# AN INVESTIGATION OF MIDDLE SCHOOL STUDENTS' NUMBER SENSE REGARDING THE PERCENT 

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#### Abstract

The purpose of this study is to examine the number sense regarding the percent of middle school students. The sample of the study consists of a total of 454 Turkish students of $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$ grade level from four different middle schools located in Kırıkkale, Turkey. The data were collected using the Test of Number Sense Regarding Percent developed by the researchers considering the components of number sense. These components are flexibility in the calculation, visual representation and the using benchmark (reference points). In the analysis of the data, students' strategies while solving percent problems were examined and analyzed using the method of descriptive analysis. As a result of the study, it was found that the number sense of the students regarding percent were very weak. The students, in general, tend to use rule-based algorithms based on memorization instead of using number sense while solving percent problems. The students preferred operational ways that took much more time even in problems related to estimation or finding an approximate value, instead of practical solutions.


Key Words: Benchmark (reference points), flexibility in calculation, number sense, percent, visual representation

## ORTAOKUL ÖĞRENCILLERİNİN YÜZDELER KONUSUNDA SAYI DUYULARININ İNCELENMESI**

## ÖZET

Çalışmanın amacı ortaokul öğrencilerinin yüzdeler konusunda sayı duyularının incelenmesidir. Araştırmanın örneklemi, Türkiye'nin Kırıkkale ilinde bulunan dört farklı ortaokulun 5, 6 ve 7 sınıflarında öğrenim gören toplam 454 öğrenciden oluşmaktadır. Veriler, sayı duyusu bileşenleri dikkate alınarak araştırmacılar tarafindan geliştirilen Yüzdeler Konusu ile İlgili Sayı Duyusu Testi ile toplanmıştır. Bu bileşenler; hesaplamada esneklik, görsel temsil ve kıyaslama (referans) noktası kullanımı bileşenleridir. Verilerin analizinde, yüzde problemlerinde öğrenciler tarafından kullanılan stratejiler incelenmiş ve bu stratejilerin kullanım sıklığı betimsel analiz yöntemi kullanılarak hesaplanmıştır. Çalışmanın sonucunda öğrencilerin yüzdeler konusunda sayı duyularının oldukça düşük olduğu saptanmıştır. Öğrenciler genellikle yüzde problemlerinin çözümünde sayı duyusu kullanımı yerine kural odaklı çözüm yollarını kullanma eğilimi göstermişlerdir. Öğrenciler, yaklaşık değeri bulma veya tahminle ilgili problemlerde bile pratik çözümler kullanmak yerine daha fazla zaman alan işlemsel yolları tercih etmişlerdir.

Anahtar Kelimeler: Kıyaslama (referans) noktası, hesaplamada esneklik, sayı duyusu, yüzde, görsel temsil

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## 1. INTRODUCTION

### 1.1. The Concept of Percent and Its Significance

Percent is one of the subjects of mathematics that we use very frequently in daily life. Examples include attractive discount rates on store windows, percentages of votes parties get in elections, and the rating of a TV show. The correct interpretation of such situations is dependent on knowing the concept of percent and learning it conceptually. Although it has such an important place in mathematics and daily life, many studies show that students and even adults find it difficult to deal with (Allinger \& Payne, 1986; Gucken, 1986; Koay, 1998; Lembke \& Reys, 1994; Parker \& Leinhardt, 1995; Van de Walle, Karp, \& Bay-Williams, 2010; Wiebe, 1986). Van de Walle et al. (2010) explain this low success rate by that the concept of percent is not developed in a meaningful way. Similarly, Koay (1998) explains the difficulty about percent by the formal education provided. According to Koay, the education given on percent obstructs the creativity and flexibility of students while approaching percent problems. In other words, providing students with rules and algorithms instead of concepts while teaching percent makes them dependent on operational rules (Allinger \& Payne, 1986).

In the middle school mathematics curriculum implemented by the Ministry of National Education (MoNE) in 2013 with the education system changed as $4+4+4$ in the school year of 2012-2013 in Turkey, it was aimed to bring $5^{\text {th }}$ grade students up to the level of being able to show fractions with the denominator of 100 with the percent symbol; relate a percent expression to a fraction and a decimal notation representing the same amount and convert such notations among each other; compare quantity expressed as fractions, decimals and percent, and find the quantity corresponding to a given percent of a certain amount. On the level of 7th grade, students are expected to find the amount corresponding to a given percent of a certain amount and find the amount for which the amount of a certain percent is given; calculate an amount as the percent of another amount; make calculations towards reducing or increasing an amount by a certain percent, and solve problems regarding the subject of percent. Considering the objectives of the curriculum, it may be seen that the subject of percent is studied in relation to fractions and decimal notations.

In the conference organized in 1989 in the editorship of Sowder and Schappelle, Trafton stated that estimation and mental computation should be used while teaching the meaning of the concept of percent. Trafton especially focused on using benchmark (reference points) like $10 \%$ and $50 \%$, being able to make mental computation with these reference points and estimate about the result using these percent.
Benchmark (reference points) used in solving percent problems and estimation skills were defined as number sense skills by Hope (1989). Considering these skills, defined in relevant studies, and objectives aimed at the middle mathematics curriculum, it is important that number sense of students is improved regarding percent. Gay and Aichele (1997) emphasized that the focus should be on number sense instead of operational skills while teaching the subject of percent.

### 1.2. Number Sense

Many studies have been conducted on what the concept of number sense is and various definitions were provided to explain number sense. In Howden's study (1989), number sense was defined as making logical inferences instead of reaching conclusions by applying the rules, and discovering that there are various ways of this. Reys, Reys, McIntosh, Emanuelsson, Johansson, and Yang (1999) defined it as finding suitable strategies and being able to conduct logical-mathematical reasoning while understanding the meanings of operations with numbers. Hope (1989) defined number sense as the skill to make logical estimation regarding the areas of usage of numbers, use the correct method of calculation, and recognize the arithmetic mistakes and number patterns.

In the book "Curriculum and Evaluation Standards for School Mathematics" by NCTM (1989), the characteristics of children with number sense were defined as; Children with number sense (1) effectively understand the meanings of numbers, (2) develop multiple relationship among numbers, (3) recognize relative magnitudes of numbers, (4) understand the effects of operations on numbers, (5) develop benchmarks (reference points) for measurements of objects in their environment.
Among the studies made in Turkey, Kayhan Altay and Umay (2013) defined number sense as; using numbers flexibly, thinking practically in operations with numbers, choosing the most effective and practical solution, creating non-standard methods suitable for some situations, using benchmark (reference points) to make the problem easier to solve in suitable situations, thinking conceptually and using different representations in fractions.

Researchers have made different classifications for number sense components after defining number sense (Greeno, 1991; Kayhan Altay \& Umay, 2013, Lembke, 1991; McIntosh, Reys, \& Reys, 1992; Reys et al., 1999; Yang, 1995). The classification used in this study is the one proposed by Kayhan Altay and Umay (2013) by considering the pioneers (Reys et al., 1999; Yang, 1995) in the field. The reason for choosing this classification is that it presents an evidence-based structure (the exploratory factor analysis) regarding the dimensions of number sense as opposed to other proposed classification. Kayhan Altay and Umay (2013) defined number sense components as (1) flexibility in the calculation, (2) conceptual thinking in fractions and (3) using benchmark (reference points) considering the results of the factor analysis. In their study, flexibility in the calculation was defined as follows: thinking practically in mathematical operations, recognizing different representations in numbers and choosing the way that will make the problem easier. Conceptual thinking in fractions is defined as being able to show fractions using visual representation forms such as number line and area model. The using benchmark (reference points) is defined as using numbers such as, $1 / 2,1,100$, $50 \%$ as reference points while making a problem easier.

### 1.3. Number Sense in Terms of Percent

It may be stated that the number of studies on number sense in Turkey has recently increased (Çekirdekçi, Şengül, \& Doğan, 2016; Gülbağcı Dede \& Şengül, 2016; İymen \& Duatepe Paksu, 2015; Yaman, 2015). Considering the studies on the field of number sense, while studies focusing on investigating the number sense of students, teachers and prospective teachers are frequently seen, studies investigating number sense skills
especially in a certain subject in mathematics are less frequent. Additionally, studies towards investigating number sense regarding percent are rare. There are not many studies in the relevant literature investigating number sense particularly regarding percent in other countries, as well.

Gay (1990) defined number sense in the subject of a percent as "understanding of the meaning of numbers expressed as a percent, developing an equivalent for percent, comparing quantities expressed as a percent, and recognizing the relative effect of finding a percent of a number" (p.2). Understanding percent was classified into five components by Lembke (1991). These components were the skills of (1) mental image/visual representation, (2) being able to relate fractions, decimals, and percents, (3) being able to use benchmark and another estimation method in percent problems, (4) mental computation and (5) being sensitivity to reasonableness of the solution. The first component is a skill related to the possibility of the concept of a percent to have a way of visual representation (such as pie charts and one-hundred square) to allow the formation of a mental schema of the problem. The second component is related to the skills of students to be able transfers among the representations of fractions, decimals, and percent. The third component is related to the skills of students to use estimation strategies and the most frequently encountered percent while solving a percent problem. Students' skill to carry out operations without using calculators, pencils or paper is related to the fourth component of mental computation. The last component is related to the skills of the students to be able to determine whether the result they have reached is suitable as a solution for the problem.

In this study, a test of number sense regarding percent was developed by considering the components of number sense defined by Kayhan Altay and Umay (2013). However, when the common characteristics of the questions under the second component of "conceptual thinking in fractions" are examined, it may be seen that these questions are similar to those under the component of "mental image/visual representation" defined in Lembke's study (1991). The common feature of these questions is the expression of the desired concepts by utilizing shapes and models. For example; being able to show fractions using a number line or area models or shading a certain percent of a shape, usage of $10 \times 10$ blank hundreds charts, etc. Thus, instead of the component of "conceptual thinking in fractions", the study used the expression of "visual representation" which we considered to be more representative as a component.

### 1.4. Rationale of the Study

Researchers have observed that students have low success rates regarding percent (Allinger \& Payne, 1986; Lembke \& Reys, 1994; Parker \& Leinhardt, 1995; Wiebe 1986). There is still ongoing debate on the most effective models and techniques to help students establish a connection between formal and informal knowledge in learning the concept of percent. It is not yet clear how students can relate the formal education provided in school to their intuitional knowledge (Lembke, 1991). In the literature, it may be seen that researchers mostly focused on skills of computation in percent problems, mistakes made by students regarding percent and methods that may be effective in teaching percent. On the other hand not much importance was paid to number sense of students regarding percent while trying to develop an effective method (Gay \& Aichele, 1997). Therefore, the purpose of this study is to investigate the number sense of $5^{\text {th }}, 6^{\text {th }}$
and $7^{\text {th }}$-grade students regarding percent. The reason for researching the concept of number sense about percent is that number sense is important in learning the subject of percent conceptually and reducing mistakes about percent. It was considered that measuring the number sense skills of students about the subject of percent will contribute to the field with the information to be reached as a result of the study, in terms of more frequent employment of implementations and activities to achieve development of number sense skills in education. The research question was therefore determined as the following:
"What are the skills expected of $5^{\text {th }}, 6^{\text {th }}$ and $7^{7 \mathrm{~h}}$-grade students who use number sense regarding the subject of percent?"

## 2. METHOD

### 2.1. Design of the Study

This study was determined by a descriptive survey design (Fraenkel \& Wallen, 2006) as it was aimed to reveal the $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$-grade students' number sense about percent.

### 2.2. The Population and Sample of the Study

The population of the study consisted of middle-grade students enrolled at schools in the province of Kırıkkale in Turkey. Four schools were randomly selected in the province of Kırıkkale. These schools are state schools under the authority of the Ministry of National Education. One of these is a rural school in the district of Delice, while others are located in the city center. Two classes were again randomly selected among all classes in each of the $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$ grades in each school. A total of 454 were included in the sample. The distribution of students based on grade level and gender are given in Table 1.

Table 1.
Distribution of Students Based on Grade and Gender

|  |  | Male |  | Female |  | Total |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{f}$ | $\mathbf{\%}$ | $\mathbf{f}$ | $\mathbf{\%}$ | f | $\mathbf{\%}$ |
| Grade | 5th grade | 64 | $27.9 \%$ | 66 | $29.3 \%$ | 130 | $28.6 \%$ |
|  | 6th grade | 85 | $37.1 \%$ | 78 | $34.6 \%$ | 163 | $35.9 \%$ |
|  | 7th grade | 80 | $34.9 \%$ | 81 | $36.0 \%$ | 161 | $35.5 \%$ |
|  | Total | 229 | $50.5 \%$ | 225 | $49.5 \%$ | 454 | $100 \%$ |

As seen in Table 1, among the students participating in the study, $28.6 \%$ were $5^{\text {th }}$ grade, $35.9 \%$ were $6^{\text {th }}$ grade, and $35.5 \%$ were $7^{\text {th }}$-grade students. Additionally, $49.5 \%$ were female, and $50.5 \%$ were male students. Accordingly, the gender and grade distributions of students were very close.

### 2.2. Data Collection Tool: The Test of Number Sense Regarding Percent

For the data collection, a tool developed by the researchers applied to $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$ graders. It was aimed to determine the number sense of students in terms of percent. The developed test is about percent and consists of 9 open-ended and 6 multiple-choice questions. The pilot study for this test was conducted with a total of 166 fifth, sixth and seventh-grade students chosen from a different public school. After its pilot study, the test had its final form. The last version of the test was given in the Appendix. The
response time was determined as one class hour. In order to determine the reliability of the test of number sense regarding percent, Cronbach- $\alpha$ reliability coefficient was calculated and found as 0.78 . A reliability coefficient of 0.78 for a 15 -item scale is highly satisfactory. Expert opinion was taken for determining the validity of the test. The questions in the measurement tool were analyzed by 5 field experts who had worked on number sense, 3 experienced teachers and 4 academicians. The experts were asked to assign scores for questions in the scale in terms of representativeness of the components of number sense defined in the relevant literature from 1 to 5 ( 1 being the lowest, 5 being the highest). The experts also analyzed the suitability of the questions for the grade level and whether they were related to the objectives in the curriculum, difficulty levels of the questions and ways of expression, and situations that might cause misunderstandings. As a result of the provided opinions, some questions were removed, and necessary adjustments were made on the remaining questions.

The classification of Kayhan Altay and Umay (2013) was taken as a basis for the formation of the test of number sense regarding percent. For the flexibility in calculation component, questions $2,5,7,8,10,11,12$ and 14 ; for the visual representation component, questions 1, 6, 9 and 13; and for the usage of benchmark (reference points) component, questions 3, 4 and 15 were used. The $6^{\text {th }}$ question developed by Lembke (1991) was adapted into Turkish in a multiple-choice format. Questions 7 and 13 were developed by using Gay's study (1990). The $10^{\text {th }}$ question was adapted from questions developed by NAEP (National Assessment of Educational Progress) in 1996. The researchers developed the other questions in the test by using the relevant literature and textbooks. The sample items of the test for the three components of number sense regarding percent are presented in Table 2.


### 2.3. Analysis of the Data

The responses of the students in the test of number sense regarding percent were analyzed based on three different cases. The students who answered the questions incorrectly or did not answer at all got 0 points, ones who used the standard, routine algorithms of rulebased solutions got 1 point, and ones who solved the question using number sense strategies got 2 points. For example, a student who "multiplied the number by the rate and divided the result by 100 to find $150 \%$ of the number 50 " received 1 point. On the other hand, the student who noticed that $150 \%$ is the sum of $100 \%$ and $50 \%$ and found without long calculations that the correct answer is the sum of the number and its half,
received 2 points. The data obtained from the number sense test for each question type (open-ended and multiple choice) based on this scoring were quantitatively analyzed.

## 3. FINDINGS

The solution methods used in the number sense test by $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$ graders were coded based on three cases as wrong or no response, standard (routine) algorithm, and calculation using number sense. The distribution of the solution methods by question is given in Table 3.

## Table 3.

Distribution of Solution Methods Used by $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$ Graders

|  | Wrong or no answer <br> $(\%)$ | Standard algorithm <br> $(\%)$ | Number sense <br> $(\%)$ |
| :--- | :---: | :---: | :---: |
| Q1 | $16.5 \%$ | $37.0 \%$ | $46.7 \%$ |
| Q2 | $48.0 \%$ | $35.7 \%$ | $16.3 \%$ |
| Q3 | $48.0 \%$ | $25.1 \%$ | $26.9 \%$ |
| Q4 | $88.8 \%$ | $6.4 \%$ | $4.8 \%$ |
| Q5 | $65.2 \%$ | $28.6 \%$ | $6.2 \%$ |
| Q6 | $28.9 \%$ | $17.6 \%$ | $53.5 \%$ |
| Q7 | $72.9 \%$ | $19.8 \%$ | $7.3 \%$ |
| Q8 | $67.8 \%$ | $22.0 \%$ | $10.1 \%$ |
| Q9 | $33.9 \%$ | $52.9 \%$ | $13.2 \%$ |
| Q10 | $92.7 \%$ | $3.3 \%$ | $4.0 \%$ |
| Q11 | $81.7 \%$ | $9.7 \%$ | $8.6 \%$ |
| Q12 | $76.2 \%$ | $15.2 \%$ | $8.6 \%$ |
| Q13 | $46.0 \%$ | $24.9 \%$ | $29.1 \%$ |
| Q14 | $67.4 \%$ | $22.2 \%$ | $10.4 \%$ |
| $Q 15$ | $76.9 \%$ | $4.6 \%$ | $18.5 \%$ |

Note. $\mathrm{Q}=$ question
Based on the data in the table (considering "wrong and no answer" percent), it may be stated that the students in had low success rates regarding percent in general. This finding supports the result of various studies conducted previously (Allinger \& Payne, 1986; Lembke \& Reys, 1994; Parker \& Leinhardt, 1995; Wiebe 1986). Additionally, it was found that usage of number sense was low and students rather used standard algorithms in their solutions. This finding agrees with those of many studies (Kayhan Altay, 2010; İymen, 2012; Mohamed \& Johnny, 2010; Şengül, Gülbağcı, \& Gerez Cantimer, 2012; Yang, 2005).
Considering the rates of using number sense regarding percent, the students used number sense skills the most in the $6^{\text {th }}$ question ( $53.5 \%$ ). This question is related to the second component of visual representation. It is a multiple-choice question related to matching the certain percent of a given number to correct quantities. The solution method with number sense by a $6^{\text {th }}$ grader and the solution method with the rule-based algorithm by another $6^{\text {th }}$ grader are given below respectively.


Figure 1. Student responses given for the solution of $6^{\text {th }}$ question
The student who solved this question with the standard method used the rule of "multiplying the given total by the percent rate and dividing the result by 100 " and multiplied 400 by 13 , then divided it by 100 . The student who used number sense, instead of making calculations, compared the numbers given with the percent symbol and matched the lowest number to the lowest percent; "The highest percent is in tent, so it is $A$. The next highest is food, so it is $B$. The next is sleeping bag, so it is $C$. The lowest is entertainment, so it is $D$."

Another question with high rates of number sense usage was the $1^{\text {st }}$ question, again related to the second component of visual representation. About half of the students (46.7\%) solved this question using number sense. The students who used number sense while shading the $30 \%$ of a square shape in the question used more familiar percent such as $25 \%$ and $50 \%$ as a reference point. The divided the shape into four equal parts and shaded a little more than its $25 \%$ (Fig. 2) :"A little more than $25 \%$ is shaded.", or divided the shape into two equal parts and shaded a little less than its $50 \%$. Students who used the standard method divided the square into 100 parts and shaded 30: "I divided the shape by 10 in two directions and shaded 30 of the shapes."


Figure 2. Student responses given for the solution of the $1^{s t}$ question
Another question in the second factor of visual representation and with high usage of number sense is the $13^{\text {th }}$ question. The rate of number sense usage by the students was $29.1 \%$ for this question. The difference of this question from other visual representation questions was the consideration of five equal circles given in the form of a set model. When the students were asked what percent of the given shape was shaded, ones using the standard method firstly found the result as the fraction $3 / 5$ and then expanded the denominator to 100 and found the percent. Students using number sense firstly used the point of $50 \%$ as a benchmark (reference point) and indicated that if the result was $50 \%$, 2.5 circles would be shaded, so it should be more than $\% 50$.


Figure 3. Student response given for the solution of the $13^{\text {th }}$ question
These three questions had shapes and were included in the component of visual representation. It agreed with the findings of the study by Gay (1990) that the students were better a finding a percent represented as a visual model.

The questions with low rates of number sense usage were the questions 4,10 and 11. The $10^{\text {th }}$ question is in the flexibility in calculation component and only $4.8 \%$ of the students used number sense while reaching a correct solution to this question. The question asked the percent of the increase in number of students in two different schools. Most students found the amount of increase by subtraction. As the increased amount was equal, they said that the increase percent was also equal. Instead of using multiplicative relationships, they compared additive results. A sixth-grade student with high number sense would think that the increase in the number of students (300) at a school with 500 students is more than half of the previous year, and the same number increase is exactly the half at a school with the initial number of students as 600 . Figure 4 shows the solution method by a student using number sense: "Kayra is right because number of students at Atatürk Middle School increased by $50 \%$, while the other increased more than $50 \%$."


Figure 4. Student response given for the solution of $10^{\text {th }}$ question
A student with high number sense was able to recognize the relationship between numbers and practically use mental calculation instead of the need for written operations. Question 11 was another question in the flexibility in calculation component where the rate of usage for number sense was low, and the percent rate was asked after giving the whole and the piece. It was seen that the students were not able to correctly identify the whole and the piece, and could not calculate the amount of increase in a magnitude as percent. The solution method by a $7^{\text {th }}$-grade student who solved the question by proportion is given below (Fig. 5): "I firstly found which percent 75 marbles was. Then I subtracted it from 60 's percent."


Figure 5. Student response given for the solution of the $11^{\text {th }}$ question
It may be seen that in the solution method of a student who correctly calculated the increase percent in number of marbles firstly identified the whole and the change correctly. The solution by a $6^{\text {th }}$-grade student who used number sense was explained as the following: "The whole is 60.15 is added onto it. 15 is $1 / 4$ of 60 , which is $25 \%$."




Figure 6. Student response given for the solution of the $11^{\text {th }}$ question
The student using number sense firstly correctly identified the whole and the amount of increase (piece), recognized the relationship as the proportion of $1 / 4$ used the equivalent notation of number correctly and stated that the percent increase is $25 \%$.

The $4^{\text {th }}$ question in the usage of benchmark (reference points) component was another question where students were unsuccessful in general, and number sense usage was low $(4.8 \%)$. When a question was asked to find a percent between two fractions, most students made rule-based calculations, made expansions to equate the denominator to 100 and tried to convert fractions into percent. Many students left the answer to the question blank after not being able to expand the denominators 28 and 40 to 100 . They did not try another method. On the other hand, a sixth-grade student with high number sense answered the question like the following: "the fraction $\frac{8}{28}$ is close to $\frac{1}{3}$, which is $33 \%$, and the fraction $\frac{18}{40}$ is close to $\frac{1}{2}$, which is $50 \%$. Therefore, $40 \%$ should be between them". In this solution method, the student did not prefer the memorized method of equating the denominator to 100 , established the meaning of concepts of fractions such as $\frac{1}{3}$ and $\frac{1}{2}$, used equivalent representations of the fractions correctly and found approximate percent values. Consequently, the student correctly guessed the percent to be found between the two fractions.

The $2^{\text {nd }}$ and $9^{\text {th }}$ questions were the ones the students used standard algorithms most frequently. These questions are ones where a certain percent of a whole is calculated. Considering the rates of solution methods for the second question, the using of standard algorithm rate $(35.7 \%)$ was higher than the rate of number sense usage ( $16.3 \%$ ). While finding a percent for a given number, the students conducted rule-based operations by establishing equations or proportions. They multiplied the given whole with the percent rate and reached the solution by dividing the result by 100. A rule-based solution method
by a $5^{\text {th }}$ grade student is given in Figure 7: "As 50 cannot be divided by 100, I multiplied it by the numerator and divided it by 100."

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Figure 7. Student response given for the solution of $2^{\text {nd }}$ question
Another method of standard algorithm used for the solution of this question is establishing proportions. A student in the $7^{\text {th }}$ grade made the following explanation: " $I$ thought, if $100 \%$ is 50, what should $150 \%$ be, and established a proportion" (Fig. 8). On the other hand, some students found it difficult to interpret $150 \%$ percent, which is higher than $100 \%$, were not able to establish a relationship between the whole and the piece, and provided explanations such as "there is no such thing as $150 \%$."




Figure 8. Student response given for the solution of $2^{\text {nd }}$ question
The response of a sixth-grade student who used number sense was as the following: "It is 75, because it should be higher by the half of the whole" (Fig. 9).


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    olacaktir Oda
    \(50: 2=25\)
    \(50+25=75\)
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Figure 9. Student response given for the solution of $2^{\text {nd }}$ question
Accordingly, the student actually recognized that $150 \%$ is equal to the sum of $100 \%$ and $50 \%$ and produced a practical solution method. Another $7^{\text {th }}$ grade student who solved the question this way validated the solution by proportion. The solution of the student is given in Figure 10: "I found the answer by using ratio/proportion. If $100 \%$ is 50, I found half of it and added it to find the $150 \%$."


Figure 10. Student response given for the solution of $2^{\text {nd }}$ question
Students' tendency rely on standard operations and rules more and they may prefer such methods due to the fact that they generally use rule-based operational skills in classes and solution methods towards improving number sense are not present in the classrooms. Although the $9^{\text {th }}$ question is in the visual representation component, only $13.2 \%$ of the students were able to use number sense. More than half of the students (52.9\%) referred to rule-based operation while solving this question. The students applied the equation rule again while finding a percent of a given number, multiplied the whole by the percent rate and divided it by 100 . Therefore, they found the $25 \%$ of the total number 20 of
squares. They found the number of squares to be shaded by using the operation $20 \times \frac{25}{100}$. The reason for the low rate of number sense usage in this question might be that the students thought of this question as a question to find a certain percent of a number independently from the shape. On the other hand, a $7^{\text {th }}$-grade student with high number sense explained the solution like the following: "One column is $25 \%$, and the whole is $100 \%$ " (Fig. 11).
9)


Verilen zekBn \% 25'ini boyayınzz. Nasil dugundadGindza açklaynniz.
Acgiklama:


Figure 11. Student response given for the solution of $9^{\text {th }}$ question
As seen in the solution method, the student thought of $25 \%$ as $1 / 4$ was able to divide the shape into four pieces mentally and shade the asked piece without making calculations.

Another question where standard algorithms were used in student solutions was the fifth question. In the operation of finding the whole of a number whose percent is given, most of the students reached the correct solution by using standard methods. While the percent of the standard algorithm used in this study was $28.6 \%$, usage of number sense was very low at $6.2 \%$. Students using the equation method found the whole by the inverse operation. They multiplied the number whose percent is given by 100 and divided the result by the percent rate. A $6^{\text {th }}$-grade student using number sense explained the reasoning in solution as the following: "The solution is 70 . This number should be higher than 63 , but not higher by 1 [considering the item C], so it cannot be 64 ." In this solution method, using number sense, the student recognized that the solution, meaning the whole number must be bigger than the piece given in the question. Among the mistakes in this question, it was found that students did not read the question carefully, made memorized operations and used a rule to find a percent of a number incorrectly, therefore multiplying 63 by 90 and dividing the result by 100 . It is considered that just memorizing rules without understanding the subject leads students to mistakes.

Considering the characteristics of the questions $7^{\text {th }}$ and $12^{\text {th }}$, which had high rates of incorrect answers and no response, they had decimal expressions. Although the seventh question was about finding a certain percent of a given number, a subject the students were rather more successful in, the students had difficulties as the required percent contained a decimal expression. A $7^{\text {th }}$-grade student missed the decimal place and actually calculated $125 \%$ of the number. While finding $12.5 \%$ of the number 40 , the result of 500 was reached, where the piece was found larger than the whole (Fig. 12): "I expanded the decimal place of the number $12.5 \%$ by a factor of 10 ." The students did not check whether the solutions were logical or not.


```
    40. }\frac{12,5}{16}=500 ब63
```



Figure 12. Student response given for the solution of $7^{\text {th }}$ question
A $6^{\text {th }}$-grade student using number sense in the solution practically found $25 \%$ of the number first. Then the student divided the result by 2 and reached the solution. The
explanation of the student is given in Fig. 13: "I divided 40 by 4, finding $25 \%$ of it. Then $I$ divided the result by 2 and found $12.5 \%$ " (Fig. 13).


```
    12,5) in: bularum.
    40:4:10:2=5
```

Figure 13. Student response given for the solution of $7^{7 \text { th }}$ question
This student who used number sense was able to not only recognize the relationship between the numbers but also use the converting of numbers among each other. In the $12^{\text {th }}$ question, $0.5 \%$ of a number is asked. Some students ignored the percent symbol in $0.5 \%$ and calculated the half of the number. An explanation by a $6^{\text {th }}$-grade student was as the following: "I can find it in a shorter way as its half is 250 already." The students did not check whether the solution was logical or not in this question either. Although a very small percent was asked in the question, students responded with number close to 500 in general. It may be stated that the students found it difficult in the study to understand decimal expressions given with percent symbols. The explanation by a $7^{\text {th }}$-grade student who used number sense was as the following: "It is 2.5. I find $1 \%$ first, which is 5. Half of 5 is 2.5 ." This student produced a more practical solution method instead of operating with decimal expression and solved more easily by using certain percent. The student conceptually understood the relationship between decimal expression and percent, and was able to use different ways of expression with success.

Another question where the percent symbol was ignored and the rate of incorrect answers or no response was higher was the question eight. Considering the strategies in correct answers, the rate of standard algorithms was $22 \%$, and number sense usage was $10.1 \%$. The rate of the students who left the answer empty or responded incorrectly was $67.8 \%$. This question is concerned with comparing a number given with a percent symbol and a different amount. It was seen that the students, instead of comparing $45 \%$ of the number 60 to the number 35, ignored the percent symbol and compared the numbers 45 and 35 . The response by a student who used this method is given below (Fig. 14): "Arda is more successful."
8) Apse Dir shoavda 60 sorunum 35 tannins dolfu cevap verlpor, Ards ice gym sinavdo 60 sorunun $* 45^{\circ}$ init

Agaklama:

$$
\frac{35}{60}<\frac{45}{60}
$$



Figure 14 Student response given for the solution of 8th question
It was observed that the students ignored the percent symbol in some questions and perceived the percent amounts as natural numbers. This might be another reason why they were unsuccessful on the subject of percent.

## 4. DISCUSSION AND CONCLUSION

The purpose of this study was to reveal the $5^{\text {th }}, 6^{\text {th }}$ and $7^{\text {th }}$-grade students' number sense about percent. Results showed that the students' success levels regarding percent were low and usage rates of number sense while solving relevant questions were very low. While solving questions about percent, the students generally tended to use rule-based algorithms instead of using number sense. Allinger and Payne (1986) also reported that students are
not very successful on this particular subject. They stated that students apply rules while solving percent problems and they try to use these rules from memory without actually understanding them. This result agrees with those in other studies showing that number sense usage is low in various subjects such as exponential numbers, decimal numbers, and integers (İymen, 2012; Kayhan Altay, 2010; Şengül et al., 2012; Yang, 2005).

The students were observed to use rule-based operations based on memorization such as setting up a proportion and finding the common denominator. Another important result of the study is that the students preferred operational methods that took much more time even in questions related to estimation or finding an approximate value, instead of practical solutions. This result supports those in studies showing the tendency of students towards rule-based strategies (Kayhan Altay, 2010; Harç, 2010; Singh, 2009; Şengül \& Gülbağcı, 2012; Şengül, et al., 2012; Yang, 2005). Moreover, according to Şengül et al. (2012), the most influential factor leading students to mistakes is their dependence on rules. As it is known, while the rule-based procedural knowledge based on memorization used in teaching the subject of percent does not establish conceptual understanding, it is a factor that is obstructing the development of number sense in students regarding percent (Allinger \& Payne, 1986). More emphasis on standard operational skills may prevent students from developing skills to understand mathematical concepts and estimation, reason, and use number sense.

Another important result of the study was that the students did not check whether the solution they reached was logical or not, that is, whether it was a suitable quantity for an answer. When $90 \%$ of a number is given, and the whole amount is asked, the expected result is that the whole is larger than the piece. However, most students found the whole smaller than the piece. This result is in parallel to those in the study by İymen (2012) who demonstrated that students did not have a sense about the meaning of the solutions they reached and they did not check their answers. The reasons for incorrect answers may be that the students did not interpret the correctness of the result and did not consider it.

In the study, it was observed that students ignored the percent symbol (\%) in some cases and made their operations based on considering percent as natural numbers. The finding that students ignored or missed the percent symbol agrees with those of studies by Risacher (1992) and Kircher (1926). Similarly, Gay (1990) also showed that the percent symbol did not make any sense for some students. At the beginning of teaching a subject, it is important that the concept is correctly placed in the minds of students first. For this, instead of using memorized operations or teaching rules, the meanings of concepts may be discussed to establish conceptual understanding. The cases where the subject of percent which is frequently used in daily life are utilized and how it is interpreted may be given to students more frequently.

According to the data, it was found that the students were more successful in finding a certain percent of a given number than finding a whole number for a given percent where the success rate was much lower. This is in parallel with Lembke's study (1991). It was observed in the study that, in a given problem situation, the students were not able to correctly identify the whole and the piece, or express the ratio of the given piece to the whole as percent. A reason for this may be that there is not much-relating percent and proportions in the "ratio-proportion" objective aimed at the curriculum. In examples given in a problem case, students' comprehension of which is the whole and which is the piece should be achieved and percent as a meaning ratio should be emphasized more.

Another finding of this study was that the students were more successful in questions in the second factor of visual representation which included models and shapes, and usage of number sense was higher in such questions. This result is in parallel with those of Kayhan Altay (2010). In Kayhan Altay's study (2010), students used number sense most frequently in questions related to showing fractions on the number line or a shape in terms of the component of conceptual thinking in fraction. The reason for higher rate of number sense usage in this may be that a question presented with models and shapes provides opportunities for the students to think deeper and more flexibly without dealing with formulas or operations. On the other hand, the students in the study found it much more difficult to find a certain percent of circles given in a set model than finding a percent of a single shape. This result is similar to those in the study by Gay (1990). This may have been caused by showing the whole as a single shape in the classroom and lack of using of different models of the whole by the teachers.

The study found that the students tended to use benchmark (reference points), and the questions where number sense was the second most frequently used were ones related to usage of reference points, following visual representation questions. Similarly, Şengül et al. (2012) also reported high rates of usage of reference points in number sense strategies by $6^{\text {th }}$ graders while solving percent questions. This finding agrees with those in studies reporting the success of students in the use of reference points such as $25 \%, 50 \%$ and $100 \%$ (Dole, Cooper, Baturo, \& Conoplia, 1997; Gay \& Aichele, 1997; Lembke \& Reys; 1994).

It was found in the study that students found it easier to interpret percent like $25 \%$ or $50 \%$, interpreted their meaning easily, but some failed to interpret percent especially higher than $100 \%$. The finding that students who use the whole-piece relationship have difficulties in interpretation of percent over $100 \%$ agrees with those of Risacher (1992). It also supports the study by Kircher (1926) who reported that students have difficulties in understanding the concept of 'the whole'.

Furthermore, the students failed in practical thinking in a way to make the operation easier, finding approximate values or making guesses, within the component of flexibility in calculation. It was found that the students tried to find a precise solution even in cases where they were expected to make simple interpretations, and took part in long operations. This result is in compliance with the result of Harç (2010) that number sense usage was low in questions related to the component of "flexibility in calculation". There should be more frequent activities of providing the students with opportunities to use mental calculation, rounding or making guesses.

According to the results of this study, the educational environment should be established to provide students with skills to use number senses instead of using rule-based algorithms in percent problems. The test of number sense regarding percent used in the study may provide clues in determining the mathematical concepts in the subject of percent students are weak, and they find difficult, improving number sense and finding out about how to design the educational system to be implemented by considering number sense components.

## REFERENCES

Allinger, G. D., \& Payne, J. N. (1986). Estimation and mental arithmetic with percent. In H. L. Schoen (Ed.), Estimation and mental computation (pp. 141- 155). Reston, VA: National Council of Teachers of Mathematics.
Çekirdekçi, S., Şengül, S., \& Doğan, M. C. (2016). 4. sınıf öğrencilerinin sayı hissi ile matematik başarıları arasındaki ilişkinin incelenmesi. [Examining the relationship between number sense and mathematics achievement of the $4^{\text {th }}$ grade students]. Qualitative Studies (NWSAQS), E0028, 11 (4), 48-66.
Dole, S., Cooper, T. J., Baturo, A. R., \& Conoplia, Z. (1997). Year 8, 9 and 10 students' understanding and access of percent knowledge. In F. Biddulph \& K. Carr (Eds.), Proceedings of the 20th Annual Conference of the Mathematics Education Research Group of Australasia, (pp. 7-11). Rotorua, New Zealand: MERGA.
Fraenkel, J. R., \& Wallen, N. E. (2006). How to design and evaluate research in education (6th edition). USA: McGrawHill, Inc.
Gay, A. S. (1990). A study of middle school students' understanding of number sense related to percent (Doctoral Dissertation). Available from ProQuest Dissertations and Theses Full Text database. (UMI No. 9119873)
Gay, S. A., \& Aichele, D. B. (1997). Middle school students' understanding of number sense related to percent. School Science and Mathematics, 97 (1), 27-36.
Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain source. Journal for Research in Mathematics Education, 22 (3), 170-218.
Gucken, J. E. (1986). Construction, validation, and use of a diagnostic test in percent for high school (Doctoral Dissertation). Available from ProQuest Dissertations and Theses Full Text database. (UMI No. 8611859)
Gülbağcı Dede, H., \& Șengül, S. (2016). İlköğretim ve ortaöğretim matematik öğretmen adaylarının sayı hissinin incelenmesi. [An Investigation of Pre-Service Elementary and Secondary Mathematics Teachers' Number Sense]. Turkish Journal of Computer and Mathematics Education, 7 (2). 285-303. doi: 10.16949/turcomat. 96275

Harç, S. (2010). 6. sınıf öğrencilerinin sayı duyusu kavramı açısından mevcut durumlarınin analizi (Yüksek Lisans Tezi). Marmara Üniversitesi, İstanbul, Türkiye.
Hope, J. (1989). Promoting number sense in school. Arithmetic Teacher, 36 (6), 12-16.
Howden, H. (1989). Teaching number sense. Arithmetic Teacher, 36 (6), 6-11.
İymen, E. (2012). 8. sinıf öğrencilerinin üslü ifadeler ile ilgili sayı duyularının sayı duyusu bileşenleri bakımından incelenmesi (Yüksek lisans tezi). Pamukkale Üniversitesi, Denizli, Türkiye.
İymen, E., \& Duatepe Paksu, A. (2015). 8. sınıf öğrencilerinin üslü ifadeler ile ilgili sayı duyularının sayı duyusu bileşenleri bakımından incelenmesi. Eğitim ve Bilim, 40 (177), 109-125. doi: 10.15390/EB. 2015.2710
Kayhan Altay (2010). İlköğretim ikinci kademe öğrencilerinin sayı duyularının; sinıf düzeyine, cinsiyete ve sayı duyusu bileşenlerine göre incelenmesi (Doktora Tezi). Hacettepe Üniversitesi, Ankara, Türkiye.

Kayhan Altay, M., \& Umay, A. (2013). İlköğretim ikinci kademe öğrencilerine yönelik sayı duyusu ölçeği'nin geliştirilmesi. [The development of number sense scale towards middle grade students]. Education and Science, 38 (167), 241-255.
Kircher, H. W. (1926). Study of percentage in grade VII A. The Elementary School Journal, 27 (4), 281-289.
Koay, P. L. (1998). The knowledge of percent of pre-service teachers. The Mathematics Educator, 3(2), 54-69.
Lembke, L. O. (1991). The development of concepts and strategies used in solving percent problems (Doctoral Dissertation). Available from ProQuest Dissertations and Theses Full Text database. (UMI No. 9133614)
Lembke, L. O., \& Reys, B. J. (1994). The development of, and interaction between, intuitive and school-taught ideas about percent. Journal for Research in Mathematics Education, 25 (3), 237-259.
McIntosh, A., Reys, B. J., \& Reys, R. E. (1992). A proposed framework for examining basic number sense. For the Learning of Mathematics, 12 (3), 2-9.
Ministry of National Education [MoNE] (2013). Ortaokul matematik dersi (5, 6, 7 ve 8. sınıflar) öğretim programı. Ankara: Milli Eğitim Basımevi. Retrieved in February 6 from http://ttkb.meb.gov.tr/program2.aspx
Mohamed, M., \& Johnny, J. (2010). Investigating number sense among students. Procedia Social and Behavioral Sciences, 317-324.
National Center for Education Statistics [NAEP] (1996). Questions tool, sample questions. Retrieved in February 6 from https://nces.ed.gov/NationsReportCard/nqt/Search
National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.
Parker, M., \& Leinhardt, G. (1995). Percent: A privileged proportion. Review of Educational Research, 65(4), 421-481.
Risacher, B. F. (1992). Knowledge growth of percent during the middle school years. (Doctoral Dissertation). Available from ProQuest Dissertations and Theses Full Text database. (UMI No. 9319088).
Reys, R., Reys, B., McIntosh, A., Emanuelsson, G., Johansson, B., \& Yang, D. C. (1999). Assessing number sense of students in Australia, Sweeden, Taiwan, and the United States. School Science and Mathematics, 99 (2), 61-70.
Singh, P. (2009). An assessment of number sense among secondary school students. International Journal for Mathematics Teaching and Learning, 1-27 (ISSN 1473-0111).
Şengül, S., \& Gülbağcı, H. (2012). Evaluation of number sense on the subject of decimal numbers of the secondary stage students in Turkey. International Online Journal of Educational Sciences, 4(2), 296.
Şengül, S.,Gülbağcı, H., \& Gerez Cantimer, G. (2012). 6. sınıf öğrencilerinin yüzde kavramı ile ilgili sayı hissi stratejilerinin incelenmesi. [Examining the number sense strategies about percent of 6th grade students] The Journal of Academic Social Science Studies, 5(8), 1055-1070. doi: http://dx.doi.org/10.9761/JASSS_407

Trafton, P. R. (1989). Reflections to the Number Sense Conference. In J. T. Sowder and B. P. Schappelle (Eds.), Establishing foundations for research on number sense and related topics: Report of a conference, (pp 74-78) San Diego, CA: San Diego State University, Center for Research in Mathematics and Science Education.

Wiebe, J. H. (1986). Manipulating percentages. Mathematics Teacher, 79, 23-26.
Van De Walle, J. A., Karp, K. S., \& Bay-Williams, J. M. (2010). Elementary and middle school mathematics: Teaching developmentally. Pearson Education, Inc.
Yaman, H. (2015). The mathematics education I and II courses' effect on teacher candidates' development of number sense. Educational Sciences: Theory \& Practice, 15 (4), 1119-1135. doi: 10.12738/estp.2015.4.2322
Yang, D. C. (1995). Number sense performance and strategies possessed by sixth and eighth grade students in Taiwan (Doctoral Dissertation). Available from ProQuest Dissertations and Theses Full Text database. (UMI No. 9705388).
Yang, D. C. (2005). Number sense strategies used by 6th grade students in Taiwan. Educational Studies, 31(3), 317-333.

## APPENDIX

## The Test of Number Sense Regarding Percent

1) Shade approximately $30 \%$ of the shape seen on the side. Explain your thought process.

2) What is $150 \%$ of the number 50 ? How do you find it? Explain your thought process.
3) İbrahim solved 48 questions correctly out of the total of 80 questions in a test. What percent of the questions did İbrahim solve correctly? Explain your thought process.
A) $30 \%$
B) $40 \%$
C) $50 \%$
D) $60 \%$
4) Which of the following is between $8 / 28$ and 18/40? Explain your thought process.
A) $20 \%$
B) $40 \%$
C) $50 \%$
D) $60 \%$
5) $90 \%$ of which of the following is 63 ? Explain your thought process.
A) 45
B) 57
C) 64
D) 70
6) The Aksoy family spent 400 TL in vacation. The graph below shows the expenditures during the vacation.


Which part on the graph shows the expenditure on entertainment? Explain your thought process.
A) A
B) B
C) C
D) D
7) What is $12.5 \%$ of the number 40 ? Explain your thought process.
8) Ayşe answered 35 out of 60 questions correctly in an exam. Arda answered $45 \%$ of the 60 questions correctly in the same exam. Compare the success rates of these two students in the exam. Explain your thought process.
9)

10) The following table shows the changes in the numbers of students for two schools according to years.

|  | Number of students in <br> 2011 | Number of students in <br> 2012 |  |
| :---: | :--- | :--- | :--- |
| Mustafa Kemal <br> Secondary School | 500 students | 800 students |  |
| Atatürk Secondary <br> School | 600 students | 900 students |  |

The comments of two students on this table is as the following:
Ayça: "The percent of increase in the number of students in both schools are equal." Kayra: "The percent of increase in the number of students at the Mustafa Kemal Secondary school is higher."
Who is right? Explain why.
11) There are 60 marbles in a bag. The number of marbles reach 75 after adding more marbles into the bag. Accordingly, what is the percent of increase in the number of marbles? Explain your thought process.
A) 10
B) 15
C) 20
D) 25
12) What is $0.5 \%$ of the number 500 ? How do you find it in a short way? Explain your thought process.
13) What percent of all circles do the shaded circles approximately constitute? Explain your thought process.
A) Lower than $50 \%$.
B) Equal to $50 \%$.
C) Higher than $50 \%$.

D) Cannot be found.
14) $75 \%$ of which number is 300 ? Explain your thought process.
15) Approximately which percent of a whole gives the fraction of $31 / 64$ ? Explain your thought process.

## GENIŞ ÖZET

## 1.Giriş

Günlük hayatta sık sık kullandığımız matematik konularından biri de yüzdedir. Matematikte ve günlük hayatta yeri önemli olmasına rağmen birçok araştırma sonucu, öğrencilerin hatta yetişkinlerin bile yüzdeler konusunda zorlandıklarını ortaya koymaktadır. (Allinger ve Payne, 1986; Gucken, 1986; Koay, 1998; Lembke ve Reys, 1994; Parker ve Leinhardt, 1995; Van de Walle, Karpand Bay-Williams, 2010; Wiebe, 1986). Bu düşük başarının sebebini Van de Walle ve diğerleri (2010) yüzde kavramının anlamlı bir şekilde geliştirilememesine bağlamaktadır. Benzer bir şekilde Koay (1998) yüzdeler konusunda yaşanan zorlukların nedenini verilen formal eğitime bağlamaktadır. Koay'a göre, yüzdeler konusunda yapılan eğitim, öğrencilerin yüzde problemlerine yaklaşımlarındaki yaratıcılıklarını ve esnekliklerini engellemektedir. Başka bir deyişle, öğrencilere yüzde öğretiminde kavramdan çok kuralların ve algoritmaların öğretilmesi onları işlemsel kurallara bağımlı hale getirmektedir (Allinger ve Payne, 1986).

Yüzde kavramının öğretiminde öğrencilerin informal ve formal bilgileri arasındaki bağlantıyı kurmalarına yardımcı olmak için en etkili model ve yöntem üzerinde tartışmalar halen sürmektedir. Öğrencilerin okulda gösterilen formal eğitimle kendi sezgisel bilgilerini ne düzeyde ilişkilendirebildikleri net değildir (Lembke, 1991). Alan yazın incelendiğinde araştırmacıların daha çok yüzde problemlerindeki hesaplamalarla ilgili becerilere, öğrencilerin yüzdeler konusunda yaptıkları hatalara ve yüzdelerin öğretiminde etkili olabilecek yöntemlere odaklandıkları ancak etkili bir yöntem geliştirmeye çalışırken öğrencilerin yüzdeler konusundaki sayı duyularına fazla önem verilmediği görülmektedir (Gay ve Aichele, 1997). Acaba yüzdeler konusunda sayı duyusunu kullanan öğrencilerden beklenen beceriler nelerdir?

Bu çalı̧̧manın amacı 5, 6 ve 7. sınıf öğrencilerinin yüzdeler konusundaki sayı duyularını incelemektir. Yüzdeler konusunda sayı duyusu kavramı ile ilgili araştırma yapılmasının nedeni, yüzdeler konusunun kavramsal olarak öğrenilmesinde ve yüzdeler konusundaki eksikliklerin, hataların azaltılmasında sayı duyusunun önemli olmasıdır. Yüzdeler konusunda öğrencilerin sayı duyusu becerilerinin ölçülmesi, araştırma sonucunda ulaşılacak bilgilerle eğitimde sayı duyusu becerisinin gelişimini sağlayacak uygulamalara ve etkinliklere daha çok yer verilmesi açısından alana katkı sağlayacağı düşünülmüştür.

## 2.Yöntem

Bu araştırma, nicel yöntemlerle toplanan veriler ile 5,6 ve 7 . sınıf öğrencilerinin yüzdeler konusunda sayı duyuları ile ilgili durumlarını ortaya çıkarmayı amaçladığından tarama modelindedir. Araştırmanın örneklemini toplam 454 öğrenci oluşturmaktadır. Araştırmacılar tarafından 5, 6 ve 7. sınıflara uygulanmak üzere geliştirilen veri toplama aracında öğrencilerin yüzdeler konusundaki sayı duyularının ne düzeyde olduğu belirlenmeye çalışılmıştır. Geliştirilen ölçme aracı yüzdeler konusu ile ilgili olup açık uçlu ve çoktan seçmeli soru tipinde toplam 15 sorudan oluşmaktadır. Yüzdeler konusunda sayı duyusu testinin güvenirliğinin belirlenmesinde Cronbach- $\alpha$ güvenirlik katsayısı hesaplanmış ve 0,78 olarak bulunmuştur. Yüzdelerde sayı duyusu testinde öğrencilerin cevapları üç farklı duruma göre değerlendirilmiştir. Öğrencilerden soruları yanlış cevaplayan veya soruyu boş bırakanlara 0 puan, standart rutin yolla yapanlara 1 puan, sayı duyusunu kullanarak çözenlere 2 puan verilmiştir.

## 3. Bulgular, Tartışma ve Sonuçlar

Araştırmadan elde edilen bulgularda öğrencilerin sayı duyusu kullanımının oldukça düşük olduğu tespit edilmiştir. Öğrenciler genellikle yüzdeler konusundaki soruların çözümünde sayı duyusu kullanımı yerine kural odaklı çözüm yollarını kullanma eğilimi göstermektedirler. Bu sonuç üslü sayılar, ondalık sayılar ve tam sayılar gibi farklı konularda yapılan çalışmalarda da öğrencilerin soru çz̈zümlerinde sayı duyusu kullanımının düşük olduğu bulgusu ile paralellik göstermektedir (İymen, 2012; Kayhan Altay, 2010; Şengül, Gülbağcı ve Cantimer, 2012; Yang, 2005). Allinger ve Payne (1986) de çalışmalarında öğrencilerin yüzdeler konusunda iyi bir performans sergileyemediklerini belirtmişlerdir. Öğrencilerin yüzde problemlerini çözerken kurallar uyguladıklarını, bu kuralları da anlamadan ezbere yapmaya çalıştıklarını ifade etmişlerdir. Bu da öğrencilerde yüzdeler konusunda kavramsal anlamanın oluşmadığını, öğrenciler tarafından işlemsel bilginin kullanıldığını göstermektedir. Öğrenci cevapları incelendiğinde öğrencilerin orantı kurma, payda eşitleme ya da denklem yöntemi gibi daha çok kural odaklı, ezbere dayalı işlemler yaptıkları görülmüştür. Tahmin etme ya da yaklaşık değeri bulma ile ilgili sorularda bile öğrencilerin pratik çözümlerden ziyade daha uzun zaman alan işlemsel yöntemleri tercih etmesi araştırmanın bir diğer önemli sonucudur. Bu sonuç, öğrencilerin kural temelli stratejilere yatkınlığını gösteren birçok araştırma sonucunu da desteklemektedir (Harç, 2010; Kayhan Altay, 2010; Singh, 2009; Şengül ve Gülbağcı, 2012; Şengül, Gülbağcı ve Cantimer, 2012; Yang, 2005).
Bu araştırmanın bir diğer bulgusu modellerin, şekillerin kullanıldığı ikinci faktör olan "görsel temsil biçimi" bileşeninde yer alan sorularda öğrencilerin daha başarılı oldukları, sayı duyusu kullanımının bu sorularda daha yüksek olduğu sonucunu ortaya koymuştur. Bu sonuç daha önce yapılmış olan Kayhan Altay (2010)'ın araştırma bulguları ile paralellik göstermektedir. Kayhan Altay (2010)'ın çalışmasında da öğrenciler en çok kesirlerde kavramsal düşünme bileşeninde yer alan kesirlerin sayı doğrusu üzerinde ve şekil üzerinde gösterimi ile ilgili sorularda sayı duyusunu kullanmışlardır. Modellerin kullanıldığı sorularda öğrencilerin sayı duyusu kullanımı yüksek olmasının bir sebebi konunun somutlaştırılması, ders kitaplarında yüzde konusunun giriş kısmında yüzlük kart gibi modellerin kullanılmasından kaynaklı öğrencilerin bu tarz sorulara aşina olması olabilir. Diğer bir sebep ise modellerle, şekillerle sorulan bir sorunun formüller ya da işlemlerle uğraşmaksızın öğrencilerin daha esnek ve derinlemesine düşünmelerini sağlayacak fırsat sunması olabilir. Ancak araştırmada dikkat çeken bir nokta; öğrencilerin küme modeli şeklinde verilen dairelerin belli bir yüzdesini bulmada tek bir şeklin belli bir yüzdesini bulmaya göre daha çok zorlanmaları olmuştur. Bu sonuç Gay (1990)'ın çalışmasının araştırma sonuçları ile de benzerlik göstermektedir. Bu durum derste konunun işlenişi esnasında bütünün tek bir şekil olarak gösterilmesinden, farklı bütün modellerinin öğretmenler tarafından kullanılmamasından kaynaklı olabilir.

Araştırmada öğrencilerin kıyaslama (referans) noktasını kullanma eğilimlerinin olduğu, görsel temsil biçimi ile ilgili sorulardan sonra sayı duyusunu kullanmada en başarılı oldukları soruların kıyaslama (referans) noktası kullanımı ile ilgili sorular olduğu ortaya konmuştur. Benzer şekilde Şengül, Gülbağcı ve Cantimer (2012)'in yaptıkları çalışmada da ilköğretim 6. sınıf öğrencilerinin yüzde problemlerini çözerken kullandıkları sayı duyusu stratejilerinde referans noktalarının kullanımının oldukça fazla olduğu belirtilmiştir. Bu bulgu öğrencilerin $\% 25, \% 50$ ve $\% 100$ gibi referans noktalarını
kullanmada başarılı olduklarını belirten çalışmaların bulguları ile de örtüşmektedir (Dole, Cooper, Baturo, ve Conoplia, 1997; Gay ve Aichele, 1997; Lembke ve Reys; 1994).

Öğrencilerin yüzde konusunda sayı duyularını geliştirebilmek için öncelikle öğretmenlere sayı duyusu kavramını öğrenebilecekleri ve zihinden işlem yapma ya da tahmin etme becerilerini kazanabilecekleri, böylece sayı duyusu becerisini nasıl geliştirmeleri gerektiğini anlayabilecekleri hizmet içi eğitimler verilebilir. Öğretmenlerin hizmet öncesinde de bu becerileri kazanmaları için lisans programı sırasında aldıkları özel öğretim yöntemleri gibi dersleri kapsamında yapılan uygulamalarda özellikle matematiğin çeşitli konularında sayı duyusu etkinliklerine yer verilmelidir. Ayrıca öğretmen adaylarını öğrencilerin yüzdeler konusunda kullandıkları sayı duyusu stratejileri hakkında bilgilenmeleri ileride öğretim ortamlarını bu beceriler üzerine yapılandırmalarında yardımcı olacaktır.
Araştırma sonuçları dikkate alındığında öğrencilerin yüzde konusunda sayı duyularını geliştirmek için sınıf içi uygulamalar da önem taşımaktadır. Bu uygulamalar kapsamında öncelikle yüzde kavramının farklı içeriklerdeki anlamları üzerinde durulmalıdır. Kavramsal anlamanın oluşması için yüzde kavramının günlük hayatta kullanımına ilişkin örnekler üzerinde durulmalı ve nasıl yorumlandığı vurgulanmalıdır.

Araştırmada geliştirilen yüzdeler konusunda sayı duyusu testi öğrencilerin zayıf oldukları, zorlandıkları yerleri tespit etmede, sayı duyusunu geliştirmede ve sayı duyusu bileşenleri dikkate alınarak uygulanacak öğretimin nasıl yapılandırılması gerektiği ile ilgili ipucu verebilir.


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