Techno-Science
Scientific Journal of Mehmet Akif Ersoy University
https://dergipark.org.tr/pub/sjmakeu

Original Research Article

hosted by Turkish JournalPark ACADEMIC

COMPLEMENTARY SOFT BINARY PIECEWISE SYMMETRIC DIFFERENCE OPERATION: A NOVEL SOFT SET OPERATION

Aslıhan SEZGİN[1*], Kubilay DAGTOROS[2]

[1] Amasya University, Education Faculty, Department of Mathematics and Science Education, Amasya, Turkey
[2] Norfolk State University, College of Science, Engineering and Technology, Faculty of Department of Mathematics, Norfolk, VA, USA

ARTICLE INFO

Article History

Received	:	23/09/2023
Revised	:	31/10/2023
Accepted	:	15/11/2023
Available online	:	31/12/2023

Keywords
Soft sets, Soft set operations, Boolean rings, Hemirings

ABSTRACT

Since Molodtsov first introduced soft set theory, a useful mathematical tool for solving problems related to uncertainties, many soft set operations have been described and used in decision making problems. In this study, a new soft set operation called complementary soft binary piecewise symmetric difference operation is defined, and its properties are examined in comparison with the basic algebraic properties of the symmetric difference operation. Moreover, it has been shown that the collection of soft sets with a fixed set of parameter together with complementary soft binary symmetric difference and restricted intersection, is a commutative hemiring with identity and also a Boolean ring.

1. INTRODUCTION

In real life, we tackle problems in a variety of disciplines, including engineering, environmental sciences, health sciences, and economics. But there is some kind of uncertainty that prevents traditional methods from being used effectively. Molodtsov [1] proposed Soft Set Theory in 1999 as a mathematical way of dealing with these uncertainties. Since then, this theory has been applied to a wide range of fields, including information systems, decision making, optimization theory, game theory, operations research, measurement theory, and some algebraic structures. The first contributions to soft set operations were made by Maji et al. [2] and Pei and Miao [3]. Subsequently, Ali et al. [4] presented and discussed several soft set operations, including restricted and extended soft set operations. The basic properties of soft set operations and the relationships between them were described in [5]. In [5], the idea of restricted symmetric difference operation was also explored and defined. An entirely new soft set operation called extended difference of soft sets was proposed by Sezgin et al. [6]. Stojanovic [7] introduced the extended symmetric difference of soft sets and studied the properties of the term. Eren [8] created an entirely new class of soft difference operation (called soft binary piecewise difference operation) and also carefully analyzed the core properties of the operation. Other soft binary piecewise operations have been defined by Yavuz [9], who also carefully analyzed their core properties. The concept of soft set operations has been extensively studied since 2003, as it is a fundamental concept of soft set theory. We refer to [10-28] for more details about soft set operations.
Semirings, first described by Vandiver [29] in 1934, consist of a set R and two associative binary operations, addition '+' and multiplication '.', with '+' distributing from both sides. Various researchers, including [30,31], have published different theories and findings on semirings, and some have studied semirings with additive inverse [32-35]. Semirings have been extensively studied recently, especially for their applications (see [36]). Semirings are extremely important in geometry, but they are also essential for solving problems in a variety of practical mathematics and informatics applications and are also important in pure mathematics. [37-45]. Hemiring means a special semiring with zero and commutative addition. Hemiring is also very important in theoretical computer science. Hemiring occurs naturally in a variety of applications in formal language theory, computer science, and automata [44–45].
This study contributes to the literature on soft set theory by describing a new soft set operation called "Complementary Soft Binary Piecewise Symmetric Difference Operation". This paper is organized as follows: Section 2 recalls the basic concepts regarding soft set theory and semirings. Section 3 gives definitions and examples of complementary soft binary piecewise symmetric difference operations. A full analysis of the algebraic properties of the new operation, including closure, associativity, unity, inverse elements, and abelian properties, is then also examined in comparison to the classical set-

* Corresponding Author: aslihan.sezgin@amasya.edu.tr

To cite this article: ASLIHAN S., DAGTOROS K., (2023). Complementary Soft Binary Piecewise Symmetric Difference Operation: A Novel Soft Set Operation. *Scientific Journal of Mehmet Akif Ersoy University*, vol. 6, no. 2-p.31-45

theoretic symmetric difference operation. In the same section, it is proved that the set of all soft sets with a fixed set of parameter together with soft binary piecewise symmetric difference and restricted intersection is both a commutative hemiring with identity and a Boolean ring. The conclusion section considers the significance of the research findings and their possible impact on the subject. Introducing new soft set operations and the extractions of algebraic properties and their implementations will provide new perspectives in dealing with problems containing parametric data.

2. PRELIMINARIES

Definition 2.1. [1] Let U be the universal set, E be the parameter set, P(U) be the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U where F is a set-valued function such that $F: A \to P(U)$.
Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$. Let A be a fixed subset of E and $S_A(U)$ be the collection of all soft sets over U with the fixed parameter set A. Clearly $S_A(U)$ is a subset of $S_E(U)$. From now on, while soft set will be designated by SS and parameter set by PS; soft sets will be designated by SSs and parameter sets by PSs for the sake of ease.
Definition 2.2. [4] (W, K) is called a relative null SS (with regard to K), denoted by $\emptyset_K$, if $W(\zeta) = \emptyset$ for all $\zeta \in W$ and (W, K) is called a relative whole SS (with regard to K), denoted by U_K if $W(\zeta) = U$ for all $\zeta \in W$. The relative whole SS U_E with regard to E is called the absolute SS over U. We shall denote by $\emptyset_\emptyset$ the unique soft set over U with an empty parameter set, which is called the empty soft set over U. Note that $\emptyset_\emptyset$ and $\emptyset_A$ are different soft sets over U [17].
Definition 2.3. [3] For two SSs (W, K) and $(Ş, T)$, (W, K) is a soft subset of $(Ş, T)$ and it is denoted by $(W, K) \tilde{\subseteq} (Ş, T)$, if $K \subseteq T$ and $W(\zeta) \subseteq Ş(\zeta), \forall \zeta \in K$. Two SSs (W, K) and $(Ş, T)$ are said to be soft equal if (W, K) is a soft subset of $(Ş, T)$ and $(Ş, T)$ is a soft subset of (W, K).
Definition 2.4. [4] The relative complement of a SS (K, W), denoed by $(W, K)^r$, is defined by $(W, K)^r = (W^r, K)$, where $W^r: K \to P(U)$ is a mapping given by $(W, K)^r = U \backslash W(\zeta)$ for all $\zeta \in W$. From now on, $U \backslash W(\zeta) = [W(\zeta)]'$ will be designated by $W'(\zeta)$ for the sake of ease.
Let " Θ " be used to denote the set operations (Namely, Θ here can be $\cap, \cup, \backslash, \Delta$), then the soft set operations can be grouped into the following categories as a summary:
Definition 2.5. [4,5] Let (W, K) and $(Ş, T)$ be SSs over U. The restricted Θ operation of (W, K) and $(Ş, T)$ is the SS (X,B), denoted by, $(W, K)\Theta_R(Ş, T) = (X, B)$, where $B = K \cap T \neq \emptyset$ and $\forall \zeta \in B, X(\zeta) = W(\zeta)\ \Theta\ Ş(\zeta)$. Here note that if $K \cap T = \emptyset$, then $(W, K)\Theta_R(Ş, T) = \emptyset_\emptyset$ [17].
Definition 2.6. [3,4,6,7] Let (W, K) and $(Ş, T)$ be SSs over U. The extended Θ operation of (W, K) and $(Ş, T)$ is the SS (X,B), denoted by $(W, K)\Theta_\varepsilon(Ş, T) = (X, B)$, where $B = K \cup T$ and $\forall \zeta \in B$,

$$X(\zeta) = \begin{cases} W(\zeta), & \zeta \in K \backslash T, \\ Ş(\zeta), & \zeta \in T \backslash K, \\ W(\zeta)\ \Theta Ş(\zeta), & \zeta \in T \cap Ş. \end{cases}$$

Definition 2.7. [8,9] Let (W, K) and $(Ş, T)$ be SSs over U. The soft binary piecewise Θ operation of (W, K) and $(Ş, T)$ is the SS (B,K), denoted by $(W, K) \underset{\Theta}{\sim} (Ş, T) = (X, K)$, where $\forall \zeta \in K$,

$$X(\zeta) = \begin{cases} W(\zeta), & \zeta \in K \backslash T \\ W(\zeta)\ \Theta\ Ş(\zeta), & \zeta \in K \cap T \end{cases}$$

Definition 2.8. [10-13] Let (W, K) and $(Ş, T)$ be SSs over U. The complementary soft binary piecewise Θ operation of (W, K) and $(Ş, T)$ is the SS (X, K), denoted by, $(W, K) \overset{*}{\underset{\Theta}{\sim}} (Ş, T) = (X, K)$, where $\forall \zeta \in K$,

$$X(\zeta) = \begin{cases} W'(\zeta), & \zeta \in K \backslash T \\ W(\zeta)\ \Theta\ Ş\zeta), & \zeta \in K \cap T \end{cases}$$

In mathematics, a semiring is used in abstract algebra to describe an algebraic structure which is more general than ring. A semiring (R,+, ・) is an algebraic structure consisting of a non-empty set R together with two binary operations usually called addition and multiplication such that (R,+) is a semigroup, (R, ・) is a semigroup and multiplication is distributive over addition from both sides. If a semiring has identity with multiplication, then it is called semiring with identity and if it has commutative multiplication, then it is called a commutative semiring. If there exists an element $0 \in R$ such that 0 ・ a =a ・ 0 =0 and 0 + a =a + 0 =a for all $a \in R$, then 0 is called the zero of R. A semiring with commutative addition and zero element is called a hemiring. For more about semirings and hemirings, we refer to [29-45].

3. ALGEBRAIC PROPERTIES OF COMPLEMENTARY SOFT BINARY PIECEWISE SYMMETRIC DIFFERENCE OPERATION

Definition 3.1. Let (Ψ, I) and $(\Omega, Ş)$ be SSs over U. The complementary soft binary piecewise symmetric difference operation of (Ψ, I) and $(\Omega, Ş)$ is the SS $(\wp, I)$, denoted by, $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, Ş) = (\wp, I)$, where $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the PS, $I=\{e_1, e_3\}$ and $Ş=\{e_2, e_3, e_4\}$ be the subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be universe set. Let (Ψ,I) and $(\Omega,Ş)$ be SSs over U defined as follows:

$(\Psi,I)=\{(e_1, \{h_2,h_5\}), (e_3,\{h_1,h_2,h_5\})\}$

$(\Omega,Ş)=\{(e_2,\{h_1, h_4,h_5\}), (e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3, h_5\})\}$.

Let $(\Psi,I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,Ş)=(\wp,I)$. Then,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Since $I=\{e_1, e_3\}$ and $I\backslash Ş=\{e_1\}$, so $\wp(e_1)=\Psi'(e_1)=\{h_1,h_3,h_4\}$. And since $I\cap Ş=\{e_3\}$ so $\wp(e_3)=\Psi(e_3)\,\Delta\,\Omega(e_3)=\{h_1, h_3, h_4,h_5\}$.

Thus, $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,Ş)=\{(e_1,\{h_1,h_3,h_4\}), (e_3, \{h_1, h_3, h_4,h_5\})\}$.

The set of elements that are in either of the sets but not in their intersection is known as the symmetric difference of two sets in classical theory. Namely, $I\Delta Ş=(I \cup Ş)\backslash(I\cap Ş)$. Now, we have:

Theorem 3.3. $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,Ş) =[(\Psi,I) \overset{*}{\underset{\cup}{\sim}} (\Omega, Ş)] \,\tilde{\backslash}\, [(\Psi,I) \cap_R (\Omega, Ş)]$.

Proof: Since the PS of the SSs of both hand side is I, the first condition for the soft equality is satisfied. Now let $(\Psi, I) \overset{*}{\underset{\cup}{\sim}} (\Omega,Ş)=(\wp,I)$ where $\forall\zeta\in I$;

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\cup\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Let $(\Psi, I) \cap_R (\Omega,Ş)=(M,I\cap Ş)$, where $\forall\zeta\in I\cap Ş$; $M(\zeta)=\Psi(\zeta)\cap\Omega(\zeta)$. Let $(\wp,I) \,\tilde{\backslash}\, (M,I\cap Ş)=(S,I)$, where for $\forall\zeta\in I$,

$$S(\zeta)=\begin{cases}\wp(\zeta), & \zeta\in I\backslash(I\cap Ş)=I\backslash Ş\\ \wp(\zeta)\backslash M(\zeta), & \zeta\in I\cap(I\cap Ş)=I\cap Ş\end{cases}$$

Thus,

$$S(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in(I\backslash Ş)\backslash Ş=I\backslash Ş\\ \Psi(\zeta)\cup\Omega(\zeta), & \zeta\in(I\cap Ş)\backslash Ş=\emptyset\\ \Psi'(\zeta)\backslash(\Psi(\zeta)\cap\Omega(\zeta)), & \zeta\in(I\backslash Ş)\cap Ş=\emptyset\\ [\Psi(\zeta)\cup\Omega(\zeta)]\backslash[\Psi(\zeta)\cap\Omega(\zeta)], & \zeta\in(I\cap Ş)\cap Ş=I\cap Ş\end{cases}$$

Thus,

$$S(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ [\Psi(\zeta)\cup\Omega(\zeta)]\backslash(\Psi(\zeta)\cap\Omega(\zeta)), & \zeta\in I\cap Ş\end{cases}$$

Hence,

$$S(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Thus, $(S,I)=(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,Ş)$.

In classical theory, $I\Delta Ş=(I\backslash Ş)\cup(Ş\backslash I)$. Now, we have:

Theorem 3.4. $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,Ş) =[(\Psi,I) \overset{*}{\underset{\backslash}{\sim}} (\Omega, Ş)] \,\tilde{\cup}\, [(\Omega,Ş) \overset{*}{\underset{\backslash}{\sim}} (\Psi, I)]$.

Proof: Since the PS of the SSs of both hand side is I, the first condition for the soft equality is satisfied. Now let $(\Psi, I) \overset{*}{\underset{\setminus}{\sim}} (\Omega, Ş)=(\wp, I)$ where $\forall \zeta \in I$;

$$\wp(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta \in I \setminus Ş \\ \Psi(\zeta) \setminus \Omega(\zeta), & \zeta \in I \cap Ş \end{cases}$$

Let $(\Omega, Ş) \overset{*}{\underset{\setminus}{\sim}} (\Psi, I)=(K, Ş)$ where $\forall \zeta \in I$;

$$K(\zeta)=\begin{cases} \Omega'(\zeta), & \zeta \in Ş \setminus I \\ \Omega(\zeta) \setminus \Psi(\zeta), & \zeta \in Ş \cap I \end{cases}$$

Let $(\wp, I) \tilde{\cup} (K, Ş)=(S, I)$, where for $\forall \zeta \in I$;

$$S(\zeta)=\begin{cases} \wp(\zeta), & \zeta \in I \setminus Ş \\ \wp(\zeta) \cup K(\zeta), & \zeta \in I \cap Ş \end{cases}$$

Hence,

$$S(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta \in (I \setminus Ş) \setminus Ş = I \setminus Ş \\ \Psi(\zeta) \setminus \Omega(\zeta), & \zeta \in (I \cap Ş) \setminus Ş = \emptyset \\ \Psi'(\zeta) \cup \Omega'(\zeta), & \zeta \in (I \setminus Ş) \cap (Ş \setminus I) = \emptyset \\ \Psi'(\zeta) \cup (\Omega(\zeta) \setminus \Psi(\zeta)), & \zeta \in (I \setminus Ş) \cap (Ş \cap I) = \emptyset \\ (\Psi(\zeta) \setminus \Omega(\zeta)) \cup \Omega'(\zeta), & \zeta \in (I \cap Ş) \cap (Ş \setminus I) = \emptyset \\ [\Psi(\zeta) \setminus \Omega(\zeta)] \cup [\Omega(\zeta) \setminus \Psi(\zeta)], & \zeta \in (I \cap Ş) \cap (Ş \cap I) = I \cap Ş \end{cases}$$

Thus,

$$S(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta \in I \setminus Ş \\ [\Psi(\zeta) \setminus \Omega(\zeta)] \cup [\Omega(\zeta) \setminus \Psi(\zeta)], & \zeta \in I \cap Ş \end{cases}$$

Therefore,

$$S(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta \in I \setminus Ş \\ \Psi(\zeta) \Delta \Omega(\zeta), & \zeta \in I \cap Ş \end{cases}$$

Hence, $(S, I)=(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, Ş)$.

Theorem 3.5.

1) $S_E(U)$ is closed under $\overset{*}{\underset{\Delta}{\sim}}$. Namely, when (Ψ, I) and $(\Omega, Ç)$ are two SSs over U, then so is $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, Ç)$ as $\overset{*}{\underset{\Delta}{\sim}}$ is a binary operation in $S_E(U)$. $S_A(U)$ is closed under $\overset{*}{\underset{\Delta}{\sim}}$, too, where A is a fixed parameter set of E.

In classical theory, $(F \Delta G) \Delta P = F \Delta (G \Delta P)$. As an analogy, we have:

2) $[(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, I)] \overset{*}{\underset{\Delta}{\sim}} (\wp, I)=(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} [(\Omega, I) \overset{*}{\underset{\Delta}{\sim}} (\wp, I)]$

Proof: Let $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, I)=(T, I)$, where $\forall \zeta \in I$;

$$T(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta \in I \setminus I = \emptyset \\ \Psi(\zeta) \Delta \Omega(\zeta), & \zeta \in I \cap I = I \end{cases}$$

Let $(T, I) \overset{*}{\underset{\Delta}{\sim}} (\wp, I)=(M, I)$, where $\forall \zeta \in I$;

$$M(\zeta)=\begin{cases} T'(\zeta), & \zeta \in I \setminus I = \emptyset \\ T(\zeta) \Delta \wp(\zeta), & \zeta \in I \cap I = I \end{cases}$$

Thus,

$$M(\zeta)=\begin{cases} T'(\zeta), & \zeta \in I \setminus I = \emptyset \\ [\Psi(\zeta) \Delta \Omega(\zeta)] \Delta \wp(\zeta), & \zeta \in I \cap I = I \end{cases}$$

Let $(\Omega,I) \underset{\Delta}{\overset{*}{\sim}} (\wp,I)=(R,I)$, where $\forall\zeta\in I$;

$$R(\zeta)=\begin{cases}\Omega'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Omega(\zeta)\,\Delta\wp(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Let $(\Psi,I) \underset{\Delta}{\overset{*}{\sim}} (R,I)=(N,I)$, where $\forall\zeta\in I$;

$$N(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\,\Delta R(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Thus,

$$N(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\,\Delta[\Omega(\zeta)\Delta\wp(\zeta)], & \zeta\in I\cap I=I\end{cases}$$

It is seen that (M,I)=(N,I).

Namely, for the SSs whose PSs are the same, $\underset{\Delta}{\overset{*}{\sim}}$ is associative. Here's what we have right now:

3) $[(\Psi,I) \underset{\Delta}{\overset{*}{\sim}} (\Omega,Ç)] \underset{\Delta}{\overset{*}{\sim}} (\wp,Ö) \neq (\Psi,I) \underset{\Delta}{\overset{*}{\sim}} [(\Omega,Ç) \underset{\Delta}{\overset{*}{\sim}} (\wp,Ö)]$.

Proof: Let $(\Psi,I) \underset{\Delta}{\overset{*}{\sim}} (\Omega,Ç)=(T,I)$, where $\forall\zeta\in I$;

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus Ç\\ \Psi(\zeta)\,\Delta\Omega(\zeta), & \zeta\in I\cap Ç\end{cases}$$

Let $(T,I) \underset{\Delta}{\overset{*}{\sim}} (\wp,Ö)=(M,I)$, where $\forall\zeta\in I$;

$$M(\zeta)=\begin{cases}T'(\zeta), & \zeta\in I\setminus Ö\\ T(\zeta)\,\Delta\wp(\zeta), & \zeta\in I\cap Ö\end{cases}$$

Thus,

$$M(\zeta)=\begin{cases}\Psi(\zeta), & \zeta\in(I\setminus Ç)\setminus Ö=I\cap Ç'\cap Ö'\\ (\Psi(\zeta)\,\Delta\Omega(\zeta))', & \zeta\in(I\cap Ç)\setminus Ö=I\cap Ç\cap Ö'\\ \Psi'(\zeta)\,\Delta\wp(\zeta), & \zeta\in(I\setminus Ç)\cap Ö=I\cap Ç'\cap Ö\\ [\Psi(\zeta)\,\Delta\Omega(\zeta)]\,\Delta\wp(\zeta), & \zeta\in(I\cap Ç)\cap Ö=I\cap Ç\cap Ö\end{cases}$$

Let $(\Omega,Ç) \underset{\Delta}{\overset{*}{\sim}} (\wp,Ö)=(K,Ç)$, where $\forall\zeta\in Ç$;

$$K(\zeta)=\begin{cases}\Omega'(\zeta), & \zeta\in Ç\setminus Ö\\ \Omega(\zeta)\,\Delta\wp(\zeta), & \zeta\in Ç\cap Ö\end{cases}$$

Let $(\Psi,I) \underset{\Delta}{\overset{*}{\sim}} (K,Ç)=(S,I)$, where $\forall\zeta\in I$;

$$S(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus Ç\\ \Psi(\zeta)\,\Delta K(\zeta), & \zeta\in I\cap Ç\end{cases}$$

Thus,

$$S(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus Ç\\ \Psi(\zeta)\,\Delta\Omega'(\zeta), & \zeta\in I\cap(Ç\setminus Ö)=I\cap Ç\cap Ö'\\ \Psi(\zeta)\,\Delta\,[\Omega(\zeta)\,\Delta\wp(\zeta)], & \zeta\in I\cap(Ç\cap Ö)=I\cap Ç\cap Ö\end{cases}$$

Here, let's consider $\zeta\in I\setminus Ç$ in the second equation. Since $I\setminus Ç=I\cap Ç'$, if $\zeta\in Ç'$, then $\zeta\in Ö\setminus Ç$ or $\zeta\in(Ç\cup Ö)'$. Hence, if $\zeta\in I\setminus Ç$, then $\zeta\in I\cap Ç'\cap Ö'$ or $\zeta\in I\cap Ç'\cap Ö$. Thus, it is seen that $(M,I)\neq(N,I)$. Namely, for the SSs whose PSs are not the same, $\underset{\Delta}{\overset{*}{\sim}}$ is not associative in the set $S_E(U)$.

In classical theory, symmetric difference operation is commutative, i.e., FΔG=GΔF. However, we have:

4) $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (\Omega,Ç) \neq (\Omega,Ç) \underset{\Delta}{\overset{*}{\sim}} (\Psi,I)$.

Proof: Let $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (\Omega,Ç)=(\wp,I)$. Then, $\forall\zeta\in I$;

$$\wp(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta\in I\setminus Ç \\ \Psi(\zeta)\,\Delta\Omega(\zeta), & \zeta\in I\cap Ç \end{cases}$$

Let $(\Omega,Ç) \underset{\Delta}{\overset{*}{\sim}} (\Psi,I)=(T,Ç)$. Then $\forall\zeta\in Ç$;

$$T(\zeta)=\begin{cases} \Omega'(\zeta), & \zeta\in Ç\setminus I \\ \Omega(\zeta)\,\Delta\Psi(\zeta), & \zeta\in Ç\cap I \end{cases}$$

Here, while the PS of the SS of left side is I; the PS of the SS of right side is Ç. Thus,

$$(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (\Omega,Ç) \neq (\Omega,Ç) \underset{\Delta}{\overset{*}{\sim}} (\Psi,I)$$

Hence, $\underset{\Delta}{\overset{*}{\sim}}$ is not commutative in $S_E(U)$. However it is easy to see that

$$(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (\Omega,I)=(\Omega,I) \underset{\Delta}{\overset{*}{\sim}} (\Psi,I).$$

That is to say, $\underset{\Delta}{\overset{*}{\sim}}$ is commutative, where the PSs of the SSs are the same.

In classical theory, $\emptyset$ is the identity element for the symmetric difference operation, i.e., $F\Delta\emptyset=\emptyset\Delta F=F$. As an analogy, we have:

5) $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} \emptyset_I= \emptyset_I \underset{\Delta}{\overset{*}{\sim}} (\Psi,I)=(\Psi, I)$.

Proof: Let $\emptyset_I=(S,I)$. Then, $\forall\zeta\in I$; $S(\zeta)=\emptyset$. Let $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (S,I)=(\wp,I)$, where $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta\in I\setminus I=\emptyset \\ \Psi(\zeta)\,\Delta\, S(\zeta), & \zeta\in I\cap I=I \end{cases}$$

Hence, $\forall\zeta\in I$; $\wp(\zeta)=\Psi(\zeta)\Delta S(\zeta)=\Psi(\zeta)\,\Delta\emptyset=\Psi(\zeta)$. Thus, $(\wp,I)=(\Psi, I)$. Note that, for the SSs whose PS is I, $\emptyset_I$ is the identity element for $\underset{\Delta}{\overset{*}{\sim}}$ in $S_I(U)$.

In classical theory, every element is its own inverse for the symmetric difference operation, i.e., $F\Delta F=\emptyset$. As an analogy, we have:

6) $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (\Psi,I)=\emptyset_I$.

Proof: Let $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} (\Psi,I)=(\wp,I)$, where $\forall\zeta\in I$;

$$\wp(\zeta)=\begin{cases} \Psi'(\zeta), & \zeta\in I\setminus I=\emptyset \\ \Psi(\zeta)\,\Delta\Psi(\zeta), & \zeta\in I\cap I=I \end{cases}$$

Here $\forall\zeta\in I$; $\wp(\zeta)=\Psi(\zeta)\,\Delta\Psi(\zeta)=\emptyset$, thus $(\wp,I)=\emptyset_I$.

This property shows us that every SS is its own inverse for $\underset{\Delta}{\overset{*}{\sim}}$ in $S_I(U)$ and also $\underset{\Delta}{\overset{*}{\sim}}$ is not idempotent in $S_E(U)$.

Remark 3.6.: By Theorem 3.5. (1), (2), (4), (5) and (6), $(S_A(U), \underset{\Delta}{\overset{*}{\sim}})$ is an abelian group with identity $\emptyset_A$.

7) $(\Psi, I) \underset{\Delta}{\overset{*}{\sim}} \emptyset_E=(\Psi,I)$.

Proof: Let $\emptyset_E=(S,E)$. Hence $\forall\zeta\in E$; $S(\zeta)=\emptyset$. Let $(\Psi,I) \underset{\Delta}{\overset{*}{\sim}} (S,E)=(\wp,I)$. Thus, $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus E=\emptyset\\ \Psi(\zeta)\,\Delta S(\zeta), & \zeta\in I\cap E=I\end{cases}$$

Hence, $\forall\zeta\in I$ $\wp(\zeta)=\Psi(\zeta)\,\Delta S(\zeta)=\Psi(\zeta)\,\Delta\emptyset=\Psi(\zeta)$, so $(\wp,I)=(\Psi,I)$. Note that, $\emptyset_E$ is the right identity element for $\underset{\Delta}{\overset{*}{\sim}}$ in $S_E(U)$.

8) $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}\emptyset_\emptyset=(\Psi,I)^r$.

Proof: Let $\emptyset_\emptyset=(S,\emptyset)$. Let $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(S,\emptyset)=(\wp,I)$, where $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus\emptyset=I\\ \Psi(\zeta)\,\Delta\, S(\zeta), & \zeta\in I\cap\emptyset=\emptyset\end{cases}$$

Hence, $\forall\zeta\in I$; $\wp(\zeta)=\Psi'(\zeta)$. Thus, $(\wp,I)=(\Psi,I)^r$.

9) $\emptyset_\emptyset\underset{\Delta}{\overset{*}{\sim}}(\Psi,I)=\emptyset_\emptyset$.

Proof: Let $(S,\emptyset)\underset{\Delta}{\overset{*}{\sim}}(\Psi,I)=(T,\emptyset)$. Since, $\emptyset_\emptyset$ is the unique SS with empty set, $(T,\emptyset)=\emptyset_\emptyset$. Note that, $\emptyset_\emptyset$ is the left absorbing element for $\underset{\Delta}{\overset{*}{\sim}}$ in $S_E(U)$.

In classical theory, $F\Delta U=U\Delta F=F'$, where U is the universal set. As an analogy, we have:

10) $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}U_I=U_I\underset{\Delta}{\overset{*}{\sim}}(\Psi,I)=(\Psi,I)^r$.

Proof: Let $U_I=(T,I)$. Then, $\forall\zeta\in I$; $T(\zeta)=U$. Let $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(T,I)=(\wp,I)$, where $\forall\zeta\in I$;

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\,\Delta T(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Thus, $\forall\zeta\in I$; $\wp(\zeta)=\Psi(\zeta)\,\Delta T(\zeta)=\Psi(\zeta)\,\Delta U=\Psi'(\zeta)$, hence $(\wp,I)=(\Psi,I)^r$.

11) $(\Psi,I)\tilde{\Delta}U_E=(\Psi,I)^r$

Proof: Let $U_E=(T,E)$. Hence, $\forall\zeta\in E$, $T(\zeta)=U$. Let $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(T,E)=(\wp,I)$, then $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus E=\emptyset\\ \Psi(\zeta)\,\Delta T(\zeta), & \zeta\in I\cap E=I\end{cases}$$

Hence, $\forall\zeta\in I$, $\wp(\zeta)=\Psi(\zeta)\,\Delta T(\zeta)=\Psi(\zeta)\,\Delta U=\Psi'(\zeta)$, so $(\wp,I)=(\Psi,I)^r$.

In classical theory, $F\Delta F'=F'\Delta F=U$, where U is the universal set. As an analogy, we have:

12) $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(\Psi,I)^r=(\Psi,I)^r\underset{\Delta}{\overset{*}{\sim}}(\Psi,I)=U_I$.

Proof: Let $(\Psi,I)^r=(\wp,I)$. Hence, $\forall\zeta\in I$; $\wp(\zeta)=\Psi'(\zeta)$. Let $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(\wp,I)=(T,I)$, where $\forall\zeta\in I$,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\,\Delta\wp(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Hence, $\forall\zeta\in I$; $T(\zeta)=\Psi(\zeta)\,\Delta\wp(\zeta)=\Psi(\zeta)\,\Delta\,\Psi'(\zeta)=U$, thus $(T,I)=U_I$.

In classical theory, $(F\Delta G)\Delta(G\Delta P)=F\Delta P$. As an analogy, we have:

13) $[(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(\Omega,\text{Ş})]\underset{\Delta}{\overset{*}{\sim}}[(\Omega,\text{Ş})\underset{\Delta}{\overset{*}{\sim}}(\wp,I)]=(\Psi,I)\,\tilde{\Delta}(\wp,\text{Ş})$.

Proof: Since the PS of the SSs of both hand side is I, the first condition for the soft equality is satisfied. Now let $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(\Omega,\text{Ş})=(\wp,I)$ where $\forall\zeta\in I$;

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in \mathrm{I}\setminus \text{Ş}\\ \Psi(\zeta)\ \Delta\Omega(\zeta), & \zeta\in \mathrm{I}\cap \text{Ş}\end{cases}$$

Let $(\Omega,\text{Ş})\ \underset{\Delta}{\overset{*}{\sim}}\ (\wp,\mathrm{I})=(\mathrm{K},\text{Ş})$ where $\forall\zeta\in\mathrm{I}$;

$$\mathrm{K}(\zeta)=\begin{cases}\Omega'(\zeta), & \zeta\in \text{Ş}\setminus \mathrm{I}\\ \Omega(\zeta)\ \Delta\wp(\zeta), & \zeta\in \text{Ş}\cap \mathrm{I}\end{cases}$$

Let $(\wp,\mathrm{I})\ \underset{\Delta}{\overset{*}{\sim}}\ (\mathrm{K},\text{Ş})=(\mathrm{S},\mathrm{I})$, where for $\forall\zeta\in\mathrm{I}$;

$$\mathrm{S}(\zeta)=\begin{cases}\wp'(\zeta), & \zeta\in \mathrm{I}\setminus \text{Ş}\\ \wp(\zeta)\ \Delta\mathrm{K}(\zeta), & \zeta\in \mathrm{I}\cap \text{Ş}\end{cases}$$

Thus,

$$\mathrm{S}(\zeta)=\begin{cases}\Psi(\zeta), & \zeta\in(\mathrm{I}\setminus\text{Ş})\setminus\text{Ş}=\mathrm{I}\setminus\text{Ş}\\ (\Psi(\zeta)\ \Delta\Omega(\zeta)', & \zeta\in(\mathrm{I}\cap\text{Ş})\setminus\text{Ş}=\emptyset\\ \Psi'(\zeta)\ \Delta\ \Omega(\zeta), & \zeta\in(\mathrm{I}\setminus\text{Ş})\cap(\text{Ş}\setminus\mathrm{I})=\emptyset\\ \Psi'(\zeta)\ \Delta\ (\Omega(\zeta)\ \Delta\wp(\zeta)), & \zeta\in(\mathrm{I}\setminus\text{Ş})\cap(\text{Ş}\cap\mathrm{I})=\emptyset\\ (\Psi(\zeta)\ \Delta\Omega(\zeta))\ \Delta\Omega(\zeta), & \zeta\in(\mathrm{I}\cap\text{Ş})\cap(\text{Ş}\setminus\mathrm{I})=\emptyset\\ [\Psi(\zeta)\ \Delta\Omega(\zeta)]\ \Delta\ [\Omega(\zeta)\ \Delta\wp(\zeta)], & \zeta\in(\mathrm{I}\cap\text{Ş})\cap\text{Ş}=\mathrm{I}\cap\text{Ş}\end{cases}$$

Thus,

$$\mathrm{S}(\zeta)=\begin{cases}\Psi(\zeta), & \zeta\in \mathrm{I}\setminus \text{Ş}\\ [\Psi(\zeta)\ \Delta\Omega(\zeta)]\ \Delta\ [\Omega(\zeta)\ \Delta\wp(\zeta)], & \zeta\in \mathrm{I}\cap \text{Ş}\end{cases}$$

Therefore,

$$\mathrm{S}(\zeta)=\begin{cases}\Psi(\zeta), & \zeta\in \mathrm{I}\setminus \text{Ş}\\ \Psi(\zeta)\ \Delta\wp(\zeta), & \zeta\in \mathrm{I}\cap \text{Ş}\end{cases}$$

Hence, $(\mathrm{S},\mathrm{I})=(\Psi,\mathrm{I})\ \tilde{\Delta}(\wp,\text{Ş})$.

In classical theory, F'ΔG'=FΔG. Now, we have the following:

14) $(\Psi,\mathrm{I})^r\ \underset{\Delta}{\overset{*}{\sim}}\ (\Omega,\text{Ş})^r=(\Psi,\mathrm{I})\ \tilde{\Delta}(\Omega,\text{Ş})$

Proof: Let $(\Psi,\mathrm{I})^r\ \underset{\Delta}{\overset{*}{\sim}}\ (\Omega,\text{Ş})^r=(\wp,\mathrm{I})$. Then, $\forall\zeta\in\mathrm{I}$,

$$\wp(\zeta)=\begin{cases}(\Psi')'(\zeta), & \zeta\in \mathrm{I}\setminus \text{Ş}\\ \Psi'(\zeta)\ \Delta\Omega'(\zeta), & \zeta\in \mathrm{I}\cap \text{Ş}\end{cases}$$

Thus,

$$\wp(\zeta)=\begin{cases}\Psi(\zeta), & \zeta\in \mathrm{I}\setminus \text{Ş}\\ \Psi'(\zeta)\ \Delta\Omega'(\zeta), & \zeta\in \mathrm{I}\cap \text{Ş}\end{cases}$$

Since $\Psi'(\zeta)\ \Delta\Omega'(\zeta)=\Psi(\zeta)\ \Delta\Omega(\zeta)$, thus, $(\wp,\mathrm{I})=(\Psi,\mathrm{I})\ \tilde{\Delta}(\Omega,\text{Ş})$.

In classical theory, for all F, $\emptyset\subseteq$ F. As an analogy, we have:

15) $\emptyset_{\mathrm{I}}\ \tilde{\subseteq}(\Psi,\mathrm{I})\ \underset{\Delta}{\overset{*}{\sim}}(\Omega,\text{Ç})$ and $\emptyset_{\text{Ç}}\ \tilde{\subseteq}(\Omega,\text{Ç})\ \underset{\Delta}{\overset{*}{\sim}}\ (\Psi,\mathrm{I})$.

In classical theory, for all F, F $\subseteq$ U. As an analogy, we have:

16) $(\Psi,\mathrm{I})\ \underset{\Delta}{\overset{*}{\sim}}(\Omega,\text{Ç})\ \tilde{\subseteq}\ \mathrm{U}_{\mathrm{I}}$ and $(\Omega,\text{Ç})\ \underset{\Delta}{\overset{*}{\sim}}(\Psi,\mathrm{I})\ \tilde{\subseteq}\ \mathrm{U}_{\text{Ç}}$.

In classical theory, FΔG = FΔP⟹G=P (Cancellation Law). As an analogy, we have:

17) $(\Psi,\mathrm{I})\underset{\Delta}{\overset{*}{\sim}}(\Omega,\text{Ş})=(\Psi,\mathrm{I})\ \underset{\Delta}{\overset{*}{\sim}}(\wp,\text{Ş})\ \Longrightarrow(\Omega,\mathrm{I}\cap\text{Ş})=(\wp,\mathrm{I}\cap\text{Ş})$.

Proof: Let $(\Psi,\mathrm{I})\underset{\Delta}{\overset{*}{\sim}}(\Omega,\text{Ş})=(\wp,\mathrm{I})$. Then, $\forall\zeta\in\mathrm{I}$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus\text{Ş}\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap\text{Ş}\end{cases}$$

Let, $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}\ (\wp,\text{Ş})=(T,I)$, where $\forall\zeta\in I$,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus\text{Ş}\\ \Psi(\zeta)\,\Delta\,\wp(\zeta), & \zeta\in I\cap\text{Ş}\end{cases}$$

Since, $(\wp,I)=(T,I)$, then for all $\zeta\in I\cap\text{Ş}$; $\Psi(\zeta)\,\Delta\Omega(\zeta)=\Psi(\zeta)\,\Delta\wp(\zeta)$, thus $\Omega(\zeta)=\wp(\zeta)$ for all $\zeta\in I\cap\text{Ş}$. Hence, $(\Omega, I\cap\text{Ş})=(\wp, I\cap\text{Ş})$. Here note that $(\Psi,I)\overset{*}{\underset{\Delta}{\sim}}(\Omega,\text{Ş})=(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\wp,\text{Ş})$ does not imply that $(\Omega,I)=(\wp,\text{Ş})$.

In classical theory, $F\Delta G\subseteq F\cup G$. As an analogy, we have:

18) $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,\text{Ş})\ \tilde{\subseteq}\ (\Psi,I)\overset{*}{\underset{\cup}{\sim}}(\Omega,\text{Ş})$.

Proof: Since the PS of the SSs of both hand side is I, the first condition for the soft subset is satisfied. Let $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,\text{Ş})=(\wp,I)$, where $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus\text{Ş}\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap\text{Ş}\end{cases}$$

Now let $(\Psi,I)\overset{*}{\underset{\cup}{\sim}}(\Omega,\text{Ş})=(T,I)$, where $\forall\zeta\in I$,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus\text{Ş}\\ \Psi(\zeta)\cup\Omega(\zeta), & \zeta\in I\cap\text{Ş}\end{cases}$$

Since for all $\zeta\in I\setminus\text{Ş}$, $\Psi'(\zeta)\subseteq\Psi'(\zeta)$ and $\forall\zeta\in I\cap\text{Ş}$, $\Psi(\omega)\,\Delta\Omega(\omega)\subseteq\Psi(\omega)\cup\Omega(\omega)$, thus for all $\forall\zeta\in I$, $\wp(\zeta)\subseteq T(\zeta)$. Hence, $(\wp,I)\ \tilde{\subseteq}\ (T,I)$.

In classical theory, $F\Delta G=\emptyset\ \Leftrightarrow F=G$. As an analogy, we have:

19) $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,I)=\emptyset_I\ \Leftrightarrow (\Psi,I)=(\Omega,I)$.

Proof: Necessity: Let $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,I)=(T,I)$. Hence, $\forall\zeta\in I$,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Since $(T,I)=\emptyset_I$, $\forall\zeta\in I$, $T(\zeta)=\emptyset$. Thus, $\forall\zeta\in I$, $\Psi(\zeta)\,\Delta\Omega(\zeta)=\emptyset$. Hence, $\forall\zeta\in I$, $\Psi(\zeta)=\Omega(\zeta)$. So, $(\Psi,I)=(\Omega,I)$.

Sufficiency: Let $(\Psi,I)=(\Omega,I)$. Then, $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,I)=\emptyset_I$.

In classical theory, $F\Delta G=F\cup G\Leftrightarrow F\cap G=\emptyset$. As an analogy, we have (20) and (21).

20) $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,I)=(\Psi,I)\overset{*}{\underset{\cup}{\sim}}(\Omega,I)\Leftrightarrow(\Psi,I)\overset{*}{\underset{\cap}{\sim}}(\Omega,I)=\emptyset_I$

Proof: Let $(\Psi,I)\ \overset{*}{\underset{\Delta}{\sim}}(\Omega,I)=(\wp,I)$ and $(\Psi,I)\overset{*}{\underset{\cup}{\sim}}(\Omega,I)=(T,I)$. Then,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap I=I\end{cases}$$

and

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\cup\Omega(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Since $(\wp,I)=(T,I)$, then $\forall\zeta\in I$, $\wp(\zeta)=\Psi(\zeta)\Delta\Omega(\zeta)=\Psi(\zeta)\cup\Omega(\zeta)=T(\zeta)$. Thus, $\forall\zeta\in I$, $\Psi(\zeta)\cap\Omega(\zeta)=\emptyset$. Hence, $(\Psi,I)\overset{*}{\underset{\cap}{\sim}}(\Omega,I)=\emptyset_I$.

21) $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,\text{Ş}) = (\Psi, I) \overset{*}{\underset{\cup}{\sim}} (\Omega, \text{Ş}) \Leftrightarrow (\Psi, I) \cap_R (\Omega, \text{Ş}) = \emptyset_{I\cap\text{Ş}}$.

Proof: Let $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,\text{Ş}) = (\wp, I)$ and $(\Psi, I) \overset{*}{\underset{\cup}{\sim}} (\Omega,\text{Ş}) = (T,I)$. Then,

$$\wp(\zeta) = \begin{cases} \Psi'(\zeta), & \zeta\in I\setminus\text{Ş} \\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap\text{Ş} \end{cases}$$

and

$$T(\zeta) = \begin{cases} \Psi'(\zeta), & \zeta\in I\setminus\text{Ş} \\ \Psi(\zeta) \cup \Omega(\zeta), & \zeta\in I\cap\text{Ş} \end{cases}$$

Since $(\wp,I)=(T,I)$, then $\forall\zeta\in I\cap\text{Ş}$, $\Psi(\zeta)\Delta\Omega(\zeta)=\Psi(\zeta)\cup\Omega(\zeta)$. Thus, $\forall\zeta\in I\cap\text{Ş}$, $\Psi(\zeta)\cap\Omega(\zeta)=\emptyset$. Hence, $(\Psi, I) \cap_R (\Omega, \text{Ş}) = \emptyset_{I\cap\text{Ş}}$.
In classical theory, $F\subseteq G \Longrightarrow F\Delta G = G\setminus F$. As an analogy, we have (22) and (23):

22) $(\Psi, I) \mathrel{\tilde{\subseteq}} (\Omega, I) \Longrightarrow (\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,I) = (\Omega,I) \overset{*}{\underset{\setminus}{\sim}} (\Psi,I)$.

Proof: Let $(\Psi, I) \mathrel{\tilde{\subseteq}} (\Omega, I)$. Then, $\forall\zeta\in I$, $\Psi(\zeta) \subseteq \Omega(\omega)$ and let $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, I) = (\wp,I)$. Then, $\forall\zeta\in I$,

$$\wp(\zeta) = \begin{cases} \Psi'(\zeta), & \zeta\in I\setminus I=\emptyset \\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap I=I \end{cases}$$

Since $\forall\zeta\in I$, $\Psi(\zeta) \subseteq \Omega(\omega)$, and $\wp(\zeta) = \Psi(\zeta)\,\Delta\,\Omega(\zeta) = \Omega(\zeta)\setminus\Psi(\zeta)$. Thus, $(\wp,I) = (\Omega,I) \overset{*}{\underset{\setminus}{\sim}} (\Psi,I)$.

23) $(\Psi, I) \mathrel{\tilde{\subseteq}} (\Omega, \text{Ş}) \Longrightarrow (\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega,\text{Ş}) \mathrel{\tilde{\subseteq}} (\Omega,\text{Ş}) \overset{*}{\underset{\setminus}{\sim}} (\Psi,I)$.

Proof: Let $(\Psi, I) \mathrel{\tilde{\subseteq}} (\Omega, \text{Ş})$. Then, $I \subseteq \text{Ş}$, and so the first condition for the soft subset is satisfied. Moreover, since $(\Psi, I) \mathrel{\tilde{\subseteq}} (\Omega, \text{Ş})$, $\forall\zeta\in I$, $\Psi(\zeta) \subseteq \Omega(\omega)$. Let $(\Psi, I) \overset{*}{\underset{\Delta}{\sim}} (\Omega, \text{Ş}) = (\wp,I)$. Then, $\forall\zeta\in I$,

$$\wp(\zeta) = \begin{cases} \Psi'(\zeta), & \zeta\in I\setminus\text{Ş}=\emptyset \\ \Psi(\zeta)\,\Delta\,\Omega(\zeta), & \zeta\in I\cap\text{Ş}=I \end{cases}$$

Let $(\Omega,\text{Ş}) \overset{*}{\underset{\setminus}{\sim}} (\Psi, I) = (T,\text{Ş})$. Then, $\forall\zeta\in\text{Ş}$,

$$T(\zeta) = \begin{cases} \Psi'(\zeta), & \zeta\in \text{Ş}\setminus I \\ \Omega(\zeta)\setminus\Psi(\zeta), & \zeta\in\text{Ş}\cap I=I \end{cases}$$

Since $\forall\zeta\in I$, $\Psi(\zeta) \subseteq \Omega(\omega)$, thus $\Psi(\zeta)\,\Delta\,\Omega(\zeta) = \Omega(\zeta)\setminus\Psi(\zeta)$. Therefore, $(\wp,I) \mathrel{\tilde{\subseteq}} (T,\text{Ş})$.
In classical theory, $F\,\Delta(F\cap G) = F\setminus G$. As an analogy, we have:

24) $(\Psi,I) \overset{*}{\underset{\Delta}{\sim}} [(\Psi, I) \overset{*}{\underset{\cap}{\sim}} (\Omega,I)] = (\Psi, I) \overset{*}{\underset{\setminus}{\sim}} (\Omega,I)$.

Proof: Let $(\Psi, I) \overset{*}{\underset{\cap}{\sim}} (\Omega,I) = (\wp,I)$. Then, $\forall\zeta\in I$,

$$\wp(\zeta) = \begin{cases} \Psi'(\zeta), & \zeta\in I\setminus I=\emptyset \\ \Psi(\zeta) \cap \Omega(\zeta), & \zeta\in I\cap I=I \end{cases}$$

Let, $(\Psi,I) \overset{*}{\underset{\Delta}{\sim}} (\wp,I) = (T,I)$, where, $\forall\zeta\in I$,

$$T(\zeta) \begin{cases} \Psi'(\zeta), & \zeta\in I\setminus I=\emptyset \\ \Psi(\zeta)\,\Delta\,\wp(\zeta), & \zeta\in I\cap I=I \end{cases}$$

Hence,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash I=\emptyset\\ \Psi(\zeta)\,\Delta[\Psi(\zeta)\cap\Omega(\zeta)], & \zeta\in I\cap I=I\end{cases}$$

So,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash I=\emptyset\\ \Psi(\zeta)\,\backslash\Omega(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Thus, $(T,I)=(\Psi,I)\ \underset{\backslash}{\overset{*}{\sim}}\ (\Omega,I)$.

In classical theory, $F\cup G=(F\Delta G)\cup(F\cap G)$. As an analogy, we have:

25) $(\Psi,I)\underset{\cup}{\overset{*}{\sim}}(\Omega,Ş)=[(\Psi,I)\ \underset{\Delta}{\overset{*}{\sim}}\ (\Omega,Ş)]\underset{\cup}{\overset{*}{\sim}}[(\Psi,I)\underset{\cap}{\overset{*}{\sim}}(\Omega,Ş)]$.

Proof: Since the PS of the SSs of both hand side is I, the first condition for the soft equality is satisfied. First let's consider right side. Let $(\Psi,I)\underset{\cup}{\overset{*}{\sim}}(\Omega,Ş)=(\wp,I)$. Then, $\forall\zeta\in I$,

$$\wp(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\cup\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Now let's consider left side. Let $(\Psi,I)\underset{\Delta}{\overset{*}{\sim}}(\Omega,Ş)=(K,I)$. Then, $\forall\zeta\in I$,

$$K(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\,\Delta\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Let, $(\Psi,I)\ \underset{\cap}{\overset{*}{\sim}}\ (\Omega,Ş)=(T,I)$, where $\forall\zeta\in I$,

$$T(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\cap\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Now, let $(K,I)\ \underset{\cup}{\overset{*}{\sim}}\ (T,I)=(S,I)$, where $\forall\zeta\in I$,

$$S(\zeta)=\begin{cases}K'(\zeta), & \zeta\in I\backslash I=\emptyset\\ K(\zeta)\cup T(\zeta), & \zeta\in I\cap I=I\end{cases}$$

$$S(\zeta)=\begin{cases}\Psi'(\zeta)\cup\Psi'(\zeta) & \zeta\in(I\backslash Ş)\cap(I\backslash Ş)=I\backslash Ş\\ \Psi'(\zeta)\cup[\Psi(\zeta)\cap\Omega(\zeta)], & \zeta\in(I\backslash Ş)\backslash\cap(I\cap Ş)=\emptyset\\ [\Psi(\zeta)\,\Delta\Omega(\zeta)]\cup\Psi'(\zeta), & \zeta\in(I\cap Ş)\cap(I\backslash Ş)=\emptyset\\ [\Psi(\zeta)\,\Delta\Omega(\zeta)]\cup[\Psi(\zeta)\cap\Omega(\zeta)], & \zeta\in(I\cap Ş)\cap(I\cap Ş)=I\cap Ş\end{cases}$$

Thus,

$$S(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\backslash Ş\\ \Psi(\zeta)\cup\Omega(\zeta), & \zeta\in I\cap Ş\end{cases}$$

Thus, $(\wp,I)=(S,I)$. This completes the proof.

In classical theory, intersection distributes over symmetric difference from both left and right side, that is, $F\cap(G\Delta P)=(F\cap G)\Delta(F\cap P)$ and $(F\Delta G)\cap P=(F\cap P)\Delta(G\cap P)$ for all F,G,P. As an analogy, we have the following two properties:

26) $(\Psi,I)\ \cap_R[(\Omega,I)\ \underset{\Delta}{\overset{*}{\sim}}\ (\wp,I)]=[(\Psi,I)\cap_R(\Omega,I)]\underset{\Delta}{\overset{*}{\sim}}\ [(\Psi,I)\ \cap_R(\wp,I)]$

Proof: Let's first consider the left side. Let $(\Omega,I)\underset{\Delta}{\overset{*}{\sim}}\ (\wp,I)=(M,I)$, where $\forall\zeta\in I$;

$$M(\zeta)=\begin{cases}\Omega'(\zeta), & \zeta\in I\backslash I=\emptyset\\ \Omega(\zeta)\Delta\wp(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Assume that $(\Psi,I)\cap_R(M,I)=(N,I\cap I)=(N,I)$, where $\forall\zeta\in I$; $N(\zeta)=\Psi(\zeta)\cap M(\zeta)$. Hence,

$$N(\zeta)=\begin{cases}\Psi(\zeta)\cap\Omega'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\cap[\Omega(\zeta)\Delta\wp(\zeta)], & \zeta\in I\cap I=I\end{cases}$$

Now let's consider the right side: $[(\Psi, I)\cap_R(\Omega,I)]\overset{*}{\underset{\Delta}{\sim}}[(\Psi, I)\cap_R(\wp,I)]$. Let $(\Psi, I)\cap_R(\Omega, I)=(K,I\cap I)$, where $\forall\zeta\in I$, $K(\zeta)=\Psi(\zeta)\cap\Omega(\zeta)$.

Let $(\Psi, I)\cap_R(\wp,I)=(T,I\cap I)$, where $\forall\zeta\in I$; $T(\zeta)=\Psi(\zeta)\cap\wp(\zeta)$. Thus, $(K,I)\overset{*}{\underset{\Delta}{\sim}}(T,I)=(L,I)$, where $\forall\zeta\in I$;

$$L(\zeta)=\begin{cases}K'(\zeta), & \zeta\in I\setminus I=\emptyset\\ K(\zeta)\Delta T(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Thus,

$$L(\zeta)=\begin{cases}(\Psi(\zeta)\cap\Omega(\zeta))', & \zeta\in I\setminus I=\emptyset\\ [\Psi(\zeta)\cap\Omega(\zeta)]\Delta[\Psi(\zeta)\cap\wp(\zeta)], & \zeta\in I\cap I=I\end{cases}$$

Hence, $(N,I)=(L,I)$.

Here note that $(\Psi, I)\cap_R[(\Omega,Ş)\overset{*}{\underset{\Delta}{\sim}}(\wp,C)]\neq[(\Psi, I)\cap_R(\Omega,Ş)]\overset{*}{\underset{\Delta}{\sim}}[(\Psi, I)\cap_R(\wp,C)]$. That is, restricted intersection distributes over complementary soft binary piecewise symmetric difference from left side only when the PSs of the soft sets are the same.

27) $[(\Psi, I)\overset{*}{\underset{\Delta}{\sim}}(\Omega,I)]\cap_R(\wp,I)=[(\Psi, I)\cap_R(\wp,I)]\overset{*}{\underset{\Delta}{\sim}}[(\Omega,I)\cap_R(\wp,I)]$

Proof: Let's consider first the left side. Let $(\Psi, I)\overset{*}{\underset{\Delta}{\sim}}(\Omega,I)=(M,I)$, where $\forall\zeta\in I$;

$$M(\zeta)=\begin{cases}\Psi'(\zeta), & \zeta\in I\setminus I=\emptyset\\ \Psi(\zeta)\Delta\Omega(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Now, let $(M,I)\cap_R(\wp,I)=(N,I\cap I)$, where $\forall\zeta\in I$; $N(\zeta)=M(\zeta)\cap\wp(\zeta)$. Thus,

$$N(\zeta)=\begin{cases}\Psi'(\zeta)\cap\wp(\zeta), & \zeta\in I\setminus I=\emptyset\\ [\Psi(\zeta)\Delta\Omega(\zeta)]\cap\wp(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Now let's consider the right side: $[(\Psi, I)\cap_R(\wp,I)]\overset{*}{\underset{\Delta}{\sim}}[(\Omega,I)\cap_R(\wp,I)]$. Let $(\Psi, I)\cap_R(\wp,I)=(K,I\cap I)$, where $\forall\zeta\in I$, $K(\zeta)=\Psi(\zeta)\cap\wp(\zeta)$.

Let $(\Omega,Ş)\cap_R(\wp,C)=(T,I\cap I)$, where $\forall\zeta\in I$; $T(\zeta)=\Omega(\zeta)\cap\wp(\zeta)$. Thus, $(K,I)\overset{*}{\underset{\Delta}{\sim}}(T, I)=(W,I)$, where $\forall\zeta\in I$;

$$W(\zeta)=\begin{cases}K'(\zeta), & \zeta\in I\setminus I=\emptyset\\ K(\zeta)\Delta T(\zeta), & \zeta\in I\cap I=I\end{cases}$$

Thus,

$$W(\zeta)=\begin{cases}(\Psi(\zeta)\cap\wp(\zeta))', & \zeta\in I\setminus I=\emptyset\\ [\Psi(\zeta)\cap\wp(\zeta)]\Delta[\Omega(\zeta)\cap\wp(\zeta)], & \zeta\in I\cap I=I\end{cases}$$

Hence, $(N,I)=(W,I)$.

Here note that $[(\Psi, I)\overset{*}{\underset{\Delta}{\sim}}(\Omega,Ş)]\cap_R(\wp,C)\neq[(\Psi, I)\cap_R(\wp,C)]\overset{*}{\underset{\Delta}{\sim}}[(\Omega,Ş)\cap_R(\wp,C)]$. That is, restricted intersection distributes over complementary soft binary piecewise symmetric difference from right side only when the PSs of the soft sets are the same.

Remark 3.7: In Remark 3.6., we show that $(S_A(U),\overset{*}{\underset{\Delta}{\sim}})$ is an abelian group with identiy $\emptyset_A$ and every element is its own inverse. Hence, we can deduce that $(S_A(U),\overset{*}{\underset{\Delta}{\sim}})$ is a semigroup. Moroever, in [3,5,17], it was proved that $(S_A(U),\cap_R)$ is a commut ative monoid with identity U_A. Hence, we can deduce that $(S_A(U),\cap_R)$ is a semigroup. Moreover, by Theorem 3.5.

(26) and (27), $\cap_R$ distributes over $\underset{\Delta}{\overset{*}{\sim}}$ from both sides when the PSs of the soft sets are the same. Therefore, $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$ is a semiring. Further, by Theorem 3.5. (4) $(F,A) \underset{\Delta}{\overset{*}{\sim}} (G,A) = (G,A) \underset{\Delta}{\overset{*}{\sim}} (F,A)$. That is to say, $\underset{\Delta}{\overset{*}{\sim}}$ is commutative in $S_A(U)$ and $(F,A) \underset{\Delta}{\overset{*}{\sim}} \emptyset_A = \emptyset_A \underset{\Delta}{\overset{*}{\sim}} (F,A) = (F,A)$ and $(F,A) \cap_R \emptyset_A = \emptyset_A \cap_R (F,A) = \emptyset_A$. That is to say, $\emptyset_A$ is the zero element of $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$. Therefore, $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$ is a hemiring. Besides, since $(F,A) \cap_R U_A = U_A \cap_R (F,A) = (F,A)$ and $(F,A) \cap_R (G,A) = (G,A) \cap_R (F,A)$ (see [3,5,17]), $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$ is a commutative hemiring with identity U_A.

Also, since $(S_A(U), \underset{\Delta}{\overset{*}{\sim}})$ is an abelian group by Remark 3.6., $(S_A(U), \cap_R)$ is a semigroup by [3,5,17] and $\cap_R$ distributes over $\underset{\Delta}{\overset{*}{\sim}}$ from both sides when the parameter sets of the soft sets are the same by Theorem 3.5. (26) and (27), we can also deduce that $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$ is a ring. Also, since $(F,A) \cap_R (G,A) = (G,A) \cap_R (F,A)$ and $(F,A) \cap_R U_A = U_A \cap_R (F,A) = (F,A)$, (see [3,5,17]), $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$ is a commutative ring with identity U_A. Moroever, $(F,A)^2 = (F,A) \cap_R (F,A) = (F,A)$ for all $(F,A) \in S_A(U)$. Thus, $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_R)$ is a Boolean ring and $(F,A) \underset{\Delta}{\overset{*}{\sim}} (F,A) = \emptyset_A$ and $(F,A) \cap_R (G,A) = (G,A) \cap_R (F,A)$ is satisfied naturally as a result of being Boolean ring.

Remark 3.8. $(S_A(U), \cap_\varepsilon)$ is a commutative monoid (and so a semigroup) with identity U_A by [5,17], $(S_A(U), \tilde{\cap})$ is a commutative monoid (and so a semigroup) with identity U_A by [10]. Also, $(S_A(U), \underset{\cap}{\overset{*}{\sim}})$ is a commutative monoid (and so a semigroup) with identity by [11]. Thus, by Remark 3.7. and by Theorem 3.5. (26) and (27), one can similary show that $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \tilde{\cap})$, $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \underset{\cap}{\overset{*}{\sim}})$ and $(S_A(U), \underset{\Delta}{\overset{*}{\sim}}, \cap_\varepsilon)$ are all commutative hemirings with identity U_A and also Boolean rings.

4. CONCLUSION

Soft sets and soft operations are powerful parametric tools when dealing with uncertain objects. Creating new soft operations and deriving their algebraic properties and implementations provide new perspectives for solving problems with parametric data. In this regard, this work presents a new form of soft set operation. This is called complementary soft binary piecewise symmetric difference operation. The basic algebraic properties of the operation have been explored. Examination of the distribution rules reveals the relationship between this new soft set operation and the restricted intersection soft set operation. It has been shown that complementary soft binary piecewise symmetric difference and restricted intersection operations with a fixed set of parameters is both a commutative hemiring with identity and a Boolean ring. Future research may develop by introducing new variants of the soft set operations. As soft set operations are powerful mathematical tools for identifying uncertain objects, researchers may propose some new cryptographic or decision-making techniques as a result of this work. The operations described in this work can also be used to revisit the study of soft algebraic structures in terms of their algebraic properties.

REFERENCES

[1] Molodtsov, D., (1999). Soft set theory-first results. Computers and Mathematics with Applications, 37 (1) 19-31.

[2] Maji, P.K., Bismas, R., Roy, A.R., (2003). Soft set theory, Computers and Mathematics with Applications, 45 (1) 555-562.

[3] Pei D. and Miao, D., (2005). From Soft Sets to Information Systems, In: Proceedings of Granular Computing. IEEE, 2 617-62.

[4] Ali, M.I., Feng, F., Liu, X., Min., W.K., Shabir, M., (2009). On some new operations in soft set theory, Computers and Mathematics with Applications, 57(9) 1547-1553.

[5] Sezgin, A., Atagün, A. O., (2011). On operations of soft sets, Computers and Mathematics with Applications, 61(5) 1457-1467.
[6] Sezgin, A. Shahzad, A., Mehmood A. (2019). New operation on soft sets: extended difference of soft Sets, Journal of New Theory, (27) 33-42.
[7] Stojanovic, N.S., (2021). A new operation on soft sets: extended symmetric difference of soft sets, Military Technical Courier, 69(4) 779-791.
[8] Eren, Ö.F., (2019). On some operations of soft sets, Ondokuz Mayıs University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Samsun.
[9] Yavuz, E., (2024). Soft binary piecewise operations and their properties, Amasya University, The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya.
[10] Sezgin, A., Sarıalioğlu M, (2024). New soft set operation: Complementary soft binary piecewise theta operation, Journal of Kadirli Faculty of Applied Sciences, (4)(1), 1-33.
[11] Sezgin A., Aybek, F., Atagün, A.O. (2023). New soft set operation: Complementary soft binary piecewise intersection operation, Black Sea Journal of Engineering and Science, 6(4) 330-346.
[12] Sezgin, A. Aybek, F., Atagün, A.O. (2023). New soft set operation: Complementary soft binary piecewise union operation, Acta Informatica Malaysia, (7) 1, 27-45.
[13] Sezgin, A., Çağman, N. (2024). New soft set operation: Complementary soft binary piecewise difference operation, Osmaniye Korkut Ata University Journal of the Institute of Science and Technology, 7(1),1-37.
[14] Yang, C.F., (2008). “A note on: “Soft set theory” [Computers & Mathematics with Applications 45 (2003), no. 4-5, 555–562],” Computers & Mathematics with Applications, 56 (7) 1899–1900.
[15] Feng, F., Li, Y.M., Davvaz, B., Ali, M. I., (2010). Soft sets combined with fuzzy sets and rough Sets: A tentative approach, Soft Computing, 14 899–911.
[16] Jiang, J., Tang, Y., Chen, Q., Wang, J., Tang, S., (2010). Extending soft sets with description logics, Computers and Mathematics with Applications, 59 2087–2096.
[17] Ali, M.I., Shabir, M., Naz, M., (2011). Algebraic structures of soft sets associated with new operations, Computers and Mathematics with Applications, 61 2647–2654.
[18] Neog, I.J, Sut, D.K., (2011). A new approach to the theory of soft set, International Journal of Computer Applications, 32 (2) 1-6.
[19] Fu, L., Notes on soft set operations, (2011). ARPN Journal of Systems and Softwares, 1, 205-208.
[20] Ge, X., Yang S., (2011). Investigations on some operations of soft sets, World academy of Science, Engineering and Technology, 75 1113-1116.
[21] Singh, D., Onyeozili, L.A., (2012a). Some conceptual misunderstanding of the fundamentals of soft set theory, ARPN Journal of systems and softwares, 2 (9) 251-254.
[22] Singh, D., Onyeozili, L.A., (2012b). Some results on distributive and absorption properties on soft operations, IOSR Journal of mathematics, 4 (2) 18-30.
[23] Singh, D., Onyeozili, L.A., (2012c). On some new properties on soft set operations, International Journal of Computer Applications, 59 (4) 39-44.
[24] Singh, D., Onyeozili, L.A., (2012d). Notes on soft matrices operations, ARPN Journal of science and technology, 2(9) 861-869.
[25] Ping, Z., Qiaoyan, W., (2013). Operations on soft sets revisited, Journal of Applied Mathematics, Volume 2013 Article ID 105752 7 pages.
[26] Jayanta, S., (2014). On algebraic structure of soft sets, Annals of Fuzzy Mathematics and Informatics, Volume 7 (6) 1013-1020.
[27] Onyeozili, L.A., T.A., Gwary, (2014). A study of the fundamentals of soft set theory, International Journal of Scientific & Technology Research, 3 (4) 132-143.
[28] Husain, S., Shamsham, Km., (2018). A study of properties of soft set and its applications, International Research Journal of Engineering and Technology, 5 (1) 363-372.
[29] Vandiver, H.S., (1934). Note on a simple type of algebra in which the cancellation law of addition does not hold, Bulletin of the American Mathematical Society, 40, (12) 914–920.
[30] Vasanthi T., and Sulochana, N., (2013). On the additive and multiplicative structure of semirings, Annals of Pure and Applied Mathematics, 3 (1) 78–84.
[31] Kaya A. and Satyanarayana, M., (1981). Semirings satisfying properties of distributive type, Proceedings of the American Mathematical Society, 82 (3) 341–346.
[32] Karvellas, P.H., (1974). Inversive semirings, Journal of the Australian Mathematical Society, 18 (3), 277–288.
[33] Goodearl, K.R., (1979). Von Neumann Regular Rings, Pitman, London.
[34] Petrich, M., (1973). Introduction to Semiring, Charles E Merrill Publishing Company, Ohio.
[35] Reutenauer C. and Straubing H., (1984). Inversion of matrices over a commutative semiring, Journal of Algebra, 88 (2) 350–360.
[36] Glazek, K., (2002). A guide to litrature on semirings and their applications in mathematics and ınformation sciences: with complete bibliography, Kluwer Acad. Publ., Nederland.

[37] Kolokoltsov, V.N., Maslov, V.P., (1997). Idempotent Analysis and its Applications, in: Mathematics and its Applications, 401, Kluwer.
[38] Aho, A.W, Ullman, J.D., Introduction to automata theory, languages and computation, Addison Wesley, Reading, MA.
[39] Beasley, L.B., Pullman, N.G., (1979). Operators that preserves semiring matrix functions, Linear Algebra Appl. 99 (1988) 199–216.
[40] Beasley, L.B., Pullman, N.G., (1992). Linear operators strongly preserving idempotent matrices over semirings, Linear Algebra Appl. 160 217–229.
[41] Ghosh, S., (1996). Matrices over semirings, Inform. Sci. 90 221–230.
[42] Wechler, W., (1978). The concept of fuzziness in automata and language theory, Akademic Verlag, Berlin, 1978.
[43] Golan, J.S., (1999) Semirings and their applications, Kluwer Acad. Publication.
[44] Hebisch, U., Weinert, H.J., (1998). Semirings: algebraic theory and applications in the computer science, World Scientific.
[45] Mordeson, J.N., Malik, D.S., (2002). Fuzzy automata and languages, theory and applications, in: computational mathematics series, Chapman and Hall, CRC, Boca Rato.

Techno-Science *Paper ID: 1365021*