Adıyaman University Journal of Science
https://dergipark.org.tr/en/pub/adyujsci
DergiPark AKADEMİK
ISSN 2147-1630
e-ISSN 2146-586X

A Numerical Application of the Chebyshev Operational Matrix Method for HIV Infection of CD4+T-cells

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Received: 28.08.2023 **Accepted:** 20.10.2023 **Published:** 31.12.2023

Abstract

In this research study, we aim to approximate a solution for the mathematical model of the Human Immunodeficiency Virus (HIV) infection of CD4+T-cells. An operational matrix method based on Chebyshev orthogonal polynomials has been adapted to obtain numerical solutions for the model of HIV infection of CD4+T-cells. The proposed numerical scheme is built on a system of a nonlinear algebraic equation, including coefficients of a finite Chebyshev series that represent the approximate solutions of the model. Results are compared to existing methods to verify the accuracy of the numerical scheme.

Keywords: Model of the HIV infection; CD4+T cells; Operational matrix method; Chebyshev polynomials; Nonlinear system of differential equations.

CD4+T Hücrelerindeki HIV Enfeksiyonunun Yayılım Modelinin Chebyshev Operasyonel Matris Metodu ile bir Nümerik Uygulaması

* Corresponding Author DOI: 10.37094/adyujsci.1351204

Öz

Bu çalışma, CD4+T hücrelerinde HIV virüsünün matematiksel yayılım modeli için yaklaşık çözümler elde etmeyi amaçlamaktadır. Nümerik çözümler Chebyshev polinomları ile operasyonel matris metodunun CD4+T hücrelerinde HIV virüsünün matematiksel yayılım modeline uygulanması ile elde edilecektir. Önerilen method modele ait nümerik çözümlerin bir Chebyshev serisi formunda yazılarak, Chebyhev serisi içindeki bilinmeyen katsayıları içeren lineer olmayan bir denklem sistemi inşa atmeyi amaçlar. Yöntemin doğruluğunu kontrol etmek için nümerik sonuçlar var olan nümerik yöntemlerle karşılaştırılmıştır.

Anahtar Kelimeler: HIV Enfenksiyon Modeli; CD4+T hücreleri; Operasyonel matris metot; Chebyshev polinomları; Lineer olmayan diferansiyel denklem sistemleri.

1. Introduction

Applied mathematics is modeled to interpret natural events. These models and their numerical solutions obtain valuable information about those events. For example, a crucial event for public health is the dynamics of HIV infection of CD4+T-cells. Firstly, Perelson developed a system of nonlinear differential equations to describe HIV infection of CD4+T-cells in 1989 [1-5]. Nowadays, humanity spends millions of dollars on the treatment of this disease.

The mathematical model of infection of HIV of CD4+T cells is given by [6-9]

$$\frac{dT}{dt} = q - \alpha T + rT(1 - \frac{T+I}{T_{\max}}) - kVT$$

$$\frac{dI}{dt} = kVT - \beta I \tag{1}$$

$$\frac{dV}{dt} = \mu\beta I - \gamma V$$

with the conditions

$$T(0) = r_1, \;\; I(0) = r_2, \; V(0) = r_3 \tag{2}$$

where $T(t)$ is denoted as the concentration of healthy CD4+ T cells, $I(t)$ infected CD4+ T cells, $V(t)$ and free H.I.V. at a time t in blood. In addition, q is the source term for uninfected CD4+

T cells, α is the natural death rate of the CD4+ T cell concentration, r is the growth rate of CD4+ T cells, k is the rate at which CD4+ T cells become infected with a virus, β is the total death rate of infected CD4+ T cells, μ is the number of virions produced by infected CD4+ T cells. γ is the death rate of the free virus. $k > 0$ indicates the infection rate, and kVT indicates the infection rate of healthy CD4+ T cells. The termination T_{max}, in the denominator of the logistic term in the equation for healthy T4 cell density, indicates the total T4 cell density that can be found stably in the blood; in other words, the carrying capacity of the blood for T4 cells. It will be assumed that each infected CD4+T cell produces. This model's global stability and a periodic solution are achieved in [10].

In this paper, we will take the numerical data as $q = 0.1$, $\alpha = 0.02$, $r = 3$ $\beta = 0.3$, $k = 0.0027$, $\gamma = 2.4$, $\mu = 10$, $T_{\max} = 1500$ and initial conditions data $r_1 = 0.1$, $r_2 = 0$, $r_3 = 0.1$. Since the mathematical model of H.I.V. infection HIV CD4+T cells (1) are nonlinear differential equations with three terms, the exact solution to this problem cannot be obtainable or nonexistent. A resolution to this problem is needed to analyze its epidemiology and stability and to predict advances in AIDS treatment. In this stage, numerical solution methods become crucial to solve Eq. (1) with the conditions. Many numerical methods for approximating H.I.V. in CD4+ T cells have been improved over the last twenty years. Ghoreishi [11] presents the homotopy analysis method for H.I.V. infection of CD4+T-cells. The homotopy analysis method accepts the solution as an infinite series with auxiliary parameters. All calculations are investigated in six terms in this method. Ongun [12] implement the Laplace Adomain Decomposition Method to get numerical results for H.I.V. infection of CD4+T-cells. To obtain approximate solutions to the H.I.V. infection of the CD4+ T cells model, Merdan [13] applied the variational iteration method. Yüzbaşı [14] developed a Bessel collocation method for finding numerical solutions of this model. Beler [15] analyzed to find approximate solutions of the proposed model by using Laguerre wavelets. In addition, we have access to more numerical papers to obtain such a class of nonlinear ordinary differential equation systems [16-29].

In this study, we have obtained the approximate solutions of the mathematical model (1) by developing the Chebyshev operational matrix method (COMM). Chebyshev polynomial is the cornerstone of numerical analysis. Those polynomials adapted almost all numerical methods. For example, in [30-32], Chebyshev polynomials combined the operational matrix method to solve the linear Fredholm-Volterra integro-differential [33], Lane-Emden equations [34-35], for fractional differential equations involving non-singular Mittag-Leffler kernel [36], fractional differential equations [37], mixed Volterra-Fredholm delay integro differential equations [38].

2. Materials and Methods

2.1. Shifted Chebyshev polynomials of the first kind

Chebyshev polynomials mainly admit to the approximation of continuous functions. Chebyshev polynomials have crucial properties to perform nearly all numerical methods [26]. We have four kinds of Chebyshev polynomials, which are defined in interval $[-1,1]$. If we choose the interval $[0,1]$, they called shifted Chebyshev polynomials [30]. While readers can find the definition of Chebyhev polynomials in many books [30-32], we want to take the recurrence relation

$$T_n^*(t) = 2(2t-1)T_{n-1}^*(t) - T_{n-2}^*(t)$$

with the following initial conditions

$$T_0^*(t) = 1,\ T_1^*(t) = 2t-1$$

Those polynomials have the following property [30-32]

$$t^n = 2^{-2n+1}\sum_{k=0}^{n}{}'\binom{2n}{k}T_{n-k}^*(t),\ 0 \le t \le 1 \tag{3}$$

and the orthogonality condition is

$$\left\langle T_j^*(t), T_k^*(t)\right\rangle_w = \int_0^1 T_j^*(t)T_k^*(t)w(t)dt = \begin{cases} \pi, & j=k=0, \\ \pi/2, & j=k\neq 0, \\ 0, & j\neq k, \end{cases}$$

where $w(t) = (t-t^2)^{-1/2}$. $\left\{T_0^*(t), T_1^*(t), T_2^*(t), \cdots, T_n^*(t)\right\}$ is an orthogonal basis of $n-$ dimensional polynomial space P_n, for $j \neq k$, $\left\langle T_j^*(t), T_k^*(t)\right\rangle_w = 0$, for $j=k$, $\left\langle T_j^*(t), T_k^*(t)\right\rangle_w > 0$. In addition, if $p_k \in P_k$, for $k<n$, then $\left\langle p_k(t), T_n^*(t)\right\rangle_w = 0$ for all $n>k$ [30-32].

Any given function $y(t) \in L^2[0,1]$ can be approximated as a sum of shifted Chebyshev polynomials in the following way [30-32]:

$$y(t)=\sum_{n=0}^{\infty}a_n T_n^*(t)$$

where

$$a_n=\left\langle y(t),T_n^*(t)\right\rangle_w=\int_0^1 y(t)T_n^*(t)w(t)dt\ ,\ n=0,1,\ldots$$

Our study aims to achieve the approximate solution of Eq. (1) as a truncated shifted Chebyshev series defined by:

$$y_N^j(t)=\sum_{r=0}^{N}a_r^j T_r^*(t) \tag{4}$$

where is used to denote the first kind of Chebyshev polynomials, a_r^j are referred to as unknown Chebyshev coefficients, and are chosen to be any positive integer.

3. Relations and Methods

3.1. Matrix relations

In this part, we shall obtain the matrix-vector form of Eq. (1). For this purpose, let us consider the truncated Chebyshev polynomials $T_N(t)$, $I_N(t)$ and $V_N(t)$ are the numerical solutions of the Eq. (1) and so those solutions can be written like this:

$$T_N(t)=\sum_{r=0}^{N}a_r T_r^*(t) \tag{5}$$

$$I_N(t)=\sum_{r=0}^{N}b_r T_r^*(t) \tag{6}$$

$$V_N(t)=\sum_{r=0}^{N}c_r T_r^*(t) \tag{7}$$

The matrix-vector shape of the numerical solution polynomials can be written as:

$$T_N(t)=\mathbf{T}^*(t)\mathbf{A} \tag{8}$$

$$I_N(t)=\mathbf{T}^*(t)\mathbf{B} \tag{9}$$

$$V_N(t)=\mathbf{T}^*(t)\mathbf{C} \tag{10}$$

where

$$\mathbf{T}^*(t) = [T_0^*(t)\ T_1^*(t) \dots T_N^*(t)]$$

$$\mathbf{A} = [\frac{1}{2}a_0\ a_1 \dots a_N]^T\ \mathbf{B} = [\frac{1}{2}b_0\ b_1 \dots b_N]^T\ \mathbf{C} = [\frac{1}{2}c_0\ c_1 \dots c_N]^T$$

and where the dimension of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ matrices are $(N+1)\times 1$, dimension of the $\mathbf{T}^*(t)$ matrix is $1\times(1+N)$.

The property Eq. (3) permits us to write the below essential relation

$$(\mathbf{X}(t))^T = \mathbf{D}(\mathbf{T}^*(t))^T \quad and \quad \mathbf{X}(t) = \mathbf{T}^*(t)\mathbf{D}^T \tag{11}$$

where

$$\mathbf{X}(t) = [1\ t \dots t^N],$$

it is a lower triangle matrix, for $i, j = 0,1,2,\dots,N$

$$\mathbf{D} = \lfloor d_{ij} \rfloor$$

where

$$d_{ij} = \begin{cases} 2^{-2(i-1)}\binom{2((i-1)}{i-j}, & j \le i \\ 0 & , j > i \end{cases}$$

Moreover, $\mathbf{D}$ is an invertible square matrix with $(N+1)\times(N+1)$ dimensional and the dimension of $\mathbf{X}(t)$ is $1\times(N+1)$.

From Eq. (11), we obtain the following matrix relation

$$\mathbf{T}^*(t) = \mathbf{X}(t)(\mathbf{D}^{-1})^T \tag{12}$$

and

$$(\mathbf{T}^*(t))^{(1)} = \mathbf{X}^{(1)}(t)(\mathbf{D}^{-1})^T$$

So, the basic matrix-vector forms of the differential of approximate solutions of Eq. (1) are

$$T_N^{(1)}(t) = \mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{A} \tag{13}$$

$$I_N^{(1)}(t) = \mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{B} \tag{14}$$

$$V_N^{(1)}(t) = \mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{C} \tag{15}$$

where

$$\mathbf{X}^{(1)}(t) = \mathbf{X}(t)\mathbf{Y} \tag{16}$$

and for $i, j = 0,1,2,...,N$,

$$\mathbf{Y} = [y_{ij}] = \begin{cases} i+1, & j = i+1 \\ 0, & otherwise \end{cases}$$

For example, $N = 3$, it can be written as

$$\mathbf{Y} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3.2. Solution method

The numerical scheme is constructed to find the unknown coefficients in Eqs. (5-7) to obtain the numerical result of Eq. (1). To constitute the numerical scheme, firstly, Eq. (1) and initial conditions are turned into a matrix-vector form with shifted Chebyshev series. Using the matrix relations in Section 3, Eq. (1) can be written in matrix form:

$$\mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{A} + [\alpha - r]\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A} + \frac{r}{T_{\max}}(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A})[\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A} + \mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{B}]$$
$$+ k(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{C})(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A}) = q$$

$$\mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{B} - k(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{C})(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A}) + \beta\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{B} = 0 \tag{17}$$

$$\mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{C} - \mu\beta\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{B} + \gamma\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{C} = 0$$

The residuals $R_i(t)$ for $i=1,2,3$ form can be written as

$$R_1(t) \approx \mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{A} + [\alpha - r]\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A} + \frac{r}{T_{\max}}(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A})[\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A} + \mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{B}] + k(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{C})(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A}) - q \quad (18)$$

$$R_2(t) \approx \mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{B} - k(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{C})(\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A}) + \beta\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{B} \quad (19)$$

$$R_3(t) \approx \mathbf{X}(t)\mathbf{Y}(\mathbf{D}^T)^{-1}\mathbf{C} - \mu\beta\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{B} + \gamma\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{C} \quad (20)$$

$3\times N$ -times nonlinear systems of the equation are obtained by applying the operational matrix method in the following form, for $i=1,2,3$

$$\left\langle R_i(t), T_n^*(t)\right\rangle = \int_0^1 R_i(t)T_n^*(t)dt = 0,\ n=0,1,\ldots,N-1 \quad (21)$$

The initial conditions Eq. (2) give us three equations:

$$\begin{aligned}[T(0)] &= \mathbf{X}(0)(\mathbf{D}^T)^{-1}\mathbf{A} = [r_1]\\ [I(0)] &= \mathbf{X}(0)(\mathbf{D}^T)^{-1}\mathbf{B} = [r_2]\\ [V(0)] &= \mathbf{X}(0)(\mathbf{D}^T)^{-1}\mathbf{C} = [r_3]\end{aligned} \quad (22)$$

As a result, we get the $3\times(N+1)$ sets of nonlinear equation systems with $3\times(N+1)$ unknowns by Eqs. (21-22). Then, finally, those systems are puzzled out by the mathematical program Maple 13, and Eqs. (5-7) coefficients are achieved.

4. Numerical Results

In this part, we applied this proposed method (PM) to the given numerical data for the proposed method N=.7. Numerical solutions are obtained by the proposed method; other numerical methods are given in Table 1 for the uninfected population T, Table 2 for infected CD+4 T-cell concentration I, and Table 3 for the concentration of free H.I.V. virus V. All tables show that PM agrees well with the solutions of other numerical results. Figures 1-2 show the uninfected population T, infected CD+4 T-cell concentration I, and concentration of free H.I.V. virus V versus time. As time increases, the uninfected population T increases, the infected CD+4 T-cells concentration I increases, and the concentration of free H.I.V. virus

decreases. The amount of infected CD+4 T-cells concentration I is slower to increase than the uninfected population T, and H.I.V. infection disease may end at any t time. In Figures 3-5, we compare the numerical results by obtained Adomian Decomposition Method, Pade approximation, Inverse Laplace transformation method, and present method. All results nearly resemble each other.

Table 1: Numerical results for T PM and other numerical methods.

t	The method in [13]	The method in [11]	The method in [12]	The method in [17]	PM N=5	PM N=7
0.0	0.1	0.1	0.1	0.1	0.1	0.1
0.2	0.2038616561	0.2088072731	0.2088073214	0.2088080849	0.208458510	0.2088072279
0.4	0.3803309335	0.4061052652	0.4061346587	0.4062405440	0.406339373	0.4062410095
0.6	0.6954623767	0.7611467713	0.7624530350	0.7644239007	0.764734581	0.7644229384
0.8	1.2759624442	1.3773198590	1.3978805880	1.4140468559	1.413686781	1.4140470895
1.0	2.382277428	2.3291697610	2.5067466690	2.5915948594	2.591645820	2.5915948594

Table 2: Numerical results for I PM and other numerical methods.

t	The method in [13]	The method in [11]	The method in [12]	The method in [17]	PM N=5	PM N=7
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2	0.624787210E-5	0.60327072E-5	0.603263436E-5	0.603270226E-5	0.6034893E-5	0.63517134E-5
0.4	0.129355222E-4	0.13159114E-4	0.131487854E-4	0.131583409E-4	0.1315659E-4	0.12751088E-4
0.6	0.203526718E-4	0.21268368E-4	0.210141719E-4	0.212237855E-4	0.2122560E-4	0.21636848E-4
0.8	0.283730212E-4	0.30069186E-4	0.279513045E-4	0.301778550E-4	0.3017857E-4	0.29847613E-4
1.0	0.369084236E-4	0.39873654E-4	0.243156231E-4	0.400378145E-4	0.4003936E-4	0.37812697E-4

Table 3: Numerical results for V PM and other numerical methods.

t	The method in [13]	The method in [11]	The method in [12]	The method in [17]	PM N=5	PM N=7
0.0	0.1	0.1	0.1	0.1	0.1	0.1
0.2	0.06187991856	0.06187996025	0.06187995314	0.06187984322	0.061874446	0.0618798985
0.4	0.03829493490	0.03831324883	0.03830820126	0.03829488777	0.038298806	0.0382948388
0.6	0.02370431860	0.02439174349	0.02392029257	0.02370455004	0.023706103	0.0237045399
0.8	0.01467956982	0.00996721893	0.01621704553	0.01468036368	0.014675339	0.0148040545
1.0	0.02370431861	0.00033050764	0.01608418711	0.00910084499	0.009100830	0.0091000845

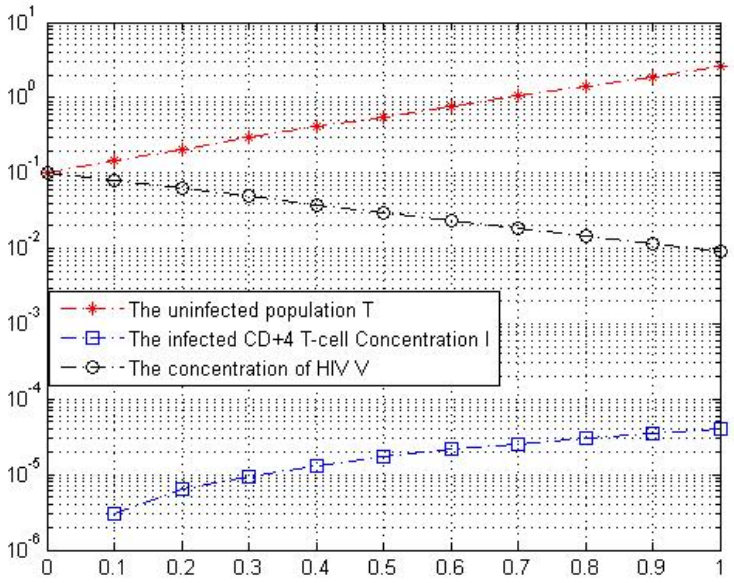

Figure 1: Comparison of the numerical results for N=5.

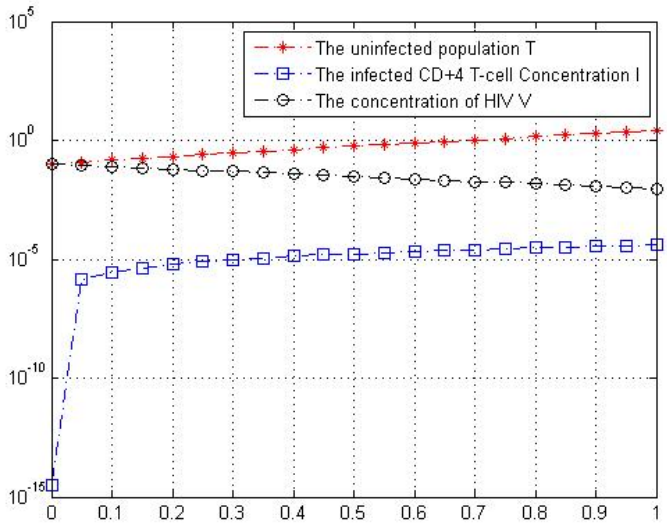

Figure 2: Comparison of the numerical results for N=7.

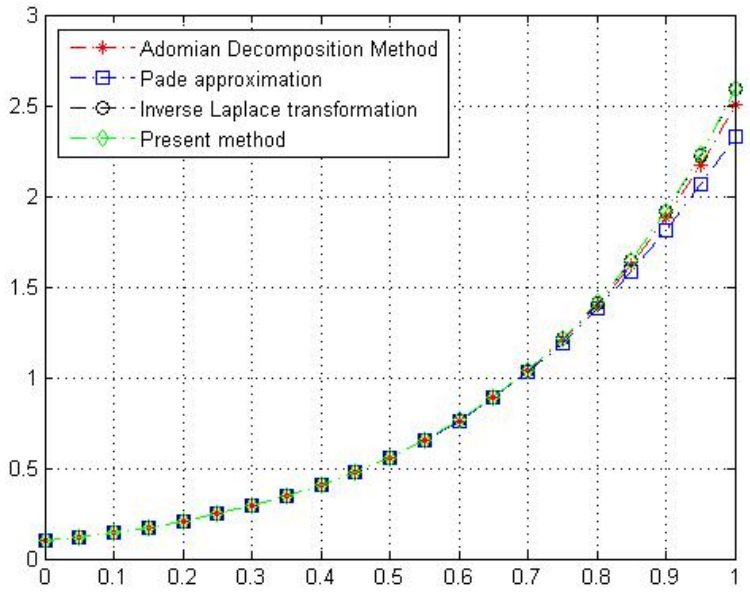

Figure 3: Comparison of numerical method solution of T

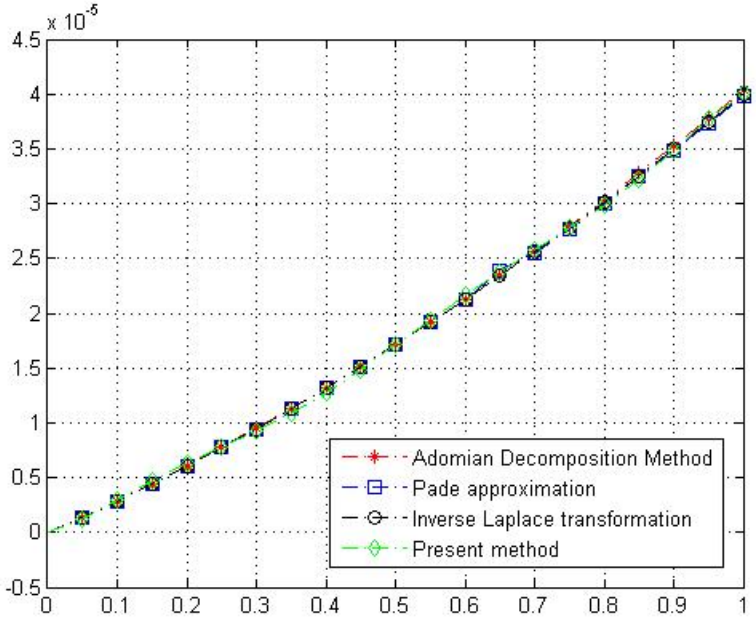

Figure 4: Comparison of numerical method solution of I

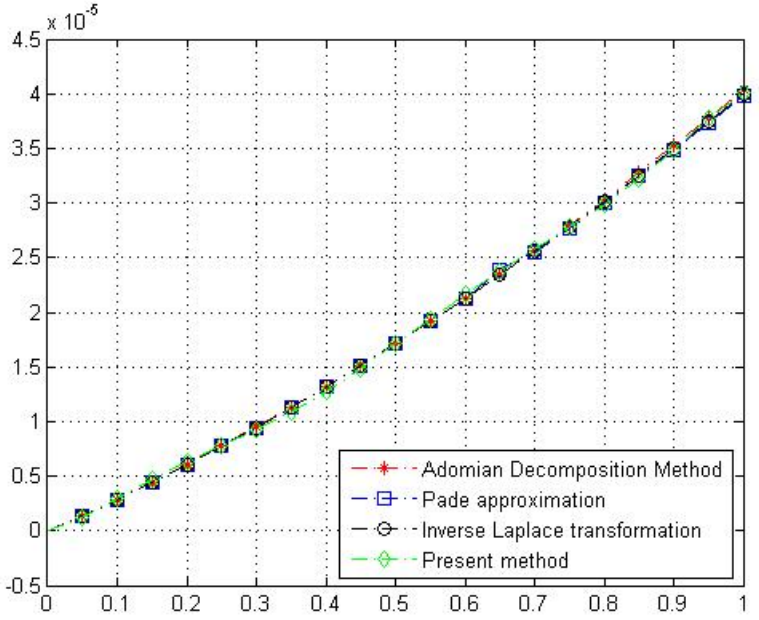

Figure 5: Comparison of numerical method solution of V

5. Conclusion

This paper uses the Chebyshev operational matrix method to solve the mathematical model of HIV infection of CD4+ T-cells. The uninfected population T infected CD+4 T-cell concentration I, and free H.I.V. virus values concentration are compared with other methods in Table 1, Table 2, and Table 3, respectively. Also, with figures, the efficiency and accuracy of the method are demonstrated. The proposed method has a lower operation, so cumulative errors are minor. Moreover, the solution code of the method is easily written in Maple. The results show that the present method is accurate compared to Bessel collocation, the Adomian decomposition, the Pade approximation, and the inverse Laplace transformation with five and seven terms.

Acknowledgments

Muğla Sıtkı Kocman University Research Projects Coordination Office has granted this paper. Project Grant Number 22/139/05/1 and title "Extending the Operational Matrix Method to Nonlinear System Models."

References

[1] Perelson, A.S., *Modelling the interaction of the immune system with H.I.V., in: C. Castillo-Chavez (Ed.), Mathematical and Statistical Approaches to AIDS Epidemiology*, Springer, Berlin, 1989.

[2] Perelson, A.S., Kirschner, D.E., De Boer, R., *Dynamics of H.I.V. infection of CD4+ T-cells*, Mathematical Biosciences, 114: 81, 1983.

[3] Zapata, H.D., Salazar C.A., Mayorga, E.M., *Mathematical model describing HIV infection with time-delayed CD4 T-cell activation*, Processes, 8(7), 782, 2020.

[4] Arafa, A.A.M., Rida, S.Z., Khalil, M., *Fractional modeling dynamics of H.I.V. and CD4+ T-cells during primary infection*, Nonlinear Biomed. Phys., 6(1), 1–7, 2012.

[5] Huo, H.F., Chen, R., Wang, X.Y., *Modelling and stability of HIV/AIDS epidemic model with treatment*, Appl. Mathe. Model. 40(13–14), 6550–6559, 2016.

[6] Wang, L., Li, M.Y., *Mathematical analysis of the global dynamics of a model for H.I.V. infection of CD4+ T cells*, Math. Biosci., 200(1), 44–57, 2006.

[7] lba-Pérez, J., Macías-Díaz, J.E., *A finite-difference discretization preserving the structure of solutions of a diffusive model of type-1 human immunodeficiency virüs*, Advances in Difference Equations, (1):1–19, 2021.

[8] Crandall, K.A., *The Evolution of H.I.V.*, Johns Hopkins University Press, Baltimore, Maryland, 1999.

[9] Culshaw, R.V., Ruan, S., *A delay-differential equation model of H.I.V. infection of CD4+ T-cells*, Mathematical Biosciences, 165, 27–39, 2000.

[10] Wang, X., Song, X., *Global stability and periodic solution of a model for H.I.V. infection of CD4+ T-cells*, Appl. Math. Model., 189, 1331–1340, 2007.

[11] Ghoreishi, M., Ismail, A.I., Alomari, A.K., *Application of the homotopy analysis method for solving a model for H.I.V. infection of CD4(+) T-cells*, Math. Comp. Model., 54, 3007-3015, 2011.

[12] Ongun, M.Y., *The Laplace adomian decomposition method for solving a model for H.I.V. infection of CD4+T cells*, Math. Comput. Model., 53(6), 597–603, 2011.

[13] Merdan, M., Gökdogan, A., Yıldırım, A., *On the numerical solution of the model for H.I.V. infection of CD4+ T cells*, Comput. Math. Appl. 62(1), 118-123, 2011.

[14] Yüzbaşı, Ş., *A numerical approach to solve the model for H.I.V. Infection of CD4+T cells*, Appl. Math. Model., 36(12), 2576-5890, 2012.

[15] Beler, A., Şimşek, G.Ö., Gümgüm, S., *Numerical solutions of the HIV infection model of CD4+ cells by Laguerre wavelets*, Math. Comp. Simulat., 209, 205-219, 2023.

[16] Mehregan, F.S., Assari, P., Dehghan, M., *On the approximate solution of dynamic systems derived from the HIV infection of CD4+ T-cells using the LRBF-collocation scheme*, Engin. Anal. Bound. Elem., 153, 39-50, 2023.

[17] Parand, K., Kalantari, Z., Delkhosh, D., *Quasilinearization-Lagrangian method to solve the H.I.V. infection model of CD4+ T cells*, SeMA J., 75(2), 271-283, 2007.

[18] Guerrero–Sánchez, Y., Umar, M., Sabir, Z., Juan, L.G., Raja, M.A.Z., *Solving a class of biological HIV infection model of latently infected cells using heuristic approach*, Discrete and Continuous Dynamical Systems - S, 14(10), 3611-3628, 2021.

[19] Yüzbaşı, Ş., Ismailov, N., *A numerical method for the solutions of the H.I.V. infection model of CD4+ T-cells*, Int. J. Biomath. 10(7), 1750098, 2017.

[20] Khan, H., Shah, R., Arif, M., Bushnaq, S., *The Chebyshev wavelet method for the numerical solution of fractional H.I.V. infection of CD4+T cells model*, Int. J. Appl. Comput. Math., 6, 34, 2020.

[21] Merdan, M., *Homotopy perturbation method for solving a model for hiv infection of CD4+ T-cells*, Istanbul Commerce University Journal of Science, 12, 39–52, 2007.

[22] Ali, N., Ahmad, S., Aziz, S., Zaman, G., *The Adomian decomposition method for solving HIV infection model of latently infected cells*, Matrix Science Mathematic, 3, 5-8, 2019.

[23] N. Doğan, *Numerical treatment of the model for H.I.V. infection of CD4+T-cells by using multistep Laplace Adomian decomposition method*, Discrete Dyna. Mature Soci., 976352, 2012.

[24] Khalid, M., Sultana, M., Zaidi, F., Khan, F.S., *A numerical solution of a model for H.I.V. infection CD4+ T-cell*, Inter. J. Inno. Scie. Rese., 16(1), 79-85, 2015.

[25] Gandomani, M.R., Kajani, M.T., *Numerical solution of a fractional order model of H.I.V. infection of CD4+T cells using Müntz-Legendre polynomials*, Int. J. Bioa., 20(2), 193-204 2016.

[26] Ullah, R., Ellahi, R., Said M.S., Din, S.T., *On the fractional-order model of HIV infection of CD4+ T*cells under the influence of antiviral drug treatment*, Journal of Taibah University for Science, 1, 50-59, 2020.

[27] Deniz, S., *Semi-analytical approach for solving a model for HIV infection of CD4+ T-cells*, TWMS J. Appl. And Eng. Math. 1,273-281, 2021.

[28] Kumbinarasaiah, S., Manohara, G., *Modified Bernoulli wavelets functional matrix approach for the HIV infection of CD4+ cells model*, Results in Control Optim., 10, 100197, 2023.

[29] Attaullah, A., Muhammad, S., Mathematical modeling and numerical simulation of HIV infection model, Results in Control Optim., 7, 100118, 2022.

[30] Mason, J.C., Handscomb, D.C., *Chebyshev polynomials*, Chapman and Hall/C.R.C., New York, 2003.

[31] Body, J.P., *Chebyshev and fourier spectral methods*, University of Michigan, New York, 2000.

[32] Epperson, J.F., *An Introduction to numerical methods and analysis*, Second Edition, Jhon Wiley & Sons, New Jersey, 2013.

[33] Öztürk, Y., Gülsu, M., *An operational matrix method to solve linear Fredholm-Volterra integro-differential equations with piecewise intervals*, Math. Scie. 15, 189-197, 2021.

[34] Öztürk, Y., Gülsu, M., *An operational matrix method for solving Lane-Emden equations arising in astrophysics*, Mathematical Methods in Applied Sciences 37, 2227-2235, 2014.

[35] Öztürk, Y., *An efficient numerical algorithm for solving system of Lane-Emden type equations arising in engineering*, Nonlinear Engineering 8, 429-439, 2019.

[36] Baleanu, D., Shiri, B., Srivasta H.M., Qurashi, *A Chebyshev spectral method based on operational matrix for fractional differential equations involving non-singular Mittag-Leffler kernel*, Advances in Difference Equations, 353, 2017.

[37] Doha, E. H., Bhrawy, A. H., Ezz-Eldien, S. S., *A Chebyshev spectral method based on operational matrix for initial and boundary value problems of fractional order*, Computers & Mathematics with Applications, 62(5), 2364–2373, 2011.

[38] Raslan, K.R., Ali, K.K., Mohammed, E:M:, Younis, J.A., Elsalam, M.A., *An operational matrix technique based on Chebyshev polynomias for solving mixed Volterra-Fredholm delay integro differential equations of variable-order,* Advan. Frac. Func. Anal., 6203440, 2023.