Examination of Mathematics Teachers' Orchestration Types Specific to Their Dynamic Geometry Software-Based Emergency Remote Teaching[1]

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ABSTRACT

The aim of this study is to examine secondary school mathematics teachers' orchestration types specific to their dynamic geometry software-based emergency remote teaching. The theoretical lens of the study is instrumental orchestration in which teachers' teaching styles with the use of technological tools are defined as orchestration types. The study was designed as a case study whose participants were two secondary school mathematics teachers (Faruk and Ela) experienced in the use of dynamic geometry software in their face-to-face teaching processes. Semi-structured interviews and observations were performed in order to collect the data in the study. The collected data were analyzed through qualitative content analysis method. The results of the study reveal that the teachers developed two new orchestration types during the emergency remote teaching process While Faruk developed the Teacher-in-sherpa-role orchestration in which the teacher performed the operations asked by the students; both teachers performed the Screen-as-board in which they used virtual pen to draw auxiliary lines, place marks and take notes on the screen. In addition, Ela adapted the Monitor-and-guide orchestration into her teaching process, in which a student-monitoring system regarding dynamic geometry activities played an essential role. The results show that various orchestration types identified in the literature do not occur in the remote teaching processes due to their characteristics regarding the physical classroom environment. It is suggested that further studies focus to examine the orchestration types developed by mathematics teachers in the post-pandemic remote teaching processes.

Keywords:[4]
Instrumental orchestration, dynamic geometry software, emergency remote teaching

INTRODUCTION

In 2020, face-to-face teaching was suspended, and remote teaching process started compulsorily in many countries around the world due to the Covid-19 pandemic. The remote teaching process during the pandemic is specifically defined as emergency remote teaching (Hodges et al., 2020). According to Hodges et al. (2020), emergency remote teaching (ERT) is "a temporary shift of instructional delivery to an alternative delivery mode due to crisis circumstances" (p. 6). During the ERT, teachers from different countries needed to learn new digital technologies that the ERT process requested (Aldon et al., 2021; Cao et al., 2021; Clark-Wilson, Robutti & Thomas, 2020; Drijvers et al., 2021; Mailizar et al., 2020; Rodriguez-Muniz et al., 2021; Şengil Akar & Kurtoğlu Erden, 2021). Among these technologies, video conferencing systems (e.g. Zoom, Google Meet, Skype) became one of the main teaching tools that are defined by Wiesemes and Wang (2010) as "a means for long-distance communication, to allow learners and teachers to link to external partners such as non-school based educational providers" (p. 29). Various studies indicate that the teachers could use the share-screen, chat, annotation, remote-control and video-recording tools of video conferencing systems to support their ERT processes (Uygan et al., 2022). On the other hand, for mathematics education, it is known that teachers were encouraged to use dynamic geometry software to enrich their teaching, prior to the ERT (Ministry of National Education, 2018; National Council of Teachers of Mathematics, 2000). Considering the ERT process, it has become an important question how mathematics teachers used dynamic geometry software and video conferencing system together in their ERT processes.

Dynamic geometry software (DGS) (e.g. Cabri, GeoGebra, and the Geometer's Sketchpad) holds a special place among digital instructional technologies that provide learners with interactive dynamic representations of geometrical objects (Laborde & Laborde, 2014; Trgalova, 2022). Here, learners have opportunities to explore the features of geometrical objects while creating and manipulating their dynamic representations (Hegedus & Moreno-Armella, 2010; Moreno-Armella et al., 2008). Moreover, various studies indicate that such learning environments enable learners to support their mathematical reasoning processes. For instance, Jones (2000) revealed that the utilization of DGS tools in geometrical construction tasks facilitated learners' deductive reasoning. In Arzarello et al. (2002) and Baccaglini-Frank's (2019) studies, it is seen that the learners who used dragging function of DGS with specific techniques could support their conjecture generation and abductive

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reasoning processes. Uygan and Bozkurt (2021) investigated a pre-service teacher's cyclic quadrilateral construction using DGS. Their results showed that DGS-mediated construction processes enabled the participant to reason about the necessary conditions to construct the most generic cyclic quadrilateral.

On the other hand, according to other studies, it might be complicated for teachers to use DGS in their teaching practices effectively (e.g. Bozkurt & Uygan, 2020; Hohenwarter et al., 2009). To give a sample, Bozkurt and Uygan (2020) examined the challenges (lesson hiccups) that a novice technology-using teacher encountered when she taught geometry with DGS-based tasks. Their results show that the teacher mainly had difficulties to follow the students' operations and lead their mathematical discussions when they developed various strategies. In addition, considering the conditions of the ERT process, teachers could encounter new challenges regarding the integration of DGS into their ERT processes and they might need to develop new teaching styles.

For the classification of teaching styles with the use of instructional technologies, Trouche (2004) introduced the conceptual lens of instrumental orchestration that is based on instrumental approach developed by Verillon and Rabardel (1995). For the next section, instrumental approach and instrumental orchestration, as conceptual framework of this study, are elucidated.

Instrumental Approach and Instrumental Orchestration

In the instrumental approach, Verillion and Rabardel (1995) model the dialectical relationship between a tool and a user around two concepts as *artefact* and *instrument*. The artefact can be considered as a physical tool, a digital technology, or a mathematical formula that a user turns to an instrument when he/she develops schemes regarding how to use it in various tasks. For the development process of an instrument, Artigue (2002) introduced the concept of instrumental genesis and this framework led researchers to investigate students' mathematics learning processes with the use of digital technologies (Buteau et al., 2020; Pittalis & Drijvers, 2023; Stewart et al., 2005). While instrumental genesis enables educators to understand how students utilize technological tools in their mathematics learning, instrumental orchestration addresses how mathematics teachers arrange the use of technological tools in their classrooms to support the students' instrumental genesis (Trouche & Drijvers, 2010). At this point teachers' orchestration processes include three main stages: didactical configuration, exploitation mode, and didactical performance (Drijvers et al., 2010; Trouche, 2004). In the didactical configuration, teachers decide which artefacts can be used during the lesson, and arrange the placement of artefacts in the classroom (Trouche, 2004). For exploitation mode, teachers plan instructional activities including the selected artefacts and decide when each artefact can be used by teacher or students during the activities (Trouche, 2004). In didactical performance, teachers conduct their teaching plans, and develop ad hoc strategies when they encounter unexpected situations during the teaching process (Drijvers et al., 2010).

The review of the related literature indicates that mathematics teachers develop different orchestration processes that are called *orchestration types* by Drijvers et al. (2010, p. 218). In Trouche's (2004) study, the researcher identified an orchestration type in which the teacher assigned a student to s*herpa*[4]*-student* role. A sherpa-student is supposed to perform the operations asked by the teacher, in which other students monitor sherpa-student's operations. Didactical configuration of this orchestration type includes the use of a projector and appropriate seating arrangement in which students can monitor sherpa-student's screen. Its exploitation mode involves the teacher's decisions regarding the operations that he/she would ask sherpa-student to perform. In the Drijvers et al.'s (2010) classification, the orchestration type identified by Trouche (2004) is called *Sherpa-at-work* orchestration. Moreover, Drijvers et al. (2010) identified five more orchestration types as *Technical demo*, *Explain-the-screen*, *Link-screen-board*, *Discuss-the-screen*, *Spot-and-show*. In *Technical demo*, the teacher introduces the technological tools and demonstrates the use of them. In the didactical configuration of *Technical demo,* the teacher selects the technological tools that would be introduced to students and plans the seating arrangement of the classroom in which students can follow teachers' operations. For the exploitation mode, the teacher plans her/his operations regarding the tools that will be introduced. In *Explain-the-screen*, the teacher provides explanations regarding mathematical ideas underlying her/his operations on the screen. While the didactical configuration of *Explain-the-screen* is similar with that of *Technical demo,* exploitation mode consists of the teacher's plans regarding her/his explanations on her/his screen. In *Link-screen-board* orchestration, the teacher uses computer and board together in which he/she makes connections between

[4]According to Trouche (2004), "the term sherpa refers to the person who guides and who carries the load during expeditions in the Himalaya mountains, and also to diplomats who prepare international conferences" (p. 305).

them. Didactical configuration of this orchestration includes the arrangement of a board in addition to computer and projector. In the exploitation mode, the teacher decides when he/she uses computer and board, and plans her operations between them. For *Discuss-the-screen* orchestration, the teacher and students discuss about properties of mathematical objects on the screen. Didactical configuration of this orchestration requests the arrangement of desks and placement of the selected tools to facilitate the classroom discussion. In exploitation mode, the teacher needs to plan classroom discussions with appropriate questions that can prompt the process. *Spot-and-show* is another orchestration type in which the teacher selects a student work and presents it (or enables the student to present it) to class. Didactical configuration of this orchestration requests a classroom arrangement in which the teacher can reach and evaluate students' work, and students can follow the teacher's (or a student's) presentation. In exploitation mode, the teacher needs to plan her/his explanations about student work, and also prepare for classroom discussions.

In addition to Drijvers et al.'s (2010) classification, another orchestration type called *Work-and-walk-by* was identified and elaborated in Drijvers' (2012) study. In this orchestration type, a teacher walks among the students, who work individually or in pairs, monitors their work, and helps them for their operations if necessary. In another study, Drijvers et al. (2013) examined the teachers' orchestrations in a technology-enriched classroom in which students were asked to work on online tasks. According to Drijvers et al.'s (2013) results, a new orchestration type was identified as *Guide-and-explain*, in which the teacher walks among the students, looks at their work, answers questions, provides explanations and continues to walk. In Tabach's (2013) study, in which 30 mathematics teachers' technology-based teaching processes were investigated, it was observed that teachers' teaching practices included two new orchestration types defined as *Monitor-and-guide* and *Discuss-tech-without-it*. In *Monitor-and-guide,* the teacher observes their students' work by walking around them or using a learning management system that shows their students' screen. Then the teacher provides the students with feedback in face-to-face communication or by sending messages through the learning management system. For *Discuss-tech-without-it* orchestration, the teacher discusses about the possible utilization of a technological tool in a task, even when the mentioned technology is not present. Similarly, Drijvers et al. (2013) defined *Board-instruction* and Tabach (2011) identified *Not-use-tech* orchestration types by observing different teaching practices in a classroom. In both orchestration types, teachers do not prefer to use technology although various technological tools were present in the classroom. In this, the use of the board is essential for the *Board-instruction* orchestration. As a summary, Table 1 presents the teaching processes corresponding to the prominent characteristics of the identified orchestration types.

Table 1. The Teaching Processes Corresponding to the Characteristics of the Orchestration Types

Orchestration Types	Teaching Process
Technical demo	Teacher introduces the technological tools and demonstrates the use of them
Explain-the-screen	Teacher explains mathematical ideas underlying her/his operations on the screen
Link-screen-board	Teacher uses computer and board together by making connections between them.
Discuss-the-screen	Teacher and students discuss about the properties of mathematical objects on the screen
Spot-and-show	Teacher selects a student's work and presents it (or allows the student to present it)
Sherpa-at-work	A student is supposed to perform the operations asked by the teacher, while other students monitor her/him
Work-and-walk-by	Teacher walks among the students and helps them for their operations if necessary
Guide-and-explain	Teacher walks among the students, answers questions, and provides explanations
Monitor-and-guide	Teacher observes their students' work by walking around them or using a learning management system, and provides them with feedback
Discuss-tech-without-it	Teacher discusses about the possible use of a technological tool in a given task, even when the mentioned technology is not present
Not-use-tech	Teacher does not prefer to use technology even when the technology is present
Board instruction	Teacher only prefers to use the board even when the technology is present

In the literature, the orchestration types are mostly investigated by considering teachers' face-to-face teaching processes. On the other hand, Uygan and Bozkurt (2023) examined the orchestration types that a teacher educator performed during his DGS-based emergency remote teaching. The results indicated that *Spot-and-show* and *Sherpa-at-work* had new characteristics during the ERT process. In the new version of *Spot-and-show*, a teacher's evaluation regarding the video-recorded lessons is essential. In this application, the teacher selects critical moments from the lesson videos and shares them with the students to remind them their previous mistakes or creative solutions. For the new version of *Sherpa-at-work*, students' collaborative group work in different video conference rooms is one of the main components, in which each group has their own sherpa-student.

Considering the research regarding the instrumental orchestration and the ERT processes, it is seen that there is a lack of findings in the literature regarding mathematics teachers' orchestrations specific to their DGS-based ERT processes. Since the ERT process is a new phenomenon for the mathematics teachers, it is crucial to reveal what types of orchestrations they develop while using new digital technologies that the ERT processes request. In addition, considering the DGS's didactical affordances for the secondary school mathematics education (Ministry of National Education, 2018), it becomes an important research topic what types of orchestrations developed by the secondary school teachers during their DGS-based ERT processes. It is believed that the obtained results could lead the educators to see the characteristics of the teaching processes involving the use of both DGS and the digital tools regarding the ERT process. Starting from this rationale, the research question is designed as follows: "What are secondary school mathematics teachers' orchestration types specific to their dynamic geometry software-based emergency remote teaching processes?"

METHOD

In this section, the research design, the background of the participants, the data collection tools, and the data analysis method are presented.

Research Design

This study is designed as a case study that is one of the qualitative research designs. According to Sharan (2009), a case study is "an in-depth description and analysis of a bounded system" (p. 40). As a case study pattern, multiple case-holistic design focuses on the analysis of two or more different cases holistically (Yin, 1984). This study focuses on ERT processes of two secondary school mathematics teachers who continued to work in the schools from different cities in Türkiye, and teach for different grades during the ERT. Since the

teachers' ERT processes are considered as two different cases to be examined holistically, the multiple case-holistic design is applied in the study.

Participants

The participants of this study are two volunteered secondary school mathematics teachers whose pseudonyms are Faruk and Ela. The criterion sampling method was applied to select the participants. The sampling criterion for the participants were (1) being experienced with the use of DGS in mathematics teaching before the pandemic, (2) being inexperienced to perform online remote teaching before the pandemic. It was known that both Faruk and Ela had participated in a project focusing on teacher training regarding the use of innovative technologies in mathematics education in 2018 summer (Bozkurt, 2022), in which the second author had worked as an educator. When the second author communicated with the teachers for their participations in this study in 2020, Faruk and Ela were continuing their profession in Mersin and Istanbul, respectively.

Faruk was a 10-years experienced secondary school mathematics teacher who worked in a public school that was in moderate level of socioeconomic status. Faruk stated that he completed an elective course regarding the technology-based mathematics teaching during his undergraduate education. In addition, Faruk expressed that he participated in an in-service professional course regarding the use of DGS in early years of his career. According to his statements, Faruk utilized GeoGebra as an instructional tool to teach various geometry topics during the face-to-face teaching. When this study started, Faruk was teaching the 7th grade students who were inexperienced in the use of GeoGebra. More than half of the students participated in Faruk's online lessons through their smart phones or tablets, and some of them could have internet connection problem in the lessons.

Ela was an 8-years experienced secondary school mathematics teacher continuing her profession in a project school with a moderate-high level of socioeconomic status in Istanbul. According to Ela's expressions, she completed an elective course regarding the use of the Geometer's Sketchpad in geometry teaching in her undergraduate education. After starting her profession, Ela completed various in-service courses organized by different educational institutions regarding the use of GeoGebra. The teacher expressed that she was also the participant of Bozkurt and Uygan's (2020) study in which her GeoGebra-based teaching regarding the quadrilaterals (for 7th grade) were examined before the pandemic. On the other hand, when this study started, Ela was teaching the 6th grade students who were experienced in the use of GeoGebra tools. Most of the students participated in Ela's online lessons through their laptops or desktop computers, and they had stable internet connections.

Data Collection

In the study, the data were collected with semi-structured observations and semi-structured interviews. For the observation of the ERT processes, the first author participated in online lesson sessions that the teachers started in the Zoom platform. The researcher observed six lessons for each teacher (completed in three days for each one) during which both teachers recorded their lessons and shared the videos with the researchers. The lesson topics that the teachers selected to teach in their DGS-based ERT processes are seen in Table 2.

Table 2. The Lesson Topics Teachers Taught in the Recorded Lessons

Day	Faruk (7th Grade)	Ela (6th Grade)
1st day (2 lesson hours)	Introduction of GeoGebra Tools	Exploration of Area Formula of Parallelogram
2nd day (2 lesson hours)	Lines and Angles (Creating Parallel Lines, Construction of Angle Bisector)	Exploration of Area Formula of Triangle
3rd day (2 lesson hours)	Lines and Angles (Additive Activities Regarding Angle Bisector)	Additive Activities Regarding Area of Parallelogram and Triangle

During the online lessons, the researcher avoided using his webcam and did not intervene in the teaching process. When the teachers performed their ERT processes, the researcher took observation notes in a semi-structured observation form that was developed by the researchers based on the components of the instrumental orchestration framework. Before and after each online lesson, the semi-structured interviews were carried out with the participants through a video conferencing system. The semi-structured interview questions focused on which technological tools the teachers selected to integrate into the lessons (didactical

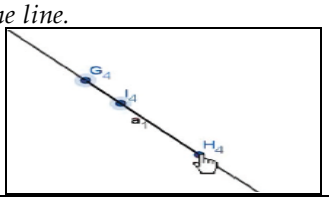
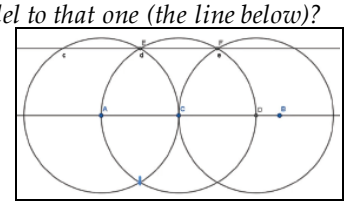
configuration) and also how they planned the utilization of the selected tools to perform their teaching (exploitation mode). The examples of the interview questions the researcher used before each lessons are "Which functions of the online platform will you utilize in your teaching process?", "Which functions of the dynamic geometry software will you utilize in your teaching process?", "Will you utilize a different technological tool or instructional material in your teaching process?", "What are the steps of your teaching?", "What roles will your selected tools have in your teaching?". The examples of the questions that the researcher used after each lesson are "Did you encounter unexpected moments during the lesson?", "What were your ad hoc actions against the unexpected moments?", "Did you achieve the planned goals?". The researcher recorded the interviews each of which took 28-45 minutes. In addition, complementary interviews were conducted for the participants' confirmation regarding their ad hoc teaching actions during the lessons. Moreover, WhatsApp conversations and e-mails including the lesson plans were used as supportive documents for the data triangulation. The semi-structured interview questions were developed by the researchers according to the concept of didactical configuration and exploitation mode. Besides, both the interview and observation forms were reviewed by an expert and university ethics committee before the data collection process.

Data Analysis

The recorded data regarding the lessons and the interviews were transcribed by the researchers. For the analysis of the didactical configuration, the instructional tools selected by the teachers were described under three themes, which are: "video conference tools (e.g. Share-screen, Annotation, Remote-control)", "DGS tools (e.g. Perpendicular line, Slider, Dragging function)" and "other tools". The "other tools" theme refers to traditional instructional tools (e.g. paper and scissors), computer hardware (e.g. keyboard, graphic tablet, virtual pen) or software (e.g. student-monitoring system) different from the video conferencing system and DGS. In the analysis of the exploitation mode, the teachers' planned actions regarding their ERT processes were described with reference to their explanations and lesson plans. For instance, "Creating points equally distanced from a line", "Explanation about the relationships between points", "Drawing with the virtual pen to support the explanations" were three successive actions planned by Faruk to teach parallel lines. For the didactical performance, the teachers' ad hoc actions were described by referring to their spontaneous intentions that were confirmed by the teachers in the interviews. As an example, when Ela spontaneously intended to show a student how to use the Perpendicular Line tool in DGS, this ad hoc action was described as "Spontaneous demonstration of the use of Perpendicular Line tool since a student requested".

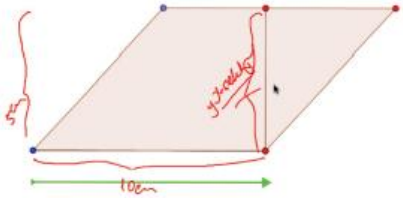
For the analysis of the orchestration types, the researchers applied qualitative content analysis method in which Creswell's (2012, p. 237) content analysis scheme was considered. At this point, the participants' teaching moments regarding their didactical performance were split and coded with the name of an orchestration type. For the consistency of the coding process, the prominent characteristics of the orchestration types identified in the literature (see Table 1) were considered as coding criteria. In this part of the analysis, the content analysis was carried out in deductive approach (Elo & Kyngas, 2007, p. 109). For instance, it was considered that the prominent characteristic of *Sherpa-at-work* is that a student is supposed to perform the operations asked by the teacher, while other students monitor her/him. For the second example, it was identified that the prominent characteristic of *Spot-and-show* is that the teacher selects a student work and presents it (or allows the student to present it). In the third example, the prominent characteristic of *Not-use-tech* is that the teacher does not prefer to use technology even when the technology is present. At this point, teachers' use of only microphone and webcam were not considered as the utilization of a technological tool for the reason that the mentioned tools did not create a difference for the teachers' mathematics teaching process comparing with the traditional teaching. To clarify the deductive content analysis process, four identified orchestration types and coding criteria are presented in Table 3.

Table 3. Examples of the Identified Orchestration Types from the Analysis with a Deductive Approach

Orchestration Type	Criterion	Data
Technical demo	Teacher introduces the technological tools and demonstrates the use of them	Teacher: *Now I'm using the Intersect tool. First, I clicked on this line and then another one. Look at it! What happened? It created a point at the intersection of them.*
Not-use-tech	Teacher does not prefer to use technology (except microphone and webcam) even when the technology is present	Teacher: *I want you to draw a triangle on it (on the folded paper). It doesn't matter what kind of triangle it is. Then, cut of this. Now we have two congruent triangles here. Well, my question is; can you create a parallelogram by using them?* 
Explain-the-screen	Teacher explains mathematical ideas underlying her/his operations on the screen	Teacher: *Can you see? I can't create a circle with these points. It can't pass through these three points because the points are collinear. What does collinear mean? It means that these are on the same line.* 
Discuss-the-screen	The teacher and students discuss about properties of mathematical objects on the screen	Teacher: *If we didn't create them (circles) congruent, would this line (the line above) be parallel to that one (the line below)?*  Berke: *Parallel? I think it would not.* Teacher: *Why not?* Dilara: *Because the gap between them (the lines) needs to remain equal. If so, they (the lines) can extend as parallel.*

During the deductive content analysis process, the data were coded by two coders (the authors). In order to test the reliability of the initial coding process, Miles and Huberman's (1994) reliability formula (percentage of agreement = consensus / (consensus + dissidence) x 100) was applied, and the agreement percentage of the coders was obtained as 74%, for which Miles and Huberman (1994, p. 64) recommend that consensus percentage is higher than 70%. In the next step, the researchers discussed about the differences between their codes and noticed that some orchestration processes of the participants could not be classified according to the components of the current orchestration types defined in the literature. For this reason, the main features of the new types of orchestrations were inferred from the data with the use of content analysis with inductive approach (Elo & Kyngas, 2007, p. 109). Finally, the researchers identified new orchestration types that the participants developed in their ERT processes. To show an example from the inductive content analysis process, the orchestration types and their identified features are presented in Table 4.

Table 4. Examples of the New Orchestration Types from the Analysis with an Inductive Approach

Orchestration Type	Main Features	Data
Screen-as-board	Teacher draws auxiliary lines, places marks, or takes notes on the pre-created figures with a virtual pen or the annotation tool	Teacher: *Now, what has short edge of the rectangle become in the parallelogram? It has become height, right?* 
Teacher-in-sherpa-role	Teacher performs the operations asked by students	Müge: *That circle should have touched the point C. It should have passed through point C.* Teacher: *[…] Where will I put the circle? Tell me.* Müge: *Its middle (centre) should be B, but also it should touch C.*

Combining the codes obtained from deductive and inductive content analysis, totally nine orchestration types were determined. In the findings section, new orchestration types as *Teacher-in-sherpa-role* and *Screen-as-board* developed by the teachers are described and interpreted. Furthermore, the *Monitor-and-guide* orchestration that Ela adapted to her ERT process and also her *Spot-and-show* and *Sherpa-at-work* orchestrations that occurred as complementary of the *Monitor-and-guide* are elucidated.

FINDINGS

The findings of this study are reported under the two headings regarding the orchestration processes of Faruk and Ela. Each participant's processes are presented with the sub-headings regarding her/his didactical configuration, exploitation mode and didactical performance. Also, their orchestration types are described within their didactical performance in which the new orchestration types they developed during their DGS based on ERT processes are explained in detail in the findings section. In the descriptions and dialogs, pseudonyms are used for students.

Faruk's Orchestration Processes

According to the findings, Faruk's orchestration processes are described with his didactical configuration, exploitation mode, didactical performance, and orchestration types.

Didactical Configuration

In Faruk's lesson plans, it was seen that he preferred to use the Zoom (video conferencing system), GeoGebra (DGS) and other tools in his teaching processes. Table 5 shows the selected instructional tools regarding the Zoom, GeoGebra and other tools in Faruk's didactical configuration.

Table 5. Instructional Tools in Faruk's Didactical Configuration

Tools	1st day (2 lesson hours)	2nd day (2 lesson hours)	3rd day (2 lesson hours)
Video conference tools	Microphone, Webcam, Share-screen, Chat, Annotation	Microphone, Webcam, Share-screen, Chat, Annotation	Microphone, Webcam, Share-screen, Chat, Annotation, Remote-control
DGS tools	Point, Line, Line segment, Ray, Intersect, Dragging function, Trace on, Animation on, Segment with given length, Perpendicular line, Parallel line, Midpoint or center, Distance or length, Angle, Angle with given size, Polygon, Circle with center through point, Circle: center & radius, Circle through three points, Compass	Point, Line, Ray, Line segment, Compass, Parallel line, Circle: center & radius, Angle, Dragging function	Line segment, Ray, Intersect, Distance or length, Angle, Polygon, Area, Perpendicular line, Compass, Dragging function
Other tools	Touchscreen computer, Mouse, Keyboard Virtual pen, Physical compass	Touchscreen computer, Mouse, Keyboard Virtual pen	Touchscreen computer, Mouse, Keyboard Virtual pen Enriched e-book (z-kitap)

According to Table 5, it becomes evident that Faruk regularly selected the virtual pen and the annotation tool to orchestrate his teaching regarding the topic of lines and angles. In addition, the findings indicated that he reduced the use of GeoGebra tools after the first day in which it was known that he planned to introduce the GeoGebra tools in the first two lesson hours. For the lessons on the second and third days, Faruk did not plan to use a tool different from those used on the first day within the Zoom and GeoGebra. On the other hand, for the last lesson, Faruk decided to integrate the enriched e-book into his teaching process.

Exploitation Mode and Didactical Performance

Based on the findings regarding Faruk's teaching processes, his exploitation mode and didactical performance are summarized in Table 6.

Table 6. Exploitation Mode and Didactical Performance in Faruk's Teaching Processes

Day	Learning Goals	Exploitation Mode	Didactical Performance
1st	Learning of the use of GeoGebra tools	Use of the selected tools on the screen Explanation and discussion about the use of the selected tools Drawing with the virtual pen to support the explanations Use the physical compass and link it with the circle tools of GeoGebra	The process was performed as planned in the exploitation mode Orchestration types: *Technical demo, Explain-the-screen, Screen-as-board, Teacher-in-sherpa-role, Discuss-the-screen, Not-use-tech*
2nd	Creating parallel lines & Construction of angle bisector	Creating points equally distanced from a line Explanation of the relationships between points Drawing with the virtual pen to support the explanations Asking students to discuss about the construction steps of the angle bisector	The teacher spontaneously assigned a student to sherpa role Orchestration types: *Screen-as-board, Explain-the-screen, Teacher-in-sherpa-role, Discuss-the-screen, Sherpa-at-work*
3rd	Exploration of angle bisectors in isosceles and equilateral triangles	Reminding the angle bisector construction Construction of special triangles Construction of angles bisectors in isosceles and equilateral triangles Drawing with the virtual pen to support the explanations Discussion about the construction steps Discussion about properties of angle bisectors in the constructed triangles	The teacher spontaneously explained the properties of angle bisector in the isosceles right angle. Orchestration types: *Sherpa-at-work, Screen-as-board, Explain-the-screen, Discuss-the-screen*

Table 6 shows that Faruk's exploitation mode regarding the first days aimed to introduce and demonstrate the use of GeoGebra tools for the reason that the students were inexperienced in the use of GeoGebra. In the didactical performance, the teacher carried out his teaching steps as he planned in the exploitation mode. For the next four lessons, Faruk's exploitation mode mainly focused on the use of the technologies to support construction tasks about angle bisector and triangles. In his didactical performance, the teacher needed to change some parts of his teaching according to the students' questions and current technological equipment. For instance, when Faruk noticed that a student who had difficulties in tasks participated in the lesson through her/his laptop, he spontaneously assigned her/him to sherpa-student role. Besides, the findings revealed that Faruk regularly utilized the virtual pen to draw auxiliary lines and place marks on the created figures that was identified as *Screen-as-board* orchestration. Moreover, the teacher developed the *Teacher-in-sherpa-role* orchestration in which he performed the operations asked by the students. In the last two hours, Faruk tended to perform Sherpa-at-work orchestration in which he provided a student with the use of remote-control tool and enabling her to make operations on teacher's screen. In the next sections, *Teacher-in-sherpa-role*, *Sherpa-at-work* and *Screen-as-board* orchestrations are exemplified with dialog parts selected from Faruk's teaching processes.

Teacher-in-sherpa-role orchestration

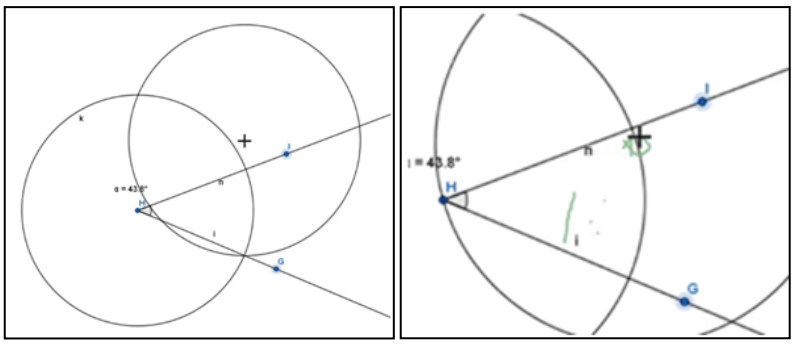
In the second day, Faruk applied construction tasks regarding parallel lines and angle bisector. To promote the discussion about the use of circles in angle bisector construction, Faruk encouraged the students to lead him how to operate with the compass tool. During the process, the students were allowed to use the annotation tool of the Zoom. By this way, they marked the critical points that they asked Faruk to click on while using the compass. Below, a part of the dialog occurring between Faruk and Melisa (pseudonym) is presented.

(Melisa):*"Let's create one more circle with the same size."*

(Teacher):*"OK, let's do. Do you want it with the same size?"*

(M):*"Yes."*

(T):*"OK, now what do you want me to click on? Where should I put it (Figure 1a)? You can mark it (with the use of annotation tool)."*
(M):*"Look at that! I have marked there, on the intersection (Figure 1b)."*
(T):*"This one?"*
(M):*"Yes, on their intersection."*



(a) **(b)**

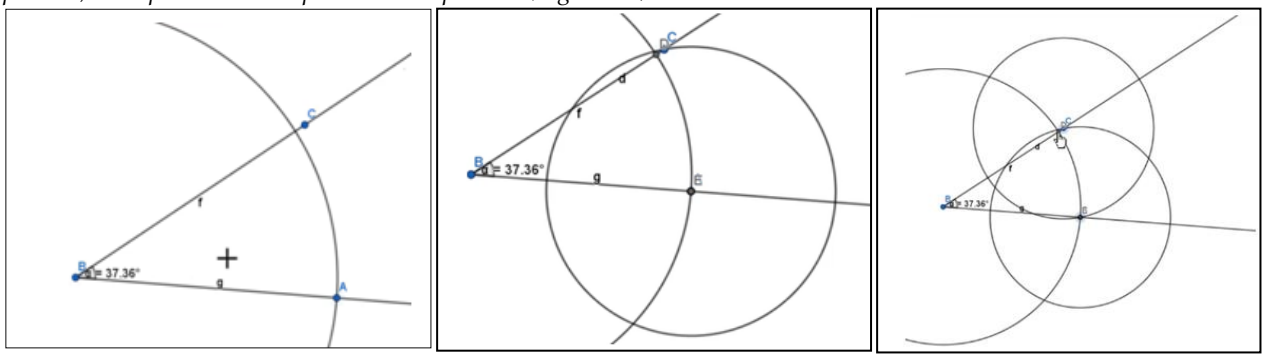
Figure 1. The Figures on the Screen in Faruk's Teacher-in-sherpa-role Orchestration

It was seen that Faruk performed *Teacher-in-sherpa-role* orchestration during the first three lessons. Then the teacher noticed that he could enable some students to share their screens or use the remote-control tool. By this way, the teacher evaluated students' performances in construction tasks, and also supported their use of the compass tool in GeoGebra. In the next section, it is described how the teacher performed Sherpa-at-work by providing a student with the use of the remote-control tool.

Sherpa-at-work orchestration

For the fifth lesson, Faruk planned to remind students how to construct angle bisector before the teaching of isosceles triangle construction. Also, before the lesson, Faruk stated that he just explored the remote-control function of the Zoom and aimed to integrate it into the next lesson. For this aim, Faruk allowed a student (Şule [pseudonym]) to use the remote-control to operate on the teacher's screen. Then the teacher asked Şule to construct angle bisector. Since Şule could not select appropriate tools, Faruk led her to select the compass and create the intersecting congruent circles in GeoGebra. A part of the dialog occurring between Faruk and Şule is seen below.

(Teacher):*"Now let's see if it works. Yes, very well! Now, select the compass. Next one, yes, this one. Now click on the point B, then point A. Now put it on the point B (Figure 2a)."*



(a) **(b)** **(c)**

Figure 2. The Geometrical Figures on the Screen in Faruk's Sherpa-at-Work Orchestration

(Teacher): *"What is this circle's role in the construction of angle bisector? Didn't we use it before to create two intersection points equally distanced (from B)? For the next, what did we do? Then, we created other circles, right? Where? Now, select the compass again. Then click on D and E."*
(Şule): *"Is D this one?"*
(Teacher): *"Yes, then click on E. Now, leave it (new circle) on E (Figure 2b). Again, click on D and E."*
(Şule): *"Then I need to leave it on D (Figure 2c)."*

In addition to orchestration types involving a sherpa, Faruk also frequently performed the *Screen-as-board* orchestration in his ERT processes. Within the orchestration type, the teacher's use of the virtual pen and the students' use of the annotation tool became prominent. In the next section, an example from Faruk's *Screen-as-board* orchestration is described.

Screen-as-board orchestration

In the sixth lesson, Faruk carried out the constructions of isosceles triangle and angle bisector in an isosceles triangle. Following the constructions, he used the virtual pen to mark the critical features of angle bisector in an isosceles triangle (Figure 3).

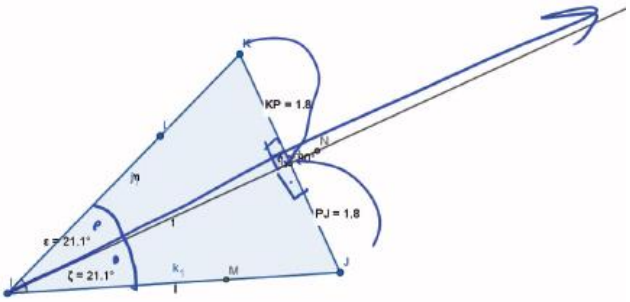
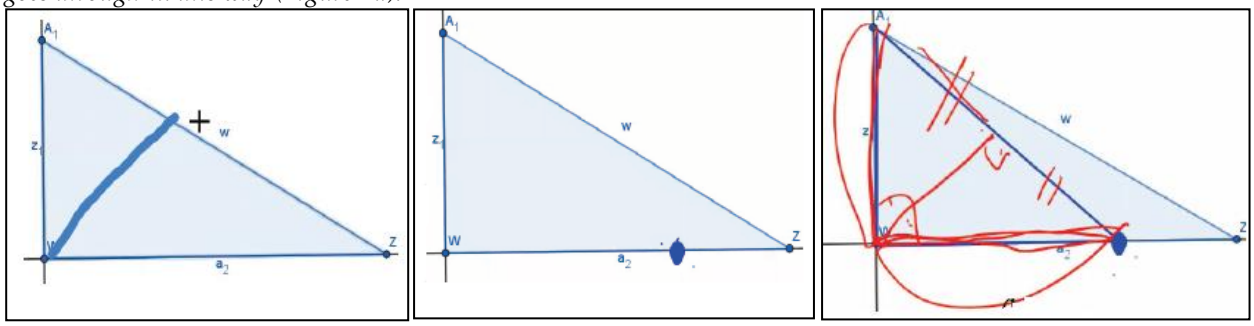


Figure 3. Use of the Virtual Pen to Mark the Critical Features of Angle Bisector in Isosceles Triangle

In the next step, one of the students (Berke [pseudonym]) asked a question regarding how to construct an isosceles right triangle in GeoGebra. At that moment, Faruk deviated from his teaching plan and constructed a right triangle (since he misunderstood Berke's question). At this point, the students discussed about properties of angle bisector in the right triangle by drawing an auxiliary line on it. Moreover, they reasoned about how to turn the created right angle to an isosceles right triangle. During the process, Faruk used the virtual pen to draw additive auxiliary lines and highlight the critical features of the isosceles right triangle. A part of the discussion process between Faruk and students (pseudonyms are used for them) is seen below.

(Berke): *"How can I draw a right triangle in GeoGebra? Also, with equal edges?"*
(Teacher): *"Let me show you. First, I select the right line tool. Then I create a line perpendicular to this one.*
(Şule)*: "A line segment from there…"*
(Teacher): *"Exactly! We can draw a line segment wherever we want (to create edges of the triangle). Then, let's use the polygon tool to connect these three points. And right triangle is here!"*
(Şule): *"Ohh it didn't work!"*
(Teacher): *"Why not?"*
(Berke): *"The edges are not equal. Let me show you. (Using the annotation tool) This line (drawn as the angle bisector) goes through in this way (Figure 4a)."*



(a) **(b)** **(c)**

Figure 4. Use of Virtual Pen and the Annotation Tool to Draw Auxiliary Lines and Place Marks

(Hilal): *"May I tell something? I think the point Z should have been further."*
(Teacher): *"For example… Here? OK, well (Figure 4b). Now, let me draw a triangle with that point. […] If these two edges are equal… When the angle bisector goes through there (drawing with the virtual pen), these two parts and the height become equal (Figure 4c), right?"*

Ela's Orchestration Processes

In this section, Ela's orchestration processes are described with her didactical configuration, exploitation mode, didactical performance, and orchestration types.

Didactical Configuration

Starting from the lesson plans, the findings reveal that Ela planned the use of the Zoom (video conferencing system), GeoGebra (DGS) and other tools. Table 7 shows the instructional tools selected by Ela regarding the Zoom, GeoGebra and other tools.

Table 7. Instructional Tools in Ela's Didactical Configuration

Tools	1st day (2 lesson hours)	2nd day (2 lesson hours)	3rd day (2 lesson hours)
Video conference tools	Microphone, Webcam, Share-screen, Chat, Annotation	Microphone, Webcam, Share-screen, Chat, Annotation	Microphone, Webcam, Share-screen, Chat, Annotation
DGS tools	Point, Line segment, Perpendicular line, Parallel line, Intersect, Dragging function, Distance or length, Polygon, Slider	Line segment, Perpendicular line, Intersect, Distance or length, Dragging function, Slider	Line, Line segment, Perpendicular line, Parallel line, Intersect, Distance or length, Polygon
Other tools	Laptop, Mouse, Keyboard, Graphic tablet, Virtual pen, Websites on GeoGebra	Laptop, Mouse, Keyboard, Graphic tablet, Virtual pen, Paper and scissors	Laptop, Mouse, Keyboard, Student-monitoring system, Enriched e-book (z-kitap),

In Table 7, it is seen that Ela generally planned to integrate the tools with the same functions into her teaching processes regarding the area of parallelogram and triangle. For the first four lesson hours, the teacher preferred to use Slider tool that provides the design of virtual manipulative in GeoGebra. Also, Ela preferred to use virtual pen, paper and scissors, enriched e-book and web resources involving student-monitoring system in which the teacher had opportunities to monitor students' work during the lesson.

Exploitation Mode and Didactical Performance

According to Ela's teaching processes, exploitation mode and didactical performance conducted by the teacher are summarized in Table 8.

Table 8. Exploitation Mode and Didactical Performance in Ela's Teaching Processes

Day	Learning Goals	Exploitation Mode	Didactical Performance
1st	Exploration of Area Formula of Parallelogram	Reminding the area formula of rectangle Creating a parallelogram by manipulating two parts of a rectangle with the use of a slider Explanation and discussion about the relationships between parallelogram and rectangle Drawing with the virtual pen to support the explanations	Spontaneous explanations about the heights of the figures by showing her gestures on webcam Spontaneous demonstration of the use of Perpendicular Line tool since a student requested Orchestration types: *Explain-the-screen, Screen-as-board, Not-use-tech, Discuss-the-screen, Technical demo*
2nd	Exploration of Area Formula of Triangle	Creating a parallelogram with two congruent triangles with the use of a paper and a pair of scissors Creating a parallelogram with two congruent triangles by using a slider Explanation and discussion about the relationships between parallelogram and triangle Drawing with the virtual pen to support the explanations	The process was performed as planned in the exploitation mode Orchestration types: *Not-use-tech, Explain-the-screen, Discuss-the-screen, Screen-as-board,*
3rd	Additive Activities Regarding Area of Parallelogram and Triangle	Sharing tasks (on creating the heights of parallelogram and triangle) with the students through the geogebra.org Following the students' work and providing feedback through the geogebra.org groups Presentations of the selected work Explanation and discussion about the student work	The teacher spontaneously assigned a student to sherpa-student role Orchestration types: *Explain-the-screen, Monitor-and-guide, Spot-and-show, Sherpa-at-work, Discuss-the-screen,*

Table 8 displays that Ela's exploitation mode for the first two days focused on the use of the slider-mediated manipulative while investigating the relationships between the area of quadrilaterals and triangles.

Ela also planned to support her manipulative-based applications with the paper-cutting activities that would be carried out without the use of technological tools except webcam and microphone. While the teacher aimed to utilize virtual pen to support her explanations in the first two days, it was detected that she performed *Screen-as-board* orchestration in her didactical performance. For the last day, she focused on the use of student-monitoring system specific to GeoGebra applications (geoegebra.org/groups), with the aim of following the students' work in the given tasks. For the tasks, the students were asked to create the heights of parallelogram and triangles on the different edges. During Ela's didactical performance regarding the use of student-monitoring system, it was inferred that she performed *Monitor-and-guide* orchestration. Moreover, it was observed that the teacher's *Spot-and-show* and *Sherpa-at-work* orchestrations followed her *Monitor-and-guide* orchestration. During *Spot-and-show* orchestration, Ela enabled the students to present their work that were selected during *Monitor-and-guide* orchestration. For *Sherpa-at-work* orchestration, the teacher led operations of the students who were observed to struggle during *Monitor-and-guide* orchestration. In the next sections, *Use-of-the-screen-as-a-board*, *Monitor-and-guide*, *Spot-and-show* and *Sherpa-at-work* orchestrations performed by the teacher are exemplified with the parts of conversation occurring in the teaching processes.

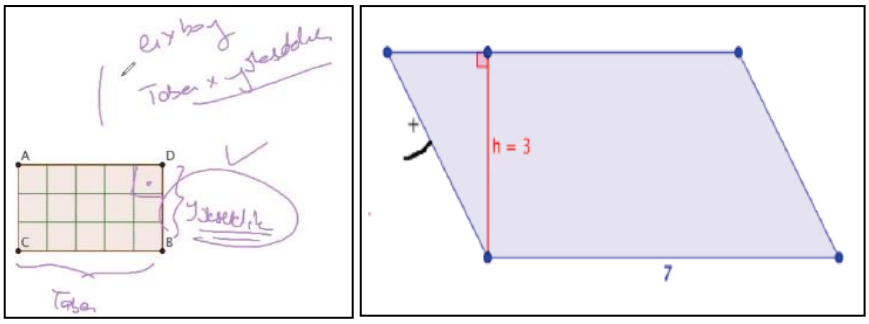
Screen-as-board orchestration

For the first day, Ela started her teaching process with a manipulative in which she used sliders to change the edges of a rectangle including unit squares. In the next step, the teacher utilized virtual pen to support her explanations regarding height and width of rectangle. During this teaching section, Ela wrote these terms on the given figure, and noted the area formula on the screen. Below, a part of a dialog between Ela and Yaprak (pseudonym) during the orchestration is presented.

(T): *"Can we call the long and short edges "width and length "(for rectangle)? Yes, we can call. Well, do we have alternative terms for them? How can we call them alternatively?"*

(Y): *"If I call one of these edges "a" and other one "b", then we can say its area is a x b."*

(T): *"Maybe. Or can I call width base, and length height (writing on the screen [Figure 5a])? Well, what does height mean?"*



(a) **(b)**

Figure 5. Use of Virtual Pen and the Annotation Tool to Take Notes and Place Marks

After the conversation about the rectangle, Ela started a new application including a manipulative in which a parallelogram could be created by moving two parts of a rectangle with a slider. When Ela asked students to find the area of the created parallelogram, one of her students (Dinçer [pseudonym]) stated that they could multiply the length of the edges as they did for area of rectangle. During his explanation, Dinçer also used the annotation tool to show the edge that he mentioned (Figure 5b). The dialog between Ela and Dinçer is seen below.

(Dinçer): *"Actually I mention about this edge."*

(Teacher): *"Show it on the screen."*

(D): *"This one (Figure 5b). Can we find the area if we multiply it with 7?"*

(T): *"Can we do?"*

(D): *"I think we can do. Because we these edges are parallel, and we could find area of rectangle by this way."*

(T): *"OK, let's try. Let's think about it now. "*

The findings regarding Ela's orchestrations reveal that she continued to use virtual pen on the second day. On the other hand, the annotation tool was not used by any students since Dinçer's use. While Ela did not prefer to use virtual pen on the third day, it was visible that she integrated a student-monitoring system specific to GeoGebra activities to her fifth and sixth lessons. The next section presents a teaching part regarding *Monitor-and-guide* orchestration in which Ela used student-monitoring system as the main tool. Also, it is described that how *Spot-and-show* and *Sherpa-at-work* orchestrations were carried out as two successors of *Monitor-and-guide.*

Monitor-and-guide orchestration

In the fifth lesson, Ela prepared GeoGebra tasks in which students were asked to create the heights of parallelogram and triangles on their different edges. Ela started the lesson by sharing her tasks with students through an online GeoGebra platform (geogebra.org/groups) that provided her with a student-monitoring system. In the system, the teacher followed students' works and used microphone to provide feedback for those creating incorrect figures. An example of Ela's feedback and the view of her screen are provided below.
(Teacher): *"Melek, your figure isn't a parallelogram as you can see. First, you should create the parallelogram correctly. The bottom and top edges look parallel, but right and left edges don't."*

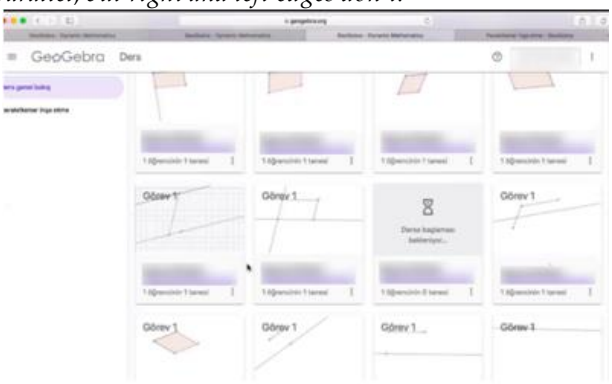


Figure 6. The View of Ela's Screen during the Monitor-and-Guide Orchestration

Following the *Monitor-and-guide*, Ela continued her teaching process by allowing the selected students to share their work with the class. This teaching preference led to performing Spot-and-show orchestration.

Spot-and-show orchestration

When time was out for the task regarding the construction of the height of the triangles, Ela asked Dinçer to share his work with the class. During the task, it was known that Dinçer created the heights of an obtuse triangle on its different edges. A part of the dialog between teacher and student can be seen below.
(T): *"Dinçer, can you share your work?"*
(D): *"OK, I will do (performing her operations on the screen)."*
(T): *"OK, can you explain to us? Which lines are perpendicular here (Figure 6)?"*
(D): *"This FI is perpendicular to DE."*
(T): *"And, is DK height of EF (Figure 8)?"*
(D): *"Yes."*

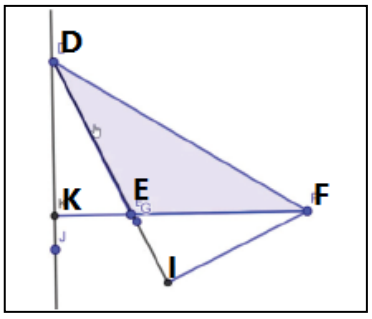


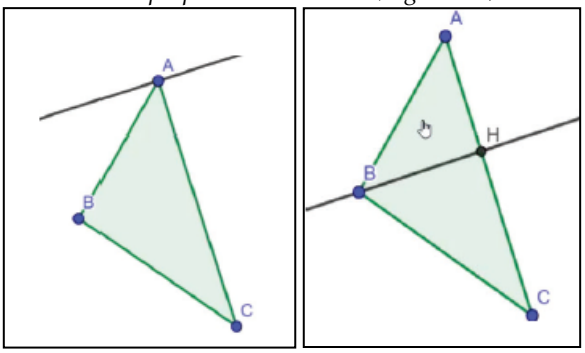
Figure 7. A Geometrical Figure on the Screen in Ela's Spot-and-Show Orchestration

In addition to Spot-and-show, Ela also tended to guide the students who had difficulties with the use of GeoGebra to complete the task. For this aim, the teacher asked some of them to share their screen and gave them directions about the use of GeoGebra tools. The next section provides an example regarding the Sherpa-at-work orchestration that Ela performed.

Sherpa-at-work orchestration

During the presentations of the work, it was observed that a student (Sinem) struggled to use the Perpendicular line tool while creating heights. At this time, Ela started to give her directions to use the tool correctly, in which Sinem (pseudonym) was in sherpa-student role. A part of the conversation is shown below.
(Teacher): *"Why did you put that perpendicular line on the point A. Which line segment is it perpendicular to now? It is AC, right (Figure 9a)?"*
(Sinem): *"Yes."*
(T): *"Which point does it (perpendicular line) need to pass through? A and C are already on that line segment. So, which point do you need to click on? It is B."*

(S): *"Do I need to click on B?"*
(T): *"Sure! Yes, you need to leave it on B. Nice! (The student performed the asked operation) Intersect tool is the next one. OK, very good. It has become the line BH perpendicular to AC (Figure 9b)."*



(a) **(b)**

Figure 8. Geometrical Figures on the Screen in Ela's Sherpa-at-Work Orchestration

In the following section, the revealed results are presented and discussed by considering the previous studies in the literature.

RESULTS, DISCUSSION and SUGGESTIONS

The results mainly reveal that two new orchestration types were developed by the teachers during their dynamic geometry software-based emergency remote teaching processes. According to Faruk's *Teacher-in-sherpa-role* orchestration, the main purpose of teacher is to use the technological tools by following directions given by students. For the didactical configuration of *Teacher-in-sherpa-role*, it is essential that the teacher shares the screen and integrates the annotation tool into the teaching process. In the exploitation mode, the teacher plans the tasks in which students can lead teacher's operations on the screen. It can be considered that the reason why Faruk employed *Teacher-at-sherpa* orchestration is that his students have not been experienced in the use of dynamic geometry software yet and the teacher has focused on the better use of time in the lessons. According to the *Screen-as-board* orchestration developed by both Faruk and Ela, the main component is that the teacher utilizes a virtual pen or annotation tool to draw auxiliary lines, place marks or take notes on the screen. For didactical configuration of this orchestration, the teachers selected appropriate tools for drawing and writing on the screen that mainly involved a virtual pen, annotation tools, and a touchscreen computer/graphic tablet. In the exploitation mode, the teacher plans what he/she draws or writes on the screen with the aim of supporting her/his teaching. Also, in Faruk's case, the students were allowed to use the annotation tool while the teacher was using virtual pen on the screen. On the other hand, Ela generally focused on her own operations instead of encouraging the students to use the annotation tool, in which she preferred to take notes and place marks more than drawing auxiliary lines with the use of virtual pen. Comparing the cases of the teachers, Faruk used the virtual pen more frequently than Ela, in which it can be argued that one of the reasons of this difference is based on the technological equipment they have. Here, it is known that Faruk integrated touchscreen computer into the lessons, while Ela needed to link a graphic tablet to her laptop for the use of the virtual pen.

During the teaching processes, Faruk tended to switch from the *Teacher-in-sherpa-role* to the *Sherpa-at-work* orchestration. Between two orchestration types, Faruk reasoned more about the exploitation of students' current technological equipment. Moreover, he also explored the use of remote-control tool during the process that led Faruk to employ *Sherpa-at-work* orchestration. Considering this result with relation to Mishra and Koehler's (2006) technological pedagogical content knowledge (TPACK) model, it can be argued that the extension of Faruk's technological knowledge regarding functions of video-conferencing system led to the evolution of his TPACK. From another perspective, according to the concept of Haspekian's (2011) professional instrumental genesis, teachers need to develop teaching schemes regarding the use of a technological tool in their lessons. At this point, teachers' teaching schemes regarding the technological tool differ from their utilization schemes that correspond to their personal instrumental genesis. Based on Haspekian's (2011) perspective, it can be asserted that Faruk tended to extend his teaching scheme regarding the use of video-conferencing system in parallel with the evolution his personal instrumental genesis. With the development of the professional instrumental genesis, it is possible that Faruk focused to employ a different orchestration type as *Sherpa-at-work*.

In Tabach's (2013) study, *Monitor-and-guide* is defined as an orchestration type in which a teacher provides students with feedback while he/she walks around the class or uses of a learning management system in which teacher can monitor students' work. In Ela's case, the results show that the teacher performed *Monitor-and-guide* orchestration with the use of a student-monitoring system specific to dynamic geometry activities and guided students by turning on the microphone. Comparing Ela's orchestration type with those presented in Tabach's (2013) study, it can be considered that Ela adapted the *Monitor-and-guide* to her dynamic geometry based remote teaching process. Also, the results indicate that Ela employed the *Spot-and-show* and the *Sherpa-at-work* as complementary elements of the *Monitor-and-guide* orchestration. In Drijvers et al.'s (2019) study, the researchers consider how teachers can arrange their orchestration types in a sequence to perform their teaching in productive ways. For Ela's case, it can be argued that *Spot-and-show* and *Sherpa-at-work* were employed in a sequence with the *Monitor-and-guide.* The obtained result can lead researchers to think about the question: "Does a sequence of orchestrations consisting of *Monitor-and-guide*, *Spot-and-show* and *Sherpa-at-work* provide teachers with productive teaching processes?"

Considering Uygan et al.'s (2022) findings, graphic tablet and touchscreen computer were one of the essential tools for many mathematics teachers during the emergency remote teaching. In addition to Uygan et al.'s (2022) results, this study reveals how the mentioned tools were used as complementary elements of dynamic geometry software-based teaching processes. For the integration of dynamic geometry software tools, it is seen that Faruk allocated the first two hours to introduce various GeoGebra tools and then provided the students with basic-level tasks in the first part of his teaching. On the other hand, Ela started her activities without the introduction of tools since her students were experienced with the use of GeoGebra. Assude et al. (2006) put forward four ways regarding teachers' technology integration based on levels of students' experiences with the use of the technology: instrumental initiation, instrumental exploration, instrumental reinforcement, and instrumental symbiosis. The first two modes involve teacher's technology introduction and basic-level tasks regarding the use of the technology, by which inexperienced students have opportunities to enhance their technical knowledge. The next two modes provide experienced students with technology-aided activities in which they are enabled to develop their mathematical knowledge. Based on Assude et al's (2006) classifications, it can be argued that the first parts of Faruk's orchestrations mainly corresponded to the instrumental initiation and the instrumental exploration modes, while Ela's orchestrations coincided with the instrumental reinforcement mode.

According to the results, Faruk or Ela did not employ *Link-screen-board*, *Board instruction Work-and-walk-by*, *Guide-and-explain* and *Discuss-tech-without-it* orchestrations (Drijvers, 2012; Drijvers et al., 2010; Drijvers et al., 2013; Tabach, 2013). Firstly, for the *Link-screen-board* and *Board instruction*, it can be considered that the affordances of the graphic tablet/touchscreen computer and the virtual pen led the teachers to use these tools as equivalents of the board and the board pen. Here, it can be deduced that the *Link-screen-board* turned to the *Screen-as-board* orchestration in Faruk and Ela's cases. Secondly, since teachers' walking around the class is the main feature of both *Work-and-walk-by* and *Guide-and-explain*, these orchestrations did not continue their existences during the teachers' emergency remote teaching. On the other hand, for the reason that a teacher might use an electronic system instead of walking around the class within *Monitor-and-guide* (Tabach, 2013), it can be argued that this orchestration type occurred in Ela's teaching process by reducing its features regarding the teacher's physical movement. Thirdly, the possible reason why the teachers did not perform the *Discuss-tech-without-it* orchestration might be that they had facilities to reach various digital technologies during the emergency remote teaching and they were experienced with the use of dynamic geometry software.

The results also displayed that Faruk and Ela occasionally integrated the physical tools as compass and paper-scissors to their teaching processes in addition to their digital tools. At this point, it can be asserted that the teachers considered the mentioned tools as complementary elements of the dynamic geometry tools. According to Faggiano, Montone and Mariotti's (2018) study, the use of both physical and digital mathematical tools in a productive synergy promote learning processes of students. Similarly, in the cases of Faruk and Ela, it is evident that they aimed at transition between the selected physical and digital tools that represent the same mathematical objects to support learning processes.

Limitations

The study focused on six online lessons that each teacher performed during three each day. The participants are two secondary school teachers of similar ages. The interview and observation processes were

conducted through a video-conferencing system for the reason that the researchers and teachers could not be in the same physical environment due to Covid-19 pandemic conditions.

Suggestions

Considering the results, various suggestions can be provided for future studies and educators. Further studies can investigate post-pandemic remote teaching processes of mathematics teachers who work in different schools at various levels, and have different professional experiences. It is also recommended that researchers conduct longitudinal studies to examine how mathematics teachers' orchestration types evolve in their remote teaching processes. In this research, the participants focused on the geometry learning domain and carried out their remote teaching with the use of dynamic geometry software. In further studies, it can be investigated what orchestrations teachers develop while teaching numbers, algebra, data processing and possibility in their remote teaching processes.

For in-service teacher training programs, trainers can aim to provide mathematics teachers with new teaching practices involving the use of both mathematical software and other innovative technologies such as video-conferencing tools and student-monitoring system. For undergraduate education, teacher-educators can provide pre-service mathematics teachers with new courses regarding the effective use of digital technologies in remote teaching. Moreover, researchers can aim to conduct design-based research to support in-service or pre-service mathematics teachers' orchestrations regarding the remote teaching.

Declarations

Conflict of Interest

There is no conflict of interest between the authors regarding the study, authorship, or publication of the article.

Ethics Approval

The ethics approval of the study was granted by the Social and Human Sciences Research and Publication Ethics Committee of Eskisehir Osmangazi University. The study was carried out in accordance with the Helsinki Declaration in 1975.

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Research and Publication Ethics Statement

The study was approved by the research team's university ethics committee of the Eskisehir Osmangazi University (Approval Number/ID: 2021-04). The authors consciously ensure that for the manuscript the following criteria are met:

- This manuscript, as original work of the authors, has not been previously published elsewhere.
- The article involves the authors' own data collection and analysis processes conducted in a truthful and complete manner.
- The results are situated within the framework of prior and existing research in an appropriate manner.
- All sources used are properly disclosed.

Contribution Rates of Authors to the Article

1st author contributed 60%, 2nd author 40%.

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