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A New Approach to the Solution of Facility Layout Problems with Filled Function Method

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Abstract: Facility layout problems are generally solved by stochastic methods in the literature. The large number of iterations used in these methods is quite costly in terms of solution time. In this study, in order to get rid of this disadvantage, the facility layout problem was solved using the filled function method, which is known to be very successful in solving global optimization problems. In order to demonstrate the effectiveness of the filled function method, the classical linear facility layout problem was handled in a non-linear manner and the problem was deliberately made more difficult. In order to use the filled function method, the facility layout problem was transformed into an unconstrained and multimodal (including more than one local minima) global optimization problem by using the hyperbolic smoothing technique and the penalty function method. Thus, in this first study in the literature where a deterministic method is combined with the solution of the facility layout problem, it is shown that the non-convex facility layout problem can be solved with the filled function method with very few iterations and short solution times.

Key words: Facility layout problem, global optimization, filled function method.

Tesis Yerleşim Probleminin Çözümüne Doldurulmuş Fonksiyon Yöntemi ile Yeni Bir Yaklaşım

Öz: Literatürdeki tesis yerleşim problemleri genellikle stokastik yöntemlerle çözülmektedir. Bu yöntemlerde kullanılan iterasyon sayısının fazlalığı çözüm süresi açısından oldukça maliyetlidir. Bu çalışmada, bu dezavantajdan kurtulmak adına tesis yerleşim problemi, global optimizasyon problemlerini çözmede oldukça başarılı olduğu bilinen doldurulmuş fonksiyon yöntemi kullanılarak çözülmüştür. Doldurulmuş fonksiyon yönteminin etkinliğini göstermek için klasik doğrusal tesis yerleşim problemi, doğrusal olmayacak şekilde ele alınmış ve problem bilinçli olarak zorlaştırılmıştır. Doldurulmuş fonksiyon yöntemini kullanabilmek adına hiperbolik yumuşatma tekniği ve ceza fonksiyonu yöntemi kullanılarak tesis yerleşim problemi kısıtsız ve multimodal (birden fazla yerel minimumun içerilmesi) bir global optimizasyon problemi haline dönüştürülmüştür. Böylece, tesis yerleşim probleminin çözümü ile deterministik bir yöntemin bir araya getirildiği literatürdeki bu ilk çalışmada, konveks olmayan tesis yerleşim problemi doldurulmuş fonksiyon yöntemi ile oldukça küçük iterasyonlar ve kısa çözüm süreleri ile çözülebileceği gösterilmiştir.

Anahtar kelimeler: Tesis yerleşim problemi, global optimizasyon, doldurulmuş fonksiyon metodu.

1. Introduction

The coordination of production tools, auxiliary facilities or workstations as a whole in terms of their physical location is called facility layout. That is, facility layout is an integration of the physical arrangement of departments, workstations, machinery, equipment, materials, common areas within an existing or proposed industry. In today's competitive global environment, optimum facility layout has become an effective tool in reducing costs by increasing efficiency. To achieve maximum capacity return, it is important that facilities have a well-organized facility layout that is optimal for all available resources. For this purpose, many techniques have been developed by scientists to achieve optimum targets Layout decision problems, which are referred to as facility layout problems or factory layout problems in the literature, are divided into four as square assignment problem, unlimited capacity facility layout, p-median and p-center [1]. Substantially, facility layout problems are based on the problem of finding the Fermat point of a triangle (Figure 1). Also, it is called the Torricelli point or Fermat-Torricelli point in the plane geometry. The problem is on "finding the point inside a triangle (not on the triangle) so that the sum of the distances from the three vertices is minimum" [2].

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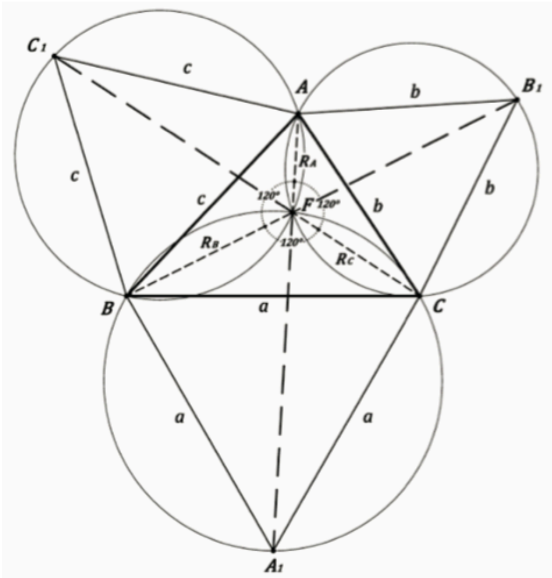

Figure 1. Torricelli construction of point of minimal sum of distances [2].

In facility layout problems, two objectives are usually tried to be optimized. The first of these purposes is to make a layout plan that will minimize the distance traveled by personnel or material between two facility locations, and the second is to make a layout plan that will maximize the predetermined proximity ratio measurements [3].

The facility layout problem deals with the efficient assignment of m facility to n location $(m \leq n)$ where material handling costs and fixed costs are tried to be minimized [4]. In other words, the main motivation in mathematical models for facility layout problem is to minimize the cost of material handling. Every facility can be a candidate for every location, all combinations of facilities and locations need to be evaluated. For this reason, the solution to the facility layout problem is considered difficult. When the number of facilities is n and the number of locations is n, there are at most combinations n!. When the number of facilities is n and the number of locations is m, if these numbers are not equal, the number of combinations to be evaluated would be $[m/(m-n)]!$ for $n < m$. Although this number is smaller than n!, it is still a large value [5-6]. Facility layout problems have been studied since the 1950s. With the developments in the field of operations research, the facility layout problem was first modeled as the *quadratic assignment problem* (**QAP**) by Koopmans and Beckmann [7]. Since the increase in the number of variables in the problem made the solution of the problem difficult, researchers have continued to search for new ways and develop new methods to solve the problem from the 1950s to the present.

In addition to the squareness of the QAP objective function, it has a non-convex structure. There are as many (n!) solution points and more than one local optimum solution in the QAP solution space. There is not a well disagned method that can find the optimum solution in the possible computation time for QAP with more than 20 facilities [8]. In fact, Meller and Gau argued that in some types of FLP, even for six locations, it is impossible to search for optimal solutions to this problem by applying precise methods [9-10]. The facility layout problem (FLP) is one of the NP (non-polynomial) problems because the solution time increases exponentially depending on the problem size [11]. Despite the great developments in computer technology and the abilities of researchers in recent years, large-scale facility layout problems cannot be solved by optimization methods, instead, solutions are sought with heuristics that make systematic approaches to problems and whose solution times are within polynomial limits.In the literature, Newton's method, the steepest regular method and conjugate gradient method are used generally for solving optimization problems involving only one local minimum. However, these methods are not suitable for problems with more than one local minimum. Because in optimization problems involving more than one local minimum, existing methods can get stuck in any existing local minimum and/or a local minimum worse than the existing local minimum. In this sense, it is necessary to turn to global optimization problems involving selective local minimums. The global optimization problem allows us to give clear answers to questions such as: 'How to escape the current local minimum?' and 'Is the current optimum solution global or not?'. In recent years, some effective methods have been proposed for solving global optimization problems. Among the existing methods, the filled function method (FFM) is particularly successful in detecting local minima repeatedly.

Facility layout problems are unimodal and their constraints are linear functions. In real world, it is generally encountered nonlinear situations. Recently, it has been observed that the use of humanoid robot technologies that exhibit non-linear movements in the industry, especially in facility layout problems, has increased. Therefore, in this study, it is shown that the classical facility layout problem can be solved with FFM by removing it from its classical state and making it more difficult by transforming it into a non-linear FLP. In fact, it is made the problem so difficult that it is solved the 22-facility settlement problem instead of the 6-facility settlement problem. The reason why is chosen 22 facilities in the problem is that it is emphasized that the solution of the facility placement problem for 22 facilities cannot be done analytically in the main works it has been examined on this subject in the literature. It should be noted here that, with the method it is proposed, the analytical solution of the problem can be achieved even if there are more than 22 facilities. However, these solutions are not included so that the study is not boring for the reader. The solution of the facility layout problem with the method proposed is outlined as follows. First of all, FLP, which contains non-linear constraints, is made unrestricted using the penalty function method. Then, using the hyperbolic smoothing technique, FLP is transformed into a non-convex optimization problem in which the derivative operator can be used. As a result of all these, in order to demonstrate the effectiveness of FFM in FLP, FLP is transformed into a multi-modal, unconstrained, non-linear and non-convex global optimization problem other than the classical case. It is shown that the solution of the hardened FLP can be achieved in a short time with the deterministic-analytical method using FFM for the first time. Before giving this solution in detail, the main structure of the facility layout, (hyperbolic) smoothing technique, penalty function method and filled function method will be briefly reminded in the following subsections.

2. Methods and Literature Review

2.1. General Algorithm for Solution of the Layout Problem

Multiple facility layout algorithms are weak in the way of the representing departments. For example; Most algorithms represented partitions as squares of equal size [12] or combinations of such squares [13-14]. Some of these algorithms-imposed constraints on the models of these composite departments in order to keep the size of the problem traceable [13]. These restrictions must be decided by the user. Because the dimensions and locations where the departments can be located should appeal to the user. On the other hand, the problem in developing algorithms that allow departments of different sizes is that the final layout created may contain irregular shapes, which is not possible for a real facility.

Linear and nonlinear optimization models have been improved for modelling multiple facility layout problems. One of the first treatment of the Euclidean distance multiple facility layout problem which was presented at references [15-17] applied a linear approach for rectilinear cost distance minimization. This approach has been generalized by Morris and Verdini [18 -19], and the l_p distance measure has been minimized to solve the multi-facility location problem.

Camp et al. [20] introduced a model for an approximate the real layout problem. In this model, points had been placed in a rectangle shape. As result, it was concluded although the areas of the points are fixed, the width/length coordinates can vary.

The model generally assumes that the transportation costs between points are known. Also, the cost of the distance of each point to the outside wall is known. In this problem, the aim is to minimize the total cost of the distance between points and between the outside walls of the points.

2.2. Smoothing Technique

The smoothing technique is used to make non-differentiable functions differentiable and also continuous. This technique can be applied by adding a small, nonzero value to the corresponding non-smooth function. The advantage of smoothing method is that optimization problems with continuously differentiable functions can be solved, which have rich theory and powerful solution methods, and a local minimizer or stationary point of the original non-smooth problem can be guaranteed by updating the smoothing parameter.

If the studies on optimization theory are examined, it is seen that smoothing functions and techniques are frequently used in these studies [for more details see 31-32]. The first study about the smoothing techniques was studied by Bertsekas to detach one of the crucial optimization problems called min-max [21]. For the same problem, another important smoothing function approach is studied in the [22]. The smoothing process is beneficial for problems which include any of minimum or any operators. The smoothing technique is controlled by a parameter which gives an opportunity to obtain a precise approximation to the original non-smooth function. And also smoothing methods are not only efficient for problems with non-smooth objective functions, but also for problems with non-smooth constraints. In this study, it is used penalty function methods and smoothing, which are well-known gradient-based optimization techniques, to solve non-uniform optimization problems. It is used the smoothing approach to locally correct the inflection points of the non-smooth function.

2.3. Penalty Function Method

With the penalty function method, the restricted programming model is made unconstrained. In this method, a penalty term is added to the objective function for each constraint and the objective function is aimed to be minimized. The added penalty is increased iteratively and continued until the minimum point for the objective function is found. In the method, the penalty added to the objective function as a result of the violation of the constraint is equal to μ times the square of the constraint function. This means to arrive at a continuous function with a continuous gradient. Considering that the constraints to be used in this study are not convex, first transforming the problem into an unconstrained problem with the penalty function method and then applying the filled function method to this unconstrained problem guarantees the finding of the local minimum point.

2.4. Use of the Filled Function Method for Solving the Facility Layout Problem

The location problem is basically an assignment problem and deals with the allocation of facilities to residential areas. If the distances and sizes between the locations of the facilities are negligible, it is not possible to place the facilities on top of each other. Facilities can be thought of as points far from each other and the problem can be addressed as a location selection problem. If the facilities are close to each other, there is a problem that the areas allocated to them overlap. It is the need to meet this constraint that makes modeling and solving the layout problem difficult. On the other hand, if the facilities to be placed are considered to be squares of equal area, the layout can be modeled as a cage structure. The layout problem, which can be thought of as a one-dimensional array in its simplest form, is too complex to be solved in a reasonable time due to its combinatorial structure. On the other hand, being stuck with the existing local minimizer and not being able to get rid of it is the main difficulty of global optimization. In this sense, many remarkable methods have been developed in global optimization theory in recent years to overcome this difficulty. Typical examples of deterministic methods are FFM [25], [26], orbital method [27], covering method [28] and tunneling method [29]. It is known that deterministic and stochastic methods have advantages and disadvantages compared to each other. Although stochastic algorithms, such as particle swarm algorithms, have the ability to search for global optimality, they may suffer from falling prematurely into local optima. Since filled function algorithms are deterministic algorithms compared to stochastic algorithms, they guarantee finding global optimization. The FFM-based algorithm finds the same result in every run and provides the exact solution. Additionally, many studies in the literature have emphasized that finding the optimal solution of FLP by applying deterministic methods is quite costly when the number of facilities is twenty or more. Meller and Gau even suggested that in some types of FLP, it is impossible to find an optimal solution to this problem by applying exact methods even for six locations [9–10].

Based on these claims, there are two main motivations for conducting this study. The first is to show that FLP can be solved in a reasonable time by a deterministic method even if the number of facilities is more than 6. The second motivation is to avoid the problem of finding random results in heuristic (or metaheuristic) algorithms, since the solution of FLP in terms of FFM is based on mathematical calculation. Thus, more successful results can be obtained by using the filled function method, which is one of the most effective deterministic methods in global optimization problems.

Using the filled function method, which is the most frequently mentioned in the literature, the above-mentioned problems were tried to be solved, based on the intended motivations, and this problem was solved in a very short time with a single iteration. Originality was achieved by proposing a new approach, the first in the literature, to solve the facility layout problem with the filled function method.

2.5. Filled Function Method

To summarize FFM in a general context, it can be explained as follows. First, a random starting point is chosen in the solution space. With the local optimization method, the local minimizer of the objective function is obtained. In the local minimizer of this objective function, a filled function with the analytic properties of the objective function is constructed. A random point that is very close to this minimizer of the objective function is then chosen as the starting point. The local minimizer of the function filled according to this point is found by the local optimization method. By taking the minimization of the resulting filled function as a starting point, a better minimum of the objective function is found. Existing minimizers are minimized until a more general minimizer (global minimum) is found (for more details see [25-26], [30]).

Remark 1. Since most of the local optimization algorithms used in FFM require gradient information, in this study combining FFM and FLP solution, the hyperbolic smoothing method is used to differentiate the objective function.

Although the definition of the filled function was first given by [25]; there are various variations of this definition in the literature by making minor changes suitable for the purpose. However, the version of this definition that is frequently used in the literature is as follows (see for more details [30].

Definition 2.5.1. (Definition 1 in [30], page 513) A continuously differentiable function $FF(x, x_1^*)$ is said to be a filled function of $f(x)$ at x_1^* (the current local minimizer of $f(x)$) if it satisfies the following properties and Equation 1:

i) x_1^* is a strict local maximizer of $FF(x, x_1^*)$ on Ω ($FF: \Omega \subseteq \mathbb{R}^n \to \mathbb{R}$),

ii) $FF(x, x_1^*)$ has no stationary point in the set
$$S_1 = \{x: f(x) \geq f(x_1^*), x \epsilon \Omega \setminus \{x_1^*\}\} \qquad (1)$$

iii) If $x_1^*$is not a global minimizer of f (x), then there exists a point x′ such that it is a local minimizer of $FF(x, x_1^*)$ on $S_2 = \{ x \in \text{int}\Omega: f(x) < f(x_1^*)\}$.

The first filled function example given by Ge Ren-Pu in 1987 in accordance with the above definition is as shownn in Equation 2.

$$P(x, r, p) = \frac{1}{r + f(x)} exp\left(\frac{\|x - x_1^*\|^2}{p^2}\right) \qquad (2)$$

However, the fact that this function contains an exponential expression and two parameters that are difficult to adjust has brought along computational difficulties. Therefore, in order to overcome these difficulties, researchers have defined new filled functions. In the literature, several studies were done with different filled function methods. In this study, filled function method was used as below.

An unconstrained global minimization problem can be briefly represented as Equations 3 and 4:

$$\min f(x) \qquad (3)$$

$$\text{s.t. } x \in \mathbb{R}^n \qquad (4)$$

where $f: \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable function. Suppose that $f(x)$ is global convex, that is $f(x) \to +\infty$, when $\|x\| \to +\infty$, therefore, there is a closed and bounded region $\Omega \subset \mathbb{R}^n$ containing all the minimizers of $f(x)$. Otherwise, unbounded global optimization problem over an unbounded region would not be solved. Thus, the Equation 3 can be written as Equations 5 and 6:

$$\min f(x) \qquad (5)$$

$$\text{s.t. } x \in \Omega \qquad (6)$$

The filled function method in this study, which was introduced by Lin, Gao ve Wang in 2014, was applied. Below is the algorithm of this filled function method:

Step 0: Choose an upper bound U bp of P (e.g., 10^6) and a constant $\rho > 0$ (e.g., $\rho = 10$); give the initial value of P, and some directions d_i, i =1, 2, · · · , $2n$, where $d_i = (0, \cdots, 1, \cdots, 0)^T$, 1 is at the i -th element of d_i , i = 1, 2, . . ., n and $d_i = -d_{i-n}$, $i = n + 1, \cdots, 2n$, where n is the dimension of the problem. Set k= 1. Choose any $x_1 \in \Omega$.

Step 1: Minimize $f(x)$ with the initial point $x_k \in \Omega$ so that a minimizer x_k^* of $f(x)$ is obtained.

Step 2: Construct

$$FF(x, x_1^*, P) = \frac{P}{1+\|x-x_1^*\|^2} h_{\frac{1}{P}}\big(f(x) - f(x_1^*)\big) \tag{7}$$

set i =1 and where h is one variable function such that c is a constant, and is defined by

$$h_c(t) = \begin{cases} c, & t \geq 0 \\ t^3 + c, & t < 0 \end{cases}$$

Step 3: If $i \leq 2n$, then set $x = x_k^* + \delta d_i$ and go to Step 4; Otherwise, go to Step 5.

Step 4: Use x as the initial point to minimize $FF(x,\ x_k^*,$ P) and denote the sequence point generated by a local optimization algorithm as x_j, j = 1, 2, · · · if ∃j_0 ∈ {1, 2, · · ·} such that $x_{j_0} \notin \Omega$, set $i = i$ +1 and go to Step 3; Otherwise, find a minimizer $x' \in \Omega$ of $FF(x,\ x_k^*,$ P) and set $x_{k+1} = x'$, $k = k$ + 1, go to Step 1.

Step 5: If P <U bp, then increase P by P = ρ × P and go to Step 2; Otherwise, the algorithm stops and x_k^* is taken as a global minimizer of $f(x)$.

3. Numerical Experiment

In this study, classical linear constraints were considered as a nonlinear constraint for FLP. The reason for contructing this is to take FLP out of its usual classical situation and make FLP a more difficult problem by randomly selecting 22 points. First of all, FLP, which contains non-linear constraints, is made unrestricted using the penalty function method. Then, using the hyperbolic smoothing technique, FLP is transformed into a non-convex optimization problem in which the derivative operator can be used. As a result of all these, in order to demonstrate the effectiveness of FFM in FLP, FLP is transformed into a multi-modal, unconstrained, non-linear and non-convex global optimization problem other than the classical case. It is shown that the solution of the hardened FLP can be achieved in a short time with the deterministic-analytical method using FFM for the first time. There are two main purposes in doing this. The first is to show that the facility layout problem can be solved in a reasonable time with a deterministic method, even if there are more than 6 facilities. The second aim is to solve this problem by using the filled function method, which is one of the most effective deterministic methods in global optimization problems. This problem was solved by using the filled function method, which is the most frequently mentioned in the literature.

In order to make the functioning of the model more understandable, it is aimed to reach the answer with very few iteration. Below, the step-by-step method to reach the global minimum will be explained. The data was constructed by us to better visualize local minima.

Problem Definiton: It was aimed to find the locations with minimum distance in a rectangular region according to the coordinates given in Table 1. Rectangular region constraints are given in Equation 9 and Equation 10. In this case, the mathematical model with objective function, such as Equation 8, is given below.

$$\underset{(x,y)\,\in\,\mathbb{R}^2}{Min}\ f(x,y) = \sum_{i=1}^{k} d\big((x,y),(x_i\,,y_i)\big) \tag{8}$$

where $d\big((x,y),(x_i\,,y_i)\big) = \sqrt{(x-x_i)^2 + (y-y_i)^2}$ for $i = 1,\ldots, k$ such that for $k = 22$.

Subject to

$$y - x^2 \leq 0 \tag{9}$$

$$\frac{(x-4)^2}{36} + \frac{(y-4)^2}{36} - 1 \leq 0 \tag{10}$$

Table 1. Coordinates of Departments

i	x_i	y_i	i	x_i	y_i
1	3	7	**12**	3	6
2	3	-7	**13**	1	5
3	-2	5	**14**	-1	4
4	-2	-5	**15**	-3	8
5	0	6	**16**	-3	7
6	0	-6	**17**	2	6
7	4	5	**18**	-2	7
8	4	6	**19**	-4	9
9	3	8	**20**	3	5
10	5	8	**21**	-3	5
11	5	9	**22**	3	-5

The steps of making the problem unconstrained and finding its global optimum are explained below.

Step 1. Based on the technique it is mentioned above, the smooth state of the objective function would be as in Equation 11.

$$g(x,y) = \sum_{i=1}^{k} \sqrt{(x-x_i)^2 + (y-y_i)^2 + \mathcal{E}} \quad (11)$$

$\mathcal{E}$ is a positive real number very close to zero and $f(x,y)$ for $k = 22$.

Step 2. With the Penalty function method, the model is made unconstrained.

$$\underset{(x,y)\in\mathbb{R}^2}{Min} \left(g(x,y) + m\left(\frac{(x-4)^2}{36} + \frac{(y-4)^2}{36} - 1\right)^2 + m(max(0, x - y^2))^2 \right) \quad (12)$$

Step 3. In this step, the global minimum point of the function given in Equation 12 was obtained using Matlab R2019b.

Step 4. The filled function x_1^* given in Equation 7 is set to the local minimum point. Minimizing this filled function with a local method it is obtained the minimizer $x_f^* = [2.9570 \quad 5.7636]$ of filled function which is certainly in a basin $B(x_2^*)$ lower than $B(x_1^*)$.

The correctness of solution with two different methods (fminsearch and fminunc) in Matlab was proved. It has been proved the accuracy of these points once again by putting them in equation 12 whether they are on an ellipse or not.

Step 5. After finding the best m value in the algorithm, x_1^* local minimum value is found. The difference between x_1^* and x_2^* is calculated after finding the x_2^* value, which is the lower basin of the x_1^* value. If the difference is less than the $\mathcal{E}$ value, the algorithm ends. The numerical results of the filled function algorithmare presented for $\mathcal{E} = 10^{-4}$ and m =200.

By these algorithm's, the global minimum point was found by reaching the answer with a single iteration.

The all results from Matlab R2019b are given in table 2. The meanings of the symbols used in these tables are as follows:

- k : the number of facilities

- *iterm* : the number of iterations in obtaining the global minimizer
- *fbest* : the best function value in 10 runs
- *fmean* : the mean of the best function value
- *feval* : the total number of the functions evaluations

Table 2. The results on the problem.

Method	**k**	**iterm**	**fbest**	**fmean**	**feval**	**fevalm**	**Global Minimizer**
FF	**22**	1	107.46	107.46	1033	1049.6	[2.9570 5.7636]

4. Conclusions and Future Works

Since facility layout problems have NP-Hard structure, the larger the problem size, the more difficult finding the optimal solution, and it also requires a lot of time to reach reliable results. Such problems cannot be solved in computation time limited by the polynomial function of the problem size. For this reason, heuristic methods that do not guarantee finding the optimum, but whose solution time remains within polynomial limits, are used in solving such problems. Most heuristics have problem-specific properties and a heuristic used for one problem cannot be used for another problem. However, interest in techniques (Anneal Simulation, Tabu Search, Artificial Neural Networks, Genetic Algorithms) that are flexible and more general in terms of their application to problems has increased in recent years. These techniques have been applied to a large number of facility layout problems and have been found to be quite powerful. However, the number of iterations in solving these FLPs is quite high and the solution time takes a very long time. Therefore, in this study, a method is proposed to reach the global optimum in a very short time with very few iterations.

Additionally, the objective function or constraints of most optimization problems encountered in the real world are nonlinear and involve more than one local minima. The facility layout problem is one of them. For problems involving a local endpoint, many local optimization methods are well known in the literature, such as the steepest descent method, Newton's method, and the conjugate gradient method. However, these methods are insufficient to find the global minimum in problems involving more than one local minimizer. Being stuck with the current local minimizer and not being able to get rid of it is the main difficulty of such problems. One of the best-known methods in the literature that can get rid of these basic difficulties is the filled function method proposed in this study.

In this study, considering FLP as a nonlinear constraint with a nonlinear objective function made the problem even more difficult. It has been made more difficult and FLP, which contains non-linear constraints, is made unrestricted using the penalty function method. Then, using the hyperbolic smoothing technique, FLP is transformed into a non-convex optimization problem in which the derivative operator can be used. Thus, FLP was constructed, which turned into a non-convex optimization problem in which the objective derivative operator could be used. As a result of all these, FLP has been transformed into a multimodal, unconstrained, non-linear and non-convex global optimization problem other than the classical case. Later, the filled function method was used due to its ability to successfully find the global reducer in multimodal global optimization problems. Thus, although there are many alternative methods to FLP, a solution was achieved in a short time with the deterministic-analytical method for the first time. Therefore, in this study, it is shown that the classical facility layout problem can be solved by FFM by removing it from its classical form and making it more difficult by converting it into a non-linear FLP.

In addition, since the solution of FLP in terms of FFM is based on mathematical calculation, the problem of finding random results in heuristic (or metaheuristic) algorithms is avoided. The FFM-based algorithm finds the same result in every run and offers an exact solution.In the following studies, it is aimed to find new filled function methods, to apply these new filled function methods to more complex facility layout problems, and to present new studies by comparing the obtained results with the previous solution methods of facility layout problems that are scientifically accepted in the literature, thus bringing a new, different and effective perspective to the solution of more advanced facility layout problems.

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