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A New Compromise Allocation Method in Stratified Random Sampling

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Abstract

Sample size of the strata is determined by the help of some allocation methods in Stratified Random Sampling. Most of the allocation methods ignore the selection cost. However, in real life applications it is very rare to come across such situations. In this study, a new compromise allocation method is proposed by adding a non-linear cost function constraint to Costa et al.(2004) method. Using this new allocation, the sample size with linear cost constraint is also obtained. The performance of the proposed method is studied utilizing the data from Statistics Canada's Monthly Retail Trade Survey (MRTS) of single establishments used by Choudhry et al. (2012).

1. INTRODUCTION

Increasing the precision of a statistic interested in is possible by decreasing the variance of this statistic, where the number of strata and their bounds determined in the sampling plan beforehand. Optimum sample size from homogenous strata can be identified to decrease the variance. Statisticians utilized from kinds of allocation methods to achieve related goal. Generally, equal, proportional, optimum, and Neyman allocation methods are applied for the determination of the sample size of the strata (Cochran, 1977). Most of the researchers ignore the unit selection cost to determine strata sample size. If some of the strata in the population have a big size or high dispersion, traditional allocation methods do not work well; therefore, compromise among two or more allocation methods have been proposed. Carroll (1970) and Fellegi (1981) are the pioneers of the power allocation approach which is one of the most preferred compromise strategy in stratified sampling designs. Bankier (1988) proposed a new power allocation method derived from Neyman and equal allocation methods and disregard the cost function. Another method compromising equal and proportional allocations and also ignoring the cost was proposed by Costa et al. (2004). Longford (2006) studied on the allocation method minimizing both variance of the estimation of the strata means and variance of the estimation of the population mean. Choudry et al. (2012) proposed a new allocation method and made a comparison to the methods mentioned above by the help of Statistics Canada's Monthly Retail Trade Survey data.

Ignoring the selection cost for strata may not be logical for real life applications. Therefore, considering the cost function in the allocation procedure makes sampling designs more reasonable. In case there is a difference in the cost of selection for strata, a researcher typically benefits from linear cost function given in Eq.(1.1).

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$$t = t_0 + \sum_{h=1}^{L} t_h n_h \tag{1.1}$$

In this equation, t is defined as the total cost for survey, t_0 is fixed cost, t_h is cost of selection for one unit from stratum h, and n_h is the sample size of the stratum h ($h=1,2,...,L$). As seen in Eq.(1.1), allocating sample size to strata is not so difficult for linear cost function and get the minimum variance estimator by this way. The linear cost function given in Eq.(1.1) is appropriate if the selection cost for one unit from stratum h is not significantly different.

On the other hand, it is not always the case that the selection of a sampling unit increases the cost function at the same rate i.e. no more extra costs for observation units after arriving to the rural area. For this reason, the selection of one unit from this stratum increases the cost function less than one unit. Just the opposite case can occur. In such cases, the cost function is in a non-linear form. Cochran (1977), Bretthauer et al.(1999), and Chernyak (2001) have defined the non-linear cost function as:

$$t = t_0 + \sum_{h=1}^{L} t_h n_h^{\alpha} \tag{1.2}$$

where α indicates the effect on the cost function for the selection of one sampling unit from stratum h. If α is smaller than 1, the selection of one sampling unit from the strata affects the cost function less than one unit and if α is larger than 1, the selection of one sampling unit from the strata affects the cost function greater than one unit. Eqs.(1.1) and (1.2) give the same results when α is equal to 1. α is a positive value determined by the researcher. If the selection cost of one unit differs among strata, then it is suggested that the cost function in Eq.(1.2) be used.

A new compromise allocation method is proposed in this paper. The present method aims to make variance minimum by adding the non-linear cost function constraint to the mentioned allocation model in Costa et al. (2004). In this proposed model, using non-linear cost function, it is also possible to obtain sample size under a linear cost constraint. This allocation method was obtained in Section 2 and the performance of the proposed model was studied by using the same data of Choudhry et al. (2012) coming from Statistics Canada's Monthly Retail Trade Survey (MRTS) of single establishments. The results for different parameters and their different levels were discussed in Section 3. Finally, the results were summarized and some remarks for the proposed model in Section 4 were given.

2. DEFINITION OF THE NEW COMPROMISE ALLOCATION

Assume that L homogenous strata are constructed in the target population. N_h, $W_h=N_h/N$, and n_h (h=1,2,…,L) respectively indicate the size, weight and sample size of stratum h. Population mean, $\bar{Y} = \sum_h W_h \bar{Y}_h$, is estimated by weighted sample mean, $\bar{y}_{st} = \sum_h W_h \bar{y}_h$. Great differences among strata sample size may cause some problems in stratified random sampling. For instance, Neyman allocation minimizes the variance of $\bar{y}_{st}$, whereas it may give less precise strata estimates with large coefficient of variations ($CV(\bar{y}_h) = C_h / \sqrt{n_h}$) under the constraint of $n = \sum_h n_h$. On the contrary, equal allocation produces efficient estimations for strata means, while it has larger CV for $\bar{y}_{st}$ estimator than Neyman allocation.

Costa et al.(2004) proposed a compromise allocation method utilizing convex combination of optimum ($n_h = nW_h = \frac{N_h}{N} n$) and equal ($n_h = \frac{n}{L}$) allocation methods. Compromise allocation of Costa et al.(2004) is given as

$$n_h^C = k(nW_h) + (1-k)\frac{n}{L} = k(\frac{N_h}{N} n) + (1-k)\frac{n}{L} \tag{2.1}$$

Here, *k* is a constant in the range of $0 \le k \le 1$. This method becomes an equal allocation when $k = 0$ and proportional allocation when $k = 1$. Eq. (2.1) needs to be modified when $\frac{n}{L} > N_h$ for some *h* in a set of strata *A*. The modified allocation is

$$\tilde{n}_h^C = k(nW_h) + (1-k)n_h^0 \qquad (2.2)$$

where $n_h^0 = N_h$ for $h \in A$ and $n_h^0 = \frac{n - \sum_{h \in A} N_h}{L - m}$ otherwise, *m* indicates the number of strata in the set of *A*.

For example; let's allocate the sample size $n = 36$ with respect to the allocation method of Costa et al.(2004) for $k = 0$, where $N_1 = 14, N_2 = 8, N_3 = 6, N_4 = 32$, $N = 60$.

For $n_h^C = k(nW_h) + (1-k)\frac{n}{L} = k(\frac{N_h}{N}n) + (1-k)\frac{n}{L}$, if $k = 0 \Rightarrow n_h^C = \frac{n}{L}$

$n_1^C = n_2^C = n_3^C = n_4^C = \frac{n}{L} = \frac{36}{4} = 9$ and

$n_1^C = 9 \Rightarrow n_1^C < N_1$, $n_2^C = 9 \Rightarrow n_2^C > N_2$, $n_3^C = 9 \Rightarrow n_3^C > N_3$, $n_4^C = 9 \Rightarrow n_4^C < N_4$.

There are two strata where $\frac{n}{L} > N_h$. In this case, two strata are included in the set of *A* and $m = 2$. Adjusted equal allocation is performed to complete the process.

$$n_h^0 = \begin{cases} N_h & h \in A \\ \frac{n - \sum_{h \in A} N_h}{L - m} & h \notin A \end{cases}$$

$h = 1 \notin A \Rightarrow n_1^0 = \frac{n - \sum_{h \in A} N_h}{L - m} = \frac{36 - (8+6)}{4-2} = 11$, $h = 2 \in A \Rightarrow n_2^0 = N_2 = 8$,

$h = 3 \in A \Rightarrow n_3^0 = N_3 = 6$, $h = 4 \notin A \Rightarrow n_4^0 = \frac{n - \sum_{h \in A} N_h}{L - m} = \frac{36 - (8+6)}{4-2} = 11$

$n = n_1^0 + n_2^0 + n_3^0 + n_4^0 = 11 + 8 + 6 + 11 = 36.$

As seen from the strata sample size, the allocation method of Costa et al.(2004) does not become an equal allocation for *k*=0 where strata size N_h (*h*=1, 2, 3, 4) are reasonably different. This feature is a useful aspect of the mentioned allocation.

The allocation method of Costa et al.(2004) ignores the cost of selection from strata. For this reason, in the rest of this section, a new compromise allocation method was produced by adding a new convex component to Eq.(2.1) considering the cost. For this purpose, objective function $\sum_h CV^2(\bar{y}_h)$ was made minimum under non-linear cost constraint $t = t_0 + \sum_h t_h n_h^\alpha$. The new allocation method was given in Theorem 2.1.

Theorem 2.1: In stratified random sampling with the non-linear cost function $t = t_0 + \sum_{i=1}^{L} t_h n_h^\alpha$, $\alpha \ge 0$, the objective function $\sum_h CV^2(\bar{y}_h)$ is minimum for a specified cost *t* when

$$n_h = \frac{\left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}}{\sum_h \left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}} n \tag{2.3}$$

If Eq.(2.3) is added to the allocation method of Costa et al.(2004) as a convex component, then the new compromise allocation method is obtained as shown in Eq. (2.4).

$$n_h = \lambda_1 \left(\frac{\left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}}{\sum_h \left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}} n \right) + \lambda_2 (nW_h) + (1-\lambda_1-\lambda_2)\frac{n}{L} \tag{2.4}$$

where $\lambda_1+\lambda_2+\lambda_3=1$ and $0 \le \lambda_1, \lambda_2, \lambda_3 \le 1$.

Proof 2.1:

$\min\left\{\sum_h CV^2(\bar{y}_h)\right\}$

Constraint: $t = t_0 + \sum_h t_h n_h^{\alpha}$

Here; if the equations $CV(\bar{y}_h) = \dfrac{C_h}{\sqrt{n_h}}$ and $T = \sum_h t_h n_h^{\alpha}$ are used, the model may be rearranged as

$\min\left\{\sum_h \dfrac{C_h^2}{n_h}\right\}$

Constraint: $T = \sum_h t_h n_h^{\alpha}$

Take λ as the Lagrangian multiplier and minimize

$$L = \left\{\sum_h \frac{C_h^2}{n_h}\right\} + \lambda\left(\sum_h t_h n_h^{\alpha} - T\right)$$

for chosen n_h.

Hence, to minimize the objective function for fixed t, we have

$$\lambda \alpha t_h n_h^{\alpha-1} = \frac{C_h^2}{n_h^2}$$

and

$$n_h = \left(\frac{C_h^2}{\lambda\alpha t_h}\right)^{\frac{1}{1+\alpha}} = \frac{\left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}}{\left(\lambda\alpha\right)^{\frac{1}{1+\alpha}}}$$

$$n = \sum_h n_h = \sum_h \frac{\left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}}{\left(\lambda\alpha\right)^{\frac{1}{1+\alpha}}} = \frac{1}{\left(\lambda\alpha\right)^{\frac{1}{1+\alpha}}} \sum_h \left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}$$

dividing n_h by n, Eq.(2.3) is obtained.

If $\alpha = 1$ and $C_h = C$ for each stratum, the new proposed method turn into Square Root allocation given in Eq.(2.5).

$$n_h = \frac{\left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}}{\sum_h \left(C_h^2/t_h\right)^{\frac{1}{1+\alpha}}} n = \frac{\left(C^2/t_h\right)^{\frac{1}{1+1}}}{\sum_h \left(C^2/t_h\right)^{\frac{1}{1+1}}} n = \frac{1/\sqrt{t_h}}{\sum_h 1/\sqrt{t_h}} n \tag{2.5}$$

New method is similar to Eq. (2.4) which is the convex combination of equal and proportional allocation. For $\lambda_1 = \lambda_2 = 0$; the proposed method is transformed into equal allocation and if the case $\frac{n}{L} > N_h$ occurs, adjusted equal allocation is performed.

3. APPLICATION

Performance of the proposed model with respect to sample size allocation for different case was analyzed in this section. To implement the allocation method of interest, the number of strata, their bounds and sample size *n* should be pre-determined. Choudry et al.(2012) proposed a Non-Linear Programming (NLP) model to decide strata sample size. For their experimental study, they utilized from the subset of the data Statistics Canada's Monthly Retail Trade Survey (MRTS) of single establishments. Strata sizes (N_h), strata population means ($\bar{Y}_h$) strata standard deviations (S_h), and strata CV_s (C_h) were given in Table 3.1 for ten strata. NLP model proposed by Choudry et al.(2012) was given in Eq. (3.1).

***Table 1**. Population values for MRTS*

Provinces	N_h	$\bar{Y}_h$	S_h	C_h
Newfoundland(NL)	909	963	1943	2.02
Price-Edward-Island(PE)	280	712	1375	1.93
New-Brunswick(NB)	1333	1368	3200	2.34
Nova-Scotia(NS)	1153	1568	4302	2.74
Quebec(QC)	11135	2006	4729	2.36
Ontario(ON)	21531	1722	6297	3.66
Manitoba(MN)	1700	1295	2973	2.30
Saskatchewan(SK)	1743	1212	3019	2.49
Alberta(AL)	5292	1698	5358	3.16
British Columbia(BC)	7803	1291	4013	3.11
Canada(CA)	52879	1654	-	-

$$\min\left\{\sum_{h=1}^{L} n_h\right\}$$
$$\text{Constraint: } CV(\bar{y}_h) \le CV_{oh} \quad h = 1, 2, ..., L \tag{3.1}$$
$$CV(\bar{y}_{st}) \le CV_o, \quad 1 \le n_h \le N_h$$

Here CV_{oh} and CV_o are specified tolerances on the coefficient of variations for the $\bar{y}_h$ and the $\bar{y}_{st}$, respectively. By means of Eq.(3.1) and MRTS parameter values in Table 3.1, Eq.(3.2) was derived which determines the sample size. For the proposed new compromise allocation model, target values for *CVs* were specified as $CV(\bar{y}_h) \le 0.15$ for the strata means $\bar{y}_h$ and $CV(\bar{y}_{st}) \le 0.06$ for the weighted sample mean $\bar{y}_{st}$. The results were given below as a result of Eq.(3.1):

$$\min\{n_1 + n_2 + n_3 + ... + n_9 + n_{10}\} \tag{3.2}$$

$n_1 \ge 150.74$, $n_2 \ge 104.19$, $n_3 \ge 205.64$, $n_4 \ge 259.66$, $n_5 \ge 241.74$,
$n_6 \ge 578.79$, $n_7 \ge 205.86$, $n_8 \ge 237.55$, $n_9 \ge 408.20$, $n_{10} \ge 407.59$

$$\frac{0.00041}{n_1}+\frac{0.00002}{n_2}+\frac{0.00238}{n_3}+\frac{0.00321}{n_4}+\frac{0.36222}{n_5}$$
$$+\frac{2.40132}{n_6}+\frac{0.00334}{n_7}+\frac{0.00362}{n_8}+\frac{0.10503}{n_9}+\frac{0.12809}{n_{10}}\leq 0.00379$$
$$1\leq n_1\leq 909,\ 1\leq n_2\leq 280,\ 1\leq n_4\leq 1153,\ 1\leq n_5\leq 11135,\ 1\leq n_6\leq 21531$$
$$1\leq n_7\leq 1700,\ 1\leq n_8\leq 1743,\ 1\leq n_9\leq 5292,\ 1\leq n_{10}\leq 7803.$$

By computing the NLP allocation for the given *CV* constraints, minimum sample size becomes *n*=3446. For MRTS data, unit selection costs between 1-20 values produced by random number generator were added and given in Table 3.2. The reason not to prefer adding nonlinear cost function to NLP model of Choudry et al.(2012) is the necessity to calculate sample size for each α value. If the cost constraint was added to this model, it would be impossible to make comparison for different α values. Besides, adding nonlinear cost constraint to Eq(3.1) would be appropriate when the proposed compromise allocation was used in real applications.

Table 2. *Selection cost for one unit from stratum h for MRTS*

Provinces	t_h
Newfoundland(NL)	6
Price-Edward-Island(PE)	8
New-Brunswick(NB)	9
Nova-Scotia(NS)	10
Quebec(QC)	16
Ontario(ON)	20
Manitoba(MN)	11
Saskatchewan(SK)	12
Alberta(AL)	15
British Columbia(BC)	16

Using the proposed compromise allocation model, overall sample size *n*=3446 was allocated to strata, and then $CV(\bar{y}_h)$, $CV(\bar{y}_{st})$ values were obtained. The new allocation method was analyzed whether $CV(\bar{y}_h)$ and $CV(\bar{y}_{st})$ indicators provide the target values. For the analysis, $\alpha=0$, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 2 values were used and λ_1, λ_2, λ_3 values were given in Table 3.3. Where $\lambda_1+\lambda_2+\lambda_3=1$ and $\alpha\geq 0$. Results were given in Tables 3.4-3.15.

Table 3. *λ_i values for i=1, 2, 3*

λ_1	0.8	0.7	0.6	0.5	0.2	0.3	0.5	0.5	0.3	0.2	0.3	0.2
λ_2	0.2	0.3	0.4	0.5	0.8	0.7	0.3	0.2	0.5	0.5	0.2	0.3
λ_3	0	0	0	0	0	0	0.2	0.3	0.2	0.3	0.5	0.5

Table 4. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.8, λ2=0.2, λ3=0*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	336	0.08	326	0.08	320	0.09	316	0.09	312	0.09	310	0.09	307	0.09	304	0.09
PE	225	0.05	236	0.04	243	0.04	248	0.04	252	0.03	255	0.03	258	0.03	261	0.03
NB	307	0.11	305	0.11	304	0.11	302	0.11	302	0.11	301	0.11	300	0.11	299	0.11
NS	372	0.11	356	0.12	344	0.12	337	0.12	331	0.12	326	0.12	323	0.12	317	0.13
QC	311	0.13	329	0.12	342	0.12	352	0.12	360	0.12	366	0.12	371	0.12	379	0.11
ON	599	0.14	591	0.14	586	0.14	582	0.14	579	0.15	576	0.15	574	0.15	571	0.15
MN	251	0.13	261	0.13	267	0.12	271	0.12	275	0.12	277	0.12	279	0.12	283	0.12
SK	269	0.13	275	0.13	279	0.13	282	0.13	285	0.13	286	0.13	288	0.13	289	0.13
AL	386	0.15	378	0.15	373	0.15	369	0.15	366	0.15	364	0.15	362	0.16	359	0.16
BC	389	0.15	388	0.15	387	0.15	386	0.15	385	0.15	384	0.15	384	0.15	383	0.15
Canada	3445	0.07	3445	0.07	3445	0.07	3445	0.07	3447	0.07	3445	0.07	3446	0.07	3445	0.07

Table 5. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.7, λ2=0.3, λ3=0*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	301	0.09	293	0.09	288	0.09	284	0.09	281	0.10	278	0.10	276	0.10	274	0.10
PE	199	0.07	209	0.06	215	0.06	220	0.06	223	0.05	226	0.05	228	0.05	231	0.05
NB	280	0.12	278	0.12	277	0.12	276	0.12	275	0.12	274	0.12	273	0.12	273	0.12
NS	335	0.12	320	0.13	311	0.13	304	0.13	299	0.13	295	0.13	292	0.13	287	0.14
QC	363	0.12	379	0.11	390	0.11	399	0.11	406	0.11	411	0.11	416	0.11	422	0.11
ON	700	0.13	693	0.13	688	0.13	684	0.13	682	0.13	680	0.13	678	0.13	675	0.13
MN	234	0.13	242	0.13	247	0.13	251	0.13	254	0.13	257	0.13	258	0.13	261	0.13
SK	249	0.14	255	0.14	259	0.14	261	0.14	263	0.14	265	0.14	266	0.14	267	0.14
AL	381	0.15	374	0.15	369	0.15	366	0.15	364	0.15	362	0.16	360	0.16	357	0.16
BC	404	0.15	403	0.15	402	0.15	401	0.15	400	0.15	400	0.15	399	0.15	398	0.15
Canada	3446	0.06	3446	0.06	3446	0.06	3446	0.06	3447	0.06	3448	0.06	3446	0.06	3445	0.06

Table 6. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ_1=0.6, λ_2=0.4, λ_3=0*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	267	0.10	260	0.10	255	0.10	252	0.10	249	0.10	247	0.10	245	0.11	243	0.11
PE	174	0.09	182	0.08	187	0.08	191	0.07	194	0.07	196	0.07	198	0.07	201	0.07
NB	252	0.13	251	0.13	249	0.13	249	0.13	248	0.13	247	0.13	247	0.13	246	0.13
NS	298	0.13	285	0.14	277	0.14	271	0.14	267	0.14	263	0.14	261	0.14	257	0.15
QC	415	0.11	428	0.11	438	0.11	446	0.10	451	0.10	456	0.10	460	0.10	466	0.10
ON	800	0.12	794	0.12	790	0.12	787	0.12	785	0.12	783	0.12	782	0.12	779	0.12
MN	216	0.14	223	0.14	228	0.14	231	0.14	234	0.13	236	0.13	237	0.13	240	0.13
SK	230	0.15	235	0.15	238	0.15	240	0.14	242	0.14	243	0.14	244	0.14	245	0.14
AL	376	0.15	370	0.15	366	0.15	363	0.16	361	0.16	359	0.16	358	0.16	356	0.16
BC	419	0.14	418	0.14	417	0.14	416	0.14	416	0.14	415	0.14	415	0.14	414	0.14
Canada	3447	0.06	3446	0.06	3447	0.06	3446	0.06	3447	0.06	3445	0.06	3447	0.06	3447	0.06

Table 7. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ_1=0.5, λ_2=0.5, λ_3=0*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	232	0.11	226	0.11	222	0.11	220	0.11	217	0.11	216	0.12	214	0.12	212	0.12
PE	148	0.10	154	0.10	159	0.10	162	0.09	164	0.09	166	0.09	168	0.09	170	0.09
NB	224	0.14	223	0.14	222	0.14	222	0.14	221	0.14	221	0.14	220	0.14	219	0.14
NS	261	0.14	250	0.15	243	0.15	239	0.15	235	0.15	232	0.16	230	0.16	226	0.16
QC	466	0.10	478	0.10	486	0.10	492	0.10	497	0.10	501	0.10	504	0.10	509	0.10
ON	901	0.11	896	0.11	892	0.12	890	0.12	888	0.12	886	0.12	885	0.12	883	0.12
MN	198	0.15	204	0.15	208	0.14	211	0.14	213	0.14	215	0.14	216	0.14	218	0.14
SK	211	0.16	215	0.15	217	0.15	219	0.15	220	0.15	221	0.15	222	0.15	223	0.15
AL	371	0.15	366	0.15	362	0.16	360	0.16	358	0.16	357	0.16	356	0.16	354	0.16
BC	434	0.14	433	0.14	432	0.14	432	0.14	431	0.14	431	0.14	430	0.14	430	0.14
Canada	3446	0.06	3445	0.06	3443	0.06	3447	0.06	3444	0.06	3446	0.06	3446	0.06	3444	0.06

Table 8. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.2, λ2=0.8, λ3=0*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	128	0.16	126	0.16	124	0.16	123	0.16	122	0.17	122	0.17	121	0.17	120	0.17
PE	70	0.19	73	0.19	74	0.19	76	0.18	77	0.18	77	0.18	78	0.18	79	0.18
NB	142	0.18	141	0.18	141	0.18	141	0.18	141	0.18	140	0.18	140	0.18	140	0.18
NS	149	0.20	145	0.21	142	0.21	140	0.21	139	0.21	138	0.21	137	0.21	136	0.22
QC	622	0.09	627	0.09	630	0.09	632	0.09	634	0.09	636	0.09	637	0.09	639	0.09
ON	1202	0.10	1200	0.10	1199	0.10	1198	0.10	1197	0.10	1196	0.10	1196	0.10	1195	0.10
MN	146	0.18	148	0.18	150	0.17	151	0.17	152	0.17	152	0.17	153	0.17	154	0.17
SK	152	0.19	154	0.19	155	0.19	156	0.19	156	0.19	157	0.18	157	0.18	158	0.18
AL	355	0.16	353	0.16	352	0.16	351	0.16	350	0.16	350	0.16	349	0.16	348	0.16
BC	479	0.13	478	0.13	478	0.13	478	0.13	478	0.13	477	0.13	477	0.13	477	0.13
Canada	3445	0.05	3445	0.05	3445	0.05	3446	0.05	3446	0.05	3445	0.05	3445	0.05	3446	0.05

Table 9. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.3, λ2=0.7, λ3=0*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	163	0.14	159	0.14	157	0.14	155	0.14	154	0.14	153	0.14	152	0.14	151	0.15
PE	96	0.15	100	0.15	103	0.15	105	0.14	106	0.14	107	0.14	108	0.14	109	0.14
NB	169	0.16	169	0.16	168	0.16	168	0.16	167	0.16	167	0.16	167	0.16	166	0.16
NS	187	0.18	180	0.18	176	0.19	173	0.19	171	0.19	169	0.19	168	0.19	166	0.19
QC	570	0.09	577	0.09	582	0.09	586	0.09	589	0.09	591	0.09	593	0.09	596	0.09
ON	1102	0.10	1099	0.10	1097	0.10	1095	0.10	1094	0.10	1093	0.10	1092	0.10	1091	0.10
MN	163	0.17	167	0.16	169	0.16	171	0.16	172	0.16	173	0.16	174	0.16	175	0.16
SK	172	0.18	174	0.17	176	0.17	177	0.17	178	0.17	178	0.17	179	0.17	180	0.17
AL	360	0.16	357	0.16	355	0.16	354	0.16	353	0.16	352	0.16	351	0.16	350	0.16
BC	464	0.14	463	0.14	463	0.14	462	0.14	462	0.14	462	0.14	462	0.14	461	0.14
Canada	3446	0.05	3445	0.05	3446	0.05	3446	0.05	3446	0.05	3445	0.05	3446	0.05	3445	0.05

Table 10. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.5, λ2=0.3, λ3=0.2*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	289	0.10	283	0.10	279	0.10	277	0.10	274	0.10	273	0.10	271	0.10	269	0.10
PE	213	0.06	220	0.06	224	0.06	227	0.06	230	0.05	232	0.05	233	0.05	235	0.05
NB	276	0.13	275	0.13	274	0.13	273	0.13	273	0.13	272	0.13	272	0.13	271	0.13
NS	315	0.13	304	0.13	297	0.14	292	0.14	289	0.14	286	0.14	284	0.14	280	0.14
QC	390	0.12	402	0.12	410	0.11	416	0.11	421	0.11	425	0.11	428	0.11	433	0.11
ON	689	0.14	684	0.14	681	0.14	678	0.14	676	0.14	675	0.14	673	0.14	672	0.14
MN	245	0.14	251	0.13	255	0.13	258	0.13	260	0.13	262	0.13	263	0.13	265	0.13
SK	257	0.14	261	0.14	263	0.14	265	0.14	267	0.14	268	0.14	268	0.14	270	0.14
AL	370	0.15	366	0.15	362	0.16	360	0.16	358	0.16	357	0.16	356	0.16	354	0.16
BC	401	0.15	400	0.15	400	0.15	399	0.15	398	0.15	398	0.15	398	0.15	397	0.15
Canada	3445	0.06	3446	0.06	3445	0.06	3445	0.06	3446	0.06	3448	0.06	3446	0.06	3446	0.06

Table 11. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.5, λ2=0.2, λ3=0.3*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	318	0.09	312	0.09	308	0.09	305	0.09	303	0.09	301	0.10	300	0.10	298	0.10
PE	246	0.04	252	0.04	257	0.03	260	0.03	262	0.03	264	0.03	266	0.03	268	0.02
NB	302	0.12	301	0.12	300	0.12	299	0.12	298	0.12	298	0.12	297	0.12	297	0.12
NS	342	0.12	331	0.13	324	0.13	319	0.13	316	0.13	313	0.13	311	0.13	307	0.13
QC	352	0.12	364	0.12	372	0.12	378	0.12	383	0.12	387	0.12	390	0.12	395	0.12
ON	583	0.15	578	0.15	575	0.15	572	0.15	570	0.15	569	0.15	568	0.15	566	0.15
MN	269	0.13	275	0.13	279	0.13	281	0.13	283	0.12	285	0.12	286	0.12	288	0.12
SK	280	0.14	284	0.14	287	0.13	288	0.13	290	0.13	291	0.13	292	0.13	293	0.13
AL	370	0.15	366	0.15	362	0.16	360	0.16	358	0.16	357	0.16	356	0.16	354	0.16
BC	385	0.15	384	0.15	383	0.15	383	0.15	382	0.15	382	0.15	381	0.15	381	0.15
Canada	3447	0.07	3447	0.07	3447	0.07	3445	0.07	3445	0.07	3447	0.07	3447	0.07	3447	0.07

***Table 12**. n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.3, λ2=0.5, λ3=0.2*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	220	0.12	217	0.12	214	0.12	212	0.12	211	0.12	210	0.12	209	0.12	208	0.12
PE	161	0.10	165	0.10	168	0.09	170	0.09	171	0.09	172	0.09	173	0.09	175	0.09
NB	221	0.14	220	0.14	220	0.14	219	0.14	219	0.14	219	0.14	218	0.14	218	0.14
NS	241	0.15	234	0.15	230	0.16	227	0.16	225	0.16	223	0.16	222	0.16	220	0.16
QC	494	0.10	501	0.10	506	0.10	509	0.10	512	0.10	515	0.10	517	0.10	519	0.10
ON	890	0.12	887	0.12	885	0.12	883	0.12	882	0.12	881	0.12	881	0.12	880	0.12
MN	210	0.15	214	0.15	216	0.15	218	0.15	219	0.15	220	0.14	221	0.14	222	0.14
SK	218	0.15	220	0.15	222	0.15	223	0.15	224	0.15	225	0.15	225	0.15	226	0.15
AL	360	0.16	357	0.16	355	0.16	354	0.16	353	0.16	352	0.16	351	0.16	350	0.16
BC	431	0.15	431	0.15	430	0.15	430	0.15	429	0.15	429	0.15	429	0.15	429	0.15
Canada	3446	0.06	3446	0.06	3446	0.06	3445	0.06	3445	0.06	3446	0.06	3446	0.06	3447	0.06

***Table 13.** n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ1=0.2, λ2=0.5, λ3=0.3*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	214	0.12	212	0.12	210	0.12	209	0.12	208	0.12	207	0.12	207	0.12	206	0.12
PE	168	0.09	171	0.09	172	0.09	174	0.09	175	0.09	175	0.09	176	0.09	177	0.09
NB	219	0.14	219	0.14	218	0.14	218	0.14	218	0.14	218	0.14	217	0.15	217	0.15
NS	230	0.16	226	0.16	223	0.16	221	0.16	220	0.16	219	0.16	218	0.16	216	0.16
QC	508	0.10	512	0.10	516	0.10	518	0.10	520	0.10	521	0.10	523	0.10	525	0.10
ON	885	0.12	883	0.12	881	0.12	880	0.12	879	0.12	879	0.12	878	0.12	878	0.12
MN	216	0.15	218	0.15	220	0.14	221	0.14	222	0.14	223	0.14	223	0.14	224	0.14
SK	222	0.15	223	0.15	224	0.15	225	0.15	226	0.15	226	0.15	226	0.15	227	0.15
AL	355	0.16	353	0.16	352	0.16	351	0.16	350	0.16	350	0.16	349	0.16	348	0.16
BC	430	0.15	429	0.15	429	0.15	429	0.15	428	0.15	428	0.15	428	0.15	428	0.15
Canada	3447	0.06	3446	0.06	3445	0.06	3446	0.06	3446	0.06	3446	0.06	3445	0.06	3446	0.06

Table 14. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ_1=0.3, λ_2=0.2, λ_3=0.5*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	306	0.09	302	0.09	300	0.10	298	0.10	297	0.10	296	0.10	295	0.10	294	0.10
PE	259	0.03	263	0.03	266	0.03	268	0.02	269	0.02	270	0.02	271	0.02	273	0.02
NB	298	0.12	298	0.12	297	0.12	297	0.12	296	0.12	296	0.12	296	0.12	295	0.12
NS	321	0.13	315	0.13	311	0.13	308	0.13	306	0.13	304	0.13	303	0.14	301	0.14
QC	380	0.12	386	0.12	391	0.12	395	0.12	398	0.12	400	0.12	402	0.12	405	0.12
ON	573	0.15	569	0.15	567	0.15	566	0.15	565	0.15	564	0.15	563	0.15	562	0.15
MN	280	0.13	284	0.12	286	0.12	288	0.12	289	0.12	290	0.12	291	0.12	292	0.12
SK	287	0.13	290	0.13	291	0.13	292	0.13	293	0.13	294	0.13	294	0.13	295	0.13
AL	360	0.16	357	0.16	355	0.16	354	0.16	353	0.16	352	0.16	351	0.16	350	0.16
BC	382	0.15	381	0.15	381	0.15	381	0.15	380	0.15	380	0.15	380	0.15	379	0.15
Canada	3446	0.07	3445	0.07	3445	0.07	3447	0.07	3446	0.07	3446	0.07	3446	0.07	3446	0.07

Table 15. *n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for λ_1=0.2, λ_2=0.3, λ_3=0.5*

	$\alpha=0$		$\alpha=0.25$		$\alpha=0.5$		$\alpha=0.75$		$\alpha=1$		$\alpha=1.25$		$\alpha=1.5$		$\alpha=2$	
	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
NL	271	0.10	269	0.10	267	0.10	266	0.10	265	0.10	265	0.10	264	0.10	263	0.11
PE	233	0.05	236	0.05	238	0.05	239	0.05	240	0.05	241	0.05	241	0.05	242	0.05
NB	271	0.13	270	0.13	270	0.13	270	0.13	269	0.13	269	0.13	269	0.13	269	0.13
NS	284	0.14	280	0.14	277	0.14	275	0.14	274	0.14	273	0.14	272	0.15	270	0.15
QC	431	0.11	436	0.11	439	0.11	442	0.11	444	0.11	445	0.11	447	0.11	448	0.11
ON	673	0.14	671	0.14	670	0.14	669	0.14	668	0.14	667	0.14	667	0.14	666	0.14
MN	263	0.13	265	0.13	267	0.13	268	0.13	269	0.13	269	0.13	270	0.13	271	0.13
SK	268	0.14	270	0.14	271	0.14	271	0.14	272	0.14	272	0.14	273	0.14	273	0.14
AL	355	0.16	353	0.16	352	0.16	351	0.16	350	0.16	349	0.16	349	0.16	348	0.16
BC	397	0.15	396	0.15	396	0.15	396	0.15	396	0.15	395	0.15	395	0.15	395	0.15
Canada	3446	0.07	3446	0.07	3447	0.07	3447	0.07	3447	0.07	3445	0.07	3447	0.07	3445	0.07

As seen from Table 3.4, 3.5, 3.6 and 3.10, when $\lambda_1 > \lambda_2 > \lambda_3$ target values for strata $CV(\bar{y}_h) \le 0.15$ and target values for the population $CV(\bar{y}_{st}) \le 0.06$ were provided almost for all α values. As seen from Table 3.11 for the case of $\lambda_1 > \lambda_3 > \lambda_2$, target values for population $CV(\bar{y}_{st}) \le 0.06$ were not obtained.

When $\lambda_2 > \lambda_1 > \lambda_3$, the target values of $CV(\bar{y}_{st}) \le 0.06$ were provided, whereas there were some cases where the strata target values of $CV_h \le 0.15$ were not provided for all α values as shown in Table 3.8, 3.9 and 3.12. As seen from Table 3.13, when $\lambda_2 > \lambda_3 > \lambda_1$,the target values of $CV(\bar{y}_{st}) \le 0.06$ were provided for all α values, however the number of the strata target values which were not provided increased.

As seen from Table 3.14 and Table 3.15, for the cases $\lambda_3 > \lambda_1 > \lambda_2$ and $\lambda_3 > \lambda_2 > \lambda_1$ respectively, target values for the population $CV(\bar{y}_{st}) \le 0.06$ were provided for all α values except for the stratum AL. In both two cases, there are some situations where target values for strata $CV(\bar{y}_h) \le 0.15$ also were not provided.

In accordance with the interpretations of the cases presented above, if more weight to λ_3 was given, which means more weight to equal allocation, both the ratio of not providing population and strata targets were seen to increase. Because of different strata sizes and strata selection costs in this application, next analysis was done for the case in which zero weight to equal allocation (λ_3=0) was given.

If $\lambda_1 > \lambda_2$ and $\lambda_3 = 0$; for example in Table 3.4 when $\lambda_1 = 0.8$ and $\alpha = 0.25$, minimum and maximum sample size in strata are $n_{PE} = 236$ and $n_{ON} = 591$, respectively. Here, the range for sample sizes is 355. Other alternative is Table 3.5 when $\lambda_1 = 0.7$ and $\alpha = 0.25$, minimum and maximum sample size in strata are $n_{PE} = 209$ and $n_{ON} = 693$, respectively. The range for sample sizes is 484 in this alternative. Third alternative is Table 3.6 when $\lambda_1 = 0.6$ and $\alpha = 0.25$, minimum and maximum sample size in strata are $n_{PE} = 182$ and $n_{ON} = 794$, respectively. The range for sample sizes is 612 for this case. If $\lambda_1 = \lambda_2 = 0.5$ and $\lambda_3 = 0$ in Table 3.7 when $\alpha = 0.25$, minimum and maximum sample size in strata are $n_{PE} = 154$ and $n_{ON} = 896$,respectively. Range for this case would be 742. Through these examples, one can see that the proposed compromise allocation model decreases the differences between strata sample sizes as the weight of proportional allocation decreases. Moreover, almost in all cases *CV* values for strata do not exceed the target value if more weight was given to λ_1. Provided that $\lambda_1 > \lambda_2$ and the difference of two linear component is not very big, both population and strata target CV values are provided. As long as more weight is given to λ_1, target value $CV_h \le 0.15$ is provided for the strata which have a big sample size and high selection cost like ON, QC and BC.

If $\lambda_2 > \lambda_1$, more weight to proportional allocation is given. Strata target values are not provided even though the population target value is provided in this situation. For Table 3.8 when $\lambda_2 = 0.8$ and $\alpha = 0.25$, minimum and maximum sample size in strata are $n_{PE} = 73$ and $n_{ON} = 1200$, respectively. The range for sample sizes is 1127 for this case. For Table 3.9 when $\lambda_2 = 0.7$ and $\alpha = 0.25$, minimum and maximum sample size in strata are $n_{PE} = 100$ and $n_{ON} = 1099$, respectively. The range for sample sizes is 999 in this case. As λ_2 increases, the range for sample sizes also increases since importance is only given to strata sizes.

4. CONCLUSION

Demands for statistics increase gradually in the world for users not only want to see global results but also domains. For instance, Eurostat asks for estimations with reasonable accuracy for areas inside a country. Most of the EU commission regulations about statistics are interested in domain estimations with certain precision. On the other hand, traditional allocation methods in stratified sampling are not adequate to provide good estimations both for global and domains. To overcome this problem, some compromise allocation strategies are needed. Bankier (1988), Costa et al. (2004), and Longford (2006) proposed some compromise models. However, none of these models used the cost function. Therefore, a new compromise allocation was proposed in this study to make the estimations of global and strata with certain precision by considering the non-linear cost function.

In the new proposed model, if one gives more weight to λ_1 than λ_2 and take λ_3 as zero, then provides the target values both for population and strata level when the strata sizes, variances and unit selection costs are different from each other. By this way, it is possible to get the optimal allocation which makes the estimator variance minimum and provides both global and domain estimates with certain precision.

The most important advantage of this model is the flexibility that is given to researchers in assigning the α and $\lambda_1, \lambda_2, \lambda_3$ values based on their needs. The proposed model makes survey studies more productive compared to classical allocation methods not using the cost function, since it handles the allocation issue in a more realistic way by using the cost. This model can also be improved for multivariate stratified random sampling for future studies.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors

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