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Investigation of Pre-service Middle School Mathematics Teachers Habits of Mind in Pattern Generalization Problems

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Abstract. In this study, it was aimed to determine pre-service middle school mathematics teachers' habits of mind. The method of the study was determined as a case study. Participants are 23 junior pre-service middle school mathematics teachers. Data were collected with four open-ended linear and quadratic pattern problems. The clinical interviews were conducted with six of the pre-service teachers. In the analysis of the data, content analysis was carried out using habits of mind framework for pattern generalization. The results of the study show that similar habits of mind revealed by pre-service teachers in problem situations. The pre-service teachers had more difficulty in balancing exploration and reflection compared to the other. It was found that the pattern types of the problems affected the habits of mind. It is suggested that more studies need to be done to reveal and foster the quality of algebra and algebraic thinking.

Keywords. Algebra, pattern, pattern generalization, algebraic thinking, habits of mind.

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Algebra is one of the important areas of mathematics education (Akkuş İspir & Palabıyık, 2011). There are several definitions of algebra in the literature by the mathematicians and mathematics educators. While Dede and Argün (2003) defined it as a language and thinking tool, Akkaya and Durmuş (2006) defined it as the science of abstraction, in which relations are examined and transformed into equations by using symbols and numbers. Kaput (1999) defined it as the order of generalization and patterns. Vance (1998) also defined it as a way of mathematical thinking. This definition created the concept of algebraic thinking, which includes the concept of algebra. Algebraic thinking is defined as the ability to represent quantitative relationships and generalize these relationships, recognize and analyse patterns (Steele & Johanning, 2004). NCTM (National Council of Teachers of Mathematics) (2000) defined algebraic thinking as using mathematical modelling to understand quantitative relationships with the help of algebraic symbols and analysing changes in daily life situations. The fact that algebra has taken its place in every field shows us the importance of learning algebra (Williams & Molina, 1998; Girit Yıldız & Gündoğdu Alaylı, 2019). Algebraic thinking is of great importance in ensuring students' success in mathematics (Amit & Neria, 2008; Eroğlu, 2021). In this way, patterns are of great importance for students to make sense of algebraic concepts and to develop algebraic thinking, this subject was given to students along with the pattern activities used in the introduction of algebra (Palank & Akkuş Ispir, 2011). Patterns are one of the main skills in learning mathematics because it requires identifying and explaining the different and similar features of concepts (Papic, 2007).

Patterns are divided into three groups: linear, quadratic and other patterns (Yeşildere & Akkoç, 2011). Linear patterns are patterns that continue by adding or decreasing units or numbers in an invariant way at each step. Quadratic patterns, on the other hand, are patterns in which the difference between steps increases or decreases at each step.

Pattern generalization activities enable students to see the relationship between mathematical concepts. In this way, students can establish relationships and make generalizations and predictions. Understanding patterns helps students support both their problem solving and abstract thinking skills (Olkun & Toluk-Ucar, 2007). Patterns play a very important role in the development of students' algebraic thinking (Özdemir, Dikici & Kültür, 2015). Zaskis and Liljedahl (2002) argued that patterns are the heart and essence of mathematics, because everything, especially algebra, is a generalization of patterns. Hargreaves, Taylor, and Trelfall (1998) point out that patterns related to numbers, geometry, and patterns help students understand the relationship between mathematical concepts. This relationship encourages mathematical thinking, which is the basis for learning more abstract thinking in the future (Ozdemir, Dikici, & Kültür, 2015).

It has been said that generalization is one of the important elements that will support the development of mathematical skills and mathematical thinking from an early age. This depends on studies with an appropriate teaching perspective to be determined (Tanışlı & Özdaş, 2009). One of the main purposes of mathematics teaching is generalization according to NCTM (2000) standards. Patterns can be the fundamental element of making generalizations. In generalizing patterns, an unchanging feature or relationship is discovered, an inference is made from this, a hypothesis is created from this inference, and this hypothesis is tested. Algebra is defined as "the generalization and order of patterns", as stated by Kaput (1999). In addition Lesley and Freiman (2004), Radford (2006) and Kaput (1999) have defined patterns and generalization-based teaching as an important element for developing algebraic thinking. Steele and Johanning (2004) stated that algebraic thinking forms the basis of pattern seeking and generalization. Friel and Markwoth (2009) see the disruption in the development of mathematical and algebraic thinking skills and the failure of these thinking skills to become a habit as one of the biggest reasons for failure in mathematics.

Pattern generalization problems in which different types of patterns are used, the common aspects of the steps in the pattern are determined, and the relationship between the steps is expressed in a way that is valid for each step (Radford, 2006). They support students' algebraic thinking by helping them make sense of the concepts of variable and function (Yakut Çayir & Akyüz, 2015; Radford, 2006).

Students' understanding of concepts and the development of problem solving skills are supported by teachers (NCTM, 2000). Therefore, in order for students to have algebraic thinking habits, teachers should be experienced in this subject and have relevant habits. Undergraduate education is very important for teachers to reveal habits of mind (Bülbül & Güven, 2020; Tolga, 2017; Zunlu, 2022).

Considering this situation, our study aimed to examine the pre-service middle school mathematics teachers' habits of mind in the process of solving pattern generalization problems. The research problem is 'Which habits of mind of pre-service middle school mathematics teachers' reveal in solving pattern generalization problems?'

Theoretical Framework

Habits of Mind: Habits of mind are defined as finding and applying the appropriate solution by performing high-level cognitive skills such as analysis, logical reasoning and critical thinking in the face of a problem situation (Leikin, 2007). In other words, it is a way of thinking that comes into play in cases where it is not possible to find out how to solve the problem and offers alternative options for solution (Costa & Kallick, 2000). In this process, the individual can recognize the relationships between concepts (Köse & Tanışlı, 2014). Habits of mind are divided into general habits and domain-specific habits of mind (Cuoco, Goldenberg & Mark, 1996). General habits of mind are basic skills such as making research, thinking, making assumptions, defining and visualizing (Korkmaz, 2015). Domain-specific habits of mind are discipline-specific habits such as mathematical, geometric, probabilistic, algebraic, analytical and scientific thinking (Bülbül & Güven, 2020). Mathematical habits of mind are defined as an element that enables student reasoning (Cuoco, Goldenberg & Mark, 1996) and the ability to reason by making abstractions when non-routine situations occur (Mark et al., 2010). Having mathematical habits of mind comes to the fore in areas such as algebra and geometry, and two sub-domains are formed as geometric habits of mind and algebraic habits of mind (Driscoll et al., 1996; Driscoll et al., 2007).

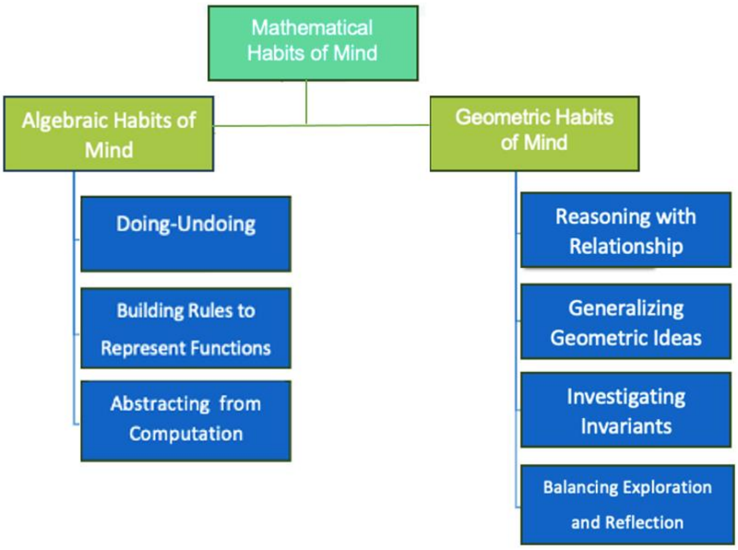


Figure 1. Mathematical Habits of Mind (Driscoll et al., 1996; Driscoll et al., 2007).

Algebraic Habits of Mind: Driscoll (1999) conceptualized two aspects of algebraic thinking, which he defined as the ability to think about functions and the effect of algebra on calculations, as Algebraic Habits of the Mind (AHoM). AHoM includes habits that an individual builds when faced with an algebraic situation (Ünveren Bilgiç & Argün, 2018). The theoretical framework of AHoM consists of three habits: "Doing-Undoing", "Building Rules to Represent Functions" and "Abstracting from Computation" (Driscoll, 1999).

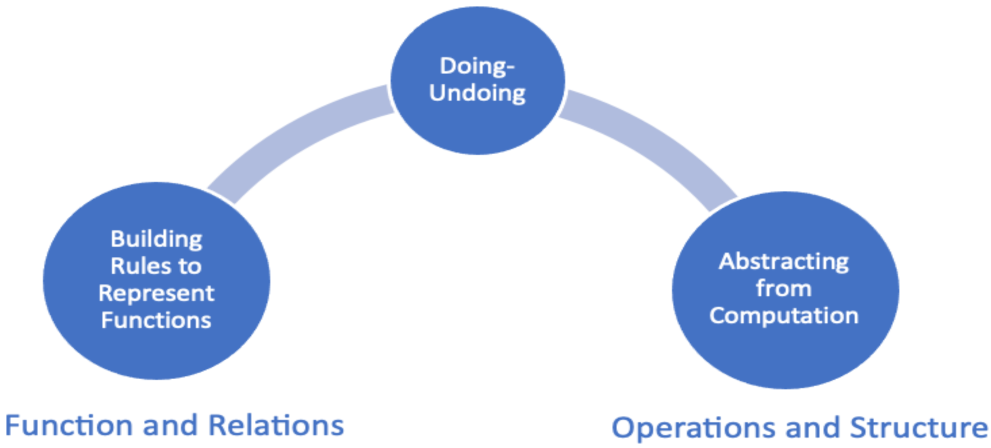


Figure 2. Algebraic Habits of Mind (Driscoll et al., 1999).

Doing-Undoing is the ability to analyze a mathematical task forward and backward in the solution process (Moyer, Huinker & Cai, 2004). The individual's ability to move from the result to the solution and provide proof is an indication that this skill is demonstrated. When solving a pattern problem, working forward from the beginning or working backwards from the result and verifying these results were revealed; so it was assumed that the doing-undoing habit was observed. For example, how many steps can be formed with the given amount in a pattern or the transformation from a shape pattern to a number pattern, from a number pattern to a shape pattern is observed. Doing-Undoing habit is addressed with the ability of understanding the problem (Schoenfeld, 2014) and this habit has some indicators. Similarly, doing-undoing habit is also inclusive for other habits, and it can occur in every problem solving situation (Eroğlu & Tanışlı, 2017). Building Rules to Represent Functions is a habit defined as seeking for patterns among the expressions given in a problem situation and making generalizations if a rule is found. It enables organizing data and determining input and output (Karyağdı, 2023). Actions such as seeking for the patterns, finding the rule of the pattern, identifying the general rule via representations can be used. In this habit, individuals try to concretize the process by thinking about different representations, looking for patterns, organizing the information given, using different representations (e.g. numerical, verbal, symbolic), and investigating the variant and invariant situations in the question. Abstracting from Computation is the habit to find operational shortcuts in solving the problem situation and to reach generalizations rather than operations. In this habit, the following behaviours are observed; generalizing by using the relationships between addition, subtraction, multiplication and division operations, transforming knowledge to develop shortcuts, knowing the meaning of the symbols and using them easily, using equivalent expressions by simplifying or complicating them depending on the need, and performing calculations on different systems. In other words, individuals think of operations in this habit independently of numbers (Driscoll, 1999).

Geometric Habits of the Mind: Geometric habits of mind are defined as ways of thinking that support the learning and application of the concepts of geometry (Köse & Tanışlı, 2014). Cuoco, Goldenberg and Mark (1996) describe it as the ability to reason, investigate geometric invariants, think of extreme values, visualize and manipulate. Driscoll et al. (2007) designed a theoretical framework based on their study "Fostering Geometric Thinking: A Guide for Teachers Grades 5-10". The theoretical framework consists of four basic components: "Reasoning with relationships", "Generalizing geometric ideas", "Investigating invariants" and "Balancing exploration and reflection".

Reasoning with relationships is the ability to determine the relations of congruence and similarity between geometric figures and objects or between themselves or with other figures and objects. Students think about how emerging relationships can help their understanding of geometric structures and problem solving processes (Uygan, 2016). The main indicators of this habit include the identification of the figures, the correct listing of their properties, relating the figures through symmetry and proportional reasoning (Driscoll et al., 2007). Generalizing geometric ideas is the ability to understand and define concepts while generalization. The questions at this point are "Does this happen in every case?" (Driscoll et al., 2007). It enables the determination under which conditions the feature or the relationship emerge (Köse& Tanışlı, 2014). The indicators of the habit of generalization are predicting general situations, checking these predictions, tabulating responses for these predictions, and making discussions according to the predictions made on the tabulating responses (Driscoll et al., 2007). Investigating invariants is considering what variant or invariant in the problem situation. Examining which features of the geometric figure change or not under such as translation, reflection and rotation can be given as examples of situations where this habit can be demonstrated (Tolga, 2017). “What are the differences between these figures?” can be an internal question of this habit. Balancing exploration and reflection habit involves using different strategies in solving the problem situation and evaluating these strategies. "What happens if I (draw a picture, add to/take apart this picture, work backward from the ending place, etc.)?" questions can be given as examples. The indicators of this habit are designing plans for a solution by making additional drawings on the figure, putting plans into action and checking them regularly to come up with new ideas (Driscoll et al., 2007).

When we examine the characteristics of the GHoM, it is concluded that there is no hierarchical structure between habits and more than one habit can be revealed at the same time in the same problem situation (Gürbüz, 2021).

Habits of Mind for Pattern Generalization: AHoM and GHoM are composed of common ways of thinking that can be observed in solving mathematical problems on different subject areas (Eroğlu, 2021). Patterns provides students with an introduction to algebra and the development of their algebraic thinking habits (Tanışlı & Özdaş, 2009; Radford, 2006). Eroğlu (2021) adapted a habits of mind framework that could be used in the teaching of the pattern of common which to suit the structure of GHoM (Driscoll et al., 2007) and AHoM (Driscoll, 1999).

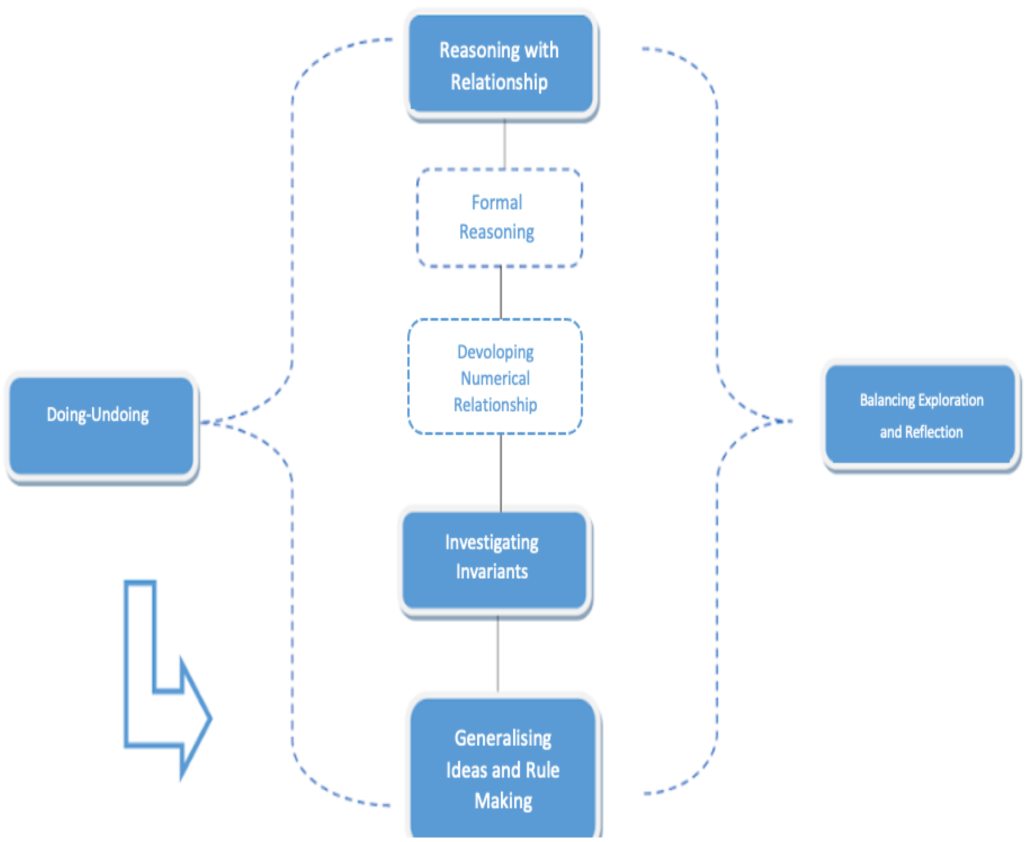


Figure 3. Habits of Mind for Pattern Generalization (Eroğlu, 2021, p.66)

According to Eroğlu's (2021) adapted theoretical framework; doing-undoing habit is defined as the ability to start from the beginning and work forwards or backwards in a pattern problem and to write symbolic relations by considering the visual structure of the pattern. Reasoning with relationships habit is characterized by the ability to determine the relationship between the components of the pattern, between the number of steps and the number of terms. Investigating invariants habit is described as the identification of variant and invariant elements in the structure of the pattern. Generalizing ideas and rule making habit is identified the ability to make generalizations regarding the further steps of the pattern by inferring the earlier steps of the pattern, and to determine whether the general rule of the pattern explained algebraically is valid at each step. Finally, balancing exploration and reflection habit is stated as the ability to try and evaluate various ways.

Method

Research Model

In this study, we aimed to examine habits of mind of preservice middle school mathematics teachers revealed while solving pattern generalization problems. In this qualitative study, the design of the research was selected as a case study. We believed that this design provides researchers to examine the case in depth and focus on the relationships between context and situation that are revealed in every aspect (Yıldırım & Şimşek, 2013). The case in this study is to examine the habits of mind revealed in pattern generalization problems of different pattern types. Since there is more than one analysis unit in this research, an embedded single-case design was used (Yin, 2003).

Study Group

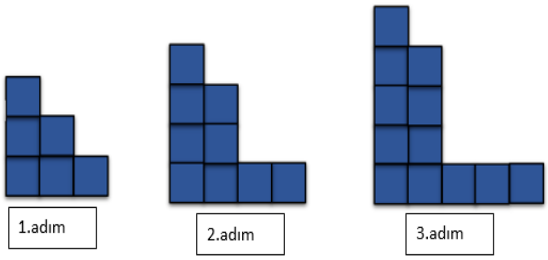
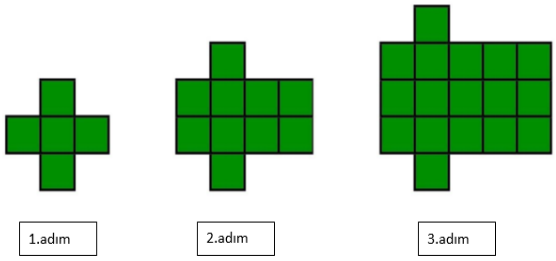
This study was conducted with 23 junior undergraduate students enrolled in the Middle School Mathematics Teacher Education Program in the academic year 2021-2022. While determining the participants, criterion sampling was used as one of the purposive sampling methods (Büyüköztürk et al., 2008). The criteria of the study were determined as being volunteered to participate in the study and being enrolled in the undergraduate level Mathematical Reasoning class where they gain experiences with pattern generalization problems.

Data Collection Tools and Processes

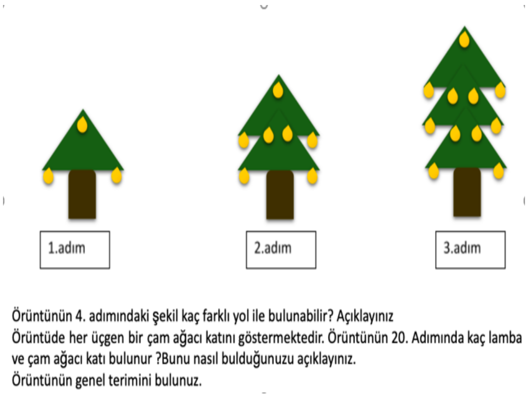
In the study, four pattern generalization problems were used as data collection tools. Two of the problems consist of linear patterns and the other two have quadratic patterns. Three of the problems were adapted from the pattern generalization problems used in the Mathematics Project PDST (Professional Development Service for Teachers) (URL-1) (2 quadratic and 1 linear). One of them was selected from Stacey (1989) study on linear pattern problems. While selecting the problems, the researchers considered different types of patterns that would most accurately reflect the habits of mind of the participants. Expert opinion was obtained and the final version of the problems revised according to the feedback received. The ethical permissions for the application was obtained and a consent form was filled out by the participants. The pre-service teachers were asked to answer the problems within 120 minutes. Before the application, the pre-service teachers were informed to explain their ways of solving the problem in detail. The problems are given in Table 1.

Table 1.

Pattern Generalization Problems

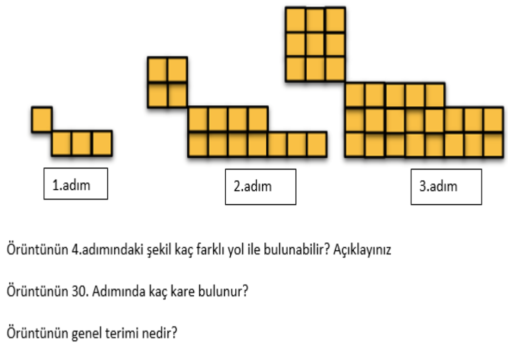
Problem 1	Problem 2
	
Örüntünün bir sonraki adımını bulmak için kaç farklı yol kullanabiliriz. Açıklayınız	Örüntünün 5. Adımını bulmak için kaç farklı yol kullanabiliriz. Açıklayınız
Örüntünün 10. adımında deseni yapmak için kaç kare gereklidir?	Örüntünün 20. adımını oluşturmak için kaç kare gereklidir?
Örüntünün genel terimini yazınız.	Örüntünün genel terimini yazınız.
Project Math (PDST, 2021)	Project Math(PDST)(2021)

Problem 3



Stacey (1989)

Problem 4



Project Math(PDST)(2021)

Köse and Tanışlı (2014) emphasized the importance of researcher observation notes and clinical interviews in addition to the worksheets to provide an in-depth explanation of the process, due to the results they obtained from the study they conducted with pre-service teachers. Starting from this point, individual clinical interviews were held with six pre-service teachers for data triangulation to understand their written statements more clearly. The pre-service teachers were given their own solutions and were expected to explain the approach that the researchers found inexplicit.

Data Analysis

The data were analysed according to the components of Eroğlu's (2021) habits of mind theoretical framework adapted from Driscoll (1999) and Driscoll et al. (2007). Internal validity was ensured by diversifying the data collection through the worksheets and clinical interview records. In order to ensure transferability, the data are described in detail. In order to ensure the consistency and the confirmability of the research, several data collection tools were used. The analysis of the data was carried out according to the theoretical framework. The analyzes were held by two coders and the inter-rater reliability percentage was found 0.86 which indicates high reliability (Miles & Huberman, 1994). The analyses were finalized while the consensus was reached between the coders.

Results

In the study, the findings of 23 pre-service teachers (PTs) regarding 4 problems with 3 stages are given below (R: Researcher, PT1, PT2….. PT23). The findings of the study are presented according to the habits in the framework. In each section, we represented the habit revealed in the patterns which are linear or quadratic.

Doing – Undoing

Linear Pattern Generalization Problems:

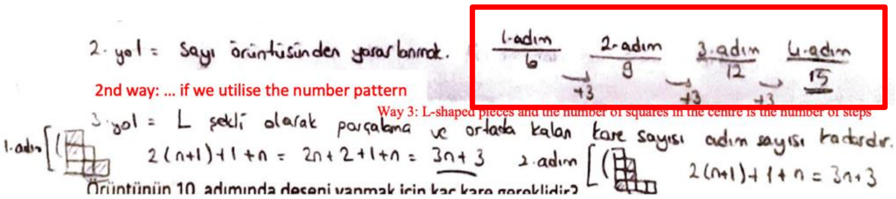


Figure 4. PT 11's solution to Problem 1.

In the first stage of the first problem, PT11 transformed the problem into a number pattern, found the general term and calculated the number of squares in the next step of the pattern using the general term. At the point of finding the number of squares, doing-undoing habit has been revealed. Then PT11 tried to generalize the pattern rule by using the visual sequence and developed a strategy based on dividing the shape into an "L" shape.

R: Why did you use a number pattern?...

PT11: It is easier and can be recognized by counting.

R: Well, can you explain what you did on the 3rd way?

PT11: I thought of the 3rd way as "L", I kept the square in the corner fixed outside and thought of it as one more square (than the previous steps)

In this problem, PT11 demonstrated this habit by working forward and backwards from the beginning.

Quadratic Pattern Generalization Problems:

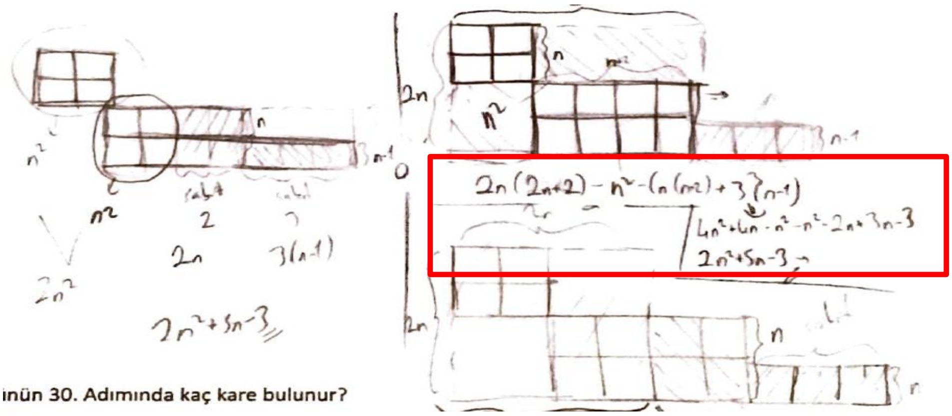


Figure 5. PT 8's solution to Problem 4.

PT8 tried to find the next step of the pattern by finding the general term in the first stage of the fourth problem. As a second way, PT8 tried to make an inference from the structure of the patterns.

R: What exactly did you do in problem 4?

PT8: It was difficult to find from the number pattern, I found n squares, I divided them into parts.

R: Can you try the number pattern?

PT8: In order to refer to the number pattern, I first look at the number of steps. It does not increase regularly, so it is difficult to find from the number pattern. I took it apart.

R: What did you notice when dissecting?

PT8: I completed (the shape) as a rectangle and focused where the invariant increase was found.

PT8 tried to focus on the structure of the pattern and transfer his/her determined information into symbolic relations as a generalization by using the abovementioned way of thinking which was coded as Doing – Undoing habit of mind.

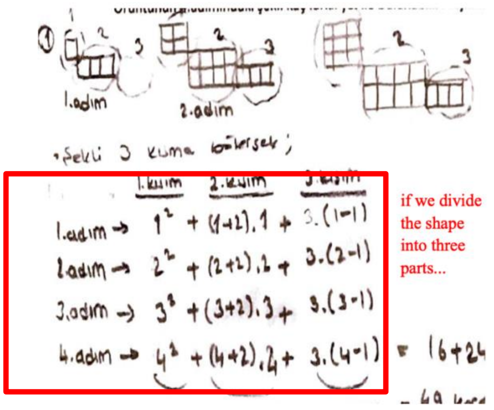


Figure 6. PT 20's solution to Problem 3.

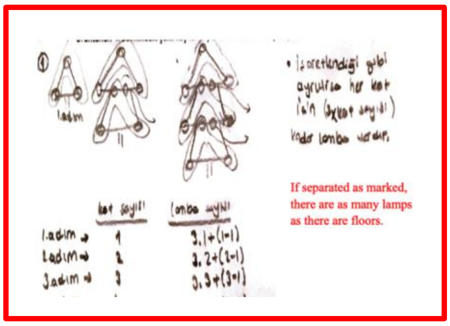


Figure 7. PT 20's solution to Problem 4.

In the first stage of the third problem, PT20 transformed the problem into a number pattern, examined the structure of the pattern within the scope of variant and invariant and tried to find it with different methods. Similarly, in the first stage of the fourth problem, PT20 also transformed the problem into a number pattern. S/he calculated the difference between consecutive terms and found the general term. PT20 calculated the number of terms in the next step (4th step) of the pattern using the general term.

Reasoning with Relationships

Linear Pattern Generalization Problems:

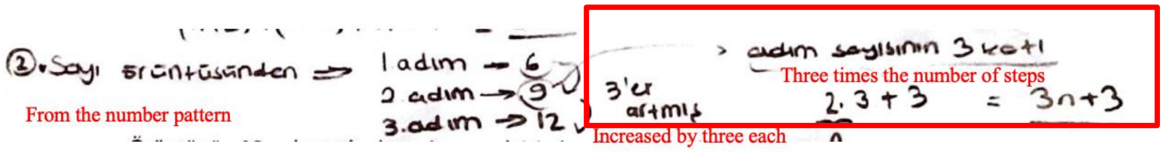


Figure 8. PT 6's solution to Problem 1.

Unlike other participants, PT6's habit of reasoning with relationships was revealed in the first problem. PT6 transformed the pattern into a number pattern and put it in a table. S/he associated the number of steps and the amount of increase with the number of terms. PT6 associated the general term with the amount of increase and this represented the indicators of the reasoning with relationships habit.

Quadratic Pattern Generalization Problems:

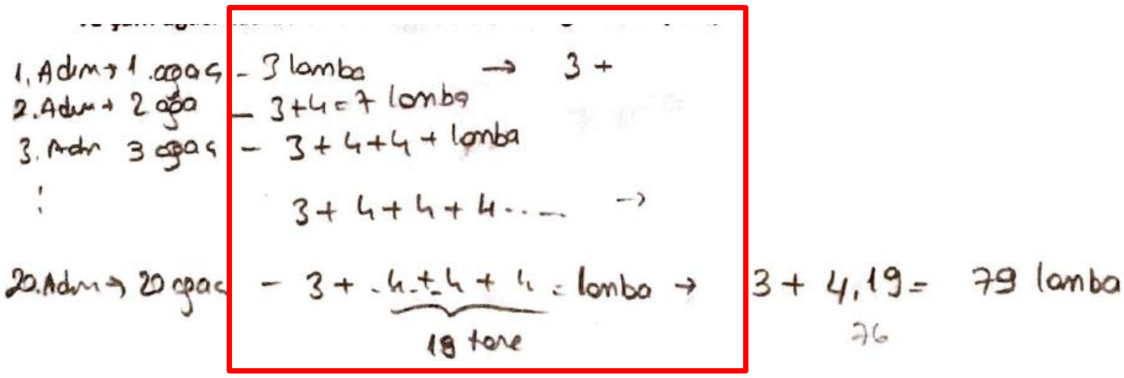


Figure 9. PT15's solution to Problem 3.

PT15 linked the amount of the triangles in each tree with the number of steps in his/her answer to the second stage of the third problem. By transforming the pattern into a number pattern form, the increase between the sequential steps was determined. PT15 established a relationship between the amount of the lamps and the number of steps such that (n-1). These are marked as an indicator of reasoning with relationships in his/her solution.

Investigating Invariants

Linear Pattern Generalization Problems:

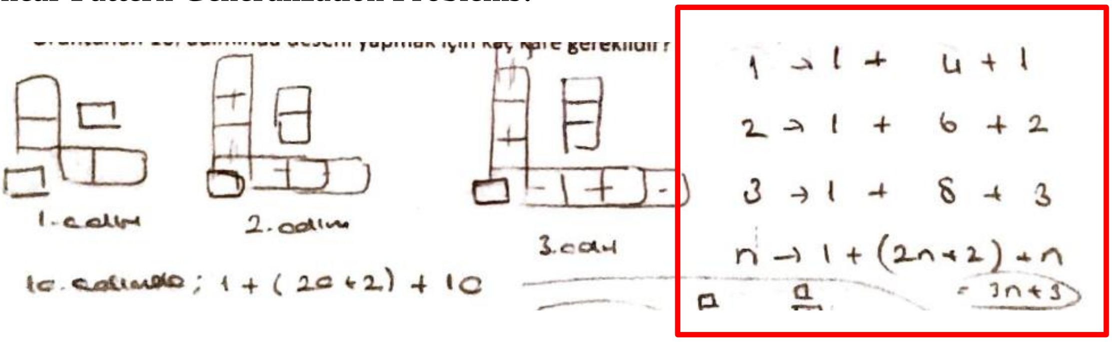


Figure 10. PT 9's solution to Problem 1.

PT9, in the second stage of the first problem, the investigating invariants habit was revealed. S/he visualized the structure of the pattern as three separate parts as given in Figure 10. S/he assumed the square located the left corner in the pattern as invariant and the remainders as growing elements.

R: Did you notice a new (solution) approach here?

PT9: In step 10, I chose one of my approach. I found it to be two more than twice the fixed number of steps. I found it from the general formula. If it was lower, I would still start from the general formula. Since I use the general term, I have shown and also found the number pattern from the path.

R: You have noticed many different ways in this pattern compared to other patterns, what is the reason for this?

PT9: The number pattern does not seem good to me, I liked this question because it is a figural pattern. I don't like the rules in number patterns, sometimes they do not make sense. It becomes more concrete with the figural patterns and I can make sense of it. I utilize the number pattern in solving the figural pattern, but it is weird to make use of the figural pattern in the number pattern and a way of solution is disappeared. Another reason I liked this pattern was that the pattern was constantly increasing when compared to the other patterns…

While identifying and working on the variants and invariants of the pattern, PT9's explanations and ways of thinking are considered to be an indicator of the habit of inversigating invariants.

Quadratic Pattern Generalization Problems:

In the abovementioned section (Figure 9), we tried to represent the problem solving process of PT15 in terms of reasoning with relationships habit. In addition to this, we noticed that PT15's approach also revealed investigating invariants habit. In each step s/he considered two sections by means that three lamps were constant and the other lamps were rounded up to four. His/her interview indicates the investigating invariants habit in a quadratic pattern problem context by him/her to investigate variants and invariants.

R: How did you proceed in the third problem?

PT15: … There were lamps, I tried dividing them. I divided the rows and columns like a square. I thought the top (lamp) was constant, and starting from this point I found a number pattern... There is a relationship of "step equals to the amount of the trees" between (amount of) the tree and the number of the steps in the pattern. I divided the lamps and the number of steps into two parts and I thought of it as two parts. There are three lamps in the first part, and four lamps in each other step. Three lamps are constant. In every step of the pattern, (total amount of) the lamps are rounded up to four. In each, the top of the triangles is ignored. As a second way, I focused on the ignored tops and replaced them with the missing ones. If there was one on the vertex of each triangle, there would be extras. I have 3 triangles, there were extra ones from each. I tried many ways, (but) this question was different.

R: What sets this question apart from others?

PT15: It seemed different to me that there were patterns consisting of triangles and lamps and different figures such as squares, there was no familiar pattern shape, and as far as I noticed, the pattern was not growing regularly.

Generalising Ideas and Rule Making

Linear Pattern Generalization Problems:

We observed in PT10's worksheet that s/he approached the first problem in two different ways. In the first one, s/he determined the last row (2n) and the first row (+1) as constant and stated that there was always the same increase in the forthcoming steps (n+2). In the second approach, s/he transformed the visual pattern into a number pattern (5, 7, 9) and formed the general formula (3n+3) based on the amount of increase (2n+3) and the constant figure in the middle (n).

During his/her problem solving process, generalizing ideas and rule making habit revealed by reaching the general term with the help of the relationship between the number of steps and the number of terms. While observing his/her process of reaching the general term from the interview records, we inferred that PT10 focused the earlier steps of the pattern at first and the invariants of each step to draw a conclusion. Finally, we noted that s/he employed earlier findings to generate further deductions to generalize the rule of the pattern.

R: Can you explain what you did in the third and fourth approach to find step 10?

PT10: I found the amount of increase through the number pattern. I determined the constant figure based on (the structure of) the first figure the previous step. There is a constant figure at the heart, and the steps continue to increase as they grow.

R: What did you consider as invariant?

ÖA10: The part with two in the first column.

R: Could we have chosen any other option?

PT10: I can take one constant (figure) from each step, I can choose the L shape which I chose in the fourth way… I can take the increase as the number of steps times two. I could have taken the top figure as an invariant and added the bottom ones one by one… I took the invariants over the visual pattern formulas *and found a relationship between the number of steps and the number of terms according to the amount of increase. I found the general term and then cross checked my answer*.

Quadratic Pattern Generalization Problems:

In this problem, PT10 considered the lamps on the ends of the pine trees as 2n and the one lamp at the top as constant, as a total of 2n+1. Afterwards s/he determined the increase in each step as 2.(n-1) and formed the general term as 4n-1 from the algebraic expression [2n+1+2.(n-1)=4n-1].

R: What kind of a path did you follow here?

PT10: I tried to go through finding the total amount of the lamps. I have counted the lamps, if the amounts of the increase are invariant… They have increased by four in each step. I thought I could find the amount of the lamps this way. Twice the number of steps come from here. I considered the lamp on the top as constant. I can get the bottom or top constant as well, or I can get all three constants like a giant pine tree.

When PT10's worksheets and his/her interview were considered together, we noticed that s/he understood the structure in the pattern, determined the relationship and definedthese relationship in an algebraic way, that indicated the habit of generalizing ideas and rule-making.

Balancing Exploration and Reflection

Linear Pattern Generalization Problems:

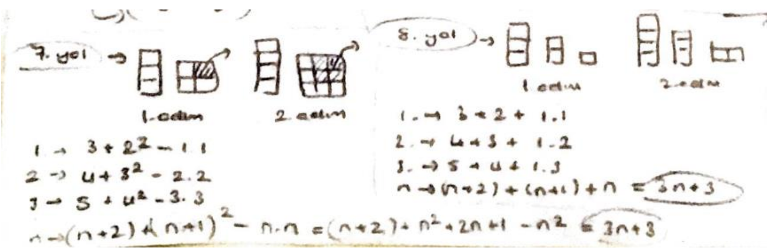


Figure 11. PT 9's solution to Problem 1.

PT9 approached more than one strategy for the first problem. Some of these strategies made it easier to reveal all habits of mind with different approaches. When these approaches are examined separately, strategies for understanding figural patterns (Figure 11) and using earlier steps to form a general term ($3+2^2$-1.1, $4+3^2$-2.2, $5+4^2$-3.3, …) are the indicators that balancing exploration and reflection habit was revealed. Also, s/he preferred at least two different approaches as visual strategies was an indicator of this habit.

Quadratic Pattern Generalization Problems:

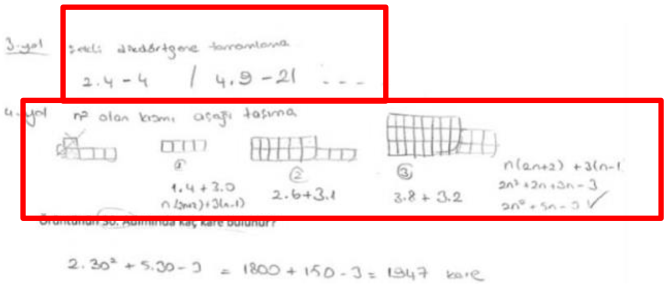


Figure 12. PT 7's solution to Problem 4.

PT7 approached this problem by dissecting the figure into smaller parts, completing the given figure into a familiar one, and transferring among representations. His/her approach showed us that several representations were used and more than one algebraic expressions were included indicating that the balancing exploration and reflection habit.

Discussion and Conclusion

This study aimed to examine the habits of mind of pre-service middle school mathematics teachers' in the process of solving pattern generalization problems in terms of the framework by Eroğlu (2021). When the results of this study were examined, all of the habits in the framework were revealed during pattern generalization problem solving similar to the study by Karyagdi (2023). Also, Tolga (2017) stated that teachers were preferred familiar strategies inevitably which resulted in the occurrence of the habits being limited.

In our study, the most common habits occurred were "reasoning with relationships" and "generalizing ideas and rule making" habits. "Doing-undoing" habit has often been seen together with "reasoning with relationships" habit. The least common habit was "balancing exploration and reflection".

Particularly, the "doing-undoing" habit was revealed in the stages that required to find each step of the pattern. For this reason, the doing-undoing habit is least common in the stages required to find the general term. While considering this habit, it was observed that the majority of the pre-service teachers focused on number patterns rather than figural patterns. In this context, the results of our study are in line with Yıldız and Gündoğdu Alaylı (2019), Barbosa and Vale (2015), Tanışlı and Köse (2011), Karyağdı (2023), Chua and Hoyles (2010), Akkan and Çakıroğlu (2012), Becker and Rivera (2006). In addition to these studies, Yeşildere and Akkoç (2011) also observed that the participants used figures only to convert them into number patterns, and therefore the figures could not be used effectively in terms of recognizing the structure of the pattern.

The “reasoning with relationships” habit was revealed in all the stages that aimed at finding the steps of the pattern, as it was in the “doing-undoing” habit. Reasoning with relationships habit has been one of the most used habits. Bülbül and Güven (2020) also obtained similar results in their study.

We observed “investigating invariants” habit in the first and second stages of the problems while trying to determine the structure of the pattern. It is also seen that pre-service teachers apply different approaches from each other and thus this resulted in various habits revealed in the same problem by each student. This result is in line with Yakut Çayir and Akyüz (2015) in this context. However, Köse and Tanışlı (2014) determined that the participants did not have original ways of thinking, which are indicators of habits of mind, and that they faced the most common approach.

“Generalizing ideas and rule making” habit observed the first and third stages of the problems. In Particular, finding the general term and generalized the patterns was revealed in the problem solving process. Within this context, the connection between fostering generalizing ideas and rule making habit and using the pattern generalization problems are crucial. As stated in Yakut Çayir & Akyüz (2015) pattern generalization problems ease individuals to identify, expand and explore the pattern so that they can be used as a transition from arithmetic to algebra.

In our study “balancing exploration and reflection” habit was revealed rarely in the pattern generalization problem solving process. Also, this habit was observed as it was embedded with other habits. Similar to our results, Tolga and Cantürk Günhan (2019) stated that this habit was the least common and the most difficult to consider. Contrary to this result, Bülbül and Güven (2020) determined that balancing exploration and reflection habit was revealed more than other habits. Based on these results, researchers (Bülbül& Güven, 2020; Tolga& Cantürk Günhan, 2019) concluded that this habit was observed with reasoning with relationships.

In terms of the types of patterns presented in the problems, we observed different habits revealed in linear and quadratic patterns. With this result, we conclude that the structure and type of the pattern used in the problem can affect the occurance of habits. Pre-service teachers found quadratic patterns more difficult and therefore we observed less developed strategies according to the habits of mind. During the interviews, the pre-service teachers stated that they could not find the quadratic type patterns familiar and they could also not reason that they could not recognize the pattern stem from its structure.

In her study Yakut Çayir (2013) found that students' performance was better in linear pattern problems when compared to quadratic ones. Similarly, Türkoğlu and Yalın (2020) and Özdemir, Dikici and Kültür (2015), Orton and Orton (1999), Akkan and Çakıroğlu (2012), Lannin (2005) found that participants had difficulty in identifying the general rule of the pattern in quadratic pattern problems.

Reccommendations

In this study, pattern generalization problems were examined within the scope of habits of mind. The subject of patterns is one of the important subjects of algebra. Algebraic thinking can be fostered through generalizations (Kieran, 2004), so that this can be achieved via pattern generalization problems. More studies need to be done to reveal and support the quality of algebra and algebraic thinking. It is thought that the increase in studies will improve the teaching of these subject. In this study, figural patterns are included. There is a need for studies to observe the pre-service teachers' and students' habits of mind revealed in order to observe the development of their ways of thinking and to foster their habits.

Further studies can investigate habits of mind in pattern generalization problems in different settings.

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Conflict of Interest

It has been reported by the authors that there is no conflict of interest.

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Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration; the participants are volunteers and know that they can give up if they do not want to participate the research. The research does not include any harmful implementation or the researchers do not obtain any special or sensitive information from participants. Necessary permissions are taken from the relevant institution

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