Osmangazi Journal of Educational Research Volume 11(1), Spring 2024

RESEARCH Open Access

Suggested Citation: Çelikkol, Ö., & Kurtuluş, A. (2024). Investigating students' problem solving achievements, the modeling steps reached and modeler types in the process of mathematical modeling activities. *Osmangazi Journal of Educational Research*, *11*(1), 1-38.

Submitted: 01/10/2023 **Revised:** 13/02/2024 **Accepted:** 08/06/2024 **DOI:** 10.59409/ojer.1369551

Investigating Students' Problem Solving Achievements, the Modeling Steps reached and Modeler Types in the Process of Mathematical Modeling Activities

***Özlem Çelikkol , **Aytaç Kurtuluş**

Abstract. The aim of this study is to identify the effect of mathematical modeling activities applied as action plans on the development of students' achievement in solving algebraic verbal problems, as well as to investigate the mathematical modeling competencies they use during these mathematical modeling activities and the modeling steps they reach. Moreover, the types of mathematical modelers that the students demonstrated during this application process were examined in the study. The study was conducted as an action research with a total of 15 7th grade students in rural areas. It was aimed to reveal the mathematical modeling competencies that students achieved during mathematical modeling activities through observations, researcher and student diaries, as well as interviews conducted with students. Content analysis was implemented to analyze the data. During the study, the mathematical modeling competencies demonstrated by the students in each mathematical modeling activity and the mathematical modeling steps they reached were identified. As a result of the study, it was confirmed that students' mathematical modeling competencies were related to competencies such as general mathematical knowledge or verbal comprehension, and it was also revealed that students' mathematical knowledge competence could be examined in more detail. It was also observed that students could achieve competencies through group work and it was found that students' success in solving the verbal algebraic problems generally improved based on their mathematical modeling competencies.

Keywords. Mathematical modelling, modelling competencies, modeller types, problem solving.

* Teacher Ministry of Education, Erzincan, Türkiye

e-mail: zlemclkkl2423@gmail.com

****** Prof. Dr. Eskisehir Osmangazi University, Faculty of Educatıon, Eskisehir, Türkiye

e-mail: agunaydi@ogu.edu.tr

Note: This article is presented as part of the master thesis.

Problem solving skill is a skill that we encounter not only in mathematics but in every field especially in this day age. In general sense, problems are the questions whose solutions are not known in advance and do not have a clear solution (MoNE, 2018). What makes the problem different is that it includes special circumstances that have not been encountered previously. Situations such as financial crises, earthquakes, negativities generated by population growth, and unplanned urbanization can indeed be considered as problems. Particularly in the present century, more complex problems are encountered that require knowledge and information from more than one discipline to solve. While knowledge of engineering field is required to solve a problem in the field of medicine, applying mathematical knowledge in the software world can be represented as an example of such situations. Zawojewski and Lesh (2003) stated that the experts in application fields where mathematics, science and technology were used intensively, such as engineering, medicine and business management, indicated that problem solving constantly changed significantly over the past 20 years.

Muller and Burkhardt (2007) stated that it was of great importance in mathematics curricula to teach students the basic skills that they would encounter in their lives or work during their school years. In this sense, mathematics education should be provided with the methods best-suited for this purpose. In this context, the interaction of future generations with problems related to real life, appropriate to their own level, rather than disconnected from the problems of daily life, turn out to be all the more important. It is a common target of mathematics educators to enable students to interpret real-world problems with figures and equations (Nemirovsky, 1996). Furthermore, as emphasized in the mathematics curriculum, "problem solving skills" should be considered within the scope of non-routine problems and just routine problems should not be satisfied with (MoNE, 2013). Non-routine problem situations can often be regarded as real-world problems. In order to address these problems at school, mathematical modeling based on real-world problems rather than classical problem solving can be debated. What differentiates the mathematical modeling problems from the routine verbal problems is that they are the problems that involve mathematization skills related to daily life, rather than being a problem that depends on the results found as a result of a few operations.

Mathematical Modeling and Problem Solving

Broadly speaking, even though models are simply representations of how an event works, they are conceptual systems that allow the definition of structures by explaining the elements, relationships between these elements, and operations with some representations (Lesh, Carmona & Post, 2002). Therefore, based on this definition, it is possible to say that models are relational and connectional structures created using signs and symbols based on situations or events. While the normal problem-solving procedure proceeds in a linear way between those given and those goals as illustrated in Figure 1, the modeling approach proceeds cyclically by dividing the problem into stages.

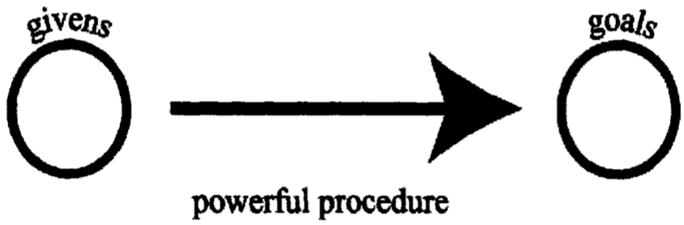


Figure 1. Information- Processing Perspective in Problem Solving (Zawojevski & Lesh, 2003).

Mathematical models are formed by establishing mathematical structures, connections and relationships over a real situation and transferring this real situation or problem to the mathematical world (Berry & Houston, 1995; Lesh & Doerr, 2003). Mathematical models are the reflection of a real situation or problem in the world of mathematics. With the connectional, relational and structural model obtained through this reflection, the problem at hand is resolved, verified and explained. This cycle generates the mathematical modelling.

According to Blume (1989, p. 93) mathematical modeling is "transforming a problem situation into its reflection in the mathematical world, where it can be worked on more appropriately than the real world and continuing this cycle." Furthermore, according to the Ministry of National Education (2013), mathematical modeling is a method that enables one to easily see the relationships between problems in life and to express, classify and generalize these relationships using mathematical language.

Blum and Ferri (2009) stated that the mathematical modeling cycle was important for students,

- Since it was important for students to help them understand the world better,
- Since it supported mathematical learning (Motivation, concept formation, comprehension, support),

- Since it contributed to the development of various mathematical competencies and appropriate attitudes,
- Since it contributed to the convenient definition of mathematics.

Blum and Ferri (2009) explicated the modeling cycle as illustrated in Figure 2. As far as this figure is concerned, they referred to a cycle that started from a problem situation in the real world and ended with transferring this situation to the mathematical world by applying the mathematical knowledge and skills; in other words, a cycle consisting of modeling, interpreting, evaluating it and explaining the results.

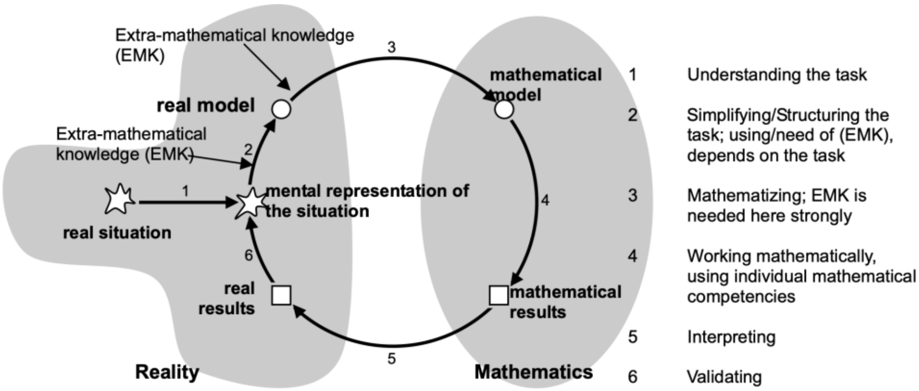


Figure 2. Mathematical Modeling Cycle in cognitive perspective (Blum & Ferri, 2009; Ferri, 2006).

At the outset, the problem situation must be comprehended by the problem solver, this is the construction of the situation model. After the problem is simplified and structured, a model that is more appropriate to the real situation of the problem is created. Particularly, the problem solver should be able to define what is worth spending time on (Blum & Ferri, 2009). After a model of the problem situation is created, it is essential to use the mathematical knowledge to ensure the transition of this real-world problem to the mathematical world. In a problem situation that has been made mathematical, mathematical results are obtained by using mathematical relations and information. Subsequently, after the obtained results are interpreted and verified, it is expected to check the real-world outputs of these mathematical results. At the very end, students are supposed to present all the results they have obtained, again using the same situation model they have started with. According to Ferri (2006), it is very important that the presentation phase takes place in revealing the details of the real-life problem that remains in mind in the next stage after the interpretation of the results.

Similarly, Silver et al. (1993) stated that there were four main stages of solving word problems. In these four stages, the first stage was understanding the structure of the mathematical problem, understanding the information given and completing the missing information; in the second stage, the appropriate process for solving the given verbal problem, planning the process, creating a mathematical plan; implementation of the solution plan created in the third stage; the fourth stage was to explain the accuracy of the answer and what it meant with both content and real situations. When all these mathematical modeling steps are considered, it is possible to say that these steps overlap with the problem solving process. There are similarities between the solutions of verbal problems and mathematical modeling activities and competencies needed when solving problems with the mathematical modeling approach, these competencies vary depending on each stage in the mathematical modelling cycle.

Mathematical Modeling Competence

Mischo and Maaß (2012) revealed that the mathematical modeling competencies were the skills that students should use when solving problems with mathematical modeling. They revealed the impact of mathematical modeling skill's field or domain competencies and ideas and specified the necessary skills for each modeling step. As far as the skills listed in Table 1 are concerned, there are skills that students should exhibit in each of the modeling steps. These personal factors can be overcome with skills corresponding to the Mathematical Modeling steps. They stated that these personal factors were related to the modeling steps. Given Table 1, it is indicated that in order to understand the model, one should initially have verbal comprehension skills and that this personal feature is effective in getting through this step.

Table 1.

Modeling Process and Personal Factors Acknowledged to be Relevant

All steps	**Personal factors considered to be related**
Step 1: Creating the model situation	Verbal comprehension outline, understanding and explaining the text
Step 2: Building a real model	Establishing logical general knowledge and representations, modeling outline and linking the general knowledge
Step 3: Creating a mathematical model	Mathematical competence
Step 4: Finding a solution with a mathematical model	Mathematical competence
Step 5: Interpreting the mathematical solution	Mathematical competence and verbal comprehension
Step 6: Verification	General information

Maaß (2006) quoted the mathematical modeling competencies from Kaiser and Blum (1997, p. 9) and defined a list of competencies as follows. As seen in Table 1, the competences concerning each of the steps in the mathematical modeling process have been revealed, and in Table 2, the modeling competencies that must be laid out in the mathematical modeling process are indicated. This list is presented in Table 2.

Table 2.

Mathematical Modeling Competencies

1. Competence to understand the real situation and build a model based on reality	• Competence to make assumptions for the problem and simplify the situation. • Competence to identify quantities that affect the situation, name them and identify the relevant variables. • Competence to establishing the connection between the variables • Competence to search for accessible information, distinguishing between the relevant and irrelevant
2. Competence to create the mathematical models from real models	• Competence to mathematize the relevant quantities and the relationships between them • If necessary, competence to simplify the relationships and connections between quantities and reduce their complexity with numbers. • Competence to choose the appropriate mathematical representations and present the situations as Figures
3. Competence to solve the mathematical problems with this mathematical model	• Competence to divide the problem into smaller parts, establish relationships with similar problems • Competence to use heuristic strategies such as rephrasing and examining the problem, vary the quantities or appropriate data. • Competence to use mathematical knowledge to solve the problem.
4. Competence to interpret the mathematical results in real situations	• Competence to interpret the mathematical results in non-mathematical contexts, • Competence to generalize the solutions developed for a specific situation • Competence to examine or talk about the solution using appropriate mathematical language.
5. Competence to validate the solution	• Competence to critically check the solution found and make reflections • Competence to revisit some parts of the model or repeat the entire modeling process, if the solution does not fit the situation, • Competence to reflect on other ways to solve the problem if solutions can be developed differently • Competence to question the model in general.

All of these competencies comprise students' behaviors and problem-solving approaches in the mathematical modeling activities. These competencies, which are structured similarly to the characteristics of being a good problem solver, are also considered in mathematical modeling activities. Eventually, the competence that students should have is the competence to see the relationship of mathematics with daily life, which can be considered as the point that mathematical modeling activities aim to reach in the end.

The mathematical modeling process dealt with in the study was first compiled from the studies of Berry and Houston (1995) and Blum and Ferri (2009). The mapping of the essential competencies in the mathematical modeling steps was made based on the studies of Mischo and Maaß (2012) and Maaß (2006). While the mathematical competencies are examined in three sections; Algebraic expression (establishing an equation), Operational (solving the equation) and interpreting the solution; the general knowledge is examined in two sections: Daily life knowledge and General mathematics knowledge. Similarly, while verbal comprehension competence represents understanding the problem, the competence to verify and explain the solution to the problem was also investigated within the scope of verbal expression competence.

Modeler Types

Students' behavioral patterns in mathematical modeling activities were also considered as the modeler types. Kaiser and Maaß (2007) referred to four different modeler types in their study, and the scheme identified by Maaß (2006) regarding these modeler types is illustrated in Figure 3. This figure presents us the clues about the behavior of students in mathematical modeling activities. The modeler types based on the attitudes towards modeling activities and attitudes towards mathematics are as follows;

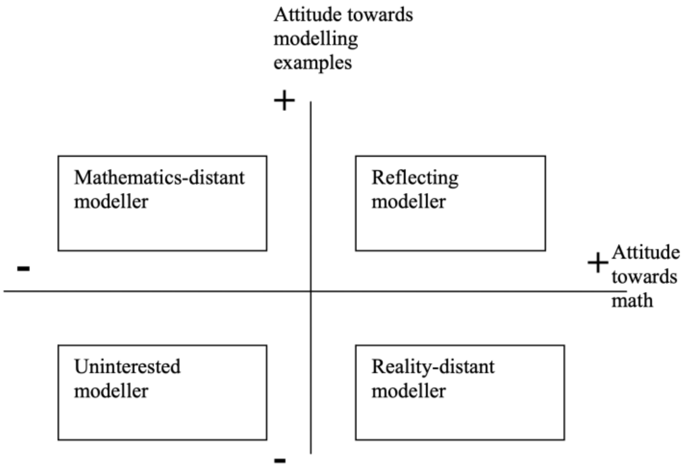


Figure 3. Modeler types based on the modeling competencies (Maaß, 2006).

Reality-distant modeler. Although they have positive attitudes towards mathematics, they are distanced in connection with reality. As an effective consequence, they build a barrier, mainly due to their lack of competence in solving problems related to the mathematical content, where they have problems with the structure of real models, their validation and, in part, the interpretation of the results.

Mathematics-distant modeler. These modeler students clearly grant the real-world problems a chance and exhibit low success in mathematics classes. These students are very enthusiastic about modeling examples. They can build real models and validate the solutions very well, too. There are flows in establishing mathematical models, finding mathematical solutions to them, and interpreting the complex solutions.

Reflecting modeler. They have positive attitudes towards the modeling examples as well as mathematics itself. They demonstrate appropriate achievement in mathematics. It is very difficult to find flaws in the modeling process.

Uninterested modeler. They are neither concerned with the real world nor with mathematics itself. There are errors in their mathematical proficiencies. When solving the modeling problems, problems arise in every modeling process.

The competencies that modelers demonstrate in mathematical modeling activities are also significant in the formation of mathematical modeler types. Furthermore, the competencies demonstrated by students in mathematical modeling steps differ in each mathematical modeling step. All these need to be addressed as a whole.

Purpose and Significance of the Study

The purpose of the study is to identify the effect of mathematical modeling activities implemented to improve students' achievement in solving verbal problems and thus to augment their success in solving the verbal problems. In the study, it is very important to investigate the progress of the 7th grade student group in solving the algebraic verbal problems that they previously had difficulties with and to identify to what extent and how these difficulties were overcome. When the studies conducted with mathematical modeling activities were reviewed, it was clear seen that the study groups of those studies, especially nationally conducted ones were generally composed of teachers and teacher candidates (Aydın Güç, 2015; Bal & Doğanay, 2014; Tuna et al., 2013; Güder, 2013; Karalı, 2013; Şen). Zeytun, 2013; Ural & Ülper, 2013; Tekin Dede &Yılmaz, 2013; Hıdıroğlu, 2012; Çiltaş, 2011; Ünveren, 2010; Korkmaz, 2010). In addition to these studies mathematical modeling studies have increased in recent years, especially with secondary school students (İnan Tutkun & Didiş Kabar, 2018; Yıldırım & Işık, 2013; Muşlu& Çiltaş, 2016; Özgen & Şeker, 2021; Çavuş Erdem & Gürbüz, 2018; Kalaycı, 2017).

The international studies, on the other hand, includes the mathematical modeling studies including primary school level (English, 2002; English & Waters, 2005). It was clearly observed that those studies generally attempted to identify the effects of mathematical modeling activities on the students' mathematics achievement and attitudes towards mathematics (Olkun et al., 2009). Furthermore, some other studies, on the other hand, focused on the solution of these mathematical modeling activities of the students. In these studies, students' behaviors in one or more mathematical modeling activities were comprehensively investigated. Nevertheless, previous studies either only investigated its effects on the quantities such as achievement and attitude quantitatively and experimentally or focused on the solution of mathematical modeling activities. In this sense, the present study makes the research distinctive by identifying the mathematical modeling competencies that students demonstrate during the mathematical modeling activities. When the difficulties in algebraic verbal problems obtained from the researcher's observations and the literature (Akgün, 2009; Aydın & Özmen, 2012; Dede, 2004; Llinares & Roig, 2008; Sezgin Memnun, 2014; Soylu, 2008;) were compared with the class, it was found that the points to be solved were; 1) Students tried to find a solution without understanding the problem, 2) Students had difficulty in identifying the unknown and establishing equations; the students failed to mathematize a problem, 3) Students operated with unknowns and knowns while solving the equation they created, 4) Students did not perceive the solution they found as a realistic solution and did not interpret or verify their solutions, 5) Students failed to explain the solution they found. The reason why mathematical modeling was chosen to solve these identified problems was that many studies confirmed that mathematical modeling activities had a positive effect on teaching problem solving (Olkun et al., 2009; Kal, 2013; English, 2002; English & Watters, 2005; English & Sriraman, 2010). It was indicated by many researchers that mathematical modeling activities were effective in teaching mathematics and especially in problem solving (Mousoulides et al., 2008; English & Doerr, 2003; Reusser & Stebler, 1997; Lester & Kehle, 2003; Işık & Yıldırım, 2014; Kal, 2013; Sandalcı, 2013). This study is significant in terms of not only improving a situation that requires intervention in teaching, but also identifying the effects of mathematical modeling activities on students in solving the algebraic verbal problems. In this sense, answers to the following questions were sought in the study;

1) How do mathematical modeling activities impact the verbal problem solving success of the 7th grade students?

2) What are the mathematical modeling steps that 7th grade students reach during the mathematical modeling activities?

3) How do the mathematical modeling competencies of 7th grade students develop during the mathematical modeling activities?

4) What are the modeler types demonstrated by the 7th grade students during mathematical modeling activities?

Method

Research Model

Since the purpose of this study was to improve students' verbal problem solving success and identify the effect of mathematical modeling steps on this process, it was implemented through the method of action research. The general purpose of action research is to ensure a development and change action in which active elements are brought together in the process, based on the existing situation where a solution or change is required, and evaluate the functionality of this action (İnan, 2011).

Prior to the study, the action process in Table 3 was conducted to establish the current situation of the students and aim for a change to improve the success of the students in solving the verbal problems due to the low achievement of the students in solving the verbal problems.

In this sense, in order to overcome the difficulties that the students experienced, especially the points where they had particular difficulty were identified through observation and this was repeated in a cycle after each action plan. In other words, the content of the mathematical modeling activity, which was the other action plan, was established accordingly, based on the observations made after each action plan was implemented and the data obtained from the student diaries. While implementing this action plan, the action committee met at each stage and the difficulties that could be encountered or the results to be achieved for each action plan were discussed and the actions were taken after these stages.

Table 3.

Procedure of the Study

Problem Statement (Preliminary diagnosis)	Data Collection	Data Analysis	Action Plan-1	Action Plan-2	Action Plan-3	Action Plan-4	Evaluation
Identifying the current situation of students Literature review Pre-test	Monitoring the students and examining their notebooks Literature review	Action planning with the data obtained from the students Deciding on and structuring other activities through student observation in each activity	Planning and implementation of the “Apple Tree” Identifying students' missing competencies as a result of the interviews about the problem	Planning and implementation of the “Speed Problem” Identifying students' missing competencies as a result of the interviews about the problem	Planning and implementation of the “Giant's Foot” Identifying students' missing competencies as a result of the interviews about the problem	Planning and implementation of the “Onion Seed” Identifying students' missing competencies as a result of the interviews about the problem	Examining student development Post-test Conducting semi-structured interviews with the students as a result of the application

Study Group

The study group consisted of a total of 15 7th grade students studying in a rural area in Ankara province. The problem situation in this group was observed by one of the researchers who was also a mathematics teacher, leading to the selection of this study group. All these students were in the same class.

Therefore, the students' levels were identified as a result of the achievement test and they were allowed to take part in mathematical modeling activities in groups of 3 students. The levels students were in were High (H), Medium (M) and Low (L). In creating the levels pre-test were considered. According to the pre-testing conducted in three levels, 6 students were classified as high, 7 students were classified as middle, and 2 students were classified as low. The students were then divided heterogeneously and homogeneously into groups of three, depending on their friendship. The students in five groups were placed as H1H2H3, H5H6M3, H4M1L2, M2M5M6 and M4M7L1.

Data Collection Tools

There was an achievement test administered to the students in an attempt obtain quantitative data. This achievement test was administered to the students before and after the research process. The content of the achievement test consisted of 10 open-ended questions based on solving the algebraic verbal problems. The test was prepared in order to identify the effect of mathematical modeling activities on problem solving success, considering the 7th grade problem solving achievements. The solution time of the achievement test was 100 minutes, allowing 10 minutes for each question.

Basically, the reason for this was to enable the students to understand and solve the problems without time limits. Opinions of two experts were taken for content validity, and the achievement test was finalized in line with the recommendations. Such a tool had to be implemented in order to quantitatively identify the effect of the application on the students' problem-solving success. The reliability of this open-ended test was calculated with the formula Consensus / (Consensus + Dissensus) *100, which calculates reliability based on the items of disagreement and agreement stated by Miles and Huberman (1994), and it was found that the reliability was 0.84. The scores students achieved from this test were classified as low, medium and high, and the students were coded and divided into groups accordingly.

Four mathematical modeling activities, two of which are researchers and two of which are adapted from different researchers, were carried out with the students for 6 weeks. Each activity was practiced for about 3 lesson hours.

In addition to the researcher's diary, student diaries and worksheets, the semi-structured interviews with students during and after mathematical modeling activities constituted the data collection tools of the study. Structured observation was conducted in the study. Before the structured observation, the solution process of the students in the mathematical modeling activities was observed in the context of the mathematical modeling competencies previously established by Mischo and Maaß (2012), which the students had to meet, as well as the mathematical modeling steps defined by Blum and Ferri (2006). The reason for choosing this observation method was that it provided reliability and validity in collecting the data (Büyüköztürk et al., 2010).

The researcher's diary consisted of recording students' in-group and individual behaviors and solutions during each activity, both in audio and text format. Furthermore, the student diaries were a resource that contained individual students' solutions and thoughts about activities, both for themselves and as a group. Similarly, through the semi-structured interviews, it was aimed to identify which paths the students followed in the activities, which mathematical connections, relationships and symbols they used and why they used them. By conducting semi-structured interviews, rich data was provided for the study and students' attention was drawn on the subject. In the semi-structured interviews, questions were asked to identify the solution keys that students followed in mathematical modeling activities. Asking these questions also contributed to reliability in terms of verifying the activity solutions in the student diaries. Furthermore, as a requirement of action research, semi-structured interviews were conducted with the students in an attempt to establish the activities to be implemented to control the progress of the students, and the modeling activities were reviewed accordingly. Through continuous interviews, the students' way of handling the problems was analyzed and the problems were arranged accordingly.

Process

Four mathematical modeling activities, lasting a total of 6 weeks, were held with the students. Students worked with their groups for the activities and made presentations to other groups at the end of each activity. Two of the four mathematical modeling activities were developed by the researchers and the other two were adapted from the previous mathematical modeling studies. Approximately 120 minutes were spent on each activity. In classrooms where mathematical modeling activities were applied, students were divided into groups of at least three people in order to understand and interpret real situations, develop a model and evaluate with their classmates, and the teacher guided them in the process of generating their own knowledge (Türker Biber & Yetkin Özdemir, 2015; English & Sriraman, 2010; Lesh & Lehrer, 2003). Hence, mathematical modeling activities were carried out by working groups of 3 students.

All the mathematical modeling activities were planned during the application process and planned in line with the mathematical knowledge and interests of the students. Four activities were implemented. During and at the end of the activity, students' progress in the mathematical modeling steps was recorded as audios. Furthermore, in each activity, the researcher's diary, observation and student diaries revealed the steps that the students had difficulty with, and the plan and content of the next activity were planned accordingly, by observing and interviewing the students. Throughout their mathematical modeling activities, students kept diaries, and after each activity, student diaries were gathered, scrutinized, and sent back to them. The four mathematical modeling activities illustrated in Table 6 were prepared or adapted based on the mathematical modeling approach.

Table 6.

Mathematical Modeling Activities İmplemented with the Students

Implementation	Purpose	Duration
Apple Tree Problem	The purpose of this activity was to make students feel that a problem situation did not always contain numerical data and develop the skills of developing a hypothesis and testing it.	3 lesson hours
Speed Problem	The purpose of this activity was to enable the students to establish the speed, distance and time variables in the problem situations.	3 lesson hours
Giant's Foot	It is an activity aimed at improving students' skills in identifying the unknowns, especially in problems involving the concept of ratio, and finding these variables by establishing the ratios.	3 lesson hours
Onion Seed	It was observed that students had difficulty especially in profit and loss problems. Moreover, onion farming was carried out in the region where the students lived and their families were also interested in agriculture. It was intended to be an activity related to daily life that included both profit and loss and drew the attention of students.	3 lesson hours

In mathematical modeling activities, both individual and group interviews were conducted with the students, how the activities would be done, the students' interest in the subject of the activity, and the weaknesses encountered in other activities were recorded. In this sense, the results of each event were shared with the action committee and it was decided how the following event should be. The action committee consisted of researchers and two mathematics teachers who have master of mathematics education science. During the activities, the students' ability to reach the mathematical modeling steps and the mathematical modeling competencies they demonstrated in the activities were also examined and the findings were recorded and interpreted.

Data Analysis

The analysis of observation data in the study was conducted using content analysis. The content analysis framework for observation data was deliberated within the framework of mathematical modeling competencies. There are basically two processes in content analysis, which are used especially in the analysis of data obtained from observations and interviews. The first of these is to identify the categories before starting the analysis and shape these categories based on the previously obtained information, theories and experiences. The second is being more familiar with the subject thanks to the data collected and new categories emerge as the analysis continues (Büyüköztürk et al., 2010).

In the literature review, the steps identified by Blum and Ferri (2009) were established as understanding the problem, making assumptions, mathematizing, solving the problem, interpreting the solution, verifying the solution and explaining-predicting-presenting. It was decided to examine the students according to these steps in mathematical modeling. Secondly, the data arranged by transcribing the audio recordings were analyzed in line with the framework established above. This analysis was implemented in order to identify at what stage of modeling the students were in the activities and what competencies they could use. In the second step, the diversification of mathematical knowledge, which was one of the mathematical modeling competencies used by students in mathematical modeling activities, was established through the observation-interviews and the student diaries examined after each activity, and the mathematical modeling step and the mathematical modeling competencies essential for this were matched again, as is the case in Table 4 below, and this new theme was also taken into consideration in the analysis of the data.

Table 4.

Mathematical Modeling Steps in the Problem Solving Process and the Required Competencies

Mathematical Modeling Steps	Competencies and Personal Factors to be Used in the Step
1. Understanding the Problem: The real-life problem or situation can be understood by the solver, and the important data is established through selection.	**Verbal comprehension:** Understanding and making sense of the problem when it is read. Competence to understand the real situation.
2.Making Assumptions: Searching the data in the problem situation; mathematical relationships, establishing connections and creating a simple verbal model for solving the problem.	**General Knowledge (General mathematics knowledge):** Identifying the variables in the problem statement using mathematical knowledge, expressing the purpose verbally in the form of a simple sentence.
3.Mathematizing: Turning the established verbal model into a mathematical model by representing the variables assigned in the problem situation such as symbols and shapes.	**Mathematical (Expressing Algebraically - Establishing an Equation):** Building a model using variables. Competence in building models based on reality and competence in creating mathematical models from real models.
4. Solving the Problem: Solving the established mathematical model with the data in the situation or problem. Going back and re-examining the variables so that assumptions can be made again, and making improvements.	**Mathematical Competence (Operational - Solving the Equation):** Substituting the numerical values using the model constructed by establishing the variables, solving the equation constructed with the general mathematical knowledge. Competence to solve the mathematical problems with this mathematical model.
5. Interpreting the Solution: Putting the solution into words to see to what extent the achieved result matches the original problem situation.	**Verbal comprehension and ability to express, Mathematical competence (Interpreting the Solution):** Explaining the result verbally by associating it with mathematics. Competence to interpret the mathematical results in real situations.

6. Verifying the Solution: Using the obtained mathematical model on other suitable data. Again, at this step, one can return to the assumption step to make retrospective corrections.	**General Knowledge (General mathematical knowledge):** Reaching the common results by substituting different values into the created model. Do not use mathematical knowledge while doing this. Solution Validation Competence
7. Explanation, Reporting: The solver makes a general presentation about the solution. They can prepare posters and presentations.	**Ability to oneself express verbally, General knowledge (Daily life knowledge):** Ability to express oneself using general mathematical knowledge to explain the operations performed.

Furthermore, the indicators in Table 5 suggested by Maaß (2006) were utilized to identify the students' mathematical modeler types.

Table 5.

Indicators of mathematical modeler types

Modeler types	**Indicators**
Distanced from reality	They exhibit a positive attitude towards mathematics and a negative attitude towards modeling activities.
Modeler with Distanced from Mathematics	They are enthusiastic in the modeling activities, but their attitude towards mathematics is negative.
Reflective Modeler	They exhibit positive attitudes towards both mathematics and mathematical modeling activities.
Uninterested Modeler	They have a positive attitude towards neither mathematics nor mathematical modeling activities.

Results

Findings and interpretations were obtained at the activity level and on a group basis.

The Effect of Mathematical Modeling on Algebraic Verbal Problem Solving Achievement

A pre-test and post-test were administered to the students in the study group. After the implementation, a post-test was conducted to identify the students' competence in order to solve the problems after performing mathematical modeling activities. By conducting dependent groups T-test, it was aimed to identify the effect of mathematical modeling activities on the students' achievement in solving the algebraic verbal problems.

Comparison of the averages of the tests performed before and after the mathematical modeling activities is as follows:

Table 7.

Pre-test and Post-test T-test and Dependent Sample Statistics

	Average	N	Standard deviation	t	p
Pre-test	24.33	15	20.607	-5.02	0.0001
Post-test	50.93	15	32.103		

As is clear in Table 7, it is possible to say that there was a significant difference between the pre-test and post-test score averages because the p/2 value obtained as a result of the dependent sample T-test was smaller than the 0.05 significance value ($p = .0001$). As far as the results obtained here are concerned, it was found that students' achievement in solving the algebraic verbal problems differed positively with the mathematical modeling activities.

Mathematical Modeling Competencies Exhibited by the Students in the Mathematical Modeling Activities

Modeling steps were examined when mathematics students participated in mathematical modeling activities. These four mathematical modeling activities are prepared according to the mathematical knowledge and connections that students have had difficulties in the previous activity.

Mathematical modeling steps reached by the students in the apple tree activity. The apple tree activity, unlike the verbal problems students encountered previously, was an activity that did not include specific quantity values and encouraged the students to use variables. In this problem, students were expected to make a mathematical solution by taking into account the values such as root, trunk and crown width. Figure 4 illustrates the mathematical modeling steps that students reached in solving this activity.

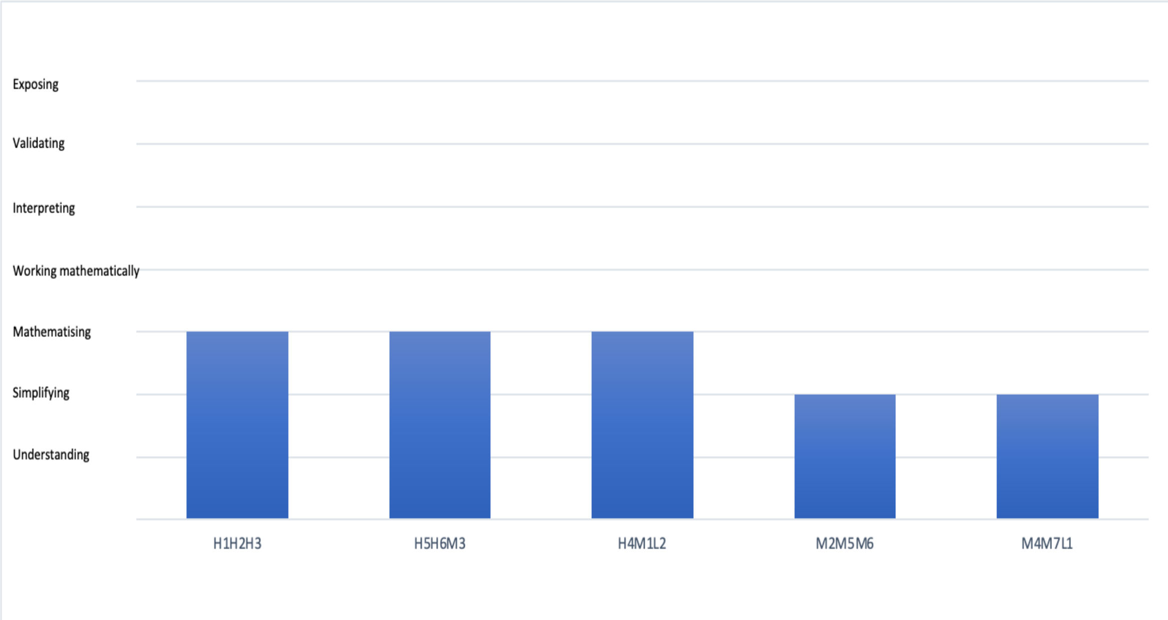


Figure 4. Mathematical Modeling Steps Reached by the Student Groups in the Apple Tree Problem.

As the Figure 4 illustrates, none of the student groups managed to reach the final stage of the problem. Below are the students' solutions and explanations:

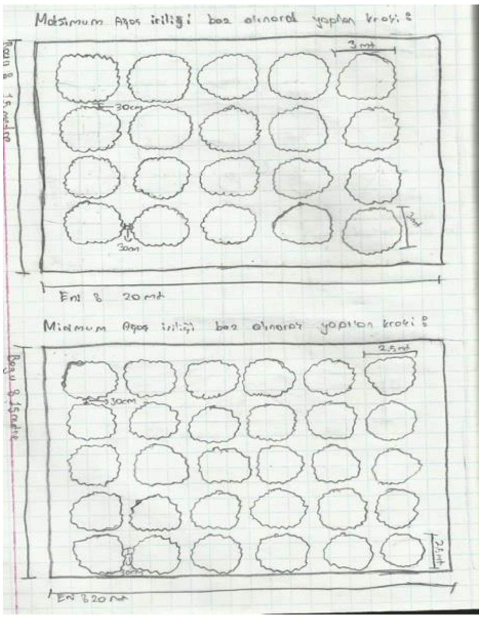


Figure 5. Drawing of the H1H2H3 Group regarding the Apple Tree Problem.

In Figure 5, which was a joint drawing of the students, the students tried to create a model to test their assumptions by drawing two different gardens on the assumption that trees could be large or small in daily life, as presented in the problem. The following conversation also demonstrates how students used the variables in this mathematization process:

"H3: The length between them also varies, but it depends on its length.

H1: I found the area, sir. It was 300 square meters. I called it n. Then I found the area of the tree, I said it was 9 square meters. I called it x. Something came out, but there are 20 at most here.

H3: The distance would be n/x, as I just said

A: What do you mean by n?

H1: n the area.

H3: Yes.

H3: Maybe we can actually figure out how many trees will fit.

H1: No, no, we should develop a formula that includes the distance, not how many trees will fit."

As far as Figure 5 above and the students' explanations are concerned, it is clear that the students tried to find out how many trees could fit in a certain area and realized that they needed to create a model using the variables in order to do this. It was stated that the students considered the area of the garden with the variable "n" and chose to represent the distance between them with the variable "x". It was pointed out that the students reached the mathematization stage, but when working with these variables, they thought that they obtained the number of trees as a result of estimating the area to the distance. With H1's warning, even though the students wanted to develop a model for finding distance, they were not successful. Therefore, the H1H2H3 group progressed to the "mathematising" stage.

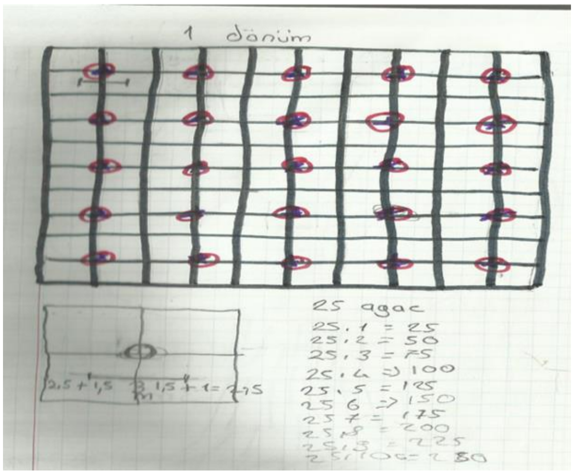


Figure 6. A Model of the Apple Tree Problem from the Worksheet of the M4M7L1 Student Group.

Considering Figure 6 above, it is clear that the M4M7L1 group tried to understand and simplify the problem from the drawing they made to place the trees in a certain area for the problem. It is possible to say that the students at this stage tried to calculate hypothetically how many trees could fit on 1 acre of land. However, it seems that they failed to mathematize the solution to the problem.

Mathematical modeling steps reached by the students in the speed activity. A Mathematical Modeling actvity was given to the students in the second week. Students' problem solving processes were examined on a group basis. The Mathematical Modeling steps that the student groups reached in solving the Speed problem were identified as illustrated in Figure 7.

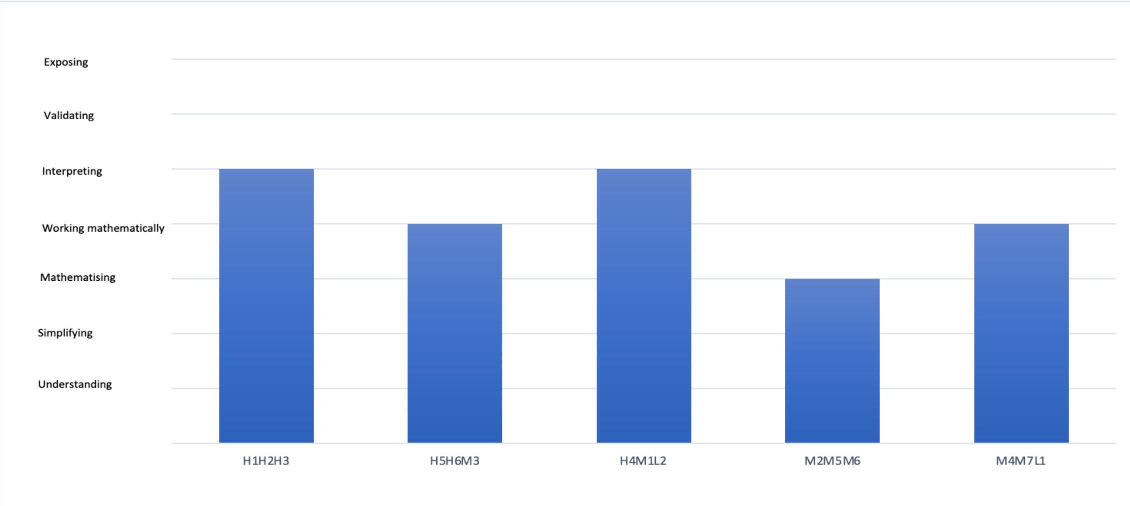
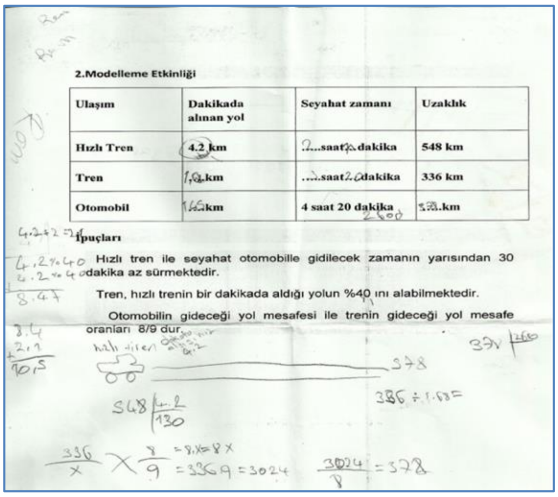


Figure 7. Mathematical Modeling Steps Reached by the Student Groups in the Speed Activity.

As Figure 7 illustrates, it is possible to say that the student groups reached higher levels of mathematical modeling in the Speed Activity. The details regarding the solutions of the student groups are presented below:



2.Modelleme Etkinliği

Ulaşım	Dakikada alınan yol	Seyahat zamanı	Uzaklık
Hızlı Tren	4.2 km	....saat....dakika	548 km
Tren	....km	....saat....dakika	336 km
Otomobil	....km	4 saat 20 dakika	....km

İpuçları

Hızlı tren ile seyahat otomobille gidilecek zamanın yarısından 30 dakika az sürmektedir.

Tren, hızlı trenin bir dakikada aldığı yolun %40 ını alabilmektedir.

Otomobilin gideceği yol mesafesi ile trenin gideceği yol mesafe oranları 8/9 dur.

Figure 8. Solution for the Activity from M3's Worksheet.

The solution made by the H5H6M3 group for the Speed Activity is presented in Figure 8. It is clear that the group tried to fill in the missing parts by using ratios, while aiming to create a model by establishing an equation. Nevertheless, it was found that the students did not use the same model to verify the model. Therefore, it is possible to say that the students only reached as far as the problem-solving/mathematical working stage.

Mathematical modeling steps reached by the students in the giant's foot activity. The giant's foot activity was given to students. The main mathematical concept to be used in solving the problem is ratio–proportion. The Mathematical Modeling Steps reached by the Student Groups in solving the Giant's Foot problem are illustrated in Figure 9 on a group basis:

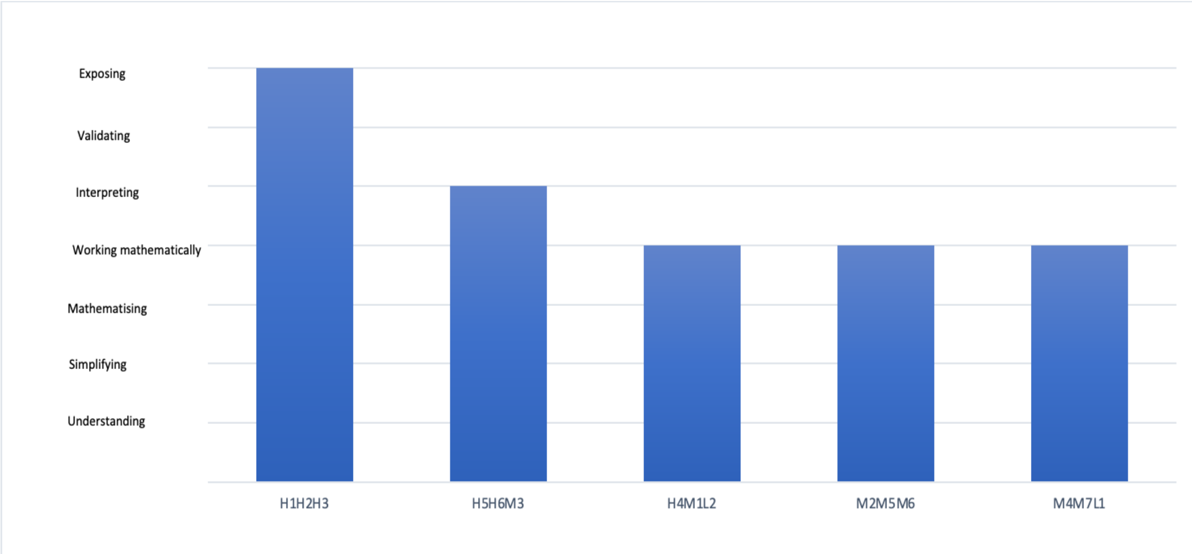


Figure 9. Mathematical Modeling Steps Reached by the Student Groups in the Giant's Foot Problem.

As far as Figure 9 is concerned, it is clear that student groups were able to reach higher levels of mathematical modeling compared to the previous activities. It is evident that especially the H1H2H3 student group was able to reach the highest level of mathematical modeling step. Considering that they could only progress as far as the mathematization step in the Apple Tree Activity, it is possible to say that the group exhibited improvement in terms of progress in mathematical modeling steps. It is possible to generalize this situation for other groups as well; in this problem, the groups reached at least the Problem Solving stage. Some of the solutions offered by the groups are listed below:

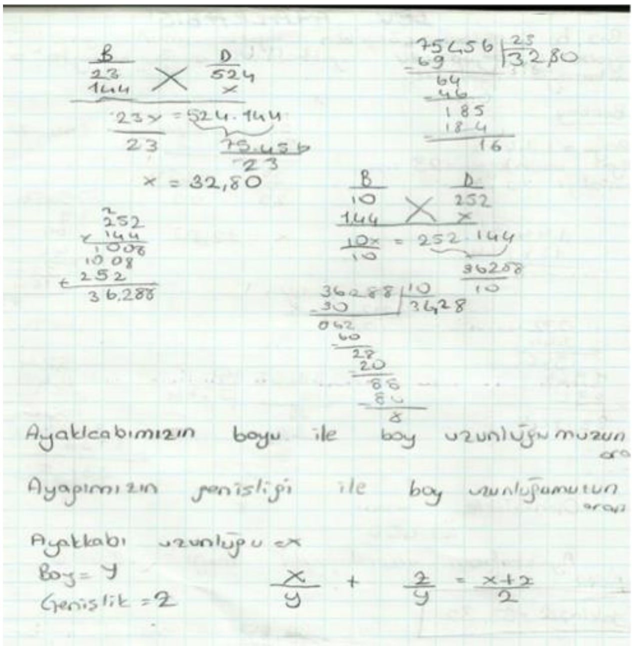


Figure 10. A Section from H3 Student's Diary regarding the Solution of the Problem.

Figure 10 illustrates a section from H3's diary regarding the solution of the problem. The students aimed to develop a model in order to solve the problem by trying to estimate the quantities given in the problem with such data as actual arm and foot length. The solution of the other students in the group, H1 and H3, shown in Figure 11 below, supports this:

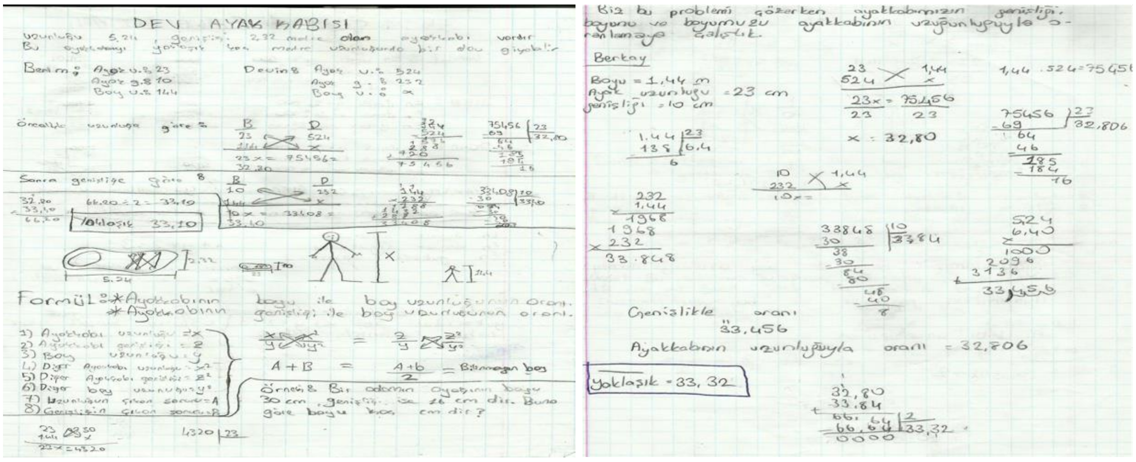


Figure 11. A Section from the Diaries of H1 and H3 on the Solution of the Problem.

As is clear in Figure 11, it is evident that the students developed mathematical models to calculate the variables by establishing a proportional connection between the real data and the data of the problem. Furthermore, it is clear to see that they reached the stage of calculating the giant's height by verifying the model they created to solve the problem situation in the activity for their own body and foot lengths. Finally, the students explained to their friends how they made this calculation and reached the last mathematical modeling step for the solution of the activity.

The H5H6M3 group's approach to the problem was slightly different. While solving the problem, the students started from the area of the shoe. They calculated the ratio between the area of the giant shoe and the foot of one of the group members.

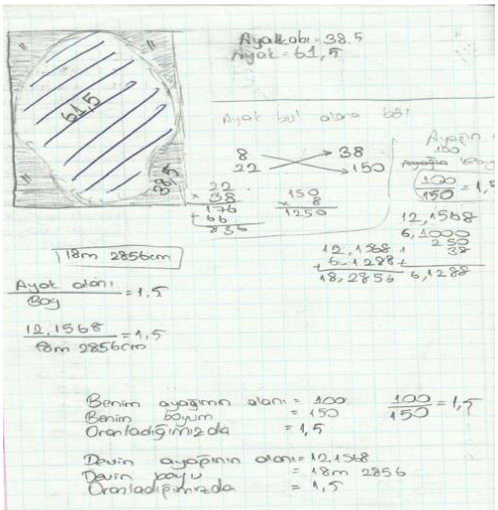


Figure 12. H6's Solution for the Giant's Foot Activity.

As is illustrated in Figure 12, the students first reached the result that the ratio of foot area to foot height was 1,5; but when asked how they calculated this area, they stated that they contemplated that it could be roughly like this.

By trying to create a model, they reached the stage of validating the model with their own foot measurements, stating that they took the giant's foot area as the product of the giant's foot width and height. Figure 13 illustrates a section from H5's diary regarding the solution of the problem:

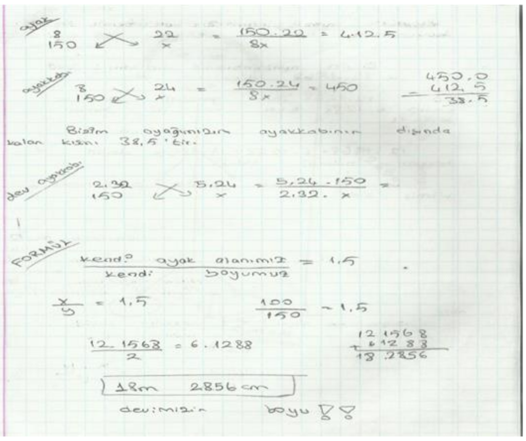


Figure 13. H5's Explanation for the Giant's Foot Activity.

According to this solution in Figure 13, it is evident that H5 tried to develop a method that demonstrated that height was related to the foot area, based on the height and foot length of herself and her group friends.

Mathematical modeling steps reached by the students in the onion seed activity. The last mathematical modeling activity was prepared by the researcher. The basis for preparing this mathematical modeling activity was that onion farming was done in the application area and the application area was in a rural area. By preparing a mathematical modeling activity in this way, it was aimed to attract the attention of the students and increased their motivation and desires. In this mathematical modeling activity, students needed to find the most suitable seed by using the existing data and table and following the mathematical modeling steps. The mathematical concepts that student groups used extensively in this problem were ratio-proportion, profit-loss, and creating a mathematical model using the algebraic expressions. Figure 14 illustrates the mathematical modeling steps that the student groups reached in this activity:

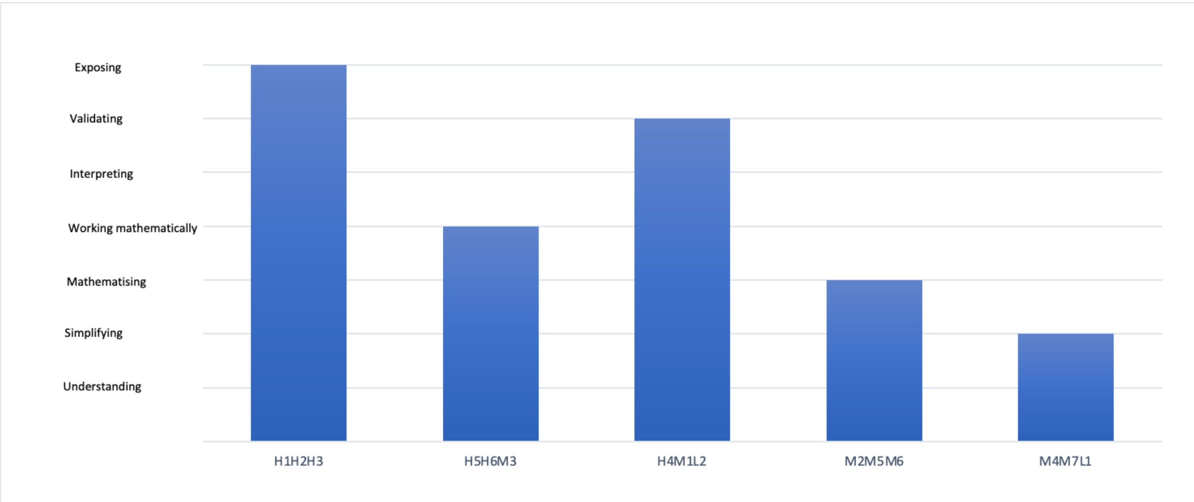


Figure 14. Mathematical Modeling Steps Reached by the Student Groups in the Onion Seed Problem.

As is seen in Figure 14, H1H2H3 student group reached the solution by completing all mathematical modeling steps. Nevertheless, it is clear that the M4M7L1 group was able to progress as far as the stage of simplifying the problem. Even though they could not reach a solution, it was observed that the M4M7L1 Group actively worked on this event. A section of the in-group discussions that the students had to solve the activity is below:

"M7: In order to be able to solve this problem, we must first find out size of the area to plant the seeds and calculate which seed is good.

L1: Yes, one by one, okay, let's start with the Yakut first.

M7: Let's first equalize the grams and get started.

M4: When 500 grams is 120 liras, 250 grams is equal to double, that's what we get. So if 500 grams is 120, wouldn't 1000 grams be 240?"

It is apparent from the students' conversations that they grasped the problem and tried to structure the problem by making assumptions for the solution.

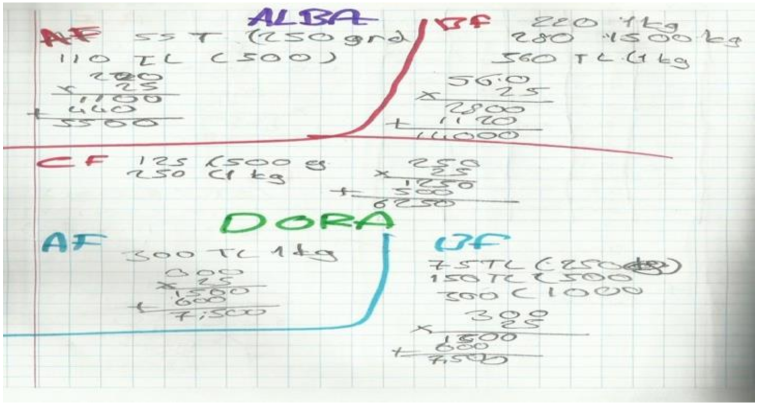


Figure 15. A section from L1's diary regarding the solution of the activity.

As clear in Figure 15 regarding the solution in L1's diary, the students tried to calculate the cost for a specific area by assigning variables using the abbreviations for each seed. Nevertheless, even though the students tried to do mathematization in this process, they had difficulty in the mathematical operation stage.

H1H2H3 student group created two different models for this activity.

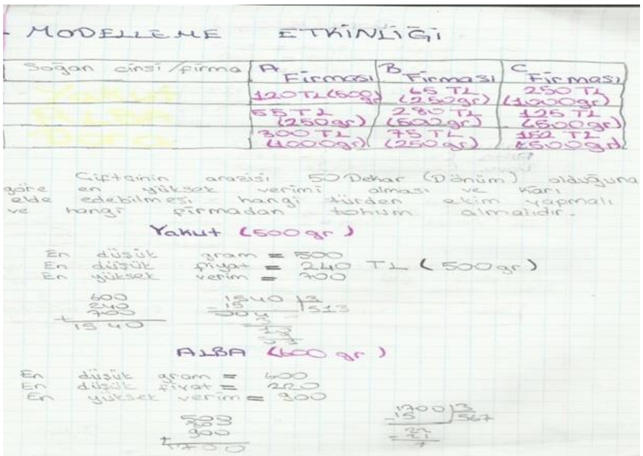


Figure 16. A Visual of Group H1H2H3's Effort to Synchronize the Units Attempting to Solve the Problem.

Figure 16 illustrates that the students tried to create a model that took into account the minimum number of grams of each seed type to be planted per acre, the lowest price and the highest harvest amount per acre, by making assumptions about the solution of the problem. Students' explanations regarding the problem situation are as follows:

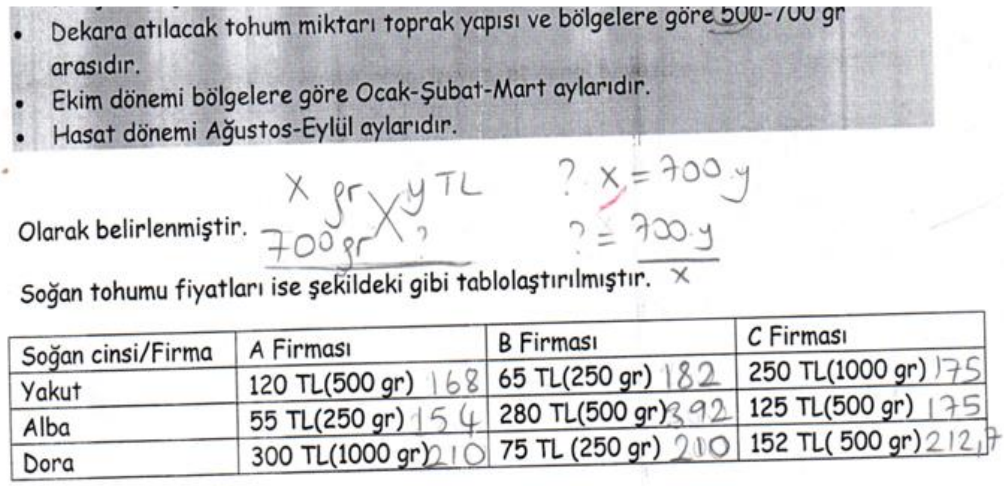
"H1: Alba is 650, Dora is 520, but since the one with the lower price is the more useful one, we will swap the two.

H3: How about this? If we call the lowest gram x...

H1: But how about something like this? The lowest gram of this is x, which is low, this is 500, and this becomes 600. Now, if the gram is low, the usefulness will increase even more, right? We can buy more grams for less money. Let's say you bought 6000 grams for 1 lira, you buy 5000 grams here.

H2: It would be better if we could find out how many grams are used per acre."

This conversation between the students is an indication that the students used the variable "x" and were at a stage of mathematization.



- Dekara atılacak tohum miktarı toprak yapısı ve bölgelere göre 500-700 gr arasıdır.
- Ekim dönemi bölgelere göre Ocak-Şubat-Mart aylarıdır.
- Hasat dönemi Ağustos-Eylül aylarıdır.

Olarak belirlenmiştir.

Soğan tohumu fiyatları ise şekildeki gibi tablolaştırılmıştır.

Soğan cinsi/Firma	A Firması	B Firması	C Firması
Yakut	120 TL(500 gr) 168	65 TL(250 gr) 182	250 TL(1000 gr) 175
Alba	55 TL(250 gr) 154	280 TL(500 gr) 392	125 TL(500 gr) 175
Dora	300 TL(1000 gr) 210	75 TL (250 gr) 210	152 TL(500 gr) 212,7

Figure 17. Model and Calculations Developed by the Group from H2's Worksheet.

As is clear in Figure 17, the students developed a mathematical model in order to solve the problem situation and calculated the seed costs required for 1 acre with the help of this model.

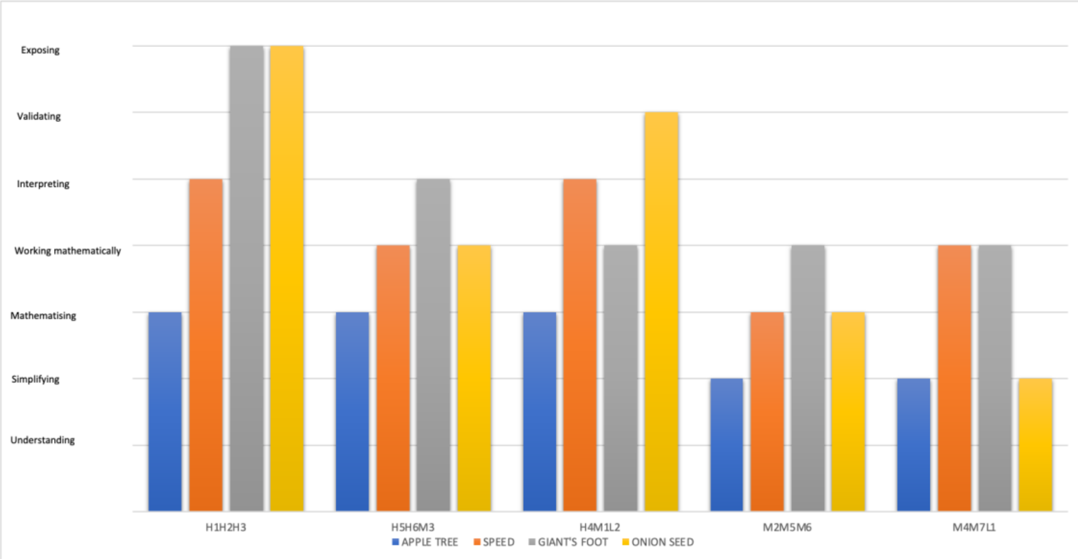


Figure 18. Mathematical Modeling Steps Reached by the Student Groups during the Implementation Process of the Activities.

Figure 18 illustrates that the mathematical modeling steps that the student groups reached throughout the activities in the implementation process. The mathematical modeling step that the student groups reached in the first activity, the Apple Tree Activity, was the mathematising. A group of homogeneous and high-achieving students, such as H1H2H3, continuously progressed through the stages of mathematical modeling and was able to reach the final stage. On the other hand, the M2M5M6 group was a homogeneous group with medium level success, but the highest level this group could achieve was the "problem solving". The H4M1L2 group, as a heterogeneous group with students from all levels, reached higher levels and made progress in general.

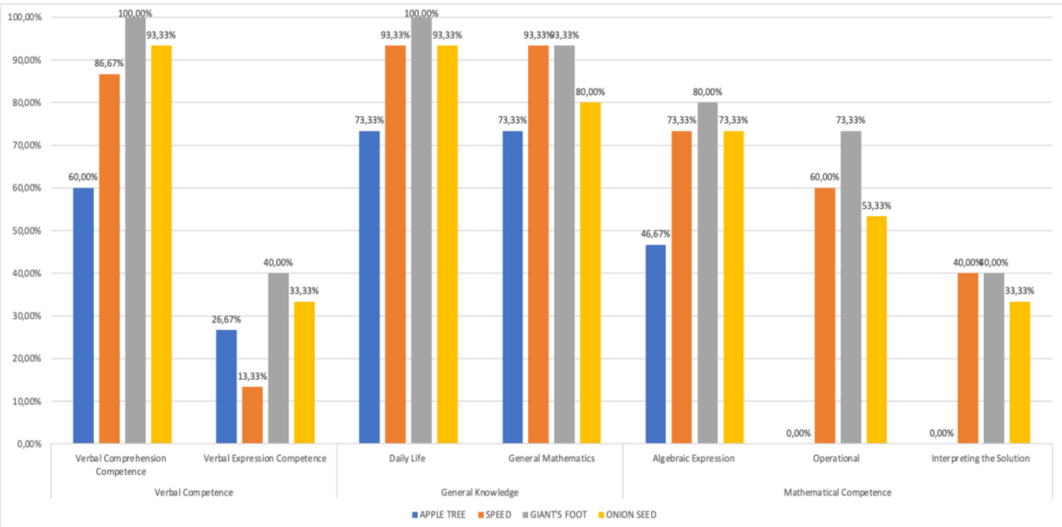


Figure 19. Percentages of the Students Achieving Mathematical Competence during the Application.

As far as Figure 19 is concerned, the mathematical competencies of the 15 students who participated in the application during the solution of the four mathematical modeling activities improved as the activities progressed. It was found that some competencies were displayed by every student in some problems. For instance, while the Verbal Comprehension Competence was demonstrated by each and every student in the Giant's Foot Activity, it is possible to say that no student exhibited the Procedural and Solution Interpretation competencies in the first activity, the Apple Tree Activity. It is clear that the development of students' Verbal Expression and Solution Interpretation competencies progressed less during the activities compared to the other competencies.

The data obtained from the semi-structured interviews conducted during the students' mathematical modeling activities and after the activity, the researcher's diary, and the transcript of the student diaries were analyzed and coded. As a result of this coding, the students' mathematical modeler types were established as illustrated in Table 8.

Table 8.

Classification of the Students According to the Mathematical Modeler Types

	Distanced from reality	Distanced from Mathematics Modeler	Reflective Modeler	Uninterested Modeler
Students	M3, M4	M2, M6, M7	H1, H2, H3, H4, H5, H6, M1, L1	M5, L2

As is illustrated in Figure 19 and Table 8, according to the mathematical modeling competencies of the students revealed in the mathematical modeling activities, it was found that most of the students were reflective modelers at the end of the application. It was further revealed that while the students with medium and low levels in mathematics were generally distanced from reality and distanced from mathematics, the students with high problem-solving achievement were in the reflective modeler type.

Discussion and Conclusion

The purpose of this study was specified as the application of mathematical modeling activities on algebraic verbal problems at the 7th grade level and identifying the effects of the application. With this idea underlying the study, the effect of the difficulties experienced by the students in solving the algebraic verbal problems on the problem solving situations of students during and after the application of mathematical modeling activities was investigated. Some particular results were achieved in the light of the results obtained both during and at the end of the application.

As far as the quantitative findings are concerned, it was revealed that the achievement of the students in solving the algebraic verbal problems in the pre-test and post-test was statistically significantly different. It is possible to say that this difference was realized positively with the post-test average. This result is similar to the results obtained by Işık and Yıldırım (2014) in their study with 5th grade students. Furthermore, it was also revealed that the students' pre-test averages were quite low (=24.33). This particular result is parallel to the idea stated by Llinares and Roig (2008) that students had difficulty in understanding the mathematical structure in verbal problems and that this was due to the lack of comprehending the question in the interviews, the operation steps not progressing smoothly, and the weaknesses in the operational dimension in lower level students. The interviews conducted with the students during the application further revealed that it was seen that the students expressed that they had difficulty in performing some of the mathematical operations.

The level of progress of the students in the modeling steps reached in the process of mathematical modeling activities has increased gradually. Similar to Tekin Dede (2015), it was determined that the mathematical modeling competencies shown by the students in the first activity were low and therefore they did not progress in the mathematical modeling steps. This is because students cannot have experience with mathematical modeling activities have been mentioned in many studies (English, 2009; Korkmaz, 2010; Özgen & Şeker, 2021; Deniz & Akgün, 2018, Sarı Uzun et. al, 2023). Although it was seen in fewer groups in later activities, especially in the first activity Apple Tree activity, it was also seen in similar studies that students struggled because there was no numerical data and tried to proceed mathematically without understanding the problem (Çoksöyler & Bozkurt, 2021; Canbazoğlu Albayrak & Tarım, 2023). In general, it was seen that students had problems in activities, understanding the problem, making assumptions by simplifying the problem, expressing it in mathematical language and working mathematically, solving problems and interpreting it in a way that is suitable for real life. However, as the activities increased, it was found that they could overcome these difficulties and reach to the upper steps (Sandalcı, 2013; Şahin, 2014). It was determined that the percentage of mathematical competences reached by the students during the mathematical modeling activities increased. For instance, during the first of the activities Apple Tree Activity, the verbal expression competencies of the students were 26.67%, while this last activity increased to 33.33 %. In addition, it has been determined that the competencies that the students have never shown can occur as the activities continue. This result shows that mathematical competences can also be improved during mathematical modeling activities. As stated by Altun (2020), mathematical modeling activities are useful in teaching mathematical competences.

Many researchers previously emphasized the importance of students working collaboratively in groups in mathematical modeling activities (Türker Biber & Yetkin Özdemir, 2015; English & Sriraman, 2010; Lesh & Lehrer, 2003; Blum & Leiß, 2007) and the benefit of working in groups was also highlighted by the students in the study, both during the activities and in the interviews held afterwards. Even though it is a special case, the fact that not all students were at the same level may make it easier to develop daily life knowledge, general mathematical knowledge, such as general algebraic expression (establishing equations), operational (equation solving), mathematical competencies such as interpreting the solution, and verbal competencies such as verbal expression and verbal comprehension. It was observed or expressed by the students that during the application, the students positively considered working cooperatively in groups and even managed to improve some of their competencies through in-group interactions. This situation is parallel to the view in Korkmaz's (2010) study that group activities were significant in terms of knowledge sharing.

In the study, the problem solving achievement level as a group in the pre-test and post-test increased the most in the H4M1L2 group. Nevertheless, the same increase was not observed in the M2M5M6 group. The fact that the students' levels were very close caused them not to be able to help one another sufficiently in the problem solving process. It is simply because a student's weakness in one competence was also observed in another one. In other words, since the students in the group did not exhibit the same competence, they approached the solution of the problem in a similar way and failed to overcome their weaknesses and share the knowledge and information among themselves. That is the reason why the group failed to progress very far. Similarly, this group had the least intra-group interaction in the activities and was able to rise to as high as the level of problem solving in the mathematical modeling steps. It is possible to say that the reason for this may be that the students were at the same level and therefore had similar mathematical modeling competencies. Therefore, it is possible to say that working cooperatively in groups in mathematical modeling activities provided a positive environment for students' problem-solving achievement, but it is possible to say that better results would be achieved if the groups consisted of heterogeneous students in terms of achievement level as much as possible.

As far as the results obtained from the application process are concerned, it was found that the students who displayed mathematical modeling competencies were more favorable for the Reflective Modeler type, one of the mathematical modeling types, while the students who did not complete the modeling steps in the modeling activities were more of the modeler types who were distanced from reality or distanced from mathematics. It is possible to say that knowledge and attitudes about both the real world and mathematics are very significant for the students to demonstrate their modeling competencies in mathematical modeling activities and complete the modeling steps.

It was found that during the application of the four mathematical modeling activities discussed in the study, students were able to climb up the top modeling steps from the first problem to the last problem. In general, it was observed that students were able to reach the highest level of mathematization in the first problem.

It is possible to conclude that mathematical modeling activities helped the students make connections between the real world and mathematics. It is also possible to add that as the activities progressed, the students achieved the mathematical modeling competencies. It was also observed that in mathematical modeling problems, students began to approach the problem from different perspectives rather than from a single perspective.

The results show that mathematical modeling activities also lead to the development of mathematical competences, the success of verbal algebraic problem solving increases, so mathematics teaching can be supported by including these applications in mathematics courses.

About Authors

First Author: Özlem Çelikkol is a teacher at Ministry of Education in Erzincan, Turkey.

Second Author: Aytaç Kurtuluş is a member of Eskişehir Osmangazi University. She works at the Faculty of Education. She is currently working at the Mathematics and Science Education Department.

Conflict of Interest

"It has been reported by the authors that there is no conflict of interest."

Funding

"No funding was received."

Ethical Standards

The study was carried out within the framework of the Helsinki Declarations. The data does not require any ethical permission.

ORCID

Özlem Çelikkol https://orcid.org/0000-0002-8504-0102

Aytaç Kurtuluş https://orcid.org/0000-0003-2397-3510

References

Akgün, L. (2009). 8. Sınıf öğrencilerinin sözel problemler ve değişken kavramı arasında ilişki kurabilme becerileri. *Mersin Üniversitesi Eğitim Fakültesi Dergisi*, *5*(2), 275-284.

Aydın Güç, F. (2015). *Matematiksel modelleme yeterliklerinin geliştirilmesine yönelik tasarlanan öğrenme ortamlarında öğretmen adaylarının matematiksel modelleme yeterliklerinin değerlendirilmesi* (Yayımlanmamış Doktora Tezi), Karadeniz Teknik Üniversitesi, Trabzon.

Aydın, F., & Özmen, Z. M. (2012). *8. Sınıf öğrencilerinin sözel problemlerde verilenler ile istenilenler arasındaki ilişkiyi belirleyebilme becerileri*. Sözlü bildiri. X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, 27-30 Haziran 2012, Niğde Üniversitesi, Niğde.

Bal, A. P., & Doğanay, A. (2014). Sınıf Öğretmenliği adaylarının matematiksel modelleme sürecini anlamalarını geliştirmeye yönelik bir eylem araştırması. *Kuram ve Uygulamada Eğitim Bilimleri*, *14*(4), 1363-1384.

Barbosa, J. C. (2006). Mathematical modelling in classroom: a socio-critical and discursive perspective. *ZDM Mathematics Education*, *38*(3), 293-301. Doi: https://doi.org/10.1007/BF02652812

Berry, J., & Houston, K. (1995). *Mathematical Modelling*. London, Gulf Professional Publishing.

Blum, W., & Ferri, R.B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, 1(1), 45-58.

Blum, W., & Leiß, D. (2007). Deal with modelling problems. *Mathematical modelling: Education, engineering and economics-ICTMA*, *12*, 222.

Blume, G. W., & Heid, M. K. (1989). New directions for mathematics instruction. 1989 Yearbook.

Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2010). *Bilimsel araştırma yöntemleri*, Pegem Akademi Yayınları: Ankara. ISBN 978-9944- 919-28-9

Canbazoğlu, H. B., & Tarım, K. (2023). İlkokul dönemi öğrencilerinin bilişsel matematiksel modelleme yeterlikleri. *SDU International Journal of Educational Studies*, *10*(1), 1-21.

Çiltaş, A. (2011). *Dizi ve seriler konusunun matematiksel modelleme yoluyla öğretiminin ilköğretim matematik öğretmeni adaylarının öğrenme ve modelleme becerileri üzerine etkisi* (Yayımlanmamış Doktora Tezi), Atatürk Üniversitesi, Erzurum.

Çiltaş, A., & Işık, A. (2013). Matematiksel modelleme yoluyla öğretimin ilköğretim matematik öğretmeni adaylarının modelleme becerileri üzerine etkisi. *Kuram ve Uygulamada Eğitim Bilimleri*, *13*(2), 1177-1194.

Çoksöyler, A., & Bozkurt, G. (2021). Bilişsel perspektif bağlamında matematiksel modelleme süreci: Altıncı sınıf öğrencilerinin deneyimleri. *Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi*, (52), 480-502.

Crouch, R., & Haines, C. (2004). Mathematical modelling: transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology*, *35*(2), 197-206. Doi: https://doi.org/10.1080/00207390310001638322

Dede, Y. (2004). Öğrencilerin cebirsel sözel problemleri denklem olarak yazarken kullandıkları çözüm stratejilerinin belirlenmesi. *Eğitim Bilimleri ve Uygulama*, 3*(6), 175- 192*.

Deniz, D., & Akgün, L. (2018). Matematiksel modellemenin matematik öğretmeni adaylarının matematik, matematiğin öğretimi ve öğrenimine yönelik inançları üzerindeki etkisinin incelenmesi. *Anemon Muş Alparslan Üniversitesi Sosyal Bilimler Dergisi*, *6*(STEMES'18), 19-25.

English, L. D. (2002). Development of 10-year-olds' mathematical modelling. In L. D., English (Eds.), *International PME Conference*, University of East Anglia, Norwich.

English, L. D. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM*, *41*, 161-181.

English, L. D., & Doerr, H. M. (2003). Perspective-taking in middle school mathematical modelling: A teacher case study. *International Group for the Psychology of Mathematics Education*, *2*, 357-364.

English, L., & Sriraman, B. (2010). Problem solving for 21st century. In B. Sriraman, L. English (Eds.), *Theories of Mathematics Education*, *Advances in Mathematics Education* (pp. 263-290), Doi: 10.1007/978-3-642-00742-2_27, Springer-Verlag Berlin Heidelberg.

English, Lyn D. and Watters, James J. (2005) Mathematical modelling with 9-year-olds. In Chick, Helen L. and Vincent, Jill L., Eds. Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education 2(1), pages pp. 297-304, Melbourne, Australia. Accessed from: http://eprints.qut.edu.au/archive/00003515

Erbaş, A. K., Ketil, M., Çetinkaya, B., Çakıroğlu, E., Alacacı, C., & Baş, S. (2014). Matematik eğitiminde matematiksel modelleme: Temel kavramlar ve farklı yaklaşımlar. *Kuram ve Uygulamada Eğitim Bilimleri, 14*(4), 1607-1627. Doi: 10.12738/estp.2014.4.2039

Erdem, Z. Ç., & Gürbüz, R. (2018). Matematik modelleme etkinliklerine dayalı öğrenme ortamında yedinci sınıf öğrencilerinin alan ölçme bilgi ve becerilerinin incelenmesi. *Adıyaman University Journal of Educational Sciences*, *8*(2), 86-115.

Eric, C. C. M. (2010). Tracing primary 6 students' model development within the mathematical modelling process. *Journal of Mathematical Modelling and Application*, *1*(3), 40-57.

Ferri, R. B. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM*, *38*, 86-95. Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2010). *Bilimsel Araştırma Yöntemleri*, Pegem Akademi Yayınları: Ankara. ISBN 978-9944- 919-28-9

Güder, Y. (2013). *Ortaokul matematik öğretmenlerinin matematiksel modellemeye ilişkin görüşleri* (Yayımlanmamış Yüksek Lisans Tezi), Fırat Üniversitesi, Elazığ.

Ikeda, T., & Stephens, M. (2001). The effects of students' discussion in mathematical modelling. In *Modelling and mathematics education* (pp. 381-390). Woodhead Publishing. Doi: https://doi.org/10.1533/9780857099655.5.381

İnan, G. 2011). Eylem araştırması: Eğitimde değişimin yaratılmasında öğretmenin gücü. *Celal Bayar Üniversitesi Sosyal Bilimler Dergisi*, *9*(02), 481-486.

Kaiser, G., & Maaß, K. (2007). Modelling in lower secondary mathematics classroom—problems and opportunities. *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, 99-108. Doi: 10.1007/978-0-387-29822-1_8

Kal, F.M. (2013). *Matematiksel modelleme etkinliklerinin ilköğretim 6. sınıf öğrencilerinin matematik problemi çözme tutumlarına etkisi* (Yayınlanmamış Yüksek Lisans Tezi), Kocaeli Üniversitesi, Kocaeli.

Kalaycı, Ö. (2017). *Ortaokul 7. sınıf öğrencilerinin bilişsel ve üst bilişsel matematiksel modelleme yeterliklerinin incelenmesi (Yayımlanmış Yüksek Lisans Tezi). Bartın Üniversitesi Eğitim Bilimleri Enstitüsü, Bartın.*

Kang, O., & Noh, J. (2012, July). Teaching mathematical modelling in school mathematics. In *12th International Congress on Mathematical Education* (pp. 8-15).

Karalı, D. (2013). *İlköğretim matematik öğretmen adaylarının matematiksel modelleme hakkındaki görüşlerinin ortaya çıkarılması* (Yayımlanmamış Yüksek Lisans Tezi), Abant İzzet Baysal Üniversitesi, Bolu.

Korkmaz, E. (2010). *İlköğretim matematik ve sınıf öğretmeni adaylarının matematiksel modellemeye yönelik görüşleri ve matematiksel modelleme yeterlikleri* (Yayımlanmamış Doktora Tezi), Balıkesir Üniversitesi, Balıkesir.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In *Beyond constructivism* (pp. 3-33). Routledge.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling, In F. K., Lester, Jr. (Eds.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* (pp.763-804). Charlotte, NC: Information Age Publishing.

Lesh, R., & Zawojewski, J. S. (2007). Second handbook of research on mathematics teaching and learning, chapter Problem solving and modeling. *Problem solving and modeling, 763, 804*.

Lesh, R., Carmona, G., & Post, T. (2002, October). Models and modeling, In: *Proceedings of the Annual Meeting [of the] North American Chapter of the International Group for the Psychology of Mathematics Education, 1*(4), Athens: PME.

Lester, F. K., Jr., & Kehle, P.E. (2003). From problem solving to modeling: the evolution of thinking about research on complex mathematical activity, In R. Lesh, & H. M., Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 501-518). Mahwah, NJ: Lawrence Erlbaum Associates.

Llinares, S., & Roig, A. I. (2008). Secondary school students' construction and use of mathematical models in solving word problems. *International Journal of Science and Mathematics Education*, *6*, 505-532. Doi: 10.1007/s10763-006-9055-6

Maaß, K. (2006). What are modelling competencies? *ZDM*, *38*(2), 113-142. Doi: 10.1007/BF02655885

Memnun, D. S. (2014). Beşinci ve altıncı sınıf öğrencilerinin sözel problemleri çözme konusundaki yetersizlikleri ve problem çözümlerindeki hataları. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *5*(2), 158-175.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Milli Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı. (2013). *Ortaokul matematik öğretim programı*. Ankara.

Milli Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı. (2018). *Ortaokul matematik öğretim programı*. Ankara.

Mischo, C., & Maaß, K. (2012). Which personal factors affect mathematical modelling? The effect of abilities, domain specific and cross domain-competences and beliefs on performance in mathematical modelling. *Journal of Mathematical Modelling and Application*, *1*(7), 2-19.

Mousoulides, N. G., Christou, C., & Sriraman, B. (2008). A modeling perspective on the teaching and learning of mathematical problem solving. *Mathematical Thinking and Learning*, 10, 293–304. Doi:10.1080/10986060802218132.

Muller, E., & Burkhardt, H. (2007). Applications and modelling for mathematics—overview. In W., Blum, P. L., Galbraith, H. W., Henn, & M., Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp.267-274). Springer US.

Muşlu, M., & Çiltaş, A. (2016). Doğal sayılarda işlemler konusunun öğretiminde matematiksel modelleme yönteminin öğrenci başarısına etkisi. *Bayburt Eğitim Fakültesi Dergisi*, *11*(2), 329-341.

Nemirovsky, R. (1996). Mathematical narratives, modeling, and algebra. In N. Bernarz, C. Kieran, & L. Lee (Eds.), *Approaches to Algebra* (pp. 197-220). Springer Netherlands.

Olkun, S., Şahin, Ö., Akkurt, Z., Dikkartın, F. T., & Gülbağcı, H. (2009). Modelleme yoluyla problem çözme ve genelleme: İlköğretim Öğrencileriyle Bir Çalışma. *Eğitim ve Bilim,* 34(151), 65-73.

Özgen, K., & Şeker, İ. (2021). 6. Sınıf öğrencilerinin farkli matematiksel modelleme problemlerindeki beceri gelişimlerinin incelenmesi. *Milli Eğitim Dergisi*, *50*(230), 329-358.

Özgen, K., & Şeker, İ. (2021). 6. Sınıf öğrencilerinin farklı matematiksel modelleme problemlerindeki beceri gelişimlerinin incelenmesi. *Milli Eğitim Dergisi*, *50*(230), 329-358.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution—The social rationality of mathematical modeling in schools. *Learning and instruction*, *7*(4), 309-327. Doi: 10.1016/S0959-4752(97)00014-5

Sandalcı, Y. (2013). Matematiksel modelleme ile cebir öğretiminin öğrencilerin akademik başarılarına ve matematiği günlük yaşamla ilişkilendirmelerine etkisi. *(Yayımlanmış Yüksek Lisans Tezi). Recep Tayyip Erdoğan Üniversitesi. Sosyal Bilimler Enstitüsü, Rize.*

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education*, *24*(2), 117-135. Doi: 10.5951/jresematheduc.24.2.0117

Soylu, Y. (2008). 7. Sınıf Öğrencilerinin cebirsel ifadeleri ve harf sembollerini (değişkenleri) yorumlamaları ve bu yorumlamada yapılan hatalar, *Selçuk Üniversitesi Ahmet Keleşoğlu Eğitim Fakültesi Dergisi, Sayı: 25, 237 -248.*

Şahin, N. (2014). İlkokul 4. sınıf öğrencilerinin model oluşturma etkinlikleri üzerindeki düşünme süreçleri. *(Yayımlanmış Yüksek Lisans Tezi). Ondokuz Mayıs Üniversitesi, Eğitim Bilimleri Enstitüsü, Samsun.*

Şen Zeytun, A. (2013), *Öğretmen adaylarının matematiksel modelleme süreçlerinin ve bu sürece etki eden faktörlere ilişkin görüşlerinin incelenmesi* (Yayımlanmamış Doktora Tezi), Orta Doğu Teknik Üniversitesi, Ankara.

Tekin Dede, A. (2015). Matematik derslerinde öğrencilerin modelleme yeterliklerinin geliştirilmesi: bir eylem araştırması. (*Yayınlanmış doktora tezi). Dokuz Eylül Üniversitesi, Eğitim Bilimleri Enstitüsü, İzmir*.

Tekin Dede, A., & Yılmaz, S. (2013). İlköğretim matematik öğretmeni adaylarının modelleme yeterliliklerinin incelenmesi. *Turkish Journal of Computer and Mathematics Education, 4*(3), 185-206.

Tomal, D. R. (2003). *Action Research for Educators: A Scarecroweducation Book*, Inc.Lanham, Maryland, and Oxford: The Scarecrow Press.

Tuna, A., Biber, A. Ç., & Yurt, N. (2013). Matematik Öğretmeni adaylarının matematiksel modelleme becerileri. *Gazi Üniversitesi Eğitim Fakültesi Dergisi, 33*(1), 129-146.

Türker Biber, B., & Yetkin Özdemir, İ. E. (2015). Matematik öğretiminde matematiksel modelleme yaklaşımı. *Cito Eğitim: Kuram ve Uygulama*, 27, 39-50.

Tutkun, M. İ., & Kabar, M. G. D. (2018). Ortaokullarda matematiksel modelleme: 7. sınıf öğrencilerinin "hava durumu" modelleme problemi ile deneyimi. *Adıyaman University Journal of Educational Sciences*, *8*(2), 23-52.

Ünveren, E. N. (2010). *İlköğretim matematik öğretmen adaylarının ispata yönelik tutumlarının matematiksel modelleme sürecinde incelenmesi* (Yayımlanmamış Yüksek Lisans Tezi), *Balıkesir Üniversitesi, Balıkesir*.

Ural, A., & Ülper, H. (2013). İlköğretim Matematik Öğretmeni adaylarının matematiksel modelleme ile okuduğunu anlama becerileri arasındaki ilişkinin değerlendirilmesi. *Kuramsal Eğitimbilim Dergisi*, *6*(2), 214-241.

Uzun, H., Ergene, Ö., & Masal, E. (2023). İlköğretim beşinci sınıf öğrencilerinin matematiksel modelleme süreçlerinin incelenmesi: Matematik Köyü'ne gidiyoruz etkinliği. *Kocaeli Üniversitesi Eğitim Dergisi*, *6*(2), 494-521.

Verschaffel, L., De Corte, E., & Vierstraete, H. (1999). Upper elementary school pupils' difficulties in modeling and solving nonstandard additive word problems involving ordinal numbers. *Journal for Research in Mathematics Education*, *30* (3), 265-285. Doi:10.2307/749836

Yenilmez, K., & Teke, M. (2008). Yenilenen matematik programının öğrencilerin cebirsel düşünme düzeylerine etkisi. *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, 9(15), 229-246.

Yenilmez, K., & Yılmaz, S. (2013). İlköğretim ikinci kademe öğrencilerinin problem çözmedeki kavram yanılgıları. *Sakarya Üniversitesi Eğitim Fakültesi Dergisi*, (15), 75-97. Retrieved From https://Dergipark.Org.Tr/En/Pub/Sakaefd/Issue/11211/133890

Yıldırım, Z. & Işık, A. (2015). The effects of mathematical modelling activities to academic achievement of the fifth grade students in mathematics course. *Kastamonu Eğitim Dergisi, 23*(2), 581-600. Retrieved from https://dergipark.org.tr/en/pub/kefdergi/issue/22599/241432

Yıldırım, Z., & Işık, A. (2015). Matematiksel modelleme etkinliklerinin 5. sınıf öğrencilerinin matematik dersindeki akademik başarılarına etkisi. *Kastamonu Eğitim Dergisi*, *23*(2), 581-600.

Yoon, C., Dreyfus, T., & Thomas, M. O. J. (2010). How high is the tramping track? Mathematising and applying in a calculus model-eliciting activity. *Mathematics Education Research Journal*, *22*(1), 141-157. Doi: 10.1007/BF03217571

Zawojewski, J. (2010). Problem solving versus modeling, R. Lesh et al. (Eds.), *Modeling Students' Mathematical Modeling Competencies* (pp.237-243). Springer US. Doi: 10.1007/978-1-4419-0561-1_20.

Zawojevski, J., & Lesh, R. (2003). A models and modeling perspective on problem solving, In R. Lesh, H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 501-518). Mahwah, NJ: Lawrence Erlbaum Associates.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in mathematics*, *63*, 89-112. Doi: 10.1007/s10649-005-9002-4

Zollman, A. (2010). Commentary on Problem Solving for the 21st Century, In L. English (Eds.), *Theories of Mathematics Education: Advances in Mathematics Education* (pp.297-301). Springer-Verlag Berlin Heidelberg. Doi: 10.1007/978-3-642- 00742-2_27.