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RESEARCH ARTICLE

AN ALGEBRAIC APPROACH FOR REFINED NEUTROSOPHIC RANDOM VARIABLES

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Abstract

In this article, we are going to extend the definition of literal neutrosophic random variables to literal refined neutrosophic random variables and we will study its new properties. We derive and present various concepts including refined neutrosophic probability distribution function in the continuous case, refined neutrosophic cumulative distribution function of continuous refined neutrosophic random variables, refined neutrosophic expected value, refined neutrosophic variance, refined neutrosophic standard deviation, refined neutrosophic mean deviation, refined neutrosophic r^{th} quartiles, refined neutrosophic moments generating function, and refined neutrosophic characteristic function. Additionally, the paper includes many solved examples and applications. This paper opens the road to generalizing many concepts in neutrosophic probability theory specially neutrosophic reliability theory, neutrosophic stochastic processes, and neutrosophic queuing theory.

Keywords

Refined neutrosophic expected value,
Refined neutrosophic variance,
Refined standard neutrosophic deviation,
Refined neutrosophic probability density function,
Refined neutrosophic cumulative distribution

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1. INTRODUCTION

Neutrosophic field of reals is a generalization of classical field reals adding an indeterminacy element(I), where $I^2 = I,\ I^n = I\ ;\ n \in N$ and I^{-1} is undefined [1, 2]. Neutrosophic has been applied in many areas, such as decision-making [3, 4], artificial intelligence and machine learning [5, 6], intelligent disease diagnosis [7, 8], communication services [9], artificial pattern recognition [10], e-learning [11], physics [12, 13], and more.

In [14], the single-valued neutrosophic probability was introduced by F. Smarandache as a function $P_N: X \to [0,1]^3$ where X is a space that contains indeterminant components, i.e., neutrosophic sample space and defined the neutrosophic probability function of event A by $P_N(A) = \big(T(A), I(neutA), F(antiA)\big) = (T, I, F)$ where $0 \le T, I, F \le 1$ and $0 \le T + I + F \le 3$. Many other studies depended on the definition that assumes a neutrosophic distribution function is a function whose parameters contain indeterminacy (see [15])

Researchers have explored the foundation of neutrosophic queueing theory as a branch of neutrosophic stochastic modeling, as discussed in [16, 17]. Furthermore, researchers presented many concepts related to neutrosophic probability theory and its applications in [18]. Additionally, they have researched neutrosophic time series prediction and modeling in various scenarios, including neutrosophic moving averages, neutrosophic logarithmic models, neutrosophic linear models, and more, as presented in [19-

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21]. These studies contribute to advancing understanding and utilizing neutrosophic principles in stochastic modeling.

Agboola proposed the idea of refined neutrosophic algebraic structures and specifically investigated refined neutrosophic groups in [22]. Since then, numerous researchers in the field of neutrosophic logic have exploredand analyzed various refined neutrosophic algebraic structures. Adeleke et al. investigated refined neutrosophic rings and refined neutrosophic subrings, presenting their fundamental properties in [23]. Additionally, in [24], Adeleke et al. studied refined neutrosophic ideals and refined neutrosophic homomorphisms, providing an exploration of their basic properties.

In this research, by incorporating an indeterminacy component, we propose a generalization of classical random variables to handle imprecision, uncertainty, ambiguity, vagueness, and enigmatic aspects. This leads to the introduction of refined neutrosophic random variables. We explore various characteristics of these refined neutrosophic random variables, such as refined expected value, refined variance, refined standard deviation, refined moments generating function, and refined characteristic function. Furthermore, we investigate the properties associated with these characteristics

2. PRELIMINARIES

This section provides some definitions of neutrosophic logic and neutrosophic probability. Preliminaries

Definition 2.1. [25] Let $X \neq \Phi$ be. A neutrosophic set $NS(A)$ it can be defined by its elements with triples of the form $\{x, (T_{NS(A)}(x), I_{NS(A)}(x), F_{NS(A)}(x)): x \in X\}$, where $T_{NS(A)}$, $I_{NS(A)}(x)$ *and* $F_{NS(A)}A(x)$ represent the degree of membership, degree of indeterminacy, and degree of non-membership, respectively, of each element $x \in X$ to the set $NS(A)$.

Definition 2.2. [26] Let F be a field, the neutrosophic field $NS(F)$ is a field that is generated by $\langle F \cup I \rangle$ and we usually denote it by $NS(F) = \langle F \cup I \rangle$.

Definition 2.3. [27] The literal neutrosophic number has the form $X(I) = x + \alpha I$ where x, α are real or complex numbers, and I is an algebraic element that satisfies $0 \cdot I = 0$ and $I^2 = I$.

Definition 2.4. [14] The single-valued neutrosophic probability of event A is $P_N(A) = \big(T(A), I(neutA), F(antiA)\big)$where T, I, F take its values in [0,1].

Definition 2.5. [22] Let $(X(I_1, I_2), +, \cdot)$ be a refined neutrosophic field where $+$ and $\cdot$ are ordinary addition and multiplication respectively we call I_1, I_2 the split indeterminacy elements of the original indeterminacy I where $I = \alpha I_1 + \beta I_2$ and $\alpha, \beta \in R$. Moreover, $I_1$and I_2 preserve the following: ${I_1}^2 = I_1, {I_2}^2 = I_2$ and $I_1 I_2 = I_2 I_1 = I_1$.

For any two elements, we define

i. $X(I_1, I_2) + Y(I_1, I_2) = x + \alpha I_1 + \beta I_2 + y + \alpha^* I_1 + \beta^* I_2 = x + y + (\alpha + \alpha^*) I_1 + (\beta + \beta^*)\, I_2$

ii. $X(I_1, I_2) \cdot Y(I_1, I_2) = (x + \alpha I_1 + \beta I_2) \cdot (y + \alpha^* I_1 + \beta^* I_2) = x \cdot y + (x\alpha^* + \alpha y + \alpha\alpha^* + \alpha\beta^* + \beta\alpha^*) I_1 + (x\beta^* + \beta y + \beta\beta^*)\, I_2$.

Definition 2.6. [28] The square root of a refined neutrosophic positive real number can be computed and defined as follows:

$$\sqrt{X(I_1, I_2)} = \sqrt{x + \alpha I_1 + \beta I_2} = \sqrt{x} + \left(\sqrt{x + \alpha + \beta} - \sqrt{x + \alpha}\right)$$

3. ON THE REFINED NEUTROSOPHIC RANDOM VARIABLES

In this section, we are going to generalize the previous definition of neutrosophic random variables to the concept of refined neutrosophic random variables taking into consideration that I is split into I_1 and I_2.

Definition 3.1. Refined Neutrosophic Random Variable

Take the classical random variable X defined on Ω the events space as follows:

$$X: \Omega \to R$$

We will define refined-neutrosophic random variable NX as follows:

$$NX: \Omega \rightarrow R(I_1, I_2)$$

and

$$NX = X + \alpha I_1 + \beta I_2$$

where I_1 and I_2 are the split components of the original indeterminacy I.

Theorem 3.2. PDF and CDF of Refined Neutrosophic Random Variables

We here prove that NCDF(Neutrosophic Cumulative distribution function) of a refined-neutrosophic random variable and NPDF(Neutrosophic Probability Density Functions), respectively, are as follows:

$$F_{NX}(x) = F_X\big(x - (\alpha I_1 + \beta I_2)\big)$$
$$f_{NX}(x) = f_X(x - (\alpha I_1 + \beta I_2))$$

PROOF.

$$F_{NX}(x) = P(NX \leq x) = P(X + \alpha I_1 + \beta I_2 \leq x) = P\big(X \leq x - (\alpha I_1 + \beta I_2)\big) = F_X\big(x - (\alpha I_1 + \beta I_2)\big)$$

and taking the derivative to the last equation concerning x we find:

$$f_{NX}(x) = \frac{dF_{NX}(x)}{dx} = \frac{dF_X\big(x - (\alpha I_1 + \beta I_2)\big)}{dx} \cdot \frac{d\big(x - (\alpha I_1 + \beta I_2)\big)}{dx} = f_X(x - (\alpha I_1 + \beta I_2))$$

Theorem 3.3. The Expectation of Refined-Neutrosophic Random Variable

The expectation of refined-neutrosophic random variable $NX = X + \alpha I_1 + \beta I_2$ is:

$$E(NX) = E(X) + \alpha I_1 + \beta I_2$$

PROOF.

Here, in the continuous case, we have:

$$\begin{aligned} E(NX) &= E(X + \alpha I_1 + \beta I_2) \\ &= \int_x (x + \alpha I_1 + \beta I_2) f(x)dx \\ &= \int_x x f(x)dx + (\alpha I_1 + \beta I_2)\int_x f(x)dx \\ &= E(X) + \alpha I_1 + \beta I_2 \end{aligned}$$

where $\int_x f(x)dx = 1$.

In discrete case:

$$\begin{aligned} E(NX) &= E(X + \alpha I_1 + \beta I_2) \\ &= \sum_x (x + \alpha I_1 + \beta I_2) f(x) \\ &= \sum_x x f(x) + (\alpha I_1 + \beta I_2)\sum_x f(x) \\ &= E(X) + \alpha I_1 + \beta I_2 \end{aligned}$$

where $\sum_x f(x) = 1$.

Preposition 3.4.

i. $E(aNX + b + \alpha I_1 + \beta I_2) = aE(NX) + b + \alpha I_1 + \beta I_2\,; a, \alpha, \beta \in R$

ii. If NX, NY are two refined-neutrosophic random variables, then

$$E(NX \pm NY) = E(NX) \pm E(NY)$$

iii. $E[(a + \alpha I_1 + \beta I_2\,)NX] = E(aV + (\alpha I_1 + \beta I_2)\,NX) = E(aNX) + E\big((\alpha I_1 + \beta I_2)\,NX\big)$

$$= aE(NX) + (\alpha I_1 + \beta I_2)\,E(NX)\,; a, \alpha, \beta \in R$$

iv. $|E(NX)| \leq E|NX|$

PROOF.

If NX is continuous:

$$|E(NX)| = \left|\int_x (x + \alpha I_1 + \beta I_2) f(x)dx\right| \leq \int_x |(x + \alpha I_1 + \beta I_2)| f(x)dx = E|NX|$$

where $|f(x)| = f(x)$ since it is a positive function. If NX is discrete:

$$|E(NX)| = \left|\sum_x (x + \alpha I_1 + \beta I_2) f(x)\right| \leq \sum_x |(x + \alpha I_1 + \beta I_2)| f(x) = E|NX|$$

Definition 3.5. The Variance of a refined-neutrosophic random variable
A variance of refined-neutrosophic variables is:

$$V(NX) = V(X)$$

Here, we can always write:

$$V(NX) = E[NX - E(NX)]^2 = E[X + \alpha I_1 + \beta I_2 - E(X) - (\alpha I_1 + \beta I_2)]^2 = E[X - E(X)]^2 = V(X)$$

Example 3.6. Let X be a classical random variable defined by its pdf as follows:

$$f_X(x) = 2x\,; 0 \leq x \leq 1$$

i. Find a PDF of $NX = X + \alpha I_1 + \beta I_2$ and prove that it's a density function.

$$f_{NX}(x) = f_X\big(x - (\alpha I_1 + \beta I_2)\big) = 2\big(x - (I_1 + I_2)\big); 0 \leq x - (\alpha I_1 + \beta I_2) \leq 1$$

$$f_{X_N}(x) = 2x - 2(\alpha I_1 + \beta I_2)\,;\ \alpha I_1 + \beta I_2 \leq x \leq 1 + \alpha I_1 + \beta I_2$$

Prove that $f_{NX}(x)$ is a density function

$$\begin{aligned}\int_{(\alpha I_1+\beta I_2)}^{1+(\alpha I_1+\beta I_2)} \big(2x - 2(\alpha I_1 + \beta I_2)\big)dx &= [x^2 - 2(\alpha I_1 + \beta I_2)x]\,|_{(\alpha I_1+\beta I_2)}^{1+(\alpha I_1+\beta I_2)}\\ &= (1 + \alpha I_1 + \beta I_2)^2 - 2(\alpha I_1 + \beta I_2)(1 + \alpha I_1 + \beta I_2) - (\alpha I_1 + \beta I_2)^2\\ &\quad +2(\alpha I_1 + \beta I_2)^2\\ &= 1 + (\alpha I_1 + \beta I_2)^2 + 2(\alpha I_1 + \beta I_2) - 2(\alpha I_1 + \beta I_2) - 2(\alpha I_1 + \beta I_2)^2\\ &\quad -(\alpha I_1 + \beta I_2)^2 + 2(\alpha I_1 + \beta I_2)^2\\ &= 1\end{aligned}$$

ii. Find the refined expected value of NX.

$$E(NX) = E(X) + \alpha I_1 + \beta I_2 = \int_0^1 2x^2 dx + \alpha I_1 + \beta I_2 = \frac{2}{3} + \alpha I_1 + \beta I_2$$

iii. Find the refined variance of NX.

$$V(NX) = V(X) = \int_0^1 \left(x - \frac{2}{3}\right)^2 2x dx = \frac{1}{18}$$

Theorem 3.7. The Mean deviation of a refined-neutrosophic random variable
The mean deviation of refined-neutrosophic random or $M.D(NX)$ is:

$$M.D(NX) = M.D(X) = E|X - E(X)|$$

PROOF.

$$\begin{aligned} M.D(NX) &= E|X_N - E(NX)|\\ &= E|X + \alpha I_1 + \beta I_2 - E(X + \alpha I_1 + \beta I_2)|\\ &= E|X + \alpha I_1 + \beta I_2 - E(X) - \alpha I_1 + \beta I_2|\\ &= M.D(X)\end{aligned}$$

Definition 3.8. The r^th quartile of a refined-neutrosophic continuous random variable:
The r^th quartile of a refined-neutrosophic random variable or Q_N^r is:

$$\int_{-\infty}^{Q_N^r} f_{X_N}(x)dx = \frac{r}{4}; r = 1,2,3$$

We call Q_N^1, Q_N^2 and Q_N^3 the on refined neutrosophic first, second, and third quartiles, respectively.

Example 3.9. Let NX be the refined-neutrosophic random variable defined in Example 3.6 and find its three quartiles. We have

$$f_{NX}(x) = 2x - 2(\alpha I_1 + \beta I_2)\,;\ (\alpha I_1 + \beta I_2) \leq x \leq 1 + (\alpha I_1 + \beta I_2)$$

Therefore,

$$\int_{\alpha I_1 + \beta I_2}^{Q_N^r} \big(2x - 2(\alpha I_1 + \beta I_2)\big)dx = \frac{r}{4}; r = 1,2,3$$

For $r = 1$, we get:

$$\int_{\alpha I_1 + \beta I_2}^{Q_N^1} \big(2x - 2(\alpha I_1 + \beta I_2)\big)dx = \frac{1}{4}$$

$$[x^2 - 2(\alpha I_1 + \beta I_2)x]_{\alpha I_1 + \beta I_2}^{Q_N^1} = \frac{1}{4}$$

$${Q_N^1}^2 - 2(\alpha I_1 + \beta I_2)Q_N^1 - (\alpha I_1 + \beta I_2)^2 + 2(\alpha I_1 + \beta I_2)^2 = \frac{1}{4}$$

$${Q_N^1}^2 - 2(\alpha I_1 + \beta I_2)Q_N^1 + (\alpha I_1 + \beta I_2)^2 - \frac{1}{4} = 0$$

Dealing with a previous refined-neutrosophic equation concerning Q_N^1 we obtain:

$$\Delta = b^2 - 4ac = 4(\alpha I_1 + \beta I_2)^2 - 4\left((\alpha I_1 + \beta I_2)^2 - \frac{1}{4}\right) = 1$$

$$(Q_N^1)_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2(\alpha I_1 + \beta I_2) - 1}{2} = \alpha I_1 + \beta I_2 - \frac{1}{2}$$

And this solution is ejected because $\alpha I_1 + \beta I_2 \leq x \leq 1 + \alpha I_1 + \beta I_2$.

$$(Q_N^1)_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2(\alpha I_1 + \beta I_2) + 1}{2} = \alpha I_1 + \beta I_2 + \frac{1}{2}$$

And this solution is accepted because $\alpha I_1 + \beta I_2 \leq x \leq 1 + \alpha I_1 + \beta I_2$.

For $r = 2$, we get:

$${Q_N^2}^2 - 2(\alpha I_1 + \beta I_2)Q_N^2 + (\alpha I_1 + \beta I_2)^2 = \frac{2}{4} = \frac{1}{2}$$

Dealing with a refined-neutrosophic equation concerning Q_N^2 we obtain:

$$\Delta = b^2 - 4ac = 4(\alpha I_1 + \beta I_2)^2 - 4\left((\alpha I_1 + \beta I_2)^2 - \frac{1}{2}\right) = 2$$

$$(Q_N^2)_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2(\alpha I_1 + \beta I_2) - \sqrt{2}}{2} = \alpha I_1 + \beta I_2 - \frac{\sqrt{2}}{2}$$

Rejected.

$$(Q_N^2)_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2(\alpha I_1 + \beta I_2) + \sqrt{2}}{2} = \alpha I_1 + \beta I_2 + \frac{\sqrt{2}}{2}$$

Accepted.

For $r = 3$, we get:

$${Q_N^3}^2 - 2(\alpha I_1 + \beta I_2)Q_N^3 + (\alpha I_1 + \beta I_2)^2 = \frac{3}{4}$$

Dealing with a refined-neutrosophic equation concerning Q_N^3 we obtain:

$$\Delta = b^2 - 4ac = 4(\alpha I_1 + \beta I_2)^2 - 4\left((\alpha I_1 + \beta I_2)^2 - \frac{3}{2}\right) = 6$$

$$(Q_N^3)_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2(\alpha I_1 + \beta I_2) - \sqrt{6}}{2} = \alpha I_1 + \beta I_2 - \frac{\sqrt{6}}{2}$$

Rejected.

$$(Q_N^3)_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2(\alpha I_1 + \beta I_2) + \sqrt{6}}{2} = \alpha I_1 + \beta I_2 + \frac{\sqrt{6}}{2}$$

Accepted.

Theorem 3.10. MGF of a refined-neutrosophic random variable
Take into account the refined-neutrosophic random $NX = X + \alpha I_1 + \beta I_2$ then its corresponding MGF will be:

$$M_{NX}(t) = e^{t(\alpha I_1 + \beta I_2)} M_X(t)$$

PROOF.

$$\begin{aligned} M_{NX}(t) &= E(e^{tNX}) \\ &= E\left(e^{t(X+\alpha I_1+\beta I_2)}\right) \\ &= E\left(e^{tX} e^{t(\alpha I_1+\beta I_2)}\right) \\ &= e^{t(\alpha I_1+\beta I_2)} E(e^{tX}) \\ &= e^{t(\alpha I_1+\beta I_2)} M_X(t) \end{aligned}$$

Preposition 3.11.

i. $M_{NX}(0) = 1$

ii. $\frac{dM_{NX}(0)}{dt} = E(NX)$

PROOF.

$$\begin{aligned} \frac{dM_{NX}(t)}{dt}\Big|_{t=0} &= \frac{de^{t(\alpha I_1+\beta I_2)} M_X(t)}{dt}\Big|_{t=0} \\ &= \frac{de^{t(\alpha I_1+\beta I_2)}}{dt} M_X(t)|_{t=0} + \frac{dM_X(t)}{dt} e^{t(\alpha I_1+\beta I_2)}|_{t=0} \\ &= (\alpha I_1 + \beta I_2) e^{t(\alpha I_1+\beta I_2)} M_X(t)|_{t=0} + M'_X(t) e^{t(\alpha I_1+\beta I_2)}|_{t=0} \\ &= (\alpha I_1 + \beta I_2) M_X(0) + M'_X(0) \\ &= (\alpha I_1 + \beta I_2) + E(X) \\ &= E(NX) \end{aligned}$$

Theorem 3.12. CF of Refined-Neutrosophic Random Variables
Let us consider the refined-neutrosophic random $NX = X + \alpha I_1 + \beta I_2$ then its CF will be:

$$\varphi_{NX}(t) = e^{it(\alpha I_1+\beta I_2)} \varphi_X(t)\, ;\, i = \sqrt{-1}$$

PROOF.

$$\begin{aligned} \varphi_{NX}(t) &= E\left(e^{itNX}\right) \\ &= E\left(e^{it(X+\alpha I_1+\beta I_2)}\right) \\ &= E\left(e^{itX} e^{it(\alpha I_1+\beta I_2)}\right) \\ &= e^{it(\alpha I_1+\beta I_2)} E\left(e^{itX}\right) \\ &= e^{it(\alpha I_1+\beta I_2)} \varphi_X(t) \end{aligned}$$

Preposition 3.13.

i. $\varphi_{NX}(0) = 1$

ii. $|\varphi_{NX}(t)| \le 1$, which means that CF always exists.

PROOF.

$$|\varphi_{NX}(t)| = \left|E\left(e^{itNX}\right)\right| \le E\left|e^{itNX}\right| = E|\cos tNX + \sin tNX| = E|1| = 1$$

iii. $\frac{d\varphi_{NX}(t)}{dt}|_{t=0} = iE(NX)$

PROOF.

$$\begin{aligned} \frac{d\varphi_{NX}(t)}{dt}\Big|_{t=0} &= \frac{de^{it(\alpha I_1+\beta I_2)} \varphi_X(t)}{dt}\Big|_{t=0} \\ &= \frac{de^{it(\alpha I_1+\beta I_2)}}{dt} \varphi_X(t)|_{t=0} + \frac{d\varphi_X(t)}{dt} e^{it(\alpha I_1+\beta I_2)}|_{t=0} \\ &= i(\alpha I_1 + \beta I_2) e^{it(\alpha I_1+\beta I_2)} \varphi_X(t)|_{t=0} + \varphi'_X(t) e^{it(\alpha I_1+\beta I_2)}|_{t=0} \\ &= i(\alpha I_1 + \beta I_2) + iE(X) \\ &= i\left(\alpha I_1 + \beta I_2 + E(X)\right) \end{aligned}$$

$$= iE(NX)$$

iv. $\frac{d^n \varphi_{X_N}(t)}{dt^n}|_{t=0} = i^n E(NX)^n$

v. $\varphi_{NX}(t) = M_{NX}(it)$

Example 3.14. Let NX be the refined-neutrosophic random variable defined in Example 3.6.

i. Find $M_{NX}(t)$.

$M_{NX}(t) = e^{t(\alpha I_1+\beta I_2)} M_X(t)$, but:

$$M_X(t) = \int_0^1 e^{tx} 2x dx = \frac{2(te^t - e^t + 1)}{t^2}$$

Therefore:

$$M_{NX}(t) = e^{t(\alpha I_1+\beta I_2)} \frac{2(te^t - e^t + 1)}{t^2} = 2\frac{te^{t(1+\alpha I_1+\beta I_2)} - e^{t(1+\alpha I_1+\beta I_2)} + e^{t(\alpha I_1+\beta I_2)}}{t^2}$$

ii. Depending on the properties of $M_{X_N}(t)$ find $E(NX)$.

$$M'_{NX}(t)$$

$$= 2\frac{t^2\left(e^{t(1+\alpha I_1+\beta I_2)} + (1+\alpha I_1+\beta I_2)e^{t(1+\alpha I_1+\beta I_2)}t - (1+\alpha I_1+\beta I_2)e^{t(1+\alpha I_1+\beta I_2)} + (\alpha I_1+\beta I_2)\right) - 2t\left(te^{t(1+\alpha I_1+\beta I_2)} - e^{t(1+\alpha I_1+\beta I_2)} + e^{t(\alpha I_1+\beta I_2)}\right)}{t^4}$$

$$= 2\frac{t\left(e^{t(1+\alpha I_1+\beta I_2)} + \left(1+(\alpha I_1+\beta I_2)\right)e^{t(1+\alpha I_1+\beta I_2)}t - \left(1+(\alpha I_1+\beta I_2)\right)e^{t(1+\alpha I_1+\beta I_2)} + (\alpha I_1+\beta I_2)e^{tI}\right) - 2\left(te^{t(1+\alpha I_1+\beta I_2)} - e^{t\left(1+(\alpha I_1+\beta I_2)\right)} + e^{t(\alpha I_1+\beta I_2)}\right)}{t^3}$$

$$= 2\frac{te^{t(1+\alpha I_1+\beta I_2)} + (1+\alpha I_1+\beta I_2)e^{t(1+\alpha I_1+\beta I_2)}t^2 - (1+\alpha I_1+\beta I_2)te^{t(1+\alpha I_1+\beta I_2)} + (\alpha I_1+\beta I_2)te^{t(\alpha I_1+\beta I_2)} - 2te^{t(1+\alpha I_1+\beta I_2)} + 2e^{t(1+\alpha I_1+\beta I_2)} - 2e^{t(\alpha I_1+\beta I_2)}}{t^3}$$

$$M'_{NX}(0) = \frac{2}{3} + \alpha I_1 + \beta I_2 = E(NX)$$

iii. Conclude $\varphi_{NX}(t)$ formula.

$$\varphi_{NX}(t) = M_{NX}(it) = 2\frac{ite^{it(1+\alpha I_1+\beta I_2)} - e^{it(1+\alpha I_1+\beta I_2)} + e^{it(\alpha I_1+\beta I_2)}}{-t^2}$$

4. DISCUSSION FOR BENEFITS AND FUTURE APPLICATIONS OF REFINED NEUTROSOPHIC RANDOM VARIABLES

Some phenomena in nature may consist of three independent categories of data which can be modeled as a random variable that follows a certain probability distribution, and therefore it can be represented as a single random variable of the type of refined neutrosophic, which facilitates calculations and arriving at predictions and results related to this phenomenon through the laws and mathematical formulas that we have proven in this research paper.

On the other hand, merging processes through one process contributes to reducing the cost of algorithms and reducing the number of large calculations in a clear and tangible way. This helps computers carry out this study and produce results more easily and without consuming computer hardware.

5. POSSIBLE CHALLENGES AND LIMITATIONS

The biggest challenge facing the application of the results of this study in the future is for computers to be introduced to the number system of refined neutrosophic numbers, and for it to be entered programmatically into the computer.

Therefore, before these results can be applied in statistical studies, computers must be introduced to this type of random variables and the mathematical formulas that we have proven must be introduced in order to provide the basic benefit from them in improving traditional algorithms and reducing their programming and computational costs.

4. CONCLUSION

In this paper, we have presented a generalized definition of what is known by refined-neutrosophic random variables, and we have successfully introduced the main concepts of its characteristics, including PDF and CDF. Our focus was on the refined-neutrosophic representation, and we proved several properties associated with it. Additionally, we demonstrated the applicability of these results in various domains, including quality control, stochastic modeling, reliability theory, queueing theory, electrical engineering, and more. In future researches, we will study the effectiveness of this generalization to many fields of probability theory, especially in stochastic processes and their applications and we will study the ability to generalize this definition to the plithogenic sets concept.

CONFLICT OF INTEREST

The author stated that there are no conflicts of interest regarding the publication of this article.

CRediT AUTHOR STATEMENT

Ahmed Hatip: Formal analysis, Writing - original draft, Visualization, Investigation, Supervision, Conceptualization.

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