Generation of Bifurcation Diagrams for Ferroresonance Characterization Using Parallel Computing

J. A. Corea-Araujo, G. Guerra, J. A. Martínez-Velasco, and F. González-Molina

Abstract— **Ferroresonance is a complex phenomenon that involves the association of nonlinear magnetizing inductances and capacitances. This nonlinear phenomenon can be initiated in many different ways, which makes its characterization very difficult. Some recent works have shown how bifurcation diagrams can be advantageously used for predicting the phenomenon at its different stages and finding safety zones for parameter selection. The present work expands the scope of some tools originally developed for assessing ferroresonance behavior by means of 3D bifurcation diagrams. The main goal of this work is to propose the application of parallel computing to speed up the generation of bifurcation diagrams.**

Index Terms—**Bifurcation Diagram, EMTP, Ferroresonance, MATLAB, Modeling, Non-linear dynamics, Parallel computing, Parametric study.**

I. Introduction

FERRORESONANCE normally refers to a series resonance that typically involves the interaction of a saturable transformer and a capacitive distribution cable or transmission line [1]. Due to the involved nonlinearities and the various situations that can lead to ferroresonance, it is difficult to predict whether it will occur or not, and the severity with which it can occur [1]. Traditionally, the analysis of ferroresonance has been difficult due to its unpredictability and the lack of an overall understanding.

A roadmap for ferroresonance analysis, going from a system modeling to a ferroresonance behavior analysis and prediction using bifurcation diagrams was presented in [2].

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Reference [2] proposed the application of 3D bifurcation diagrams for complementing ferroresonance characterization, mainly when more than one parameter can significantly affect the behavior of the ferroresonant nonlinear circuit.

The generation of bifurcation diagrams is however a very tedious task since a high number of simulations has to be run in order to obtain a rigorous representation of the whole phenomenon; that is, a complete parametric study of a ferroresonant scenario can require thousands of runs. In addition, a rather short time step (i.e. less than 10 micro-seconds) has usually to be considered due to the nonlinearities involved in this phenomenon. This means that for a rather small system model several days of computer time can be necessary for completing the parametric study.

A simple solution to this problem is the application of parallel computing. Indeed, several parametric studies can be simultaneously run using a multicore installation to cover the whole range of parameter values that are of concern.

The approach followed in this work is basically that presented in [2]; it is based on a MATLAB procedure that uses the ATP version of the EMTP to carry out the transient simulations: the MATLAB procedure drives ATP within a multicore installation and collect the generated information to obtain the bifurcation diagram that will characterize the behavior of the test system.

This document has been organized as follows. A short summary on ferroresonance characterization is first presented in Section 2. The tools developed for parametric studies using a multicore installation and the postprocessing tasks that will generate bifurcation diagrams are detailed in Section 3. The remaining sections are dedicated to present some test cases that will illustrate the way in which bifurcation diagrams are generated and their usefulness for ferroresonance analysis.

II. Characterization And Simulation of Ferroresonance

In many scenarios, more than one ferroresonant state is possible, and the operation may jump in and out of ferroresonance modes depending on switching angle or nonlinear circuit parameters. Recorded waveforms corresponding to actual events and results from numerical simulations have led to the classification of ferroresonance states into four different modes [10]:

- ***Fundamental mode:*** Voltages and currents are periodic with a period *T* equal to the power frequency period.

- ***Subharmonic mode:*** The signals are periodic with a period *nT* that is a multiple of the source period.
- ***Quasi-periodic mode:*** This mode is not periodic and exhibits a discontinuous spectrum.
- ***Chaotic mode:*** The signals show an irregular and unpredictable behavior.

Given the difficulties for distinguishing the normal transient state from the ferroresonant transient state in a ferroresonant circuit, this classification corresponds to the steady state condition, once the initial transient state is over.

Ferroresonance can be characterized by using either a spectral study method based on Fourier's analysis or a stroboscopic analysis obtained by measuring current *i* and voltage *v* at a given point of the system and plotting in plane *v-i* the instantaneous values at instants separated by a system period. The stroboscopic method, also known as *Poincaré Map* [11], can be used to differentiate between ferroresonance states (i.e., a quasi-periodic state from a chaotic state). A *bifurcation diagram* records the locations of all the abrupt changes in a system signal (e.g., voltage) when a system parameter is quasi-statically varied.

The bifurcation diagram is an alternative to traditional parametric and sensitivity methods for understanding system behavior in ferroresonance analysis. Techniques applied to the calculation of a bifurcation diagram may be based on the principle of continuation [3], [7], experimentation, or time-domain simulation [4], [6]. Reference [2] used time-domain simulation and an ATP–MATLAB link to obtain both 2D and 3D bifurcation diagrams.

If time-domain simulation is used to analyze and predict ferroresonance, it is important to keep in mind that simulation results have a great sensitivity to models and errors in the parameters of nonlinear components. The transformer model is probably the most critical part of any ferroresonance study. Different models and different means of determining the parameters are required for each type of core. Several topology transformer models based on the principle of duality have been presented in the literature [12] - [16]. By default, the transformer model must have an accurate representation of all nonlinear inductances of the core, including hysteresis effects [17] - [19]. Internal capacitances of the transformer might be also considered since they could be crucial in the final development of a ferroresonant process.

Since no standard model is available in any simulation package, tests suggested in the literature cannot be always performed and no standard test has been developed for determining the parameters specified in some models [13], [15], so their use is presently rather limited. Although much effort has been made on refining models for transformers and performing simulations using transient circuit analysis programs such as EMTP and like [20], determining nonlinear parameters is still a challenge.

Other components to be considered, besides the transformer, are the study zone that must be represented in the model, the source impedance, the transmission or distribution line(s)/cable(s), and any other capacitance not already included from the previous components.

Source representation is not generally critical; if the source does not contain nonlinearities, it is sufficient to use the steady-state Thevenin impedance and the open-circuit voltage. Lines and cables may be represented as *RLC* coupled pi equivalents, cascaded for longer lines/cables. Shunt or series capacitors may be represented as a standard capacitance, paralleled with the appropriate resistance. Transformer stray capacitances may also be incorporated either at the corners of an open-circuited delta transformer winding or midway along each winding.

Since ferroresonance is a nonlinear phenomenon, the conditions with which the phenomenon is initiated will have a significant impact; therefore, residual fluxes, initial capacitance voltages, angles of sources and switches cannot be neglected, see [21], [22]. The latter aspect was analyzed in [2] by varying the switch angle and the phase shift of the source. Both values play a very important role and can decide the final state of the ferroresonance and the length of the transient between the normal operation and the ferroresonant final state [23].

III. Parametric Studies Using A Multicore Installation

A 2D bifurcation diagram records on a plane the locations of all abrupt changes experienced by the final operating state of the test system while one or more system parameters are quasi-statically varied [24]. The 3D bifurcation diagram is based on the variation of two system parameters, stacking as many 2D planes as values given to the second parameter range. The implementation of 3D bifurcation diagrams applies the brute-force method, which consists of repeating time-domain simulations followed by frequency-domain sampling of the same output to determine its periodicity. The procedures for obtaining 2D and 3D bifurcation diagrams were presented in [2].

Parametric studies required to obtain a bifurcation diagram may involve thousands of simulations and take a significant amount of computer time. This time can be significantly reduced by distributing the tasks among several cores. Fig. 1 presents a schematic diagram of the procedure implemented in MATLAB to run ATP in a multicore environment.

A template of the input file corresponding to the test case is firstly elaborated. This template includes the ATP feature Pocket Calculator Varies Parameters (PCVP), normally used to perform parametric studies. By means of the MATLAB procedure, the PCVP section is varied as many times as required to cover the whole range of the first parameter; PCVP is also used to vary the second parameter while the first one remains constant. The user has to specify the range of values and the number of runs for each parameter.

The diagram presented in Fig. 1 shows a procedure with three main parts: (1) data input preparation (aimed at editing input files); (2) data handling and generation of plots; (3) conversion of simulation results produced by ATP (i.e. the so-called PL4 files) to MATLAB format (i.e. MAT files). Note the directions in which the information is flowing: the MATLAB code generates the final input files, distributes them between cores, controls the ATP runs and the data conversion, reads and manipulates simulation results once they have been translated into in MATLAB code, and finally takes care of the generation of bifurcation diagrams. Data conversion and further post-processing steps of this procedure take advantage of the previous developments, used for the work presented in reference [2]. For this expanded version, additional MATLAB code was needed to edit ATP input files and distribute them among cores.

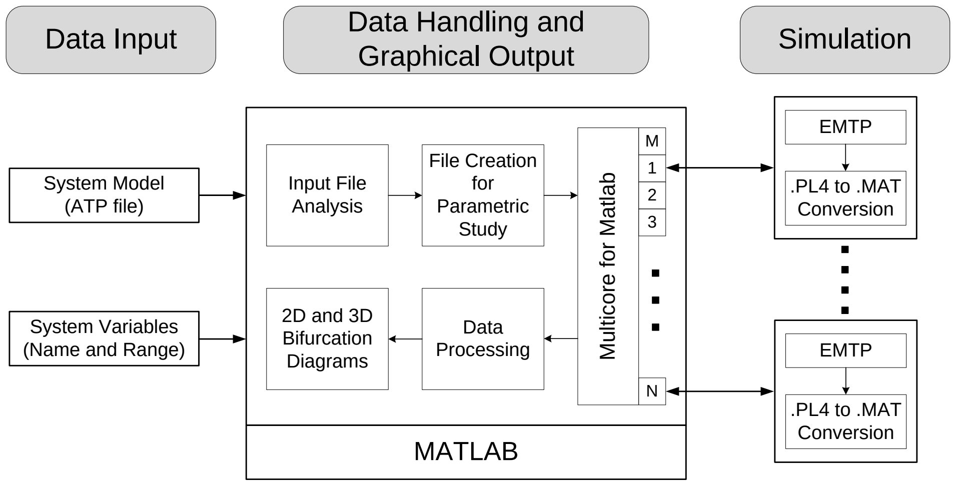


Fig. 1. Flow chart for a 3D plot implementation.

Another important aspect was the size of the files generated by ATP when using PCVP. Since a PL4 file for each combination of the two parameters that are varied to obtain the 3D diagram is needed, the size of the information that has to be manipulated is huge (e.g. if every parameter is varied 200 times, up to 40000 PL4 files will have to be manipulated). To avoid the storage and manipulation of such a quantity of files, the simulations are progressively made; for instance, if 200 runs are to be controlled from PCVP, the same file is run 4 times and the parameter is varied only 50 times within each run. Remember that the ultimate goal is to obtain both 2D and 3D bifurcation diagrams for ferroresonance characterization.

IV. 4. Case Studies

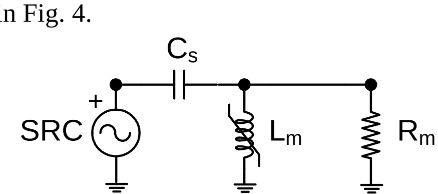
A. *Case 1: An Illustrative Example*

The basic circuit of this example is shown in Fig. 2a. The circuit is composed of a 25 kV, 50 Hz ideal voltage source, a capacitance of 0.1 µF and a saturable inductance (see Fig. 2b) paralleled with a resistance of 40 Ω [25]. The shifting angle of the voltage source is -90º, being zero both the initial capacitance voltage and the residual flux in the saturable inductance. The simulation is carried out without introducing any switching event.

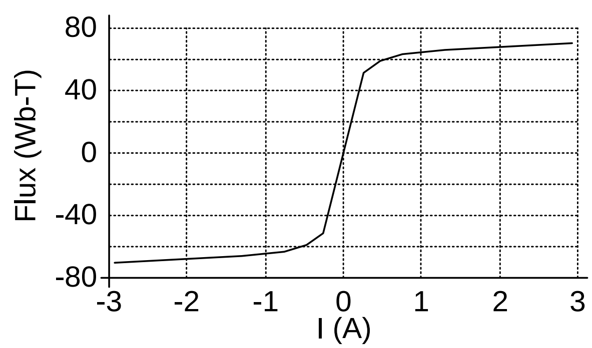
Using the above parameters, the resulting voltage in the inductor is that shown in Fig. 3a; it can be easily identified as a periodic ferroresonant signal.

Consider now that the value of the resistance is infinite; that is, assume a lossless circuit. Fig. 3b shows the new time-domain response. Notice, however, that the ferroresonant signal in Fig. 3b cannot be easily characterized, since it is not possible to distinguish between a chaotic and a quasi-periodic mode.

The 3D diagram elaborated according to the procedure proposed in [2] and created by slowly varying C_s (parameter K1), from 0 to 1 µF, and R_m (parameter K2), from 0 to 40 kΩ, is shown in Fig. 4.



a) Diagram of the test circuit



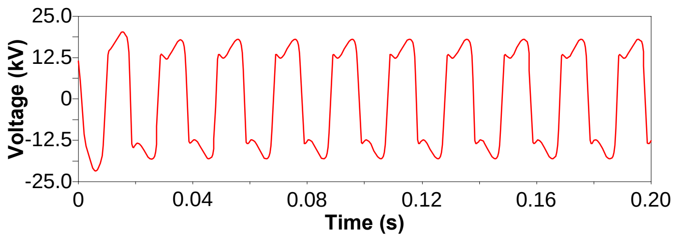
b) Non-linear inductance characteristic

Fig. 2. Case 1: Test circuit.

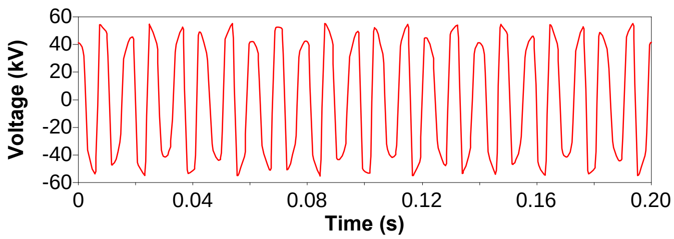
This plot is basically the same that was previously presented in [2].

Note that even though ferroresonance is present in almost any match C_s-R_m, there are two intervals (i.e., C_s = 0 ÷ 0.2 µF and $R_m \leq$ 10 kΩ) in which the signal is completely damped. Notice also that the surrounded light blue area remains below the nominal voltage.

The 3D bifurcation diagram summarizes a parametric study and can be used to select design parameters for newer systems and predict beforehand whether in a ferroresonant event the values will remain or not in a non-destructive zone. Besides, the diagram can be easily applied for analyzing the impact on given equipment by indicating in a chosen color (e.g., red color) the range of values for which equipment failure could occur.



a) Ferroresonant signal – Lossy circuit, R_m=40 kΩ and C_s=0.1 μF



b) Ferroresonant signal – Lossless circuit, R_m=∞ and C_s=0.1 μF

Fig. 3. Ferroresonance (lossless) test circuit. Simulation results.

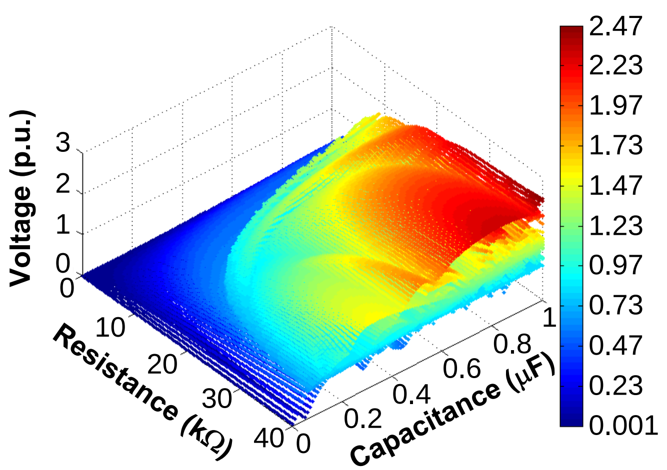


Fig. 4. Case 1: 3D bifurcation diagram.

B. *Case 2: Ferroresonant Behavior of a Voltage Transformer*

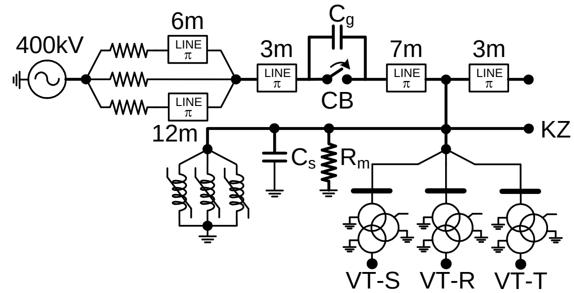
This study is based on a work presented in [8], and deals with a ferroresonance case that involves inductive voltage transformers (VTs) in a substation equipped with circuit breaker grading capacitors.

Fig. 5a shows the diagram of the model implemented in ATPDraw, based on the BCTRAN model; Fig. 5b shows some relevant information for ferroresonant studies.

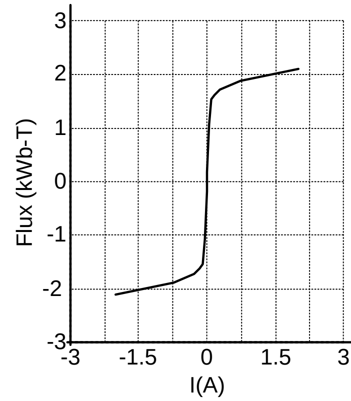
The system consists on a 400 kV busline and a branch feeding a transformer bank with no residual flux. The cable lengths and the grading capacitance of the circuit breaker CB are shown in the figure. More information about the test system is provided in the Appendix.

All capacitances are initially discharged, the phase angle of the source is 0º and the closing of the switch is performed at t = 0.3 s.

A catastrophic incident was reported when the three phase breaker (CB) was opened [8].



a) Diagram of the test circuit



Current (A)	Flux (Wb-T)
0.00073	180.058
0.00578	1080.14
0.00838	1530.28
0.01540	1620.73
0.02730	1710.55
0.07416	1890.61
0.20000	2100.00

b) Saturation curve of the voltage transformer (VT-S, VT-R, VT-T)

Fig. 5. Case 2: Test system.

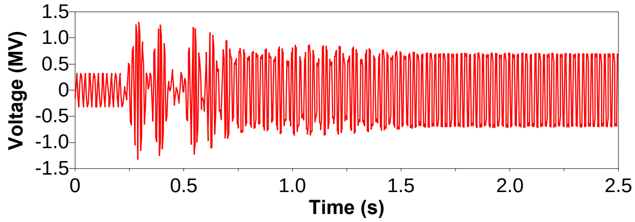
The operation left the grading capacitance connected in series to the nonlinear inductance of a transformer bank connected to the same line. The ferroresonant signal is presented in Fig. 6a. After a study aimed at analyzing the influence of the value C_s, it was discovered that for a range of values the result could be much different, see Fig. 6b. The stray capacitance C_s and the grading capacitance C_g are recognized as major influence parameters in the ferroresonant system. In this example, the values of C_s and C_g are respectively 490 pF and 600 pF [8].

The 3D diagram depicted in Fig. 7 was again constructed following the procedure presented in [2] using C_s and C_g as parameters. The z axis gives the per-unit value of the voltage at node KZ (phase B), and referred to the source voltage. One can easily note that with most combinations of C_s and C_g, the resulting mode of ferroresonance will not exceed the 1 p.u. value, except for those values of C_s lower than 200 pF, an interval that can unfold chaotic mode with voltage values of up to 4 p.u. This plot is again the same that was presented in [2].

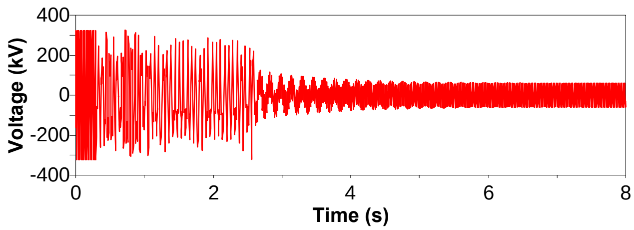
C. *Case 3: Ferroresonance in a Five-Legged Transformer*

This case is based on a series of studies conducted in the early 1990's and aimed at synthesizing experimental work and modeling of five-legged core transformers driven to a ferroresonance condition. Originally, the National Rural Electric Cooperative Association (NRECA) funded a study in

1986 and 1987 to assess the problem on rural electric cooperative systems.



a) C_s=490 pF and C_g=600 pF



b) C_s=2590 pF and C_g=600 pF

Fig. 6. Case 2: Simulation results.

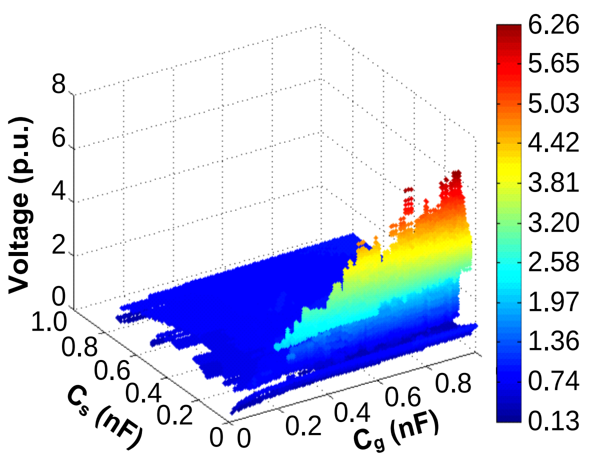


Fig. 7. Case 2: 3D bifurcation diagram.

Several reports were released describing overvoltage condition problems related to five-legged core transformers operating with the grounded-wye connection [26]. This situation induced the need to develop a transformer model that would efficiently analyze the causes of the phenomenon [27]. After some validation work, the problem under study could be categorized as a ferroresonance phenomenon [28]. An ATP model of a five-legged core transformer was later proposed in [29]. That study was aimed at analyzing the ferroresonant response of the transformer to changes in the capacitance of the cable between the transformer and the source.

Fig. 8 shows the model implemented in ATPDraw and the characteristics of the saturable inductances. More information about the test system is provided in the Appendix.

This case analyzes the effects in the ferroresonance final stage provoked by the cable capacitance and the source voltage regulation (±10%). Some simulation results showing the ferroresonance state in phase A are presented in Fig. 9.

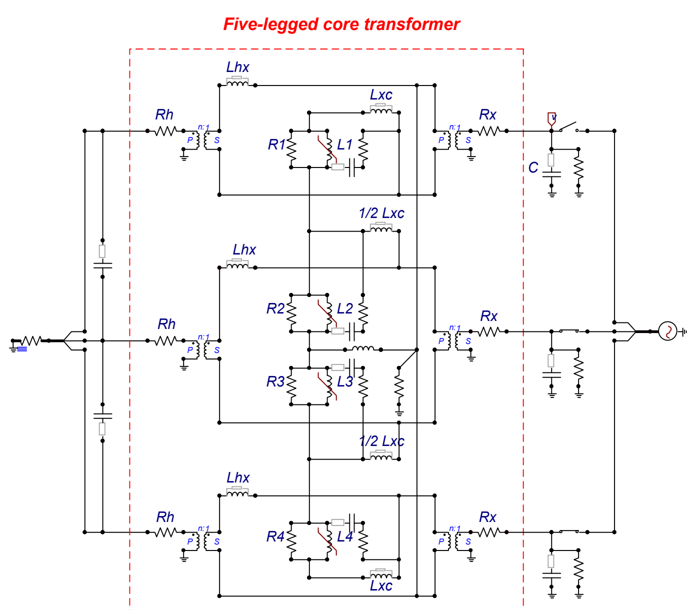
Since the ferroresonant situation starts along with the simulation, the switch between the source and the phase A is left open (see Fig. 8), and the initial capacitance voltage and the residual flux are set to zero. After running a series of simulations it was possible to understand how the capacitance value, the switching angle and the source voltage were directly affecting the final state of the ferroresonance. Thus, the capacitance has been varied from 0 to 30 μF, while the source voltage has been varied from 432 to 528 V. All tests were carried out assuming a no load condition (represented by means of a very high resistance).

An extensive parametric study has been again resumed using the 3D bifurcation diagram; the diagram has been obtained from 90000 runs. Fig. 10 show the resulting 3D diagram for the source voltage variation. From this plot it is possible to easily locate those potentially destructive zones as well as zones where the oscillation peaks are harmless for equipment and protections (i.e. they are not higher than 0.5 p.u.).

V. An Extension of Bifurcation Diagrams

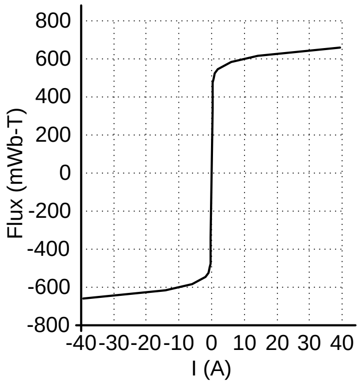
An extension of the bifurcation diagram can be made by introducing the idea of a 4D diagram, which may consist of varying three individual parameters and collecting the voltage peaks resulting of the induced ferroresonance. The peaks selected are then colored according to its severity; this may enhance the map by avoiding the use of an axis representing the peaks. The example selected to illustrate the advantage of a 4D diagram is based on the first case study presented in Section IV.A. The parameters to be evaluated are: resistance R_m, capacitance C_s, and the power source variation (±10%).

The "fourth axis" is the colored range given to the output voltage. Fig. 11 shows the resulting map. Dark red areas represent zones with ferroresonance peaks above twice the rated value.



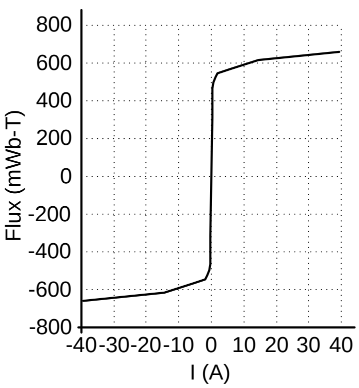
a) Diagram of the test system.

Fig. 8. Case 3: System model in ATPDraw.



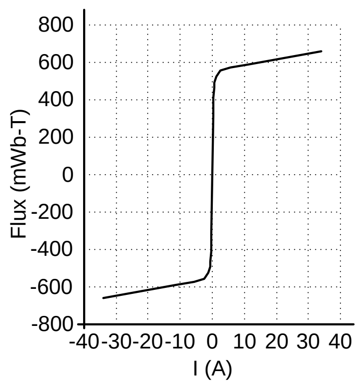
Current (A)	Flux (Wb-T)
0.2048	0.3423
0.3253	0.4330
0.4611	0.4726
0.6594	0.4975
1.0484	0.5223
1.9703	0.5470
5.9339	0.5810
14.299	0.6147
39.392	0.6573

b) Saturation curve of the inductance L_1



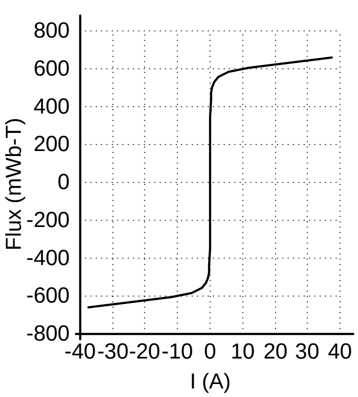
Current (A)	Flux (Wb-T)
0.1950	0.3136
0.3100	0.4133
0.4550	0.4646
0.6581	0.4936
1.1030	0.5204
2.0099	0.5423
5.6543	0.5687
14.323	0.6147
39.418	0.6573

c) Saturation curve of the inductance L_2



Current (A)	Flux (Wb-T)
0.2000	0.3109
0.3200	0.4103
0.4976	0.4643
0.7182	0.4930
1.2090	0.5231
2.4080	0.5562
5.5097	0.5742
11.331	0.5897
34.121	0.6592

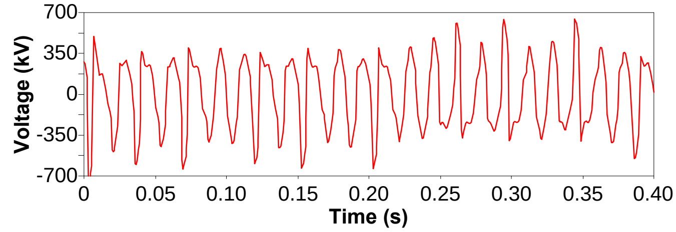
d) Saturation curve of the inductance L_3



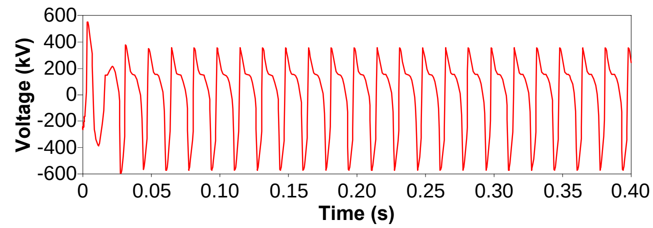
Current (A)	Flux (Wb-T)
0.0264	0.3433
0.3308	0.4348
0.4041	0.4767
0.6844	0.5005
1.1574	0.5274
2.3642	0.5541
5.6238	0.5820
11.906	0.6043
37.472	0.6591

e) Saturation curve of the inductance L_4

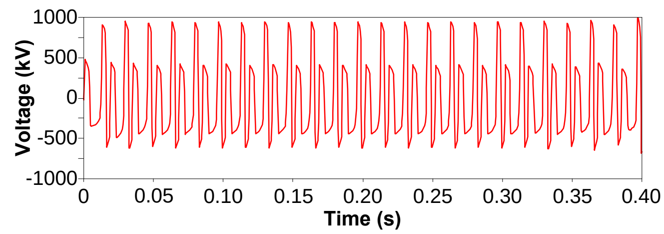
Fig. 8. Case 3: System model in ATPDraw (cont.).



a) Shift angle = −90º, Capacitance = 15 μF, Source voltage = 480 V



b) Shift angle = −90º, Capacitance = 10 μF, Source voltage = 440 V



c) Shift angle = 0º, Capacitance = 25 μF, Source voltage = 500 V

Fig. 9. Case 3: Phase A – Some ferroresonant signals.

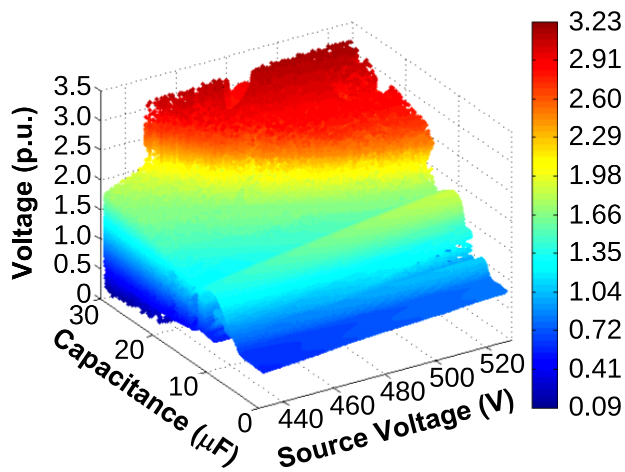


Fig. 10. Case 3: 3D bifurcation diagram.

To facilitate the interpretation, the map can be separated in sub-plots containing the range of values of interest. Fig. 12 introduces the section of the map for those peaks colored in red and corresponding to the highest ferroresonance voltages.

VI. Conclusion

Parametric studies can be very useful to detect parameter ranges of concern in some transient phenomena. The CPU time required to carry out some of these analyses can be too long, mainly if a short or very short time step must be considered and a high number of runs is necessary to cover the full range of parameter values.

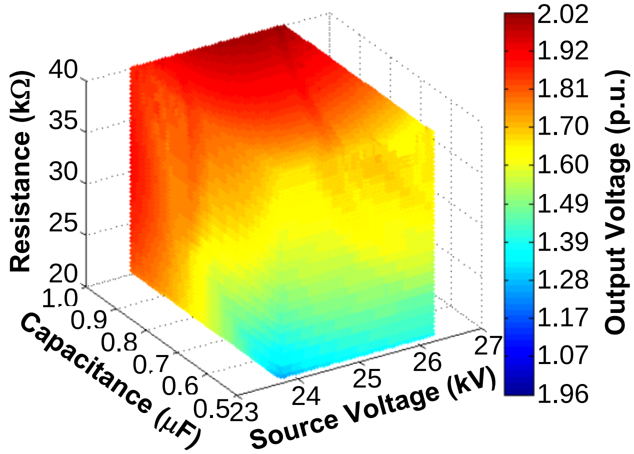


Fig. 11. Case 1: 4D bifurcation diagram.

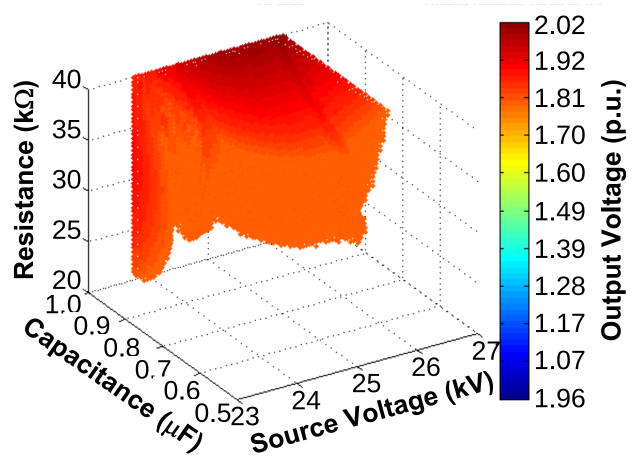


Fig. 12. Case 1: Selected segments of the 4D bifurcation diagram.

An efficient option is the usage of a multicore installation where the runs are shared among the cores. The work presented here is based on a previous work that proposed 3D bifurcation diagrams as a technique for a good comprehension of the behavior of a power system while driven into a ferro-resonant state. This technique can also be used to establish the roadmap of a hypothetical ferroresonant situation, pointing the safe zones in which the parameter under study should be selected.

This paper has presented the application of a parallel MATLAB procedure for creating bifurcation diagrams that can characterize ferroresonant situations. Since the reduction of the simulation time is obviously proportional to the number of cores, some studies can easily become affordable by using a few dozens of cores. Some useful information about the cases studied in this paper is provided in Table I. In order to understand the advantage of this approach it can be mentioned that all the cases studied here required several days of single-CPU time to obtain all the information needed for creating the corresponding 3D plot. The 4D map has been obtained after 125000 simulations and contains up to 6125000 points. The total simulation time was around 6 hours.

It is important to point out the factors influencing the duration of the total simulation time, which are: The amount of simulations, individual simulation time and the integration time step.

Finally, it is important to emphasize the influence of the transformer model on the results and therefore the bifurcation diagrams obtained with the present methodology. Reference [30] shows that for single-phase transformers an accurate ferroresonance study can only be carried out when the saturable cores are represented by means of a hysteretic core. All case studies presented in this paper were carried out by using a non-hysteretic representation of transformer cores; therefore, results presented here should be used with care.

Table I – Simulation Times – 50 cores

Test Case	Parameters	Tmax	Time step	Runs	CPU time
1	R_m: 0 ÷ 40 kΩ; C_s: 0 ÷ 1 μF	1 s	1 μs	200×200	6031 s
2	C_s: 0 ÷ 1 ηF; C_g: 0 ÷ 1 ηF	8s	10 μs	200×200	8621 s
3	V: 432 ÷ 528 V; C: 0 ÷ 30 μF	1s	1 μs	300×300	35558 s

APPENDIX

A. *Case 2 – Main Parameters*

Power source (ideal): L-L Voltage = 400 kV; Frequency = 50 Hz.

Cables: They are represented by constant-parameters PI models. Their lengths vary between 3 and 12 m.

Circuit breaker: It is represented by a grading capacitance, C_g, with variable value.

Voltage transformers (*VTs*): Main characteristics

- Single-phase three-winding transformer
- Ratings: 400/0.1/0.033 kV, 0.1/0.1/0.05 kVA.
- Short-circuit test values: $Z_{sh,1w\text{-}2w}$ = 13.46 %, $Z_{sh,1w\text{-}3w}$ = 20 %, $Z_{sh,2w\text{-}3w}$ = 20 %.
- No load test losses: R_m = 182 MΩ.
- Saturation curve shown in Fig. 7.

Stray capacitance (C_s): Variable value.

B. *Case 3 – Main Parameters*

Power source: L-L Voltage = 230 kV; Frequency = 60 Hz; Per phase short-circuit impedance = 0.2116 Ω + *j*4.3801 Ω.

Bus-line B1-B2: Per phase values are as follows

Line Filter 1: R = 2.17 Ω; X = 16.72 Ω; C = 0.938 μF.

Line Filter 2: R = 1.84 Ω; X = 16.74 Ω; C = 1.31 μF.

Bus-lines interconnections: They are represented by a PI arrangement of capacitances with the following values

Series capacitance C_Y = 0.001108 μF.

Parallel capacitance C = 0.005316 μF.

Circuit breaker: It is represented by a grading capacitance, C_g, with variable value.

Bus-line A1-A2: It is represented by a stray capacitance, C_s, with variable value.

Voltage transformers (*V13F, V33F*):

- YNyn0 three-phase two-winding transformers
- Rated voltages: 230/0.115 kV.

- Short-circuit test: $R_{sh,1w}$ = 7490 Ω, $X_{sh,1w}$ = 0.001 Ω; $R_{sh,2w}$ = 0.0463 Ω, $X_{sh,2w}$ = 0.1642 Ω.
- No load test (steady state): Current = 0.00478 A; Flux = 0.38 W-T; Core loss resistance R_m = 9.52 MΩ.
- Saturation curve shown in Fig. 12.

Substation transformer (*SST2*):

- YNy0 Three-phase two-winding transformer, grounded through a 2.4 Ω impedance.
- Rated voltages = 230/4.16 kV
- Short-circuit test: $R_{sh,1w}$ = 12 Ω, $X_{sh,1w}$ = 322.69 Ω; $R_{sh,2w}$ = 0.0012 Ω, $X_{sh,2w}$ = 0.1056 Ω.
- No load test (steady state): Current = 0.02485 A; Flux = 0.04 W-T; Core loss resistance: R_m = 3.11 MΩ.
- Saturation curve shown in Fig. 12.

Load (*connected to V13F and V33F*): Per phase values R = 163.2 Ω; X = 0.268 Ω.

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