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Research article

B.-Y. Chen-Type Inequalities for Three Dimensional Smooth Hypersurfaces

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(Dedicated to Professor Bang-Yen CHEN on the occasion of his 80th birthday)

Abstract

By J.F. Nash's Theorem, any Riemannian manifold can be embedded into a Euclidean ambient space with dimension sufficiently large. S.-S. Chern pointed out in 1968 that a key technical element in applying Nash's Theorem effectively is finding useful relationships between intrinsic and extrinsic elements that are characterizing immersions. After 1993, when a groundbreaking work written by B.-Y.Chen on this theme was published, many explorations pursued this important avenue. Bearing in mind this historical context, in our present project we obtain new relationships involving intrinsic and extrinsic curvature invariants, under natural geometric conditions.

Keywords: *Principal curvatures, smooth surfaces, smooth hypersurfaces.*

AMS Subject Classification (2020): *Primary: 53B20 ; Secondary: 53C21.*

1. Introduction

It might seem surprising that some elementary inequalities could provide an insight into the problem of the best possible immersion of a space into another ambient space. This idea definitely deserves an accessibile illustration. By J.F. Nash's Theorem (going back to 1956 [19], a historical period depicted in the *Beautiful Mind* movie), any Riemannian manifold can be embedded into a Euclidean ambient space with dimension sufficiently large.

While working on the proof of this important result, John F. Nash, Jr. was motivated by the hope that such a tool, when proved, will usher in a powerful pathway to investigate the deformation of space, by allowing one to regard any abstract Riemannian manifold as a subspace of some Euclidean space. S.-S. Chern pointed out (see e.g. p.13 in [12]) that a key technical element in applying Nash's Theorem effectively is finding useful relationships between intrinsic and extrinsic elements that are characterizing immersions. In a fragment inviting reflection even today, Chern and Osserman wrote [13]: "First, Ricci made the surprising discovery that there are simple necessary and sufficient conditions on a two-dimensional metric for it to be realizable on a minimal surface in $\mathbb{E}^3$. For higher-dimensional minimal submanifolds, various necessary conditions on the metric have been given by Pinl-Ziller [20] and Barbosa-Do Carmo [2], but they are clearly far from sufficient." Many decades later, there is still work to do.

In the visionary paper [10] (see relation (3.6) in Lemma 3.2), B.-Y. Chen proved that for a submanifold M^n in a space form $R^{n+m}(c)$ of constant sectional curvature c the scalar curvature satisfies at a point the fundamental inequality

$$\delta(2) = scal - \inf(sec) \leq \frac{n^2(n-2)}{2(n-1)}|H|^2 + \frac{(n+1)(n-2)}{2}c, \tag{1.1}$$

where $|H|$ represents the magnitude of the mean curvature vector, and $\inf(sec)$ represents the infimum of all the scalar curvature taken over all 2-planes at that respective point. Recall that for any orthonormal

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basis $e_1, ..., e_n$ of the tangent space T_pM in a Riemannian manifold M^n, the scalar curvature is defined to be $scal(p) = \sum_{i<j} sec(e_i \wedge e_j)$. The quantity $\delta(2)$ is today called Chen's first curvature invariant. Chen's inequality reminded above is important because it illustrated the kind of relationships it would be interesting to obtain: between intrinsic geometric quantities, by one hand (the terms in the left), and extrinsic geometric quantities by the other (in the right). This is the kind of relations we are interested in finding out.

In this spirit, we are interested to obtain a new relationship involving intrinsic and extrinsic curvature invariants, in the spirit of B.-Y. Chen's fundamental inequalities (for the most general form, see [11], relations (13.28) in Theorem 13.3, et al.).

Let $\sigma : U \subset \mathbb{R}^n \to \mathbb{R}^{n+1}$ be a hypersurface given by the smooth map σ. The interest in the geometry of hypersurfaces rose to a wider attention after a series of works by É. Cartan following [6]. Let p be a point on the hypersurface. Denote $\sigma_k(p) = \frac{\partial \sigma}{\partial x_k}$, for all k from 1 to n. Consider $\{\sigma_1(p), \sigma_2(p), ..., \sigma_n(p), N(p)\}$, the Gauss frame of the hypersurface, where N denotes the normal vector field. We denote by $g_{ij}(p)$ the coefficients of the first fundamental form and by $h_{ij}(p)$ the coefficients of the second fundamental form. Then we have

$$g_{ij}(p) = \langle \sigma_i(p), \sigma_j(p) \rangle, \qquad h_{ij}(p) = \langle N(p), \sigma_{ij}(p) \rangle.$$

The Weingarten map $L_p = -dN_p \circ d\sigma_p^{-1} : T_{\sigma(p)}\sigma \to T_{\sigma(p)}\sigma$ is linear. Denote by $(h^i_j(p))_{1 \leq i,j \leq n}$ the matrix associated to Weingarten's map, that is:

$$L_p(\sigma_i(p) = h_i^k(p)\sigma_k(p),$$

where the repeated index and upper script above indicates Einstein's summation convention. Weingerten's operator is self-adjoint, which implies that the roots of the algebraic equation

$$\det(h^i_j(p) - \lambda(p)\delta^i_j) = 0$$

are real. The eigenvalues of Weingarten's linear map are called principal curvatures of the hypersurface. They are the roots $k_1(p), k_2(p), ..., k_n(p)$ of this algebraic equation. The mean curvature at the point p is

$$H(p) = \frac{1}{n}[k_1(p) + ... + k_n(p)],$$

and the Gauss-Kronecker curvature is

$$K(p) = k_1(p)k_2(p)...k_n(p).$$

The hypersurface is said to be minimal if the geometric quantity $H(p)$ vanishes at every point p.

2. The Particular Context of Three-Dimensional Hypersurfaces

The three dimensional smooth hypersurfaces represent a really particular case. Let $\sigma : U \subset \mathbb{R}^3 \to \mathbb{R}^4$ be a hypersurface given by the smooth map σ. Let p be a point on the hypersurface. Denote $\sigma_k(p) = \frac{\partial \sigma}{\partial x_k}$, for all k from 1 to 3. Consider $\{\sigma_1(p), \sigma_2(p), \sigma_3(p), N(p)\}$, the orthonormal Gauss frame of the hypersurface, where N denotes the normal vector field to the hypersurface at every point.

The quantities similar to κ_1 and κ_2 in the geometry of surfaces are the principal curvatures, denoted $\lambda_1, \lambda_2, \lambda_3$. They are introduced as the eigenvalues of the so-called Weingarten linear map, as we will describe below. Since our discussion is focused on the curvature quantities of three-dimensional smooth hypersurfaces in $\mathbb{R}^4$, we start by introducing these quantities. Similar to the geometry of surfaces, the curvature invariants in higher dimensions can also be described in terms of the principal curvatures. The mean curvature at the point p is

$$H(p) = \frac{1}{3}[\lambda_1(p) + \lambda_2(p) + \lambda_3(p)],$$

and the Gauss-Kronecker curvature is

$$K(p) = \lambda_1(p)\lambda_2(p)\lambda_3(p).$$

In Riemannian geometry, a third important curvature quantity is the scalar curvature ([11], p.19) denoted by $scal(p)$, which intuitively sums up all the sectional curvatures on all the faces of the trihedron formed by the tangent vectors in the Gauss frame:

$$scal(p) = \sec(\sigma_1 \wedge \sigma_2) + \sec(\sigma_2 \wedge \sigma_3) + \sec(\sigma_3 \wedge \sigma_1) = \lambda_1\lambda_2 + \lambda_3\lambda_1 + \lambda_2\lambda_3.$$

The last equality is due to the Gauss equation of the hypersurface $\sigma(U)$ in the ambient space $\mathbb{R}^4$ endowed with the Euclidean metric.

The question of which quantities are intrinsic and which extrinsic is settled e.g. in [17], p.33, in the following way. Denoting by σ_i the elementary symmetric functions of n variables, the curvatures of the hypersurface are given by $K_i = \sigma_i(\lambda_1, \dots, \lambda_n), 1 \le i \le n.$ In [17] it is pointed out that the principal curvatures are determined up to a sign common to all of them, because the Weingarten map is determined up to a sign, according to the choice of the normal. It follows that the curvatures of odd order are determined up the a sign, whereas the ones of even order are determined uniquely. In the case when we have just a 3-dimensional surface in $\mathbb{R}^4$, we have $n = 3$, and K_1 leads to the mean curvature, while K_3 corresponds to the Gauss-Kronecker curvature. It is the scalar curvature that is intrinsic, and we are interested in isolating it aside in one term of an inequality. This research idea pursues the direction outlined in works like e.g. [5, 14, 21, 22, 23, 24].

3. Chen-Type Inequalities Under a Bonnet-Type Curvature Condition

In 1855, Ossian Bonnet [3] investigated the compactness of a surface with Gaussian curvature bounded below by a positive constant. This condition attracted the interest of S. Myers [18], who produced an interesting theorem with topological consequences in Riemannian geometry [16]. The classical Myers condition is about the Ricci curvature being bounded below by a constant away from zero. Note that the classical Myers' Theorem [18] asserts the sufficient conditions to determine the compactness of the Riemannian manifold; for a recent interpretation in terms of Cauchy-Schwarz inequality, see [4]. In our investigation we will return to a Bonnet-type curvature condition on the sectional curvatures, with the goal of obtaining inequalities between intrinsic and extrinsic geometric quantities of a hypersurface.

We prove the following.

Theorem 3.1. *Let (M^3, g) be a three-dimensional smooth hypersurface isometrically embedded into a four-dimensional real space endowed with the canonical metric. Suppose on (M^3, g) the sectional curvature at every point is bounded below by $\varepsilon > 0$. Denote the scalar curvature $scal$, its mean curvature H, and its Gauss-Kronecker curvarture K. Then at every point $p \in M$:*

$$\varepsilon^2 scal \ge 3\varepsilon KH - K^2 + \varepsilon^3. \tag{3.1}$$

Equality holds at a point p if and only if at p the sectional curvatures are all equal to ε.

Proof. Let p be a point of the hypersurface M immersed into $\mathbb{R}^4$ endowed with the canonical metric. Let e_1, e_2, e_3 an orthonormal frame at p, that diagonalizes the Weingarten operator. Suppose the principal curvatures at p are $a = \kappa_1(p), b = \kappa_2(p), c = \kappa_3(p)$. The the sectional curvature on the plane spanned by e_i and e_j is $\kappa_i\kappa_j$, for $i \ne j$, $i, j \in \{1, 2, 3\}$. Then the hypothesis is equivalent to $ab \ge \varepsilon$, $bc \ge \varepsilon$, $ac \ge \varepsilon$.

Henceforth

$$(ab - \varepsilon)(bc - \varepsilon)(ca - \varepsilon) \ge 0,$$

which by a straightforward calculation yields

$$(abc)^2 + \varepsilon^2(ab + bc + ca) - abc\varepsilon(a + b + c) \ge \varepsilon^3.$$

This last relation turns into

$$K^2 + \varepsilon^2 \cdot scal - 3\varepsilon KH \ge \varepsilon^3,$$

which is the relation we claimed. □

It would be interesting to see what happens if we impose a Bonnet-type curvature restriction for the principal curvatures on the three-dimensional hypersurface.

Theorem 3.2. *Let (M^3, g) be a three-dimensional smooth hypersurface isometrically embedded into a four-dimensional real space endowed with the canonical metric. Suppose on (M^3, g) the principal curvatures at every point are bounded below by $\varepsilon > 0$. Denote the scalar curvature $scal$, its mean curvature H, and its Gauss-Kronecker curvarture K. Then at every point $p \in M$:*

$$\varepsilon scal \le K + 3\varepsilon^2 H - \varepsilon^3. \tag{3.2}$$

Equality holds at a point p if and only if at p the sectional curvatures are all equal to ε.

Proof. Let p be a point of the hypersurface M immersed into $\mathbb{R}^4$ endowed with the canonical metric. Let e_1, e_2, e_3 an orthonormal frame at p, that diagonalizes the Weingarten operator. Suppose the principal curvatures at p are $a = \kappa_1(p), b = \kappa_2(p), c = \kappa_3(p)$. The condition we have is that at every point $a \geq \varepsilon > 0$, $b \geq \varepsilon > 0$, $c \geq \varepsilon > 0$.

The condition we have is

$$(a - \varepsilon)(b - \varepsilon)(c - \varepsilon) \geq 0.$$

Then, by a direct calculation

$$abc - \varepsilon(ab + bc + ca) + \varepsilon^2(a + b + c) \geq \varepsilon^3.$$

In terms of curvature invariants, this is

$$K - \varepsilon \cdot scal + \varepsilon^2 \cdot 3H \geq \varepsilon^3.$$

This last relation can we rewritten in the form presented in the statement. □

4. Darij Grinberg's Inequality for Convex Smooth Hypersurfaces

The argument used in this section appears in [1], p.176-177 and belongs to Darij Grinberg. Pursuing the line of thinking inspired by the study of the relationships between intrinsic and extrinsic geometric quantities, we feel this inequality has a particularly interesting geometric meaning, which we intend to investigate. Therefore, we obtain the following.

Theorem 4.1. *Let (M^3, g) be a three-dimensional convex smooth hypersurface isometrically embedded into a four-dimensional real space endowed with the canonical metric. Denote the scalar curvature $scal$, its second fundamental form by h, and its Gauss-Kronecker curvarture K. Then at every point $p \in M$:*

$$scal \leq K + \frac{||h||^2 + 1}{2} \tag{4.1}$$

Equality holds at a point p if and only if at p the principal curvatures are all equal to 1.

Proof. Let p be a point of the hypersurface M immersed into $\mathbb{R}^4$ endowed with the canonical metric. Let e_1, e_2, e_3 an orthonormal frame at p, that diagonalizes the Weingarten operator. Suppose the principal curvatures at p are $a = \kappa_1(p), b = \kappa_2(p), c = \kappa_3(p)$. The convexity condition started in the hypothesis means that at every point $a \geq 0$, $b \geq 0$, $c \geq 0$. Without any loss of generality we can assume that $c \leq a$ and $c \leq b$. As in [1], p. 176, consider the function

$$f(a, b, c) = a^2 + b^2 + c^2 + 2abc + 1 - 2(ab + bc + ca).$$

A direct calculation yields

$$f(a, b, c) - f(\sqrt{ab}, \sqrt{ab}, c) = (\sqrt{a} - \sqrt{b})^2 \left(a + b + 2\sqrt{ab} - 2c\right) \geq 0,$$

where the last step is due to the assumption that c is smaller than a and b. On the other hand, it turns out that $f(t, t, c) \geq 0, \forall t \in \mathbb{R}$, which in particular holds for $t = \sqrt{ab}$. The argument is that

$$f(t, t, c) = c^2 + 2t^2c + 1 - 4tc = (c - 1)^2 + 2c(t - 1)^2 \geq 0,$$

where in the last step it is essential that $c \geq 0$. The equality holds for $a = b = c = 1$. □

The inequality (4.1) can be rephrased as follows.

Corollary 4.1. *Let (M^3, g) be a three-dimensional convex smooth hypersurface isometrically embedded into a four-dimensional real space endowed with the canonical metric. Denote the scalar curvature $scal$, its second fundamental form by h, and its Gauss-Kronecker curvarture K. Then at every point $p \in M$:*

$$4scal \leq 2K + 9H^2 + 1 \tag{4.2}$$

Equality holds at a point p if and only if at p the principal curvatures are all equal to 1.

Proof. To see this now relation, note that $||h||^2 = a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca) = (3H)^2 - 2scal$. The calculation follows immediately. □

5. Three Dimensional Smooth Hypersurfaces in Space Forms with Bonnet-Type Curvature Conditions

Consider Σ a hypersurface of $(\bar{M}, \bar{g})$ and denote by $\iota : \Sigma \to \bar{M}$ an isometric immersion. If $\bar{\nabla}$ and ∇ are the Levi-Civita connections of $\bar{M}$ and Σ, respectively, then the Gauss formula and Weingarten formula are, respectively

$$\bar{\nabla}_X \iota Y = \iota \nabla_X Y + g(AX, Y)\xi,$$
$$\bar{\nabla}_X \xi = -\iota AX,$$

where locally ξ represents a choice of the unit normal to Σ, X, Y are tangential vector fields to Σ, and A is the shape operator.

Suppose the ambient space $(\bar{M}, \bar{g})$ is a space form of constant sectional curvature ψ. Denoting by R the curvature tensor of Σ, the Gauss equation is

$$R(X,Y)Z = \psi[g(Y,Z)X - g(X,Z)Y] + g(AY,Z)AX - g(AX,Z)AY.$$

The principal curvatures κ_i are the eigenvalues of the shape operator. From the Gauss equation follows immediately that for e_1, e_2, e_3 an orthonormal frame at p, that diagonalizes the shape operator we have (see e.g. [15], pp.70-71)

$$R(e_i, e_j)e_j = (\psi + \kappa_i \kappa_j)e_i$$

which yields immediately that

$$sec(e_i \wedge e_j) = \psi + \kappa_i \kappa_j.$$

In this context we have the following.

Theorem 5.1. *Let (M^3, g) be a three-dimensional smooth hypersurface isometrically embedded into a four-dimensional real space form of constant sectional curvature ψ. Denote the scalar curvature $scal$, its mean curvature by H, and its Gauss-Kronecker curvature K. Suppose at every point $p \in M$ and in every scalar direction the sectional curvature is bounded below by ε. Then at every point $p \in M$:*

$$(\psi - \varepsilon)^2(scal - 3\psi) \geq 3K(\psi - \varepsilon)H - K^2 - (\psi - \varepsilon)^3. \tag{5.1}$$

Equality holds at a point p if and only if at p the sectional curvatures are all equal to ε.

Proof. Denote $a = \kappa_1(p), b = \kappa_2(p), c = \kappa_3(p)$. Since $sec(e_i \wedge e_j) \geq \varepsilon$, it follows that at p we have three conditions $\psi + ab \geq \varepsilon$, $\psi + bc \geq \varepsilon$, and $\psi + ca \geq \varepsilon$. Then we have

$$(\psi + ab - \varepsilon)(\psi + bc - \varepsilon)(\psi + ca - \varepsilon) \geq 0.$$

That is

$$(\psi - \varepsilon)^3 + (abc)^2 + (\psi - \varepsilon)^2(ab + bc + ca) - abc(\psi - \varepsilon)(a + b + c) \geq 0.$$

Now we take into account that $scal(p) = 3\psi + ab + bc + ca$, as a direct consequence of Gauss equation, while the mean curvature is $3H = a + b + c$, and the Gauss-Kronecker curvature is $K(p) = abc$. A direct calculation yields the claimed inequality. □

With a stronger curvature condition, we obtain an extrinsic upper bound for the scalar curvature, as it is to be expected for Bang-Yen Chen-type inequalities, as follows.

Theorem 5.2. *Let (M^3, g) be a three-dimensional smooth hypersurface isometrically embedded into a four-dimensional real space form of constant sectional curvature ψ. Denote the scalar curvature $scal$, its mean curvature by H, and its Gauss-Kronecker curvature K. Suppose at every point $p \in M$ the principal curvatures are bounded below by ε. Then at every point $p \in M$:*

$$\varepsilon \cdot scal \leq 3\varepsilon\psi + K + 3H\varepsilon^2 - \varepsilon^3. \tag{5.2}$$

Equality holds at a point p if and only if at p the principal curvatures are all equal to ε.

Proof: The argument follows the same lines as the derivation of (3.2), with the notable difference that in this geometric context of a space form t $scal(p) = 3\psi + ab + bc + ca$, which accounts for an additional term in the right hand side. □

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Author's contributions

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