Fuzzy VIKOR Method for Dynamic MADM Problem Solution in ESI 5-Level Triage System

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Keywords
Dynamic MCDM, Uncertainty, Fuzzy Theory, Emergency Severity Index, Triage Patients.

Abstract
Multiple Attribute Decision Making (MADM) tools make preference decisions over multiple attributes' alternatives available, which in most cases conflict among themselves. The classic MADM includes techniques that consider a set of fixed and predefined attributes when making a decision. However, the majority of real-world decisions occur in dynamic and unstable scenarios. Therefore, classic MADM will not be the answer to our problems in the real world and uncertainty. This paper addresses a flexible framework for dynamic MADM, based on the concept of fuzzy sets theory and the VIKOR method to provide a rational, scientific and systematic process for prioritizing patients in the Emergency Department (ED), under a fuzzy environment where the uncertainty, subjectivity, and vagueness are addressed with linguistic variables parameterized by triangular fuzzy numbers. Finally, the computational results are discussed in detail. Dynamic decisions arise in many applications, including military, medical, management, sports and emergency situations. Therefore, this study can affect a wide range of applied fields.

Research Article
Submission Date : 11.09.2023
Accepted Date : 24.11.2023

1. INTRODUCTION

As existing systems become more complex, the importance of dynamic systems has become much wider than in the past. Decision making in dynamic systems can cause growth, survival and even destruction of systems. Today, most real-world decisions are made in a dynamic environment (Jassbi et al., 2014). Therefore, it is necessary to create a suitable framework for these types of decisions. In this paper, we seek to find a framework for this type of decision making. In classical models of multiple criteria decision making (MCDM), it is assumed that when making a decision, the decision maker has predefined a fixed set of criteria and presented fixed alternatives with a clear picture of all

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available alternatives (Peng & Tzeng, 2013). In dynamic situations, the problem is that decisions are made in a constantly changing (dynamic) environment (Alinezhad & Taherinezhad, 2020), and the available alternatives can change over time. In most studies conducted in the field of multiple attribute decision making (MADM), the decision-making matrices in them are static (Alinezhad et al., 2023; İşler & Çalık, 2022; Norouziyan, 2022; Amini et al., 2016). That is, the weight of attributes and the value of alternatives for each attribute belong to a period of time, and the change of these items during the past or future periods was less considered. While it is quite evident that all the information related to MADM matrices can change over time and their values are not necessarily constant during several time periods. Therefore, these items can be seriously effective in the decision-making process and ranking of alternatives.

Decision-making with the above conditions, which we call dynamic decision-making under uncertainty, is used in many fields, including military, medical, management, sports, and emergency situations. In hospitals and medical centers, the triage system refers to the process of prioritizing patients based on the severity of the disease in order to perform the best treatment measures in the shortest possible time (Sabry et al., 2023). Our main problem is to provide a framework for decision-making in one of the common types of triages (American 5-level triage). The proper triage will increase the quality of patient care services, increase satisfaction, reduce the waiting time and stay of patients, reduce deaths, and increase the efficiency of emergency departments in parallel with reducing related costs (Sabry et al., 2023).

The term triage was first used in 1800 by one of Napoleon's army doctors named Doctor Dominique Jean Larry to prioritize and treat wounded soldiers in war. From the early 1990s, several countries started designing and providing triage systems until the five-level triage systems were created and introduced in the late 1990s and early 2000s (Travers et al., 2002). Among these systems, the triage system of Australia, Canada, Manchester and the emergency severity index (ESI) gained the most acceptance. The triage process becomes meaningful when, firstly, there are resources for providing services, secondly, the relative balance between the supply and demand of resources is not established, and thirdly, a specific plan for prioritizing patients is defined (Sabry et al., 2023). The ESI system is an American 5-level triage system that was invented in 1999 by two emergency medicine specialists named Richard Warez and David Eitel. The ESI triage structure is one of the 5-level triage methods in which patients are divided based on the two criteria of disease severity and the facilities required by the patient. Currently, ESI triage seems to be the most appropriate triage system. This system has been revised three times and currently the fourth edition is available (Gilboy et al., 2012).

In the ESI algorithm, there are four decision points as shown in Figure 1 (Gilboy et al., 2012):

- Decision point A: "Is the patient dying or does he need immediate and life-saving intervention?" In this case, it is placed at level 1.
- Decision point B: "Shouldn't the patient wait?" (Including: high-risk symptoms, impaired consciousness, pain, severe distress), which in this case is placed at level 2.
- Decision point C: In the absence of conditions A and B, the facilities needed by the patient are estimated in the emergency room to determine the patient's task. The patient's need for two or more emergency facilities, if vital signs are not disturbed, puts the patient at level 3. The patient's need for one of the emergency facilities places the patient on level 4, and the patient who does not need to use emergency facilities is placed on level 5.
- Decision point D: If the facilities needed by the patient are two or more according to the definition, at this stage, the patient's vital signs should be considered for classification. If there is a

disturbance in the vital signs, the patient will return to level 2, and otherwise, the patient will be divided into level 3.

In the following, after identifying the problem and its precise definition, the first step in conducting the research is to study the literature of the subject in order to gain knowledge and determine the place of the current research among the studies. The next step is problem modeling with the help of MADM techniques and modeling methods in fuzzy conditions. Then, the proposed model will be implemented in a case study that is an improvement in the way of prioritizing patients in the emergency department of the hospital to determine its efficiency and effectiveness. At the end, the obtained results are analyzed and a final summary is made.

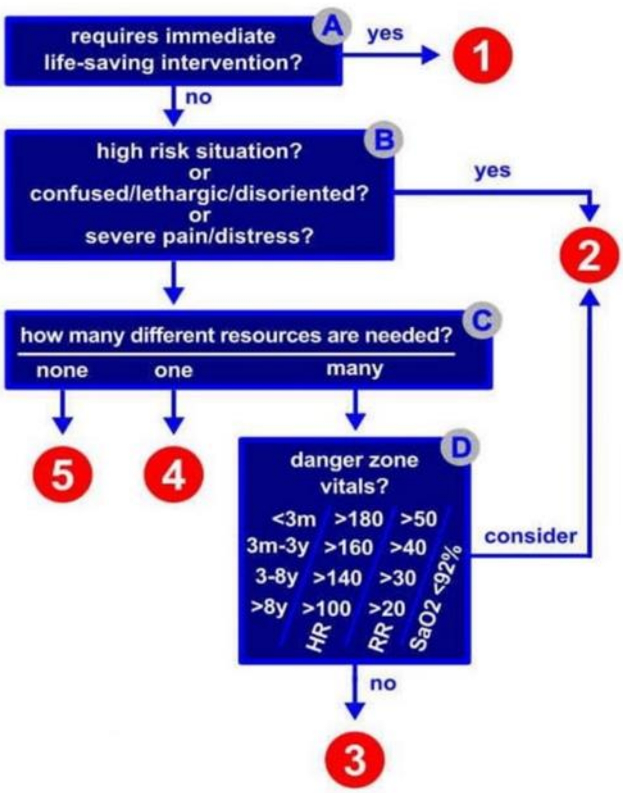

Figure 1. The ESI triage algorithm (Gilboy et al., 2012)

2. LITERATURE REVIEW

Decision making is a long-standing field that has been widely researched by various decision-making tools (Taherinezhad & Alinezhad, 2023; Taherinezhad & Alinezhad, 2022; Alinezhad & Taherinezhad, 2021; Khalili & Alinezhad, 2018; Sarrafha et al., 2014; Kiani Mavi et al., 2010; Alinezhad et al., 2007). For example, specifically in the area of decision making using non-dynamic MADM, we can refer to İşler and Çalık's (2022) study. İşler and Çalık (2022) proposed the use of the WASPAS technique to select Islamic financial trades, focusing on the problem of "investment according to Islamic principles". In addition, they used the entropy method to determine the weights of criteria. In another research, Norouziyan (2022) focused on a petrochemical case study, using analytic hierarchy process (AHP) and VIKOR methods to determine the weights of criteria and ranking the alternatives, respectively. Also, Ramadan and Özdemir (2022) prioritized Istanbul rail system projects using Fuzzy AHP and PROMETHEE. The important point is that the criteria did not change in any of the decision-making stages in the mentioned studies and were constant. In other words, these MADM problems are non-dynamic. While this paper focuses on a dynamic problem. Therefore, in order to review the detailed and scientific literature, we limited the search for articles to the field of dynamic MADM (DMADM). For the first time, Brehmer (1992) examined decision-making under conditions in which decisions are not independent and the state of the surrounding world changes (dynamically), and presented a general method based on control theory as a means of organizing research in this field. Badiru et al., (1993) presented a decision support system based on the simulation and Analytic Hierarchy Process (AHP) method, which is called dynamic decision-making and can be used to implement dynamic decision-making scenarios. Lin et al., (2008) presented a dynamic decision-making model whose main structure is based on the TOPSIS method. Also in it, integration and integration of the concepts of gray numbers and the Minkowski distance function have been done in order to deal with uncertain information. Wei (2009) investigated the problem of dynamic intuitionistic fuzzy MADM in which all attributes' values are expressed as intuitionistic fuzzy numbers or interval values of intuitionistic fuzzy numbers. In addition, he has presented some geometric cumulative operators such as the Dynamic Intuitionistic Fuzzy Weighted Geometric (DIFWG) operator and the Uncertain Dynamic Intuitionistic Fuzzy Weighted Geometric (UDIFWG) operator to collect uncertain dynamic intuitionistic fuzzy information. Chen and Li (2011) presented a dynamic MADM model based on Triangular Intuitionistic Fuzzy Numbers (TIFN) to solve DMADM problems, where all decision information was in TIFN form. Hu and Yang (2011) also proposed a method based on cumulative prospect theory and pair set analysis to solve stochastic dynamic decision-making problems in which the weight information of the criteria is completely unknown and the values of the criteria are in the form of discrete random variables. Campanella and Ribeiro (2011) introduced a flexible framework for solving the DMADM problem based on the classical model, which can be applied to any dynamic decision-making process. This framework aims to solve the above problem by expanding the classic MCDM model in a flexible way. Wang et al., (2015) presented an interval dynamic reference point-based method for Emergency Decision Making (EDM) problems. The above method uses a method similar to TOPSIS, which is a popular decision-making technique, to rank the alternatives. Lourenzutti and Krohling (2016) developed the TOPSIS technique and presented the Group Modular Random TOPSIS (GMo-RTOPSIS) method for group decision making with heterogeneous information and in a dynamic environment. In this method, each decision maker can independently define the set of attributes, the weight vector, and the basic factors effective in ranking the alternatives, as well as the type of information for each attribute.

In the following, we will review the research done on the problem of prioritizing emergency department patients through decision tools. Chen et al., (2010) presented an analytical framework for Dynamic Multiple Criteria Decision Analysis (DMCDA) problems as an extension of classical static MCDA. Their research process was such that an overview of MCDA was done and an introduction to DMCDA was stated. Then, various design aggregation strategies and an analytical framework of DMCDA were described in detail. Finally, an emergency management case study was provided using data from the Emergency Management Australia (EMA) database to demonstrate the feasibility of the proposed analysis method. Ashour and Okudan (2012) believe that the triage process relies on the interaction of the nurse with the patients and then classifying them based on the severity of the disease. They used the Fuzzy AHP algorithm and Multi-Attribute Utility Theory (MAUT) to rank patients according to their attributes including chief presenting complaint, age, sex, pain intensity and vital signs. Also, this algorithm has been applied to a sample of clinical data set from Susquehanna Health's William Sport. In addition, Chang (2014) also presented a scientific and systematic framework based on the concepts of fuzzy sets and using the VIKOR method to evaluate the quality of hospital services under conditions of uncertainty. By reviewing previous researches, we find that none of them have focused on the evaluation of ESI triage systems in dynamic conditions, while considering the necessity of prioritizing patients and its widespread use in emergency departments, the need to conduct such a study is strongly felt. Therefore, according to this need and gap, the main contribution of this paper in the literature will be the use of the fuzzy VIKOR method in the dynamic environment of American 5-level triage (ESI).

3. MATERIAL AND METHODS

3.1. VIKOR Method

Opricovic and Tzeng (2004) developed the VIKOR method for optimizing MCDM problems in complex systems. This method focuses on ranking and selecting from a set of alternatives and determines compromise solutions to the problem with conflicting attributes, so that it is able to help decision makers to reach a final decision. Following previous research, Opricovic and Tzeng (2007) presented an extension of the VIKOR method to solve decision problems with conflicting and disproportional criteria (different measurement units). In this paper, the combination of the VIKOR method with the theory of fuzzy sets and linguistic variables is used to overcome the uncertainty in the ranking of alternatives. In addition, the group opinions of the decision-makers are used in such a way that the weights of the importance of each of the decision-makers in the final choice are different.

3.2. Triangular Fuzzy Numbers & Linguistic Variables

The basic theory of triangular fuzzy numbers is described by Dubois (1980), Klir and Folger (1988), and Klir and Yuan (1995), where a fuzzy number is considered as a normalized and convex fuzzy set. The triangular fuzzy number $\tilde{n}$ is represented as a triplet set $\tilde{n} = (n_1, n_2, n_3)$ and shown as in Figure 2.

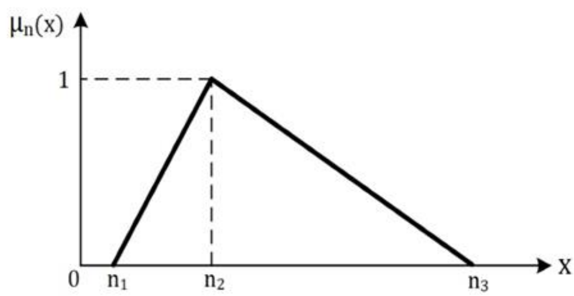

Figure 2. View of triangular fuzzy number $\tilde{n}$ (Chen, 2000)

The membership function of triangular fuzzy numbers is defined as Equation 1:

$$\mu_{\tilde{n}}(x) = \begin{cases} \dfrac{x - n_1}{n_2 - n_1} & , \quad n_1 \leq x \leq n_2 \\ \dfrac{x - n_3}{n_2 - n_3} & , \quad n_2 \leq x \leq n_3 \\ 0 & , \quad otherwise \end{cases} \tag{1}$$

Where n_1 and n_3 are the lower and upper limits of the fuzzy number $\tilde{n}$, respectively, and n_2 is the middle limit of $\tilde{n}$. Fuzzy numbers play an important role in quantitatively formulating fuzzy variables, and fuzzy variables can be linguistic variables. Figure 3 shows an example of linguistic variables in fuzzy form. In determining the membership function of linguistic variables, one of the variables is assumed as the base variable and the membership function is determined for it. Then the membership function of other linguistic variables is obtained using special relations based on the base variable. Each base variable is defined based on physical variables or numerical variables (Zadeh, 1983).

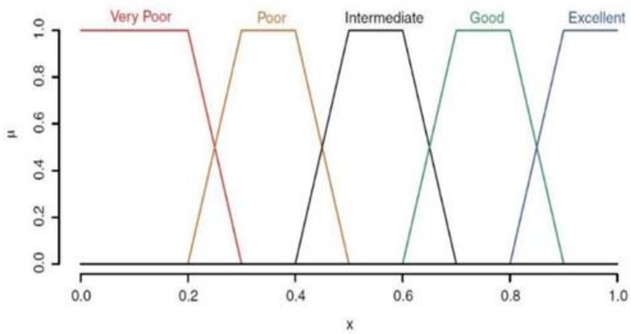

Figure 3. An example of linguistic variables in the form of trapezoidal fuzzy numbers (Zadeh, 1983)

3.3. Proposed Model

In this part, the problem is modeled with the VIKOR method, which is the basis of this research. Appropriate common functions have also been used to calculate scores in modeling. The modeling is the same for the five different levels of triage (ESI) and they differ only in the shared functions. Supposedly, for the first level, which is the level of emergency patients, we have used a stronger sharing function than other levels to calculate scores. In the fifth level, which is related to outpatients and is more crowded than other levels, weaker sharing functions can be used. Figure 4 shows the dynamic decision-making diagram in the ESI system. The dynamics of the problem is determined using a maintenance policy, which we will talk about later.

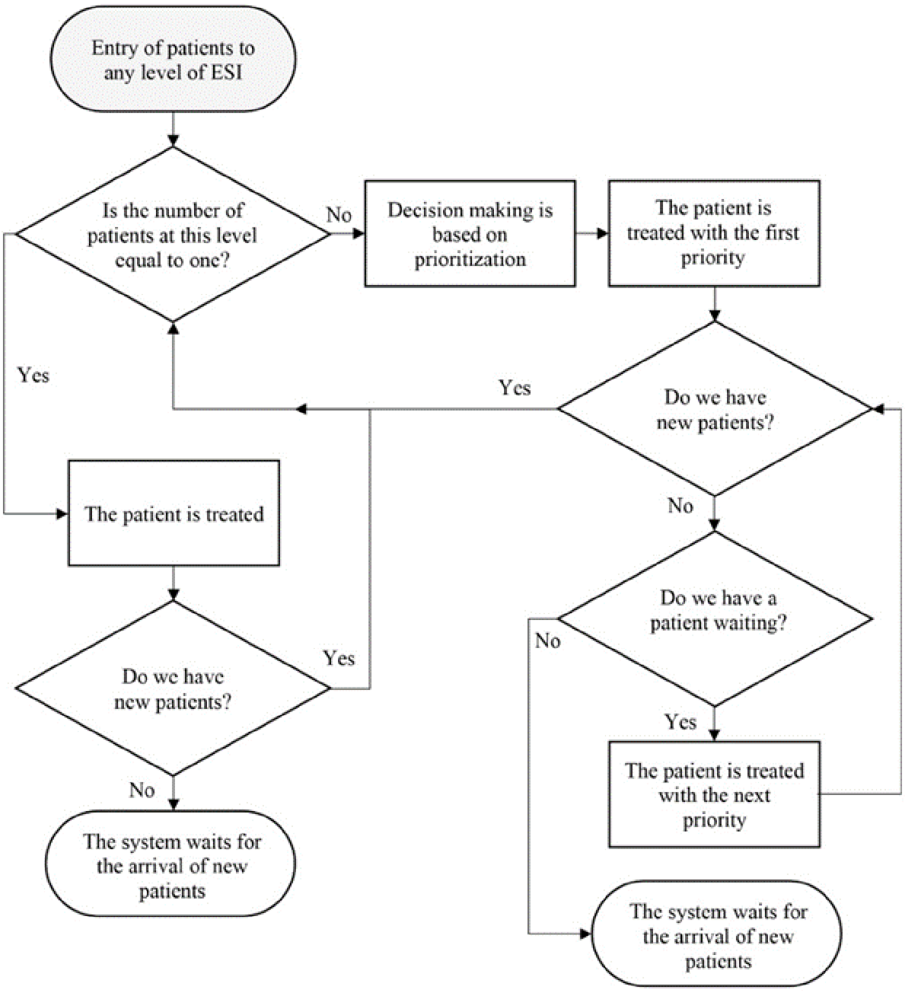

Figure 4. Prioritizing patients according to the dynamic cycle in the ESI system

3.3.1. Symbols Definition

Before stating the proposed model, we will introduce the symbols and some policies considered in it:

P_t: The set of patients available at the moment t
C_t: The set of criteria available at moment t
$\widetilde{W}_c$: Weight vector of criteria (considered as fuzzy numbers)
t_{ij}: The time of entering the ith patient from the jth level for treatment ($j = 1, 2, 3, 4, 5$)
p_{ij}: Patient i of the level j ($j = 1, 2, 3, 4, 5$)
t_n: Time of next patient arrival (new arrival)
H_t: The set kept to the next iteration in the tth iteration
U_t: The performance function in the tth iteration
R_t: Ranking of alternatives in the tth iteration in the first stage of decision-making (VIKOR method ranking)
E_t: Evaluation function in the tth iteration
O_t: Ranking of the alternatives in the tth iteration in the second stage of decision-making (final ranking)

D_E: Shared function considered in the second stage of decision making

3.3.2. Maintenance Policy

Because the current research model is implemented in a dynamic environment, it is necessary to define a maintenance policy. That is, a criterion for selecting a subset of current and past alternatives that are taken to the next iteration. The set maintained by the next iteration can be defined in different ways. One of these definitions is given in Equation 2:

$$H_t = \begin{cases} p_{ij} \notin H & , \quad if \ : \ t_{ij} \le t_n \\ p_{ij} \in H & , \ otherwise \end{cases} \tag{2}$$

Where t_{ij} is the time of entry of the ith patient from the jth level (the level means the same 5 levels of ESI triage) for treatment and t_n is the time of arrival of a new patient. As long as t_{ij} is smaller than t_n, the patient p_{ij} is directed for treatment, otherwise, it will be a member of the maintained set and will be evaluated again for prioritization with new patients. According to the above definition, the patients who were not treated in the tth iteration will be members of the H set in the $(t+1)$th iteration.

3.3.3. Dynamic Decision-Making Process

In this research, the end is not considered for the decision-making process. It means that the patient can enter the emergency department at any moment. So, the system will always be in a decision cycle. Usually, the task of prioritization is carried out by an experienced triage nurse. The iterations consist of two main stages, which can be seen in Figure 5.

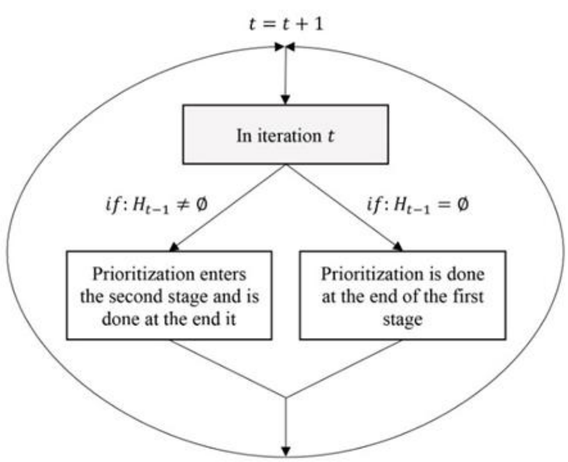

Figure 5. The cycle of iterations in the dynamic decision-making process

t stands for iterations in the dynamic decision process. In each iteration, the first stage is performed first. Then, if the maintenance set is empty in the previous iteration, prioritization is formed at the end of the first stage. Otherwise (if the maintenance set is not empty in the previous iteration), we enter the second stage and then perform the prioritization. Now, according to the above definitions and Figure 5, we describe the dynamic decision-making process. The first stage consists of 7 steps:

- Step 1: In this step, the decision-maker must express his opinion about the patients according to the measures that have been specified in advance and assign a score to the patient according to the tables in step 2. It should be noted that the evaluation criteria must also be specified in advance.
- Step 2: In this step, we convert the qualitative variables selected in the previous step into quantitative variables using the following tables. Using Table 1, the importance of the weight of each criterion can be converted into quantitative variables in the fuzzy environment.

Table 1. Linguistic labels for the weight importance of each criterion

Linguistic labels	Fuzzy equivalent
Very low	$(0, 0, 0.1)$
Low	$(0, 0.1, 0.3)$
Medium low	$(0.1, 0.3, 0.5)$
Medium	$(0.3, 0.5, 0.7)$
Medium high	$(0.5, 0.7, 0.9)$
high	$(0.7, 0.9, 1)$
Very high	$(0.9, 1, 1)$

Blood pressure, respiration, temperature and pulse ranges are different for different ages (Stewart, 2003). By using Tables 2 to 7, the qualitative variables of the alternatives can be converted into quantitative variables in the fuzzy environment.

Table 2. Linguistic labels for ranking alternatives based on measures of blood pressure and respiratory status

Linguistic labels	Linguistic variables in short	Fuzzy equivalent
Normal	N	$(0, 1, 3)$
Low	L	$(1, 3, 5)$
Medium low and Medium high	$ML - MH$	$(3, 5, 7)$
Low and High	$L - H$	$(5, 7, 9)$
Very low and very high	$VL - VH$	$(7, 9, 10)$

Table 3. Linguistic labels for ranking alternatives based on degree of consciousness criteria

Linguistic labels	Linguistic variables in short	Fuzzy equivalent
Very low	VL	$(0, 1, 3)$
Low	L	$(1, 3, 5)$
Medium	M	$(3, 5, 7)$
High	H	$(5, 7, 9)$
Very High	VH	$(7, 9, 10)$

Table 4. Explanation of linguistic labels based on the level of consciousness

Level of consciousness	Description
Alert	The patient is fully awake.
Pain & Voice	The patient responds to sound or painful stimulation.
Voice	The patient’s eyes open while talking.

Pain	The patient does not respond to sound stimulation but responds to painful stimulation.
Unresponsive	The patient is unresponsive and does not respond to sound or painful stimulation.

Table 5. Linguistic labels for ranking alternatives based on criteria of pain intensity and required patient actions

Linguistic labels	Linguistic variables in short	Fuzzy equivalent
Alert	A	$(0, 1, 3)$
Pain & Voice	$P - V$	$(1, 3, 5)$
Voice	V	$(3, 5, 7)$
Pain	P	$(5, 7, 9)$
Unresponsive	U	$(7, 9, 10)$

Table 6. Linguistic labels for ranking alternatives based on fracture degree criteria

Linguistic labels	Linguistic variables in short	Fuzzy equivalent
Degree 0	D_0	$(0, 1, 3)$
Degree 1	D_1	$(1, 3, 5)$
Degree 2	D_2	$(3, 5, 7)$
Degree 3	D_3	$(5, 7, 9)$
Degree 4	D_4	$(7, 9, 10)$

Table 7. Explanation of linguistic labels based on the criterion of degree of fracture

Degree of fracture	Description
Degree 0	Fracture can be seen only as a crack.
Degree 1	Despite the fracture, the skin remains healthy and does not get injured.
Degree 2	A fracture causes the skin to tear, but it is not associated with a wound.
Degree 3	A fracture causes the skin to tear, but it is associated with a wound.
Degree 4	A fracture causes damage to other organs such as veins and nerves.

- Step 3: According to the previous two steps, the weight matrix of the criteria and the fuzzy decision matrix are in the form of Equation 3:

$$\widetilde{W}_c = \{\widetilde{w}_1 , \widetilde{w}_2 , \widetilde{w}_3 , \dots , \widetilde{w}_n \} \quad \approx \quad \forall\, \widetilde{W}_i = (\widetilde{w}_{i1} , \widetilde{w}_{i2} , \widetilde{w}_{i3}) \tag{3}$$

Where $\widetilde{W}_c$ is the weight of the nth criterion in the form of a triangular fuzzy number, and the elements of the decision-making matrix in Equation 4 are also triangular fuzzy numbers:

$$\widetilde{D}_t = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \quad \Longrightarrow \quad \forall\, \tilde{x}_{ij} = \left(\tilde{a}_{ij} , \tilde{b}_{ij} , c_{ij} \right) \tag{4}$$

$\tilde{x}_{mn}$ is the mth alternative score according to the nth criterion.

- Step 4: In this step, the largest and smallest number of each column is determined from Equation 5:

$$\tilde{f}^* = \max_i\{x_{ij}\} \quad , \quad \tilde{f}^- = \min_i\{x_{ij}\} \tag{5}$$

- Step 5: Average level of regret (S) and maximum regret (R) for each patient are calculated from Equation 6:

$$S_i = \sum_j^n \left(w_j \times \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \right) \quad , \quad R_i = Max_j \left(w_j \times \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \right) \tag{6}$$

Where S_i represents the relative distance of the ith alternative from the positive ideal solution (the best combination) and R_i represents the maximum regret of the ith alternative from the positive ideal solution.

- Step 6: Now, for the final evaluation of patients, the VIKOR index (Q) is calculated from Equation 7:

$$Q_i = \left(\nu \times \frac{(S^* - S_i)}{(S^* - S^-)} \right) + \left((1 - \nu) \times \frac{(R^* - R_i)}{(R^* - R^-)} \right)$$

$$S^* = Min\ \{S_i\} \quad , \quad S^- = Max\ \{S_i\}$$

$$R^* = Min\ \{R_i\} \quad , \quad R^- = Max\ \{R_i\} \tag{7}$$

ν is a number between zero and one and it is usually considered 0.5. The closer the value of ν is to one, it indicates that the decision maker is more interested in using the weighted value of utility and the involvement of all criteria than the maximum utility (Opricovic & Tzeng, 2007).

- Step 7: Any alternative that has a lower value of Q_i will have a higher priority for selection. At the end of these 7 steps, an efficiency matrix according to Equation 8 will be obtained and a VIKOR index (Q) will be obtained for each alternative.

$$U_t = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{bmatrix} \tag{8}$$

At the end of this stage, the set H_{t-1} is decisive. If the set H_{t-1} is empty, the evaluation function for each patient will be equal to the efficiency function of that patient. By defining the evaluation function, the ranking (R_t) is created and the maintained collection is also determined in the next iteration (H_t). If the set H_{t-1} is not empty, we go to the second step.

At this step, due to the fact that the maintenance set is not empty, we use Equation 9 to calculate the evaluation function:

$$E_t(p) = \begin{cases} U_t(p) & , p \in P_t \\ D_E\big(E_{t-1}\ (p)\, , U_t(p)\big) & , p \in H_{t-1} \end{cases} \tag{9}$$

Where if the patient is a member of the P_t set and not a member of the H_{t-1} set, the same efficiency function is used to calculate the evaluation function, and if the patient is a member of the H_{t-1} set, we will use the shared function D_E. In order to calculate the shared function D_E, we have described various functions from the family of t-norms as follows (Equation 10 to 15). t-norms are introduced as an operator to combine distribution functions on statistical metric spaces (Schweizer & Sklar, 2005).

Each of these functions has its own characteristics, but their common characteristic is that they are a reduction function. The function of Equation 10 is the weakest and the function of Equation 15 is the strongest. In the numerical example section, it is explained which common function will be suitable for which level.

$$Minimum: \quad D_E\left(E_{t-1}(p), U_t(p)\right) = min\{E_{t-1}(p), U_t(p)\} \tag{10}$$

$$Product: \quad D_E\left(E_{t-1}(p), U_t(p)\right) = E_{t-1}(p) . U_t(p) \tag{11}$$

$$Lukasiewicz: \quad D_E\left(E_{t-1}(p), U_t(p)\right) = \max\{0, E_{t-1}(p) + U_t(p) - 1\} \tag{12}$$

$Nilpotent\ Minimum:$

$$D_E\left(E_{t-1}(p), U_t(p)\right) = \begin{cases} \min\{E_{t-1}(p), U_t(p)\} & , \quad E_{t-1}(p) + U_t(p) > 1 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{13}$$

$Hamacher\ Product:$

$$D_E\left(E_{t-1}(p), U_t(p)\right) = \begin{cases} 0 & , \ E_{t-1}(p) = U_t(p) = 0 \\ \dfrac{E_{t-1}(p) . U_t(p)}{E_{t-1}(p) + U_t(p) - E_{t-1}(p) . U_t(p)} & , otherwise \end{cases} \tag{14}$$

$$Drastic\ Product: \quad D_E\left(E_{t-1}(p), U_t(p)\right) = \begin{cases} E_{t-1}(p) & , \quad U_t(p) = 1 \\ U_t(p) & , \quad E_{t-1}(p) = 1 \\ 0 & , \quad otherwise \end{cases} \tag{15}$$

Now, with the evaluation function specified, we rank the alternatives and determine the retention set for the next iteration. Figure 6 shows a summary of the important steps in the decision-making process.

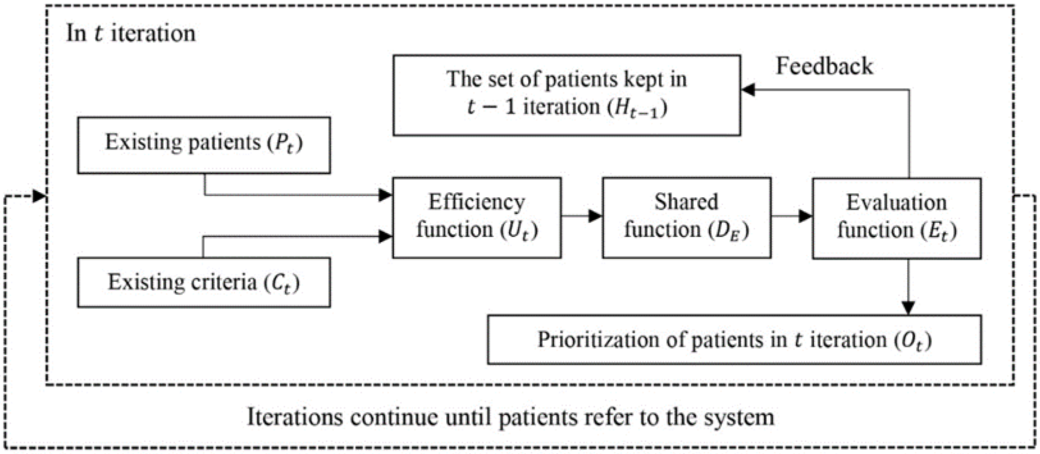

Figure 6. Operations performed in each iteration in the dynamic decision model

4. RESEARCH IMPLEMENTATION IN CASE STUDY

The investigation of this research is related to the prioritization of patients who visit the emergency department on a normal day. This problem is implemented in the Excel environment and prioritizes patients in each iteration. Patients of one of the ESI triage levels are considered and prioritized. Also, when prioritizing patients, the shared functions mentioned in the previous section are also compared and the best function is selected for the proposed model. The assumptions of the model are as follows:

- The arrival of patients is considered in time intervals. Here, patients are compared and prioritized in 15-minute intervals.
- The number of criteria and their weights can be changed in each iteration.
- Patients' conditions can be changed in each iteration.

The data of 20 studied patients are presented in Table 8. The prioritization of up to four iterations is checked as follows:

First iteration ($t = 1$): At time t_1, four patients have referred to the triage section in the emergency department. To evaluate patients, we have considered four criteria: blood pressure, respiratory status, level of consciousness and pain intensity. The weight of each of the criteria as well as the information related to the patients is given in the form of linguistic labels (triangular fuzzy numbers) in Tables 9 and 10. According to the calculations and prioritization, among the patients, patient P_3 is selected for treatment. It is assumed that no new patient has been referred to the triage system until the completion of the treatment of patient P_3. Therefore, patient P_4 is also treated. Until the completion of the treatment of patient P_4, no new patient has entered the system, so patient P_1 is also directed to the treatment department. During the treatment of patient P_1, two new patients refer to the system. Therefore, the patient with the fourth priority P_2 is considered as a member of the maintained set, and along with these two new patients, prioritization is done again.

Table 8. Data and information of patients in the study department

	Criteria				
	Blood pressure (C_1)	**Respiratory status (C_2)**	**level of consciousness (C_3)**	**Intensity of pain (C_4)**	**Actions required (C_5)**
W_j	H (0.7, 0.9, 1)	VH (0.9,1, 1)	MH (0.5, 0.7, 0.9)	M (0.3, 0.5 0.7)	MH (0.5, 0.7, 0.9)
P_1	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(9, 10, 10)	(5, 7, 9)
P_2	(9, 10, 10)	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(7, 9, 10)
P_3	(9, 10, 10)	(7, 9, 10)	(5, 7, 9)	(5, 7, 9)	(9, 10, 10)
P_4	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(1, 3, 5)	(1, 3, 5)
P_5	(9, 10, 10)	(1, 3, 5)	(5, 7, 9)	(7, 9, 10)	(7, 9, 10)
P_6	(7, 9, 10)	(3, 5, 7)	(1, 3, 5)	(7, 9, 10)	(5, 7, 9)
P_7	(1, 3, 5)	(1, 3, 5)	(0, 1, 3)	(3, 5, 7)	(3, 5, 7)
P_8	(5, 7, 9)	(7, 9, 10)	(5, 7, 9)	(1, 3, 5)	(0, 1, 3)
P_9	(7, 9, 10)	(1, 3, 5)	(3, 5, 7)	(7, 9, 10)	(7, 9, 10)
P_{10}	(1, 3, 5)	(0, 1, 3)	(1, 3, 5)	(5, 7, 9)	(5, 7, 9)
P_{11}	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(0, 1, 3)	(3, 5, 7)
P_{12}	(3, 5, 7)	(0, 1, 3)	(1, 3, 5)	(1, 3, 5)	(3, 5, 7)
P_{13}	(1, 3, 5)	(3, 5, 7)	(7, 9, 10)	(5, 7, 9)	(9, 10, 10)
P_{14}	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(7, 9, 10)	(7, 9, 10)
P_{15}	(3, 5, 7)	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(1, 3, 5)
P_{16}	(1, 3, 5)	(3, 5, 7)	(3, 5, 7)	(9, 10, 10)	(5, 7, 9)
P_{17}	(7, 9, 10)	(1, 3, 5)	(9, 10, 10)	(5, 7, 9)	(0, 1, 3)
P_{18}	(3, 5, 7)	(5, 7, 9)	(7, 9, 10)	(3, 5, 7)	(0, 1, 3)
P_{19}	(7, 9, 10)	(0, 1, 3)	(1, 3, 5)	(7, 9, 10)	(3, 5, 7)
P_{20}	(1, 3, 5)	(7, 9, 10)	(5, 7, 9)	(9, 10, 10)	(5, 7, 9)

Table 9. Fuzzy weight values of criteria and patients' information in the first iteration

	C_1	C_2	C_3	C_4
W_j	H (0.7, 0.9, 1)	VH (0.9,1, 1)	MH (0.5, 0.7, 0.9)	M (0.3, 0.5 0.7)
P_1	(1, 3, 5)	(7, 9, 10)	(1, 3, 5)	(5, 7, 9)
P_2	(5, 7, 9)	(3, 5, 7)	(0, 1, 3)	(1, 3, 5)
P_3	(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(0, 1, 3)
P_4	(7, 9, 10)	(1, 3, 5)	(7, 9, 10)	(9, 10, 10)

Table 10. Prioritizing patients in the first iteration

	S_i	R_i	Q_i	R
P_1	1.319	0.875	0.659	3
P_2	1.744	0.700	0.661	4
P_3	1.408	0.569	0.328	1
P_4	0.775	0.975	0.503	2

Second iteration ($t = 2$): Due to the dynamic nature of the system, a new criterion (C_5: Actions required) has been added to the other criteria. The fuzzy values of criteria weight and patients' information are presented in Table 11 and the prioritization of patients in the second iteration is presented in Table 12.

Table 11. Fuzzy weight values of criteria and patients' information in the second iteration

	C_1	C_2	C_3	C_4	C_5
W_j	H (0.7, 0.9, 1)	VH (0.9,1, 1)	MH (0.5, 0.7, 0.9)	M (0.3, 0.5 0.7)	MH (0.5, 0.7, 0.9)
P_1/P_2	(5, 7, 9)	(3, 5, 7)	(0, 1, 3)	(1, 3, 5)	(5, 7, 9)
P_2	(3, 5, 7)	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(1, 3, 5)
P_3	(5, 7, 9)	(1, 3, 5)	(9, 10, 10)	(7, 9, 10)	(7, 9, 10)

Table 12. Prioritizing patients in the second iteration

	S_i	R_i	Q_i	R	E_1	E_2	E_3	E_4	E_5	E_6	O_1	O_2	O_3	O_4	O_5	O_6
P_1	2.361	0.925	0.738	3	0.661	0.488	0.399	0.661	0.535	0	3	2	1	3	2	1
P_2	1.852	0.875	0.479	1	0.479	0.479	0.479	0.479	0.479	0.479	1	1	2	1	1	2
P_3	1.326	1.100	0.607	2	0.607	0.607	0.607	0.607	0.607	0.607	2	3	3	2	3	3

In this iteration, in order to better compare the results, we have used the mentioned six shared functions (Equations 10 to 15, respectively). By using functions one and four, patient P_1 will become a member of the maintained set, and by using functions two, three, five and six, patient P_3 will become a member of the maintained set. According to the obtained results, evaluation functions one and four are not suitable functions for evaluating the most urgent level of the ESI standard, because this level requires a stronger common (descending) function to reduce the VIKOR index (Q) of patients. Patients who have been waiting for previous courses. It is assumed that no new patient will come to the system until the treatment of the patient with the first priority is completed, and two new patients will be admitted during the treatment of the patient with the second priority. Therefore, evaluation functions one and four have been removed, and the patient with the third priority of this iteration (P_3) along with two newly arrived patients will go to the third iteration and will be evaluated using functions two, three, five and six.

The third iteration ($t = 3$): In this iteration, another criterion is added to the criteria of the evaluation system (C_6: Degree of fracture). The fuzzy values of criteria weight and patient information are presented in Table 13 and the prioritization of patients in the third iteration is presented in Table 14. According to the calculations, using functions two and five, patient P_1 (patient P_3 in the previous iteration) is considered a member of the maintained set, and using functions three and six, patient P_2 is a member of the maintained set. Therefore, due to the reasons stated in the previous section, functions two and five are not suitable functions for evaluation and are removed. Assuming that when treating the patient with the second priority (P_3), we have a newly arrived patient, the patient with the third priority (P_2) goes to the next iteration together with the new patient, and they are evaluated by using functions three and six.

Table 13. Fuzzy weight values of criteria and patients' information in the third iteration

	C_1	C_2	C_3	C_4	C_5	C_6
W_j	H (0.7, 0.9, 1)	VH (0.9,1, 1)	MH (0.5, 0.7, 0.9)	M (0.3, 0.5 0.7)	MH (0.5, 0.7, 0.9)	ML (0.1, 0.3, 0.5)
P_1/P_3	(5, 7, 9)	(1, 3, 5)	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)	(3, 5, 7)
P_2	(7, 9, 10)	(3, 5, 7)	(1, 3, 5)	(7, 9, 10)	(5, 7, 9)	(7, 9, 10)
P_3	(5, 7, 9)	(7, 9, 10)	(3, 5, 7)	(1, 3, 5)	(7, 9, 10)	(0, 1, 3)

Table 14. Prioritizing patients in the third iteration

	S_i	R_i	Q_i	R	E_2	E_3	E_5	E_6	O_2	O_3	O_5	O_6
P_1	2.404	0.925	0.765	3	0.465	0.372	0.512	0	3	1	3	1
P_2	2.359	0.838	0.451	2	0.451	0.451	0.451	0.451	2	3	2	3
P_3	2.230	0.888	0.434	1	0.434	0.434	0.434	0.434	1	2	1	2

The fourth iteration ($t = 4$): According to the dynamics of the system, in this iteration, the weight of the second criterion (respiratory status) and the third criterion (level of consciousness) has decreased compared to the previous period. The fuzzy weight values of criteria and patients' information are presented in Table 15 and the prioritization of patients in the fourth iteration is presented in Table 16.

Table 15. Fuzzy weight values of criteria and patients' information in the fourth iteration

	C_1	C_2	C_3	C_4	C_5	C_6
W_j	H (0.7, 0.9, 1)	MH (0.5, 0.7, 0.9)	M (0.3, 0.5, 0.7)	M (0.3, 0.5 0.7)	MH (0.5, 0.7, 0.9)	ML (0.1, 0.3, 0.5)
P_1/P_2	(7, 9, 10)	(3, 5, 7)	(1, 3, 5)	(7, 9, 10)	(5, 7, 9)	(7, 9, 10)
P_2	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)	(3, 5, 7)	(7, 9, 10)	(1, 3, 5)

Table 16. Prioritizing patients in the fourth iteration

	S_i	R_i	Q_i	R	E_3	E_6	O_3	O_6
P_1	2.766	0.875	0.756	2	0.341	0	2	1
P_2	2.047	0.675	0.244	1	0.244	0.244	1	2

Based on the results obtained from functions three and six, function six is more suitable. Since level one is the most sensitive and urgent level, therefore, the strongest shared function (namely function six) is chosen for evaluating patients. In the next iterations, patients are prioritized and treated using this function. For the way of prioritizing patients at other ESI triage levels, it is possible to use other weaker shared functions that were introduced in the previous section, based on the amount of urgency and the sensitivity of the patients' waiting time. The proposed framework for solving dynamic decision-making problems is implemented as a case study in the Emergency Department (ED) of Edalatian in Mashhad city in Iran. Figure 7 shows the flow process diagram of patients in Edalatian emergency center.

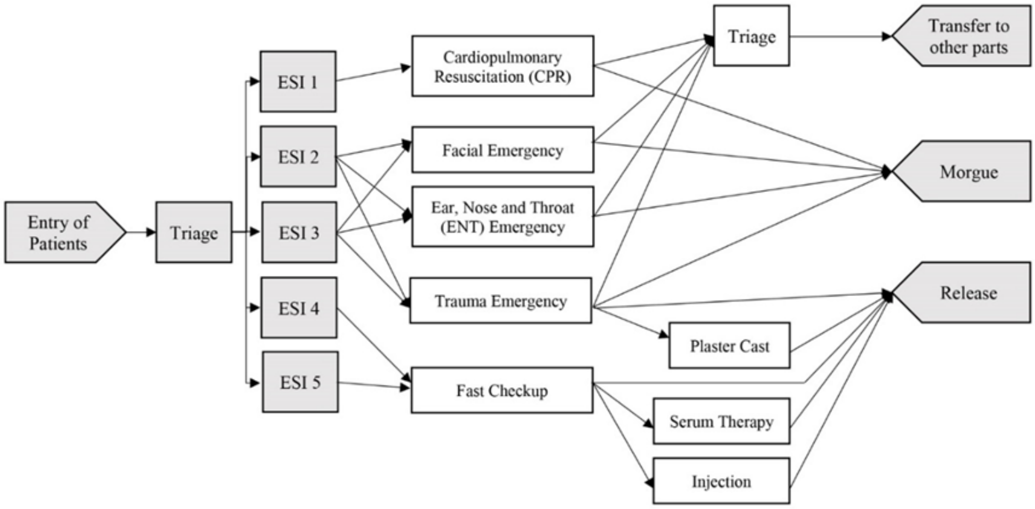

Figure 7. Flow chart of patients in Edalatian emergency center

In order to calculate the waiting time of patients in the normal state and compare it with the state where we use the proposed framework, a computer simulation has been done using Arena software. In order to determine the necessary statistical distributions, information has been collected from the documentation office of the nursing unit of the triage department. This information includes times between patient arrivals as well as service time in different emergency departments for 100 patients. Using the data input analyzer menu in the Arena software, the corresponding statistical distributions have been obtained, which are mentioned below. The process of entering patients has a beta distribution with different parameters for each level, which is presented in Table 17. The distribution of service time to patients in different parts of the emergency department is also presented in Table 18.

Table 17. Statistical distribution of triage levels (Time in minutes)

ESI Levels	Distribution	Description
ESI 1	BETA	$270 + 297 * BETA(0.866, 0.889)$
ESI 2	BETA	$84.5 + 184 * BETA(0.721, 0.785)$
ESI 3	BETA	$64.5 + 57 * BETA(0.955, 0.987)$
ESI 4	BETA	$51 + 39 * BETA(0.931, 0.961)$
ESI 5	BETA	$29 + 46 * BETA(1.08, 1.25)$

Table 18. Statistical distribution of service in different parts of the emergency (Time in minutes)

Part	Distribution	Description
CPR	BETA	$4.5 + 5 * BETA(0.851, 0.952)$
Triage	BETA	$0.5 + 6 * BETA(1.08, 0.977)$
Facial Emergency	BETA	$14.5 + 16 * BETA(1.17, 1.09)$
ENT Emergency	BETA	$20.5 + 16 * BETA(1.09, 1.13)$
Trauma Emergency	BETA	$14.5 + 31 * BETA(1.07, 1.04)$
Plaster Cast	BETA	$12.5 + 9 * BETA(1.09, 1.18)$
Fast Checkup	BETA	$2.5 + 5 * BETA(0.998, 1.06)$
Serum Therapy	BETA	$41.5 + 20 * BETA(0.854, 0.867)$
Injection	BETA	$2.5 + 4 * BETA(1.28, 1.13)$

5. DISCUSSION ON RESULTS

The aim of the simulation performed in this research is to estimate the waiting time of patients in different parts of the emergency department. Here is the Cardiopulmonary Resuscitation (CPR) part for comparison and analysis. The results of the simulation can be seen in Table 19.

Table 19. Arena software outputs

Part of Emergency	Waiting time (minutes)
CPR	12.11
Triage	1.73
Facial Emergency	17.90
ENT Emergency	8.59
Trauma Emergency	14.96
Plaster Cast	4.09
Fast Checkup	24.83
Serum Therapy	0.08
Injection	0.19

As the results show, patients will wait for an average of 12.11 minutes in the CPR part. In the static model where the queue type is FIFO (First In, First Out), the patient who is in serious condition must also wait in the queue and has no priority over other patients in the queue. While in the dynamic model, based on the presented prioritization, critical patients are treated sooner and their waiting time should naturally be reduced compared to the static state. Table 20 shows the average waiting time of patients in the CPR part along with their prioritization.

The first part of the table, which includes the arrival and service times of the patients, is obtained in such a way that we have reduced the simulation execution speed in the Arena software so that these times can be determined. The second part, which is related to iterations and prioritization of patients, is obtained from the implementation of the VIKOR method in Excel software. As it can be seen, decision-making has been done in 15-minute intervals and 9 iterations have occurred, and the output of the program presents 9 priorities. The obtained results show that the average waiting time of patients in the CPR part is 8.31 minutes, which has significantly decreased compared to the static state.

6. CONCLUSION AND FUTURE STUDIES

A proper triage system is a system that can perform the process of prioritizing patients in the best way in the shortest possible time. Although the type of triage system has a special effect on its performance, sometimes even the best systems are confused in prioritization. This is due to the inherent nature of triage. In the real world, the criteria at each decision point of the triage process are unstable and dynamic and can change constantly. If a scientific method for dynamic triage management is not developed, this issue will show its first effect on patients' waiting time.

Table 20. Waiting time (minutes) of patients in CPR part in dynamic mode

Patients	Arrival Time	Start of Service	End of Service	Waiting Time	t_1		t_2		t_3		t_4		t_5		t_6		t_7		t_8		t_9	
					E	O	E	O	E	O	E	O	E	O	E	O	E	O	E	O	E	O
P_1	6:04:22	6:16:01	6:21:03	11:39	0.77	**2**	0.00	**1**														
P_2	6:09:15	6:09:15	6:16:01	0:00	0.23	**1**																

P_3	6:16:29	6:21:03	6:29:40	4:34	0.10	**2**														
P_4	6:21:09	6:36:50	6:42:35	15:41	0.89	**4**	0.00	**1**												
P_5	6:25:50	6:29:40	6:36:50	3:50	0.74	**3**														
P_6	6:34:13	6:42:35	6:48:48	8:22			0.04	**2**												
P_7	6:38:58	6:59:43	7:05:11	20:45			1.00	**4**	0.83	**3**										
P_8	6:43:48	6:48:48	6:53:03	5:00			0.28	**3**	0.00	**1**										
P_9	6:49:24	6:53:03	6:59:43	3:39					0.04	**2**										
P_{10}	6:55:13	7:05:11	7:10:10	9:58					0.91	**4**	0.00	**1**								
P_{11}	7:03:11	7:10:10	7:18:46	6:59							0.08	**2**								
P_{12}	7:08:23	7:29:51	7:35:53	14:46							1.00	**4**	1.00	**3**						
P_{13}	7:14:41	7:18:46	7:23:09	4:05							0.33	**3**	0.00	**1**						
P_{14}	7:18:01	7:23:09	7:29:51	5:00									0.56	**2**						
P_{15}	7:33:01	7:35:53	7:44:54	2:52											0.09	**1**				
P_{16}	7:38:28	7:53:48	8:01:16	15:20											0.84	**3**	0	**1**		
P_{17}	7:44:25	7:44:54	7:53:48	0:29											0.78	**2**				
P_{18}	7:50:54	8:01:16	8:07:33	10:22													0.10	**2**	0.00	**1**
P_{19}	7:57:35	8:15:46	8:20:10	18:11													1.00	**3**	0.99	**3**
P_{20}	8:05:33	8:07:33	8:15:46	2:00															0.32	**2**

Due to the importance of this problem in hospitals, many algorithms have been presented to reduce the waiting time. In the present study, a dynamic algorithm based on MADM techniques and mathematical modeling of the problem was presented in order to prioritize patients in the emergency department. Also, in order to reduce the VIKOR index (Q) values of the patients belonging to the maintenance set, appropriate shared functions were used in the modeling of the problem.
By simulating the Mashhad Edalatian emergency center and estimating the average waiting time, the results of the proposed dynamic model were compared and analyzed with static models. The results showed that the waiting time in the dynamic algorithm was significantly reduced compared to the static algorithm. Therefore, the presented dynamic algorithm has a better capability and ability to reduce waiting time than static algorithms. Considering the extent of the subject of this research, it is suggested for future studies that by using other shared (cumulative) functions, the waiting time in dynamic mode and other static modes should be investigated. Specifically, the ideas that can be considered as future contributions in the literature of this field are as follows:

- Focusing on other types of triage systems and selecting appropriate emergency departments as a case study.
- Using dynamic group decision-making methods with a focus on solving other problems of triage systems such as: Increasing patient satisfaction, proper accommodation for patients, reducing costs.
- Using uncertainty methods (including neutrosophic, grey, probabilistic planning, robust optimization, etc.) for more realistic simulation of triage systems in order to more scientifically match the models with the real world.

Conflicts of Interests

The authors declared that there is no conflict of interest.

Contribution of Authors

The authors involved in this study are Ali Taherinezhad*, Alireza Alinezhad, and Saber Gholami; All authors contributed to the idea, design, resources, data collection, literature review, methods implementation and analysis and interpretation sections of the study.

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