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# ANALYSIS OF NATURAL FREQUENCIES FOR EARLY DAMAGE DETECTION IN CFRP SHAFTS USING RSM-BASED FREQUENCY CONTOUR PLOTS

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## ABSTRACT

This study explores early damage detection in carbon fiber-reinforced polymer shafts by analyzing natural frequencies. Modern engineering components often face increased flexibility and high stress levels, leading to cracks in rotating parts, which can result in premature failures. To address this issue, modal analysis, specifically natural frequency analysis, is employed to identify deviations caused by cracks. Cracks alter stiffness and mass distribution, leading to shifts in natural frequencies. The study employs finite element models to simulate various crack locations and depths, normalizing them with respect to shaft diameter and length. A cantilever shaft configuration is utilized with refined mesh structures near the transverse crack. The analysis leverages the frequency contour method with response surface methodology to visualize how crack depth and location influence normalized natural frequencies. Results indicate that crack depth has a significant impact on natural frequencies, while crack location has a subtler effect. Combining depth and location produces the most pronounced frequency variations. The corresponding  $R^2$  values for the normalized first and third natural frequencies are 95.69% and 96.32%, while for the normalized second natural frequency this value is 75.70%. The study also demonstrates the use of frequency contour curves for accurate crack detection, with the 1st and 3rd natural frequencies being reliable indicators.

**Keywords:** CFRP shafts, Crack detection, Natural frequency analysis, Finite element modeling, Frequency contour method.

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## 1. INTRODUCTION

Modern machine components prioritize multifunctionality, aiming to achieve attributes such as high strength, durability, lightweight design, and cost-effectiveness [1–4]. However, this pursuit often leads to the formation of cracks in rotating parts due to increased flexibility and high-stress levels [5]. These cracks play a critical role in the failure of rotating machine components [6]. In recent years, there has been growing interest in understanding the dynamic behavior and analysis of cracked rotors using a variety of damage detection techniques. Particularly, excessive vibrations in rotating shafts contribute to premature component failures, necessitating vibration reduction strategies [7]. Additionally, the presence of cracks intensifies vibrations, underscoring the necessity to accurately detect their depths and positions to

prevent catastrophic failures and facilitate proactive maintenance measures. The unique characteristics of cracks, causing fluctuations in vibration frequencies and amplitudes, encourage focused research on crack detection and localization, which holds special significance in applications like propeller shafts to avert potential catastrophic failures [8-9]. The appearance of a transverse crack in a shaft elevates the risk of collapse. Although the presence of a crack may not result in sudden failure, it significantly impacts dynamic behavior. Detecting cracks, especially in rotating shafts with an overhang, poses a challenge due to minor frequency changes unless the crack depth approaches 50% of the shaft diameter, beyond which sudden failure occurs [10]. In recent years, numerous numerical and experimental studies have examined the effects of various crack types—

transverse, longitudinal, slant, breathing cracks, and notches [11–13]. Researchers have employed diverse methods to identify the presence of cracks within structures. Green and Casey proposed two theoretical analyses, global and local asymmetry crack models, to determine the characteristics of the system response directly attributable to the presence of a transverse crack in a rotating shaft. They showed that the 2X harmonic component of the system response is the primary response characteristic resulting from the appearance of a crack. They demonstrated that the unique characteristics of the system response can be used as target observations for the monitoring system [5]. Tlaisi et al. carried out experimental and numerical investigations to determine the presence of cracks in a cylindrical overhanging shaft with a propeller at the free end. The results showed that using the rate of change of frequencies, modal amplitudes, as a function of crack depth ratio will indicate the presence of cracks in the shaft from a crack depth ratio of 0.2 [10]. Long et al. proposed a multiscale method for analyzing the nonlinear vibration of a cracked beam subjected to harmonic excitation. They revealed the relationship between the nonlinear vibration of the cracked beam and the system parameters. The study showed that the nonlinear response of a beam with a breathing crack is affected by structural and crack parameters [11].

Modal analysis, commonly known as natural frequency analysis, serves as a technique to identify the natural vibration frequencies and corresponding mode shapes of mechanical structures. Applying this method to rotating shafts enables the characterization of their dynamic behavior and the detection of deviations resulting from cracks or other structural irregularities [14]. This analysis is based on the idea that the vibrational response of a structure is affected by stiffness, mass distribution and damping properties. Cracks in shafts induce localized changes in stiffness and mass distribution, thus modifying shaft vibrational behavior. These modifications manifest as shifts in the natural frequencies of shaft vibration modes. The extent of frequency shifts depends on factors like crack depth, location, and type.

This study is dedicated to the investigation of carbon fibre reinforced polymer (CFRP) shafts,

aiming to facilitate early detection of damage through the use of RSM (response surface methodology) based frequency contour plots. The main focus of this research is to perform a comprehensive analysis of natural frequencies to enable timely detection of potential structural damages. To achieve this, a series of numerical models were developed using the finite element method (FEM) to simulate cracked shafts. These models covered various locations and depths of the cracks, allowing a comprehensive investigation of their effects. By extracting the first three natural vibration frequencies, important information on the dynamic behavior of cracked structures was obtained. However, the main objective was to assess the influence of crack localization and depth on the changes in these fundamental modes of vibration. This was achieved using a frequency contour approach based on RSM.

## 2. MATERIALS & METHOD

### 2.1. Finite element model for health and cracked shaft

Figure 1 presents the shaft designed for FEM analysis, accompanied by the embedded crack structures. The shaft possesses dimensions measuring 500 mm in length and 30 mm in diameter. In the context of this study, crack depths ranging from 3 mm to 15 mm were considered, ensuring that the maximum crack depth remained equivalent to half the diameter of the shaft. Furthermore, precise crack locations were pinpointed at intervals of 100 mm, 200 mm, 300 mm, and 400 mm from the fixed end of the shaft. The crack depth ratio has been normalized with respect to the shaft diameter, and the crack location has been normalized with respect to the shaft length. The crack modeling analysis was executed by employing a cantilever shaft configuration, securely fixed at a single end, as visually depicted in Figure 1. This figure also visually represents the mesh structures established for both the healthy and cracked conditions of the shaft. Significantly, a finer mesh, 1 mm was applied to the area housing the transverse crack, while the remaining regions were discretized using element sizes of 5 mm. The determination of the optimal mesh size was informed by a comprehensive convergence study, undertaken with the objective of attaining heightened energy efficiency. Figures 2a and b show the mesh size dependent results obtained for healthy and cracked models, respectively.

Firstly, the optimum element size was determined as 5 mm for the healthy model and 1 mm for the damaged surfaces. ANSYS Modal Analysis has been employed for the determination of natural frequencies.

The mechanical properties of the chosen unidirectional CFRP material are provided in Table 1 for finite element analysis. Material data for CFRP was used from ANSYS material library.

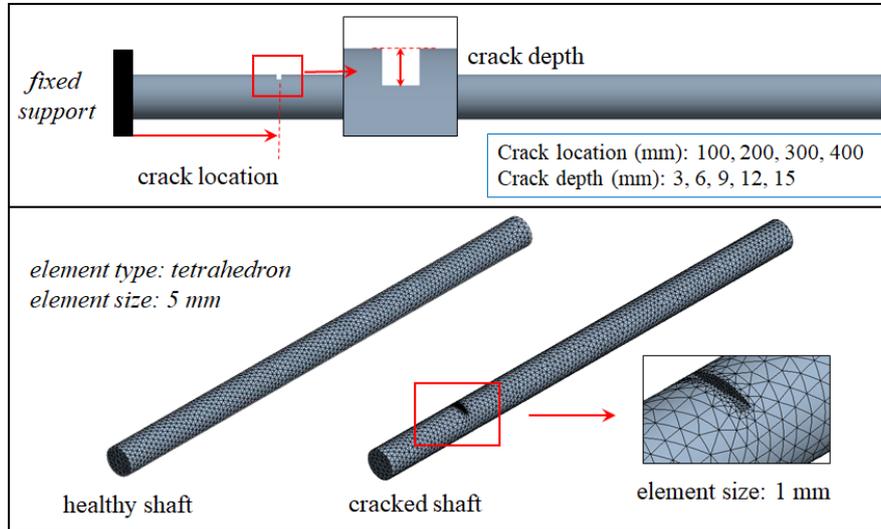


Figure 1. 3D modelling of CFRP shafts

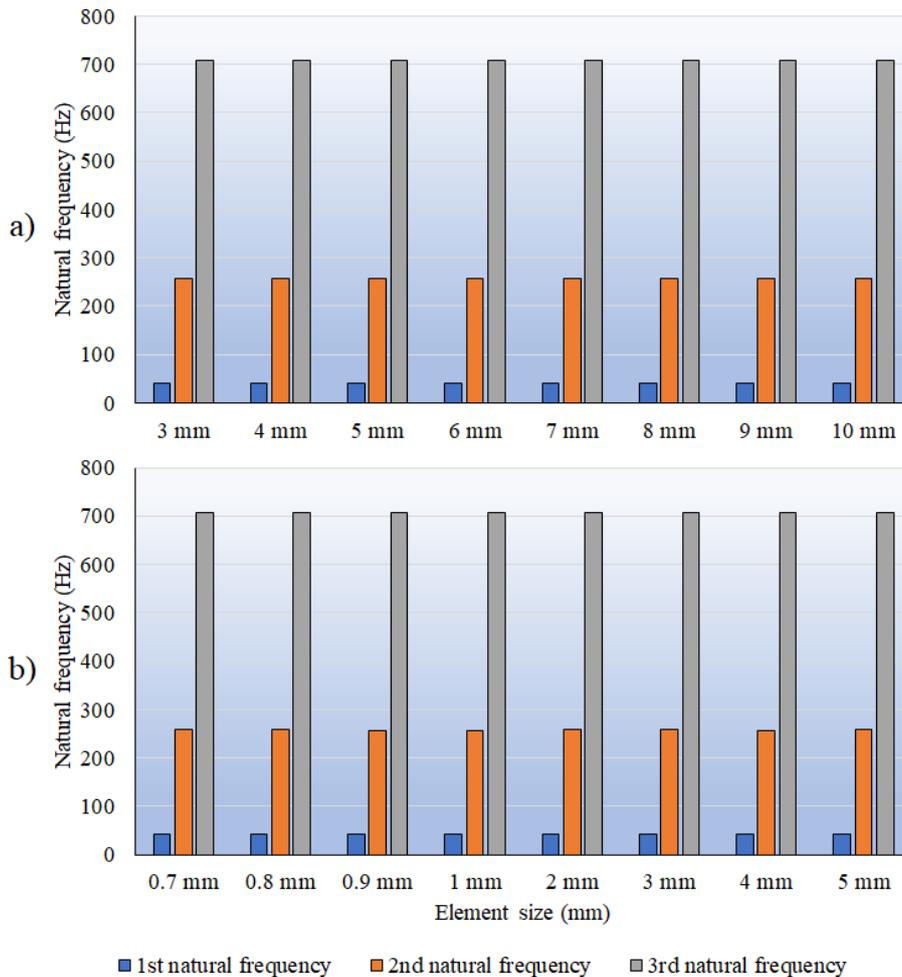


Figure 2. a) Mesh convergence for healthy model b) Mesh convergence for normalized crack location: 0.2 and crack depth ratio:0.1

**Table 1.** Material properties of unidirectional CFRP

Material	$E_x$ (GPa)	$E_y$ - $E_z$ (GPa)	$\nu_{xy}$ - $\nu_{xz}$	$\nu_{yz}$	$G_{xy}$ - $G_{xz}$ (GPa)	$G_{yz}$ (GPa)	$\rho$ (g/cm <sup>3</sup> )
CFRP-UD	209	9.45	0.27	0.4	5.5	3.9	1.54

**2.2. RSM based frequency contour method**

In the frequency contour method, normalized frequencies for a specific beam or shaft are graphed based on both normalized crack position and depth. It is important to note that the normalized natural frequency is influenced not only by the crack's depth and position but also by the number of vibration modes. When contour curves for the first and second vibration modes are plotted along the same axis for a shaft with a crack, it is observed that these curves can intersect at multiple points, indicating the presence of a crack. To accurately determine the precise crack location, the addition of a third frequency contour curve is necessary. The intersection point among these three contours indicates the position and size of the crack [15]. In this study, frequency contour analysis is conducted using an RSM-based statistical approach, employing Minitab 19 for implementation. RSM was selected to optimize the process of crack detection and to elucidate the intricate relationships among crack depth, location, and natural frequencies. The analysis parameters established for this research are outlined in the provided Table 2. RSM enables the systematic exploration of the parameter space by modeling nonlinear interactions and dependencies between input parameters and responses. By fitting a mathematical model to experimental data, RSM aids in identifying optimal combinations of crack depth and location. These combinations maximize discrepancies in natural frequencies, thereby enhancing accuracy in crack identification.

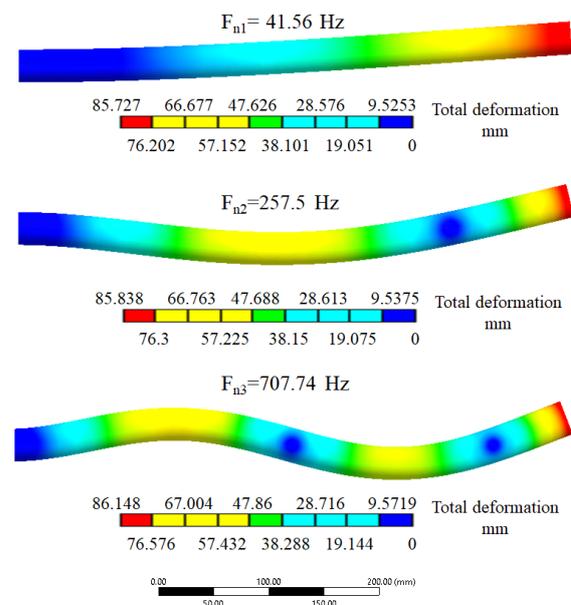
**Table 2.** Crack detection parameters for RSM design

Factors	Levels	Parameters
Normalized crack location	4	0.2, 0.4, 0.6, 0.8
Crack depth ratio	5	0.1, 0.2, 0.3, 0.4, 0.5

**3. RESULTS AND DISCUSSION**

The first 3 natural frequency values obtained for the healthy shaft after modal analysis were 41.56 Hz, 257.5 Hz and 707.74 Hz,

respectively. These values will also be used to normalize the natural frequency values to be obtained from the After performing modal analysis on the healthy shaft, it was determined that the initial three natural frequency values were as follows: 41.56 Hz, 257.5 Hz, and 707.74 Hz, respectively. These particular frequency values hold significant importance, as they will be used as references for normalizing the natural frequency data obtained from shafts containing cracks. Figure 3 provides a visual representation of the natural vibration modes associated with the first natural frequency. This visualization technique is commonly used in structural dynamics and vibration analysis to provide the dynamic behavior of mechanical systems [14]. To conduct a comprehensive analysis of the impact of cracks on shaft performance, a total of 20 analyses were carried out for each unique crack location and depth. The results of these analyses, specifically the first three normalized natural frequency values, are summarized in Table 3.



**Figure 3.** First three natural frequency of healthy shaft and mode shapes

**Table 3.** Numerical analysis results

Normalized crack location	Crack depth ratio	Normalized 1st natural frequency	Normalized 2nd natural frequency	Normalized 3rd natural frequency
0.2	0.1	0.995500481	0.9999612	0.9990260
0.2	0.2	0.981159769	0.9999223	0.9956384
0.2	0.3	0.954307026	0.9995728	0.9889054
0.2	0.4	0.910370549	0.9989903	0.9774155
0.2	0.5	0.840976901	0.9979418	0.9579787
0.4	0.1	0.998195380	0.9974370	0.9983344
0.4	0.2	0.992276227	0.9888936	0.9928435
0.4	0.3	0.980774783	0.9727778	0.9826946
0.4	0.4	0.960563041	0.9463322	0.9665749
0.4	0.5	0.925000000	0.9050522	0.9423954
0.6	0.1	0.999711261	0.9963108	0.9979815
0.6	0.2	0.998363811	0.9840783	0.9913050
0.6	0.3	0.995620789	0.9610112	0.9792081
0.6	0.4	0.990447546	0.9225661	0.9604206
0.6	0.5	0.980389798	0.8610539	0.9332345
0.8	0.1	1.000288739	0.9991845	0.9970076
0.8	0.2	1.000721848	0.9965438	0.9870704
0.8	0.3	1.001179018	0.9913013	0.9677465
0.8	0.4	1.001491819	0.9816318	0.9339544
0.8	0.5	1.001419634	0.9631471	0.8765897

Table 4 presents the coefficients of determination obtained separately for each of the three responses after the application of RSM. The coefficient of determination, denoted as  $R^2$ , expresses the percentage of variance in the dependent variable explained by the independent variables.  $R^2$  takes values between 0 and 1, where values close to 0 indicate that the model is insufficient to explain the data, while values close to 1 indicate that it fully explains the data. The corresponding  $R^2$  values for the normalized first and third natural frequencies are 95.69% and 96.32%, while for the normalized second natural frequency this value is 75.70%. Similarly, the adjusted  $R^2$  values for the normalized first and third natural frequencies are 94.51 and 95.00%, while for the normalized second natural frequency it is 67.02. These values are particularly satisfactory for the first and third vibration modes and the second vibration mode also reaches an acceptable level of coherence. Therefore, it is reasonable to

conclude that the established model provides a sufficiently comprehensive explanation. Similarly, Banerjee et al. reported that the regression model developed for crack prediction agreed reasonably well with the values [15]. However, when the  $R^2$  (pred) value is analyzed, a significant decrease is observed, especially in the second case. The  $R^2$  (pred) values for the normalized first and third natural frequencies are 86.98% and 88.28%, while for the normalized second natural frequency they are 29.57%.  $R^2$  (pred) represents a version of  $R^2$  adapted for predictions, which aims to show how the model will respond to predictions that need to generalize beyond the actual data. Unlike  $R^2$ , which focuses on the extent of data description,  $R^2$ (pred) provides an estimate of how well the model is likely to perform when making future predictions. Accordingly, it is concluded that the data obtained from the 2nd vibration mode are more insufficient than the 1st and 3rd data in crack detection.

**Table 4.** Determination coefficients

Response	R <sup>2</sup>	R <sup>2</sup> (adj)	R <sup>2</sup> (pred)
Normalized 1 <sup>st</sup> natural frequency	95.96%	94.51%	86.98%
Normalized 2 <sup>nd</sup> natural frequency	75.70%	67.02%	29.57%
Normalized 3 <sup>rd</sup> natural frequency	96.32%	95.00%	88.28%

The frequency contour plots generated in the study are derived from a response surface model that visually represents the complex interactions between crack depth, location, and natural frequencies. As shown in Figure 4, these plots vividly illustrate regions of high sensitivity by highlighting the crack depths and location ranges that cause the most significant changes in natural frequencies. The response surface model presents the data in a graphical format, facilitating the understanding of the interdependencies and non-linear effects that shape the response of the system. It can be seen that different contour curve distributions appear for the three different responses. This observation emphasizes the multiplicity of responses and the importance of considering the contour patterns involved. This phenomenon can be attributed to the complex nature of the structural response to varying crack characteristics. In the first scenario, it was observed that an increase in the crack depth ratio leads to a decrease in the normalized natural frequency. This phenomenon is in line with the intuitive understanding that larger crack depths will lead to greater stiffness reduction in the structure, which in turn will cause a decrease in the natural frequency. In contrast, the effect of normalized crack location on the normalized natural frequency is more limited. This subtle relationship implies that changes in crack location can have a relatively subtle effect on the overall dynamic behavior of the system. Ergene showed that the natural frequency value of the beam is expected to decrease due to the decrease in the stiffness of the beam with increasing crack depth, but the fibre angle and the size of the crack depth can

change this situation and an increase in natural frequency values can also be seen [16]. On the other hand, Cunedioğlu et al. conducted free vibration analysis of cracked cantilever sandwich beams and found that natural frequency values decreased with increasing crack depth [17]. In the second scenario, the effectiveness of crack location occurs predominantly at the ends of the shaft, while an increase in crack depth reduces the normalized natural frequency. This observation highlights the importance of crack localization in certain regions of the structure. It suggests that cracks located near the ends of the shaft may lead to more pronounced changes in the natural frequencies due to their proximity to the nodal points. Furthermore, the decrease in natural frequency with increasing crack depth underlines the importance of accurately assessing crack depth when assessing structural health. Finally, in the third scenario, an increase in both crack location and depth contributes to a decrease in normalized natural frequency. This result reinforces the understanding that the cumulative effects of cracks on structural behavior are exacerbated when both depth and location vary. The interaction between these two parameters highlights the potential for a more significant variation in natural frequencies and underlines the need for a holistic approach to damage assessment. Owalabi et al. also demonstrated the effectiveness of using contour plots for crack location in beams. Because they stated that a crack will definitely belong to a contour line for each mode and it will be sufficient to measure the lowest three natural frequencies in a beam [18].

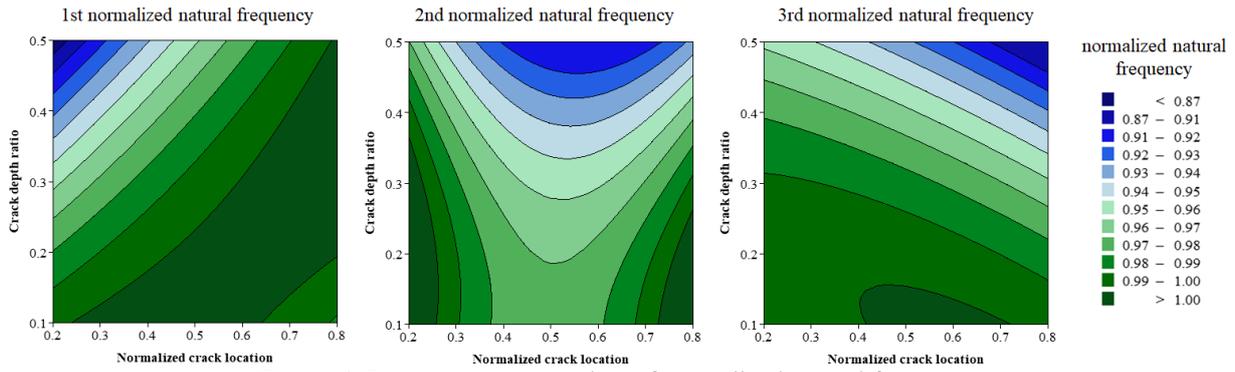


Figure 4. Frequency contour plots of normalized natural frequency

Figure 5 illustrates the use of frequency contour curves to predict both crack location and size in three different crack scenarios. In the case where the crack depth ratio is 0.5 and the normalized crack location is 0.4, the normalized natural frequency contours corresponding to curves 1 and 3 in the developed model intersect exactly at the point of interest. It should be noted that among the natural frequency curves plotted, curve 2 does not appear within the limits of the given value range. However, it can be noticed that the accuracy of the prediction of the presence of cracks lies primarily in the 1st and 3rd natural frequencies. In the next scenario, contour curves were plotted with a crack depth ratio of 0.2 and a normalized crack location of 0.8. The graph shows that the 2nd natural frequency curves intersect in a different way, while the 3rd frequency curve shows a very close alignment at the crack location. The pairs of contour curves, which are distorted due to natural uncertainty caused by measurement

and modeling errors, intersect at three different points and collectively form a triangular configuration [15]. This triangle was defined as the boundary center of the formation and the location for determining the location and depth of the fracture. Moving to the third scenario, where the crack depth ratio is equal to 0.4 and the normalized crack location takes the value 0.2, the frequency contour curves show the intersection of the 1st and 3rd frequencies, with the 2nd natural frequency coming very close to this intersection point. Similarly, Banerjee et al. showed that there is a smaller amount of error when the location and depth of the crack are estimated by the frequency contour plot method [15]. Tlaisi et al. stated that the third mode shape can be used as a good indicator of the presence of a crack in the shaft, giving a much higher variation in these mode shapes than the changes in frequencies due to the presence of the crack [10].

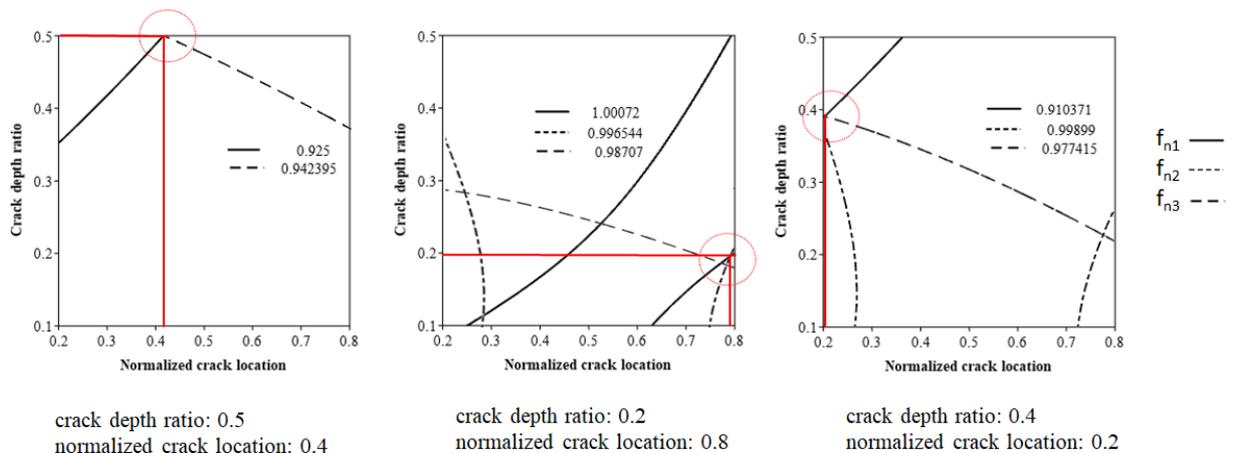


Figure 5. Crack prediction using frequency contours

4. CONCLUSIONS

In this study, the analysis of natural frequencies in CFRP shafts for early damage detection using RSM and frequency contour plots has been

conducted. The following key conclusions can be drawn:

1. Detecting cracks in rotating machine components, especially CFRP shafts, is of paramount importance to prevent catastrophic failures. The study emphasizes the role of natural frequency analysis in identifying deviations caused by cracks.
2. Cracks in shafts lead to localized changes in stiffness and mass distribution, resulting in shifts in natural frequencies. The extent of these frequency shifts depends on crack depth, location, and type.
3. Finite element models were used to simulate various crack locations and depths in CFRP shafts. This comprehensive approach allowed for a detailed investigation of crack effects.
4. The study introduced the frequency contour method, leveraging RSM to visualize the influence of crack depth and location on normalized natural frequencies. This method proved effective in highlighting regions of high sensitivity for crack detection.
5. The analysis revealed that the 1st and 3rd natural frequencies are particularly reliable indicators of crack presence, making them valuable tools for early damage detection.
6. The study demonstrated that crack depth primarily affects natural frequencies, while crack location has a subtler influence. The most significant frequency variations are observed when both depth and location vary, underscoring the need for a holistic approach to damage assessment.
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