement, at least five basic structures should be learned: a) segmentation, b) unit repetition, c) area conservation, d) structuring of the sequence, and e) linear measurement. The individual needs to understand that the structure to be calculated is a limited region and that the units should not overlap or be covered so that there are no gaps when covering the region (partitioning). Afterwards, with the repeated use of a unit of the same type and appropriate size determined, that region needs to be covered (unit iteration). The same type and suitable unit of measurement means a unit suitable for the structure of the covered plane, covering it entirely and leaving no spaces. Covering perpendicularly intersecting polygons such as rectangles and squares gives an idea of the area covered by circles or triangles but covering them with a square gives more precise information (Freudenthal, 1983, as cited in Zacharos, 2006). For the individual to cover it correctly with the appropriate unit at this point, it is also essential to know that the shape's area will not change regardless of the structure of the shape as long as the piece decreases or not (conservation of area). In this way, the area to be calculated can change the area to be covered more quickly.

After covering the region with units, the individual must determine how many units are in the row and column and understand that the region is a two-dimensional structure (Constructing the array). By covering a rectangle with appropriate and equivalent units, calculating how many units correspond to each line and how many from each line brings systematic counting to the fore (Outhred & Mitchelmore, 2000). Finally, the individual needs to understand how to obtain the total number of units covering the area by multiplying two dimensions and making the necessary association. It is essential to see the column-row coordination in the transition of area measurement to multiplicative dimension and to associate it with the product of the side lengths (Huang & Witz, 2013). Thus, it is possible for students to conceptually understand the multiplication process in the area formula (Outhred & Mitchelmore, 2000; Van De Walle et al. 2014). In this manner, it is possible to understand mathematically the "area=length x width" algorithms based on rote measurement in traditional teaching. It is essential to plan by considering these issues in the curriculum and course contents prepared within the scope of teaching the subject of area measurement.

Our mathematics curriculum includes area measurement gains from the 3rd grade (MEB, 2018). In the program, a progress process in the form of covering the area of shapes with non-standard material, covering with unit square, establishing the relationship between the area relation of square and rectangle and the number of unit squares, measuring using standard units, and finally applying and interpreting this information is discussed (MEB, 2018). In this context, a teaching approach similar to the explanations above is adopted in the program. However, the effectiveness of this teaching style adopted in the program in student learning depends on how much it is carried into the classroom environment. Because researches indicate that students have misconceptions about measuring the area and make incorrect measurements (Çavuş Erdem, 2018; Çelik, 2023; Gelici, 2022; Gürefe, 2018; Huang & Witz, 2013; Kamii & Kysh, 2006; Kidman & Cooper, 1997; Orhan, 2013; Olkun Çelebi et al., 2014; Tan Şişman & Aksu, 2009; 2016). Kamii and Kysh (2006) revealed in their studies that students do not think of the unit square as a unit of measurement for the area. Kidman and Cooper (1997) stated that students made mistakes while calculating the rectangle area by adding the side lengths. Tan Şişman and Aksu (2009, 2016) stated that half of the students in their study did not have area conservation, and they confused area and environment. Orhan (2013), in his study, similarly concluded that students did not have area conservation; they needed help finding the circumference of a polygon whose area measure was given, and procedural knowledge was emphasized. Huang and Witz (2013) stated that the conceptual understanding of the area measurement formula directly affected the area measurement performance, and students who used the area measurement formula by heart exhibited poor performance. In their study, Olkun et al. (2014) concluded that students did not accept the unit square as a unit of area and tend to use formulas. Gürefe (2018), in her study, in which she determined the strategies used by students in area measurement problems, concluded that students commonly used formulas in triangle and rectangle-related problems, and they tended to use multi-step strategies when measuring area in rhombus and trapezoid. Kaya (2019) concluded that the students did not fully know the area concept and that the procedural dimension was emphasized. Gelici (2022) stated that the students had an incorrect concept image in measuring the area. They confused the area and the environment. Çelik (2023) stated in his study that students had less difficulty covering a shape with unit squares, and they could not associate the measurement process with the unit used when measuring the area of a polygon.

Revealing the problems in the basic subjects of mathematics is important in order to plan future learning by solving these problems (Önal & Aydın, 2018). Because unless errors are corrected, they lead to misconceptions (Yenilmez & Yaşa, 2008) and increase the possibility of failure in mathematics. Therefore, early detection of student errors is very important. Based on this idea, this research aims to determine the level of student knowledge and skills regarding area measurement, which is an important sub-learning area of the mathematics basic education program, and to reveal their learning deficiencies. In the research, student knowledge and skills regarding different basic concepts of area measurement such as square unit, area measurement skill, conversion of standard area measurement units, circumference-area relationship were discussed together. Thus, it is aimed to present a general picture of student information on the subject of area measurement. There are a limited number of studies that examine student knowledge of the basic concepts of area measurement together (Tan Şişman & Aksu, 2009). Therefore, it is thought that the study will contribute to the literature. The research also asked questions about how students could calculate area measurement both with the help of unit square and area relation, thus aiming to determine student awareness about two important components that are critical in the conceptualization of area measurement, as stated by Stephan and Celement (2001). Another important issue in learning measurement is to understand the measurement unit correctly (Yenilmez & Pargan, 2008). Learning the measurement unit also includes the conversion skill of standard measurement units (MEB, 2018). This skill is directly related to the student's awareness of the quantity of the unit of measurement. It is obvious that a student who does not know the size of the measurement unit cannot learn unit conversion and the use of units in area measurement at a conceptual level (Van De Walle et al., 2014). Therefore, studies that determine students' knowledge about the amount of area measurement units were thought to be important. It is thought that this research, which examines students' knowledge and skills regarding the conversion of standard area measurement units and their awareness of the quantity of these units, will contribute to the literature in this sense.

Thus, the problem of the research is "“What are the knowledge and skills of seventh grade secondary school students on area measurement?" and within the scope of this problem, answers are sought to the sub-problems presented below.

1. What are the knowledge and skills of seventh grade secondary school students regarding measuring the area of spaces covered by unit squares?

2. What are the knowledge and skills of seventh grade secondary school students regarding standard area measurement units?

3. What are the knowledge and skills of seventh grade secondary school students regarding measuring the area of geometric shapes (square, rectangle, parallelogram, triangle, trapezoid)?

Method

Research Design

The case study method was adopted in this research. Case study is a research method in which the situation is described by collecting detailed and in-depth information about a system through limited situations (Creswell, 2020). While a single case about the problem or research topic addressed in the case study can be examined, results can be obtained by examining more than one case study within the framework of the same problem. In this research, where the descriptive feature of the case study (Yin, 2009) came to the fore, the knowledge and skills of middle school students on area measurement were examined in detail through open-ended questions on 92 students.

Participants

The research participants comprised seventh-grade students in the second semester of the 2018-2019 academic year. The achievements of the area measurement subject are included in the curriculum from the third grade, and it is planned to give a large part of the acquisitions until the seventh grade (MEB, 2018). Since it was aimed to present a general picture of students' area measurement information, seventh-grade students were thought to be the most appropriate sample group for secondary school. In the research conducted with 92 students, 52 were female, and 40 were male. Participants were studying in three different secondary schools located in a city center. A class from each school was determined, and all students studying in that class participated in the research. While determining the classes, attention was paid to ensure that the mathematics grade point average was at a medium level, and that students with high (between 80-100 points), medium (50-80 points) and low academic achievement scores (between 20-50 points) were included in a balanced manner. Thus, an attempt was made to ensure data diversity by creating heterogeneous groups. To that end, classes were determined by considering the first semester grade point average of the mathematics course and the opinions of the mathematics teacher, and the application was carried out with a total of 92 students from three classes.

Data collection

In the research, the "Area Knowledge Evaluation Form" in the study of Çavuş-Erdem (2018) was used to measure the students' area measurement knowledge and skills. The questions in the form and the related acquisitions are presented in Table 1.

Table 1 The Acquisitions Related to the Questions in the Area Knowledge Evaluation Form

Question	Related Outcome
Question1	Being able to determine that the areas of the shapes are the number of unit squares covering this area and to compare the areas of the shapes
Question2	Calculating the area of a rectangle, using square centimeters, and square meters
Question3	Being able to recognize standard area measurement units
Question4	Solving problems related to the area of a parallelogram.
Question5	Comparing the areas of polygons
Question6	Interpreting the side-length-area relationship, calculating the area of the square
Question7	Solving problems related to the area of a triangle
Question8	Being able to solve problems related to the area of a trapezoid.

The form consists of eight open-ended questions. The questions in the form were determined by within the scope of the achievements in the seventh grade and previous years' curriculum. Before the application, a short explanation was given to the students about the questions in the form, and they were asked to write down the reason for the question along with the answer and to explain what they thought transparently without worrying about whether it was a wrong or correct statement. All of the applications were carried out under the researcher's supervision, and the application was completed in 40 minutes to enable the students to think about the questions sufficiently.

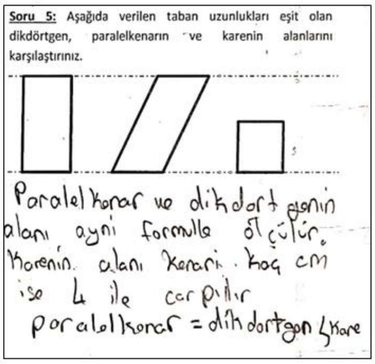
Data analysis

The scoring scale developed by Gürbüz & Birgin (2012) and presented in Table 2 was used to analyze the data consisting of open-ended questions.

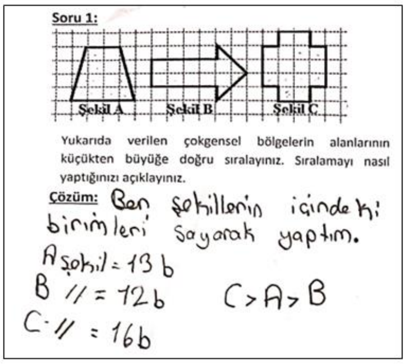
Table 2 Open-Ended Questions Scoring Scale

Levels of Understanding	Explanation	Evaluation Criteria Stage 1 – Stage 2	Score
Correct explanation	Answers that include all aspects of a valid explanation	Correct answer – correct explanation	5
		Wrong answer correct explanation	4
Partially correct explanation	Answers that do not include all aspects of the valid explanation	Correct answer – partially correct explanation	3
		Wrong answer - partially correct explanation	2
Wrong explanation	Answers with inaccurate information	Correct answer – wrong explanation	1
		Wrong answer – wrong explanation	0
No explanation	Correct, incorrect or blank answers with no explanation	Correct answer - no explanation	1
		Wrong answer - no explanation	0
		No answer - no explanation	0

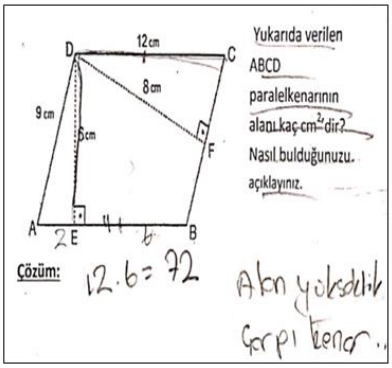
Each student's answer was coded separately according to the categories above. In order to detail the analysis framework, sample student answers and explanations are presented below (Figure 1). In the first example, the student gave a correct answer, but the explanations he wrote for both the area relationship of the rectangle and parallelogram and the area relationship of the square were wrong. Thus, the student's answer was coded in the "Correct answer - Wrong explanation" category. In the second example, the student gave the wrong answer. However, he stated that he made calculations by counting units when calculating the areas of regions and made a correct statement. However, since the student used the unit expression instead of unit square, the student's answer was coded in the "Wrong answer - Partially correct explanation" category. In the third example, the student correctly calculated the area of the parallelogram, but expressed the area relationship as the product of height and side instead of height and base. Thus, the student answer was coded in the "Correct answer- Partly Correct explanation" category.



The area of a parallelogram and rectangle is measured by the same formula. The area of the square and its side in cm are multiplied by 4.
Parallelogram = Rectangle > Square



I did it by counting the units in the shapes.



Area, height times side

Figure 1 Student Answers

For the data analysis, assistance of an expert researcher was received. Accordingly, 19 student solution sheets, which constituted 20% of the data, were individually coded by two researchers. Afterwards, the researchers came together and compared the coding in detail. For the coding agreement percentage, Miles and Huberman's (1994) coder reliability formula ([Compatible codes/ (Consistent codes + Incompatible codes)] x100) was applied, and the agreement percentage was determined as 134 compatible codes-152 total codes). Incompatible codes were evaluated, and after a consensus was reached, the researcher carried out the analysis process alone. Each student paper was coded according to the scoring scale, and the coding results presented with frequency and percentage values were supported by sample student solutions based on each question. Translations of student explanations in the solutions are presented below the visual of the solution.

Results

In the study, student answers were evaluated both based on questions and students' performances. The distribution of the student's answers and the scores they got from the form is presented in Table 3.

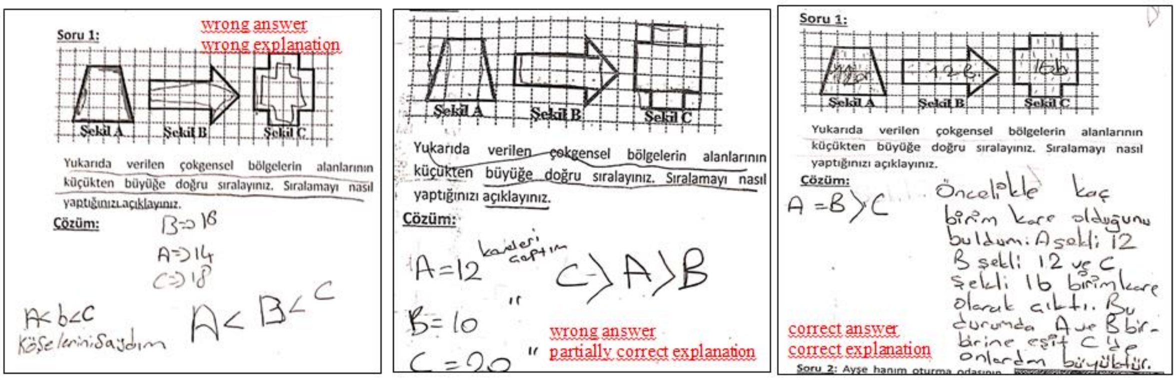
Table 3 Distribution of Student Answers

	Q1		Q2		Q3		Q4		Q5		Q6		Q7		Q8		Mean
	f	%	f	%	F	%	f	%	f	%	f	%	f	%	f	%	%
Correct answer-correct explanation	22	24	8	9	50	54	14	15	9	10	1	1	26	29	16	17	20
Wrong answer-correct explanation	13	14	0	0	0	0	1	1	0	0	4	4	3	3	6	7	4
Correct answer-partially correct explanation	0	0	5	5	12	13	1	1	15	16	1	1	1	1	0	0	5
Wrong answer-partially correct explanation	22	24	31	34	4	4	25	27	10	11	21	23	15	16	15	16	19
Correct answer-wrong explanation Correct answer-no explanation	1	1	1	1	9	10	2	2	14	15	0	0	0	0	0	0	4
Wrong answer-wrong explanation Wrong answer-no explanation No answer-no explanation	34	37	47	51	17	19	49	54	44	48	65	71	47	51	55	60	48

	Unsuccessful (0-8 p)		Must be developed (9-16p)		Acceptable (17-24p)		Good (25-32p)		Very Good (33-40p)	
	f	%	F	%	f	%	f	%	f	%
Total	32	35	29	32	21	23	8	9	2	2

Considering the information in the table, it was noted that the average percentage of students that gave correct answers with correct explanations was 20%, and the cases where they made correct explanations but gave wrong answers by making mistakes was four percent on average. While the average of the students who gave partially correct explanations with the correct answer was five percent, the average of those who gave partially correct explanations with the wrong answer was 19%. While students who gave correct answers but had incorrect or incomplete explanations were in the four percent group, students that gave both answers and explanations incorrectly or missing were the group with the highest percentage with an average of 48%. Based on the question, it was noted that the students gave correct answers only in the third question, with 54% of the correct explanation. The most significant percentage of the other seven questions belonged to the level with incomplete or incorrect answers and explanations. Considering the students' individual scores in the evaluation form, 35% of the students received weak scores and were unsuccessful. 32% of the students were in the must-be-developed group, and 23% scored at an acceptable level. Very few students could score in the good (nine percent) and very good (two percent) categories. In light of this information, it was possible to note that the students exhibited an unsuccessful performance in area measurement in general. The questions will be discussed individually, together with sample student solutions, to detail the cases where students were successful and unsuccessful.

In the first question, to compare the areas of regions consisting of unit squares, 24% of the students gave the correct answer with the correct explanation. 14% of the students counted the unit squares when comparing the areas but made a calculation error. Instead of counting unit squares, 24% of the students tried to calculate the area by converting the given shapes into familiar quadrilaterals such as rectangles and found incorrect results. On the other hand, 37% of the students were unsuccessful in this question and gave wrong answers and explanations. Sample student answers to the situations mentioned below are presented.



I counted the corners

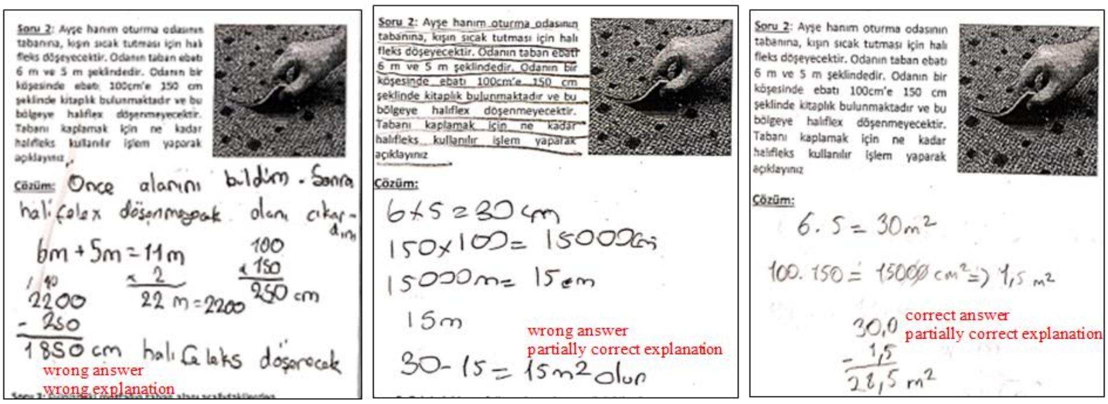
I multiplied the squares

First of all, I found out how many square units there are. Shape A turned out to be 12 square units, shape B 12 square units, and shape C 16 square units. A and B are equal to each other, and C is greater than them.

Figure 2 Student Answers to the First Question

Considering the student explanations, it was noted that the student in the first image was trying to calculate the circumference of the shapes to calculate the area. In the second image, the student calculated the area by converting all shapes into rectangles. The result was incorrect because student should have paid more attention to some unit squares. In the third image, it was noted that the student reached the correct answer by using the unit squares. The students who reached the correct answer accepted the area as the number of unit squares covering a region; in this sense, they had conceptually correct information.

The second question of the study was one that students needed help with. Only 14% of the students reached the correct answer in calculating the area of the rectangle and unit conversion, while the majority, 51%, answered the question incorrectly or left it unanswered. On the other hand, 34% of the students showed a partially correct solution approach. Sample student solutions to the question are presented in Figure 3.

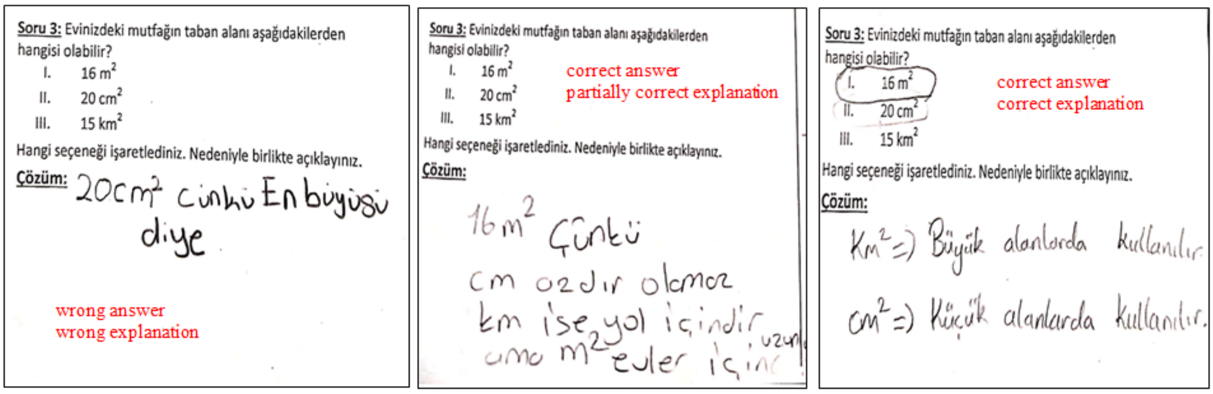


First, I found the areas. Then I removed the area that would not be carpeted.

Figure 3 Student Answers to the Second Question

When the students' answers were examined, it was seen that the student who gave the wrong answer added the side lengths to find the floor area of the room and made the unit conversion as m-cm correctly. The student confused the area with the circumference. In the second and third images, the students correctly calculated the area of the rectangle. Students who correctly calculated the area of the rectangle in this question constituted 49% of the group. However, students made the unit conversion incorrectly, therefore the students in the group who partially gave correct explanations gave wrong answers. In the second example, it was seen that the student also uses m and cm as units of area measurement. The findings obtained from this question showed that students need to improve in standard area measurement units and conversion between units.

The third question was about using area measurement units in daily life. In the question in which the students showed the most successful performance, 19 of the group gave the wrong answer, and 54% gave the correct answer with the correct explanation. 13% of the students who gave partially correct explanations gave correct answers, four percent gave incorrect answers, and the students who made a wrong explanation with the correct answer or did not make any explanation constituted 10% of the group. Sample student solutions are presented in Figure 4.



20 cm². Because it is the biggest

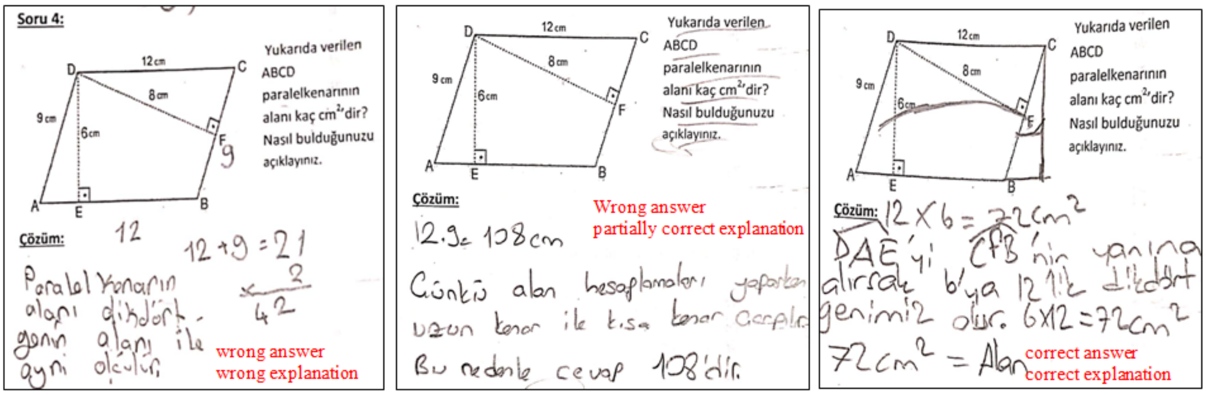
It's not possible because it's too small in cm. km is for the road. m² is for houses

km², It is used in large areas. cm², It is used in small areas.

Figure 4 Student Answers to the Third Question

When the students' answers were examined, it was seen that the student in the first image did not know the sizes of the standard area measurement units. Although the students in the second and third images gave the correct answer, it was seen that the student in the second image reached the correct answer based on the standard length measurements. Since the number of students who gave wrong answers to this question was relatively low, it was possible to say that, in general, students had information about the equivalents of standard area measurement units in daily life.

In the fourth research question, the students were asked to calculate the area of the parallelogram. Only 15% of the students gave the correct answer with the correct explanation, while 54% needed help to calculate the area of the parallelogram. Although 27% of the students gave wrong answers, they tried to calculate the area by showing a more accurate approach than the unsuccessful 54% group. Sample student answers are presented in Figure 5.



The area of a parallelogram is measured the same as the area of a rectangle.

Because when calculating the area, the long side is multiplied by the short side. Therefore, the answer is 108.

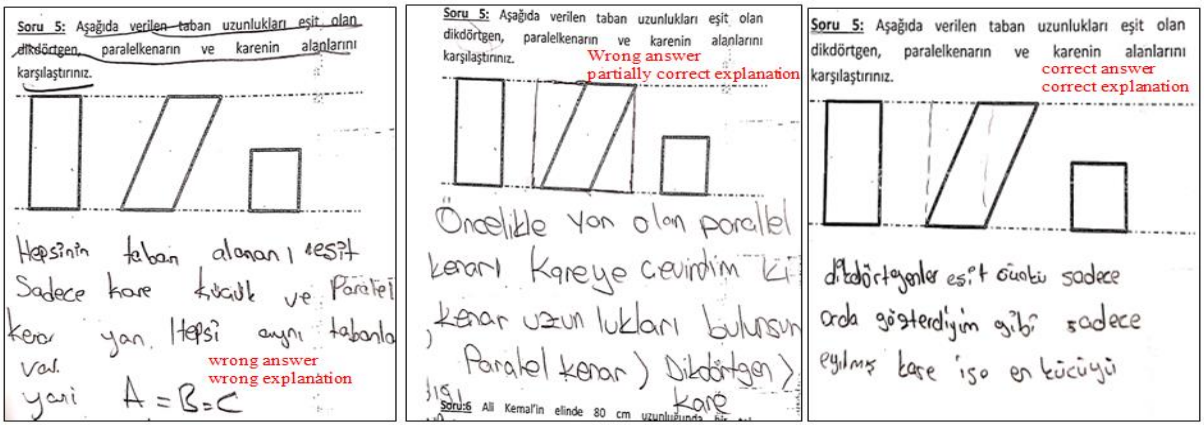
If we put the DAE triangle next to the CFB triangle, we have a 6 by 12 rectangle.

Figure 5. Student Answers to the Fourth Question

When the students' answers were examined, it was seen that the student in the first image confused the area of the parallelogram with its circumference and calculated the area with the circumference relation. In this way, students who confuse the area with the circumference and try to find the area in polygons by adding the side lengths constitute 27% (25 students) of the whole group. In the second image, the student calculated the area of the parallelogram as the product of the long and short sides and therefore answered the question incorrectly. Students calculating the area of a parallelogram as the product of its two sides constitute 27% of the group. In the last image, the student converted the parallelogram into a rectangle, calculated the area, and reached the correct answer.

The fifth question of the research was about area conservation and area relations. In total, 31% of the students gave the correct answer, 10% made a correct explanation, 16% made a partially correct explanation, and 15% gave an incomplete or incorrect explanation.

11% of the students gave a partially correct explanation with an incorrect answer, and 48% answered the question incorrectly with an incomplete or incorrect explanation. Sample student answers are presented below (Figure 6).



All of them have equal base area. Only the square is smaller, and the parallelogram is on its side. They all have the same base.

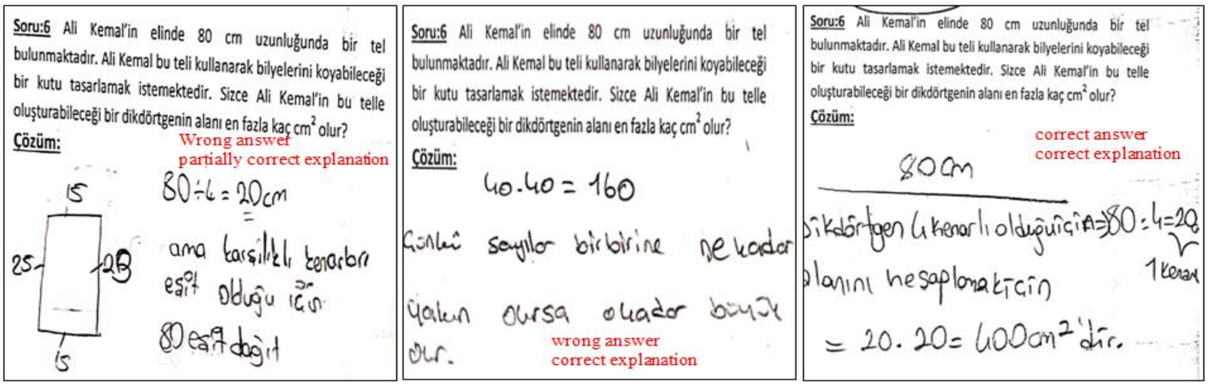
First of all, I turned the side parallelogram into a square to find the side lengths.
Parallelogram > rectangle > square

The rectangles are equal because it is just bent as I showed there, while the square is the smallest.

Figure 6. Student Answers to the Fifth Question

In the first image, the student should have paid more attention to the height while calculating the area, accepted the area of the three shapes as equal, and miscalculated the question. In the first image, the student made such a mistake because he did not know the mathematical equivalent of the area. Looking at the answer in the second image, it was seen that the student interpreted the areas after transforming the parallelogram into a familiar shape. However, while making this transformation, the student ignored the area conservation as in the third image and made a mistake by enlarging the shape from both sides. In the third image, the student transformed the parallelogram into a rectangle and reached the correct answer. Some students gave numerical values to all shapes and tried to find the answer. Students who gave the same values to parallelograms and rectangles with the same height (e.g. eight to the long side and four to the short side) calculated the area of both shapes as the product of the two sides and answered the question correctly. However, since the students made the mistake of calculating the area of the parallelogram as a rectangle, the students who found the correct answer to the question by calculating this way were coded in correct answer-partially correct explanation category. Students who found the correct answer to the question by calculating this way constitute 16% of the group. The findings obtained in this question coincided with the findings in the area of the parallelogram.

The sixth research question was within the scope of the circumference -length-area relationship, and it was determined as the question in which the students performed at the lowest level. The students who gave the correct answer with the correct explanation constituted only one percent of the group, the students who found the wrong answer and gave the correct explanation four percent of the group, and the students who could not give the correct answer but followed a partially correct solution formed 23% of the group. 71% of the group answered the question incorrectly. Sample student solutions are presented in Figure 7.

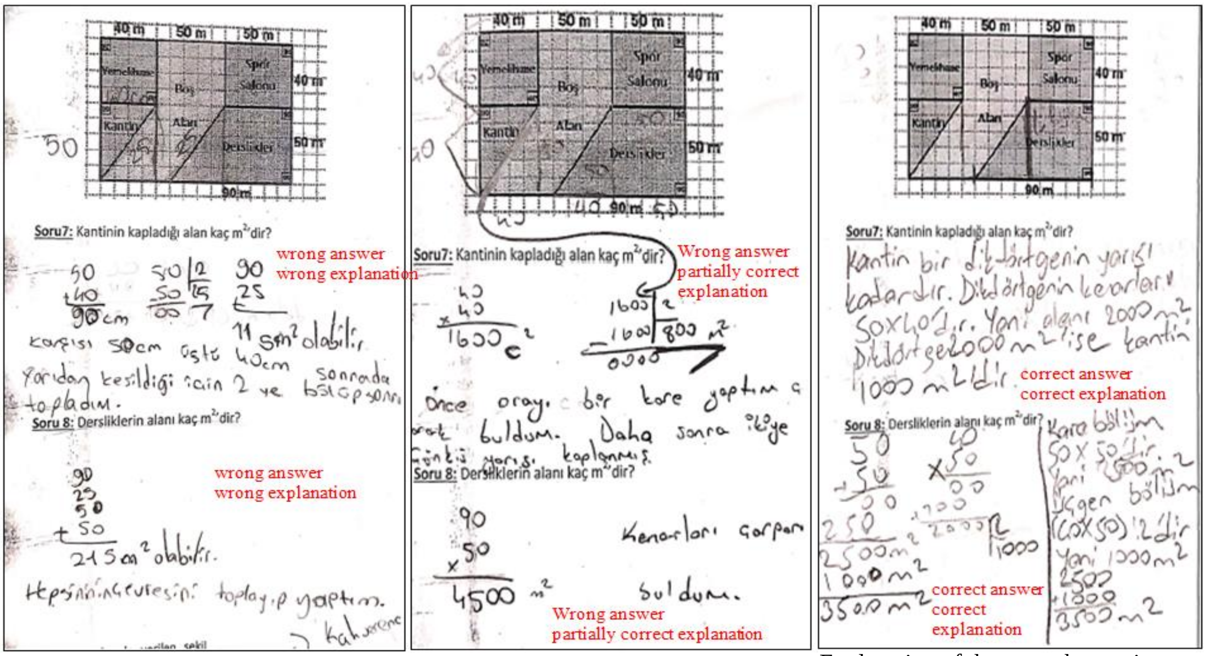


Since opposite sides are equal, distribute 80 equally.	Because the closer the numbers are to each other, the larger they are.	Since the rectangle has 4 sides, 80:4 = 20 cm (1 side) to calculate the area, 20 x20 = 400 cm^2

Figure 7. Student Answers to the Sixth Question

Figure 7 gives the answers of the students who followed the correct strategy. In the first image, the student showed a correct approach to finding the side lengths, but since the question asked for a rectangle, student tried to keep the opposite side lengths equal instead of all side lengths. The student did not accept the square as a rectangle. The students, who were in the wrong answer partly correct explanation category, tried to answer the question by giving values to complete the two side lengths to 80 and similar solutions. In the second image, the student made a correct explanation, valued the lengths of the two sides as 80, and found the wrong answer. In the third image, the student correctly explained and calculated the area. There was one student in the application group who gave the correct answer with the correct explanation. From this point of view, the students had difficulties and failed in this problem based on the area-circumference relationship.

The last two research questions were to determine student skills in determining areas of the triangle and trapezoid. Students who answered the seventh question correctly with a correct explanation were determined as 29% and 17% for the eighth question. While the students who made a correct explanation but gave a wrong answer were 3% for the seventh question and 7% for the eighth question, Iin comparison, the students who gave a partially correct explanation with an incorrect answer were 16% for both questions. The students who answered the questions incorrectly, made wrong explanations, and did not explain were determined as 51% in the seventh and 60% in the eighth questions. Sample student answers to two questions are presented in Figure 8.



Explanation of the seventh question: The opposite side is 50 cm, the upper side is 40 cm, then since it was divided in half, I divided it by 2 and added it.
Explanation of the eighth question: I summed all circumferences

Explanation of the seventh question: First I made it a square, then I found it by multiplying. Then I divided it in half. Because it covered half of it.
Explanation of the eighth question: I found it by multiplying the sides.

Explanation of the seventh question: The canteen is half of a rectangle. The sides of the rectangle are 50x40. That is 2000 m^2. If the rectangle is 2000 m^2, the canteen is 1000 m^2.
Explanation of the eighth question: Square section is 50x50 = 2500 m^2. Triangular section (40 x50)/ 2. So it is 1000 m^2.

Figure 8. Student answers to the seventh and eighth questions

Looking at the student answers, the student who gave the wrong answer in the first image calculated the area of the right-angled triangle by adding the lengths of the right-angled sides and dividing it by two. The student, who made a mistake by adding the sides instead of multiplying the side lengths, calculated the circumference of the right-angled trapezoid instead of the area in the eighth question and expressed the result in area units. This student confused the area with the circumference. In the second image, the student calculated the area of the perpendicular triangle with the correct method but found an incorrect result because student determined the side lengths incorrectly. Student incorrectly calculated the right-angled trapezoid area by multiplying the side lengths. In the third image, the student correctly calculated the area of both polygons and acted with the strategy of dividing and completing while calculating. Student found the area of the right-angled triangle by completing the rectangle in both questions. While calculating the area of the right-angled trapezoid, student divided the shape into more familiar shapes, such as triangles and rectangles. Students who calculated in this way constituted 17% of the whole group. Based on student answers, while calculating the area of triangles and trapezoids, students generally make calculations by completing or dividing the shape into a rectangle or square.

Discussions, Conclusions and Suggestions

This study examined middle school student's knowledge and skills in measuring areas. The results showed that the students had an overall unsuccessful performance in measuring area. While 35% of the students were in the unsuccessful category, the students in the excellent category made up two percent of the whole group. The evaluation based on questions determined that students' answers to all questions except the third question were concentrated on the "wrong answer-wrong explanation." In the study, a question was asked to the students, aiming to find the area of shapes that did not resemble the polygons they know, such as square, rectangle, and parallelogram, by counting the unit squares. However, most of the students had difficulty finding the area of the region by counting the unit squares, and it was determined that some students tried to calculate the area with the formula by completing the area to familiar rectangular and square shapes instead of counting the unit squares. While there was an easier way to find the area by counting the unit squares, the fact that the students tried to find the area with the formula brought to mind the idea that they did not know the concept of unit square or perceive the area as the amount occupied by a region. As Kamii and Kysh (2006) stated, students did not accept the unit square as a unit of area measurement. Olkun et al. (2014) stated in their study that students use formulas instead of calculating unit squares and area. The fact that these students thought of the area concept as two lengths multiplied rather than covering a region might be another reason for the results obtained. It is stated in studies that students limit the concept of area-to-area measurement (Çavuş Erdem, 2018; Huang & Witz, 2013). The main reason for this might be that the unit square was not emphasized much in the teaching of the subject, the teaching was formula-oriented, and the examples were presented on this axis because it was known that the problems arising from teaching and the teacher's lack of knowledge could be an essential factor in the students' mistakes in measuring area (Kidman & Cooper, 1997). Learning the concepts of area measurement with their mathematical properties correctly in studies is an effective method for eliminating errors (Çavuş Erdem, 2018). Therefore, in order to prevent the limited perception of students, it would be a correct approach to create the perception that the area is covered with equivalent units, together with an understanding based on algorithms, and to transform it from column-row coordination to a systematic counting to multiplicative dimension (Clement & Stephan, 2004; Outhred & Mitchelmore, 2000).

Students' lack of understanding of unit squares, one of the basic concepts of area measurement, also affects their knowledge of standard area measurement units (Çavuş Erdem, 2018). Similarly, this study concluded that the students had deficiencies in standard area measurement units, and they made mistakes in the conversion between units. It was determined that the students divided the square centimeter by 1000 or 100 to convert the square meter, and some students wrote m and cm as the unit of measurement for the area. It is stated in the studies that students make mistakes in the transformations between units and have difficulties in terms of which number to multiply or divide by unit transformations (Doğan Çoşkun, 2017). Dealing with the relationship between units in an operational way is one of the main reasons for this situation (Bragg & Outhred, 2000). In textbooks, the transformation between units is made with a ladder analogy (Çağlayan et al., 2021). This method can cause students to convert by rote. In order to avoid such difficulties in transformations between units, it can be an effective method to show the size of the units and their relationship with each other through concrete objects (Olkun & Toluk Uçar, 2009). It can be suggested that unit squares such as m^2 and cm^2, which are large enough to be displayed in the classroom environment, should be shown with concrete materials, and the transformation should be explained through these materials. Thus, it can be ensured that students have an idea about the size of the units. In the study, it was determined that there was a group of 20% of students who had erroneous information about the size of the area measurement units. Most of the students had correct information about the sizes of standard unit squares. Students might have considered length measurement units when associating units of area measurement with examples in daily life. The statements "km is for the road, cm is less" in the students' answers supported this idea. Length measurement is the basis of area measurement, and for a correct understanding of area measurement, length measurement must also be understood correctly (Çetin, 2020; Outhred & Mitchelmore, 2000). The research results showed that this situation is also valid for measurement units.

In order to examine the area measurement skills of the students in the research, questions were asked to calculate the area of the parallelogram, triangle, and trapezoid. It was observed that students made different mistakes while calculating. Some students added the side lengths to calculate the polygon area, while others mistakenly multiplied the two sides instead of the base and the height. The fact that the students added the side lengths showed that they confused the circumference and the area. In many studies, it is emphasized that students confuse the circumference and the area and similarly add the side lengths of the shapes to calculate the area (Baturo & Nason, 1996; Gelici, 2022; Simon & Blume, 1994; Smith, et al., 2016; Tan Şişman & Aksu, 2009). The fact that the area and the circumference were measurable properties of polygons, that both subjects were taught simultaneously, and that it was a formula-based teaching caused difficulties in this regard (Van De Walle et al., 2014). Another mistake identified by the students was that they multiplied the side lengths while calculating the area of the polygons. It could be argued that students made mistakes by overgeneralizing the area relation of square and rectangle (Schifter & Szymaszek, 2003). Students who were more successful in calculating the area of a rectangle had lower levels of success in calculating the area of a parallelogram, triangle, and trapezoid. Students were known to calculate the rectangle's area more easily (Gürefe, 2018). Baturo and Nason (1996) stated that students learned the area formula of the rectangle correctly without questioning why and how. In squares and rectangles, unlike other polygons, the height, which is the base and auxiliary element of the figure, is also the length of the side, which is the essential element of the figure. This situation could make the area calculation of rectangles and squares more memorable for students. At the same time, area calculation of rectangles and squares takes place before other polygons in the curriculum. The fact that it is the first subject that students encounter might also be another reason for the overgeneralization in area calculation. Another reason was that in calculating the area of this square and rectangle, the formula was expressed and taught as "the product of two sides" or "length × width" instead of "base x height." From this point of view, students might be mistaken as "the lengths of the sides are multiplied when calculating the area of polygons," this may cause the concept of area to be limited to the perception of "width x height." Kamii and Kysh (2006) stated that teaching with unit squares would be effective in preventing the limited perception of "width x height" in students. It could be effective to emphasize that the two lengths multiplied in the area formula of a rectangle, the base of the polygon and the height of that base, were also the two perpendicular sides of the polygon.

Another result obtained in the study about measuring area was that the students followed a strategy of dividing or completing into rectangles and squares while calculating the area of triangles, parallelograms, and trapezoids. Gürefe (2018) similarly stated that students made calculations based on formulas in triangles and rectangles and follow multi-step strategies in parallelograms and trapezoids. In the field of teaching, the calculation of parallelograms, a calculation method based on rectangular conversion, was discussed (Çağlayan et al., 2021; Çetin, 2020). In the area formula of the triangle, the square and rectangle were similarly used when explaining the reason for dividing into two. This result was thought to be due to education.

In the study, students had difficulty calculating the area of a polygon given its perimeter. Similarly, Orhan (2013) stated in his study that students had difficulty in finding the perimeter of a polygon whose area was given, while Çelik (2023) stated that while students were trying to create rectangles with equal and different areas, they could not establish a relationship depending on the size of the unit. Chappell and Thompson (1999) stated that students thought polygons with the same area would have the same perimeter. The area and circumference mentioned above could confuse students (Gelici, 2022), and not making the correct association between the area and the environment caused difficulties (Martin & Strutchens, 2000). How students interpreted the relationship between the side length and the area was also essential. Because students might think there was a linear relationship between the side length and the area and between the circumference and the area (Çavuş Erdem, 2018; Moreira & Content, 1997), this thinking also led to errors. One of the main reasons students made such mistakes was that formulas were handled operationally without being conceptually understood because correctly using the area formula did not mean that the concept of area was learned (Fauzan, 2002). The fact that there were students who correctly calculated the area of the rectangle but could not calculate the area of the shape given the circumference supported this idea.

Regarding this question, it was also determined that some students stated that the square was not a rectangle. Studies involving students who did not accept the square as a particular form of rectangle emphasized that this situation might have cognitive and pedagogical origins (Monaghan, 2000). Studies showing that teacher candidates had similar perceptions (Horzum, 2018) strengthened the possibility that this misconception in students stemmed from teaching.

In summary, in this study, it was determined that students had difficulties in transforming units, and they were more successful in calculating the area of a rectangle than in calculating the area of a parallelogram, triangle, or trapezoid. Students who calculated the area using unit squares covering a region was the majority. However, students who did not use unit squares in calculating the area made up one-third of the group. Other results obtained from the research were that students tended to multiply the two sides of the polygon while measuring area and that some students confused area measurement with circumference measurement. Not learning the area conceptually and formula-based teaching were the main reasons for these mistakes. In teaching the subject, emphasizing that the area is the amount that covers a region and the role of the unit square in determining this amount, then switching from the counting form to the multiplicative form, which is a shorter way, thus creating area relations, would support students' correct learning and improve their area measurement skills. There may be other factors that are effective in the occurrence of detected errors. It was recommended to conduct research with different student groups to reveal these factors that cause errors in epistemological, pedagogical, or psychological dimensions.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

No conflict of interest.

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CRediT author statement

The study was single authored and the whole process was carried out by the corresponding author

Research involving Human Participants and/or Animals

The study involves human participants. The author signed the declaration stating that there was no need for ethics committee approval and submitted it to the journal because the manuscript is a study produced from the data gathering before 2020.

Ortaokul Öğrencilerinin Alan Ölçme Bilgi ve Becerilerinin İncelenmesi

Özet:

Bu çalışmada, ortaokul öğrencilerinin alan ölçme bilgi ve becerilerinin incelenmesi amaçlanmıştır. Betimsel yöntemin kullanıldığı araştırmada veriler, yedinci sınıfta öğrenim gören 92 öğrenciden, 8 adet açık uçlu soru ile yazılı olarak toplanmıştır. Öğrencilerin cevapları açık uçlu soruları puanlama ölçeği ile analiz edilmiştir. Araştırmada, alan ölçme birimlerinin büyüklüğünü doğru bir şekilde tahmin eden öğrencilerin çoğunlukta olduğu, fakat standart alan ölçme birimlerinin dönüşümünde zorlandıklarını göstermektedir. Alanı, bir bölgeyi kaplayan birim karelerin sayısı olarak hesaplayabilen öğrenciler çoğunluktadır. Dikdörtgenin alanını ölçmede başarılı olan öğrencilerin, paralelkenar, üçgen ve yamuğun alanını ölçmedeki başarısı düşük olarak belirlenmiş ve öğrencilerin alan ölçerken çokgenin iki kenarını çarpmaya meyilli oldukları gözlenmiştir. Ayrıca bazı öğrencilerin alanı çevre ile karıştırdıkları belirlenmiştir.

Anahtar kelimeler: alan ölçme, alan ölçme becerisi, geometri öğretimi, ortaokul öğrencileri.

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