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RESEARCH ARTICLE

THE TRAVELING WAVE SOLUTIONS OF THE CONFORMABLE TIME-FRACTIONAL ZOOMERON EQUATION BY USING THE MODIFIED EXPONENTIAL FUNCTION METHOD

Aslı ALKAN [1,*], Tolga AKTÜRK [2], Hasan BULUT [3]

[1] Department of Mathematics, Faculty of Science, Fırat University, Elazığ, Turkey
[2] Department of Mathematics and Science Education, Faculty of Education, Ordu University, Ordu, Turkey
[3] Department of Mathematics, Faculty of Science, Fırat University, Elazığ, Turkey

ABSTRACT

The present study focuses on the acquisition of traveling wave solutions associated with the conformable time-fractional Zoomeron equation through the utilization of the modified exponential function method (MEFM). The solution functions derived from mathematical computations encompass hyperbolic, trigonometric, and rational functions. Various graphical representations, such as 2D, 3D, contour graphs, and density graphs, are utilized to visually depict the distinct features of the solution functions derived from the determination of suitable parameters.

Keywords: The wave solution, Zoomeron equation, The conformable derivative

1. INTRODUCTION

Nonlinear partial differential equations (NPDEs) have a significant role in various scientific disciplines, such as optics, hydrodynamics, economics, meteorology, plasma physics, and engineering. The existing body of literature presents a range of approaches for acquiring solutions to these equations. Several approaches have been proposed in the literature for solving various types of differential equations. These methods include the (G'/G) expansion method, the new function methods, the generalized Kudryashov method, the sine-Gordon expansion method, the $\partial -$ dressing method, the homotopy perturbation method. In the present investigation, the modified exponential function method (MEFM) is implemented to address the problem of solving a nonlinear conformable time-fractional Zoomeron equation (CTFZE). New interactions between traveling wave solutions have been identified [1-11].

CTFZE can be defined as follows [9, 13-15]:

$$\frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}\left[\frac{u_{xy}}{u}\right] - \frac{\partial^2 u}{\partial x^2}\left[\frac{u_{xy}}{u}\right] + 2\frac{\partial^\alpha u}{\partial t^\alpha}[u^2]_x = 0, 0 < \alpha \leq 1, \tag{1}$$

where $u = u(x, y, t)$.

The organization of the article is as follows: In Section 2, the methodology of modified expansion function method has been provided. In Section 4, an application about the conformable time-fractional Zoomeron equation is presented. Section 5 presents the final remarks of the study.

*Corresponding Author: alkanasli47@gmail.com
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2. THE METHODOLOGY OF MODIFIED EXPONENTIAL FUNCTION METHOD

This section presents an overview of MEFM.

Handle the subsequent NPDE

$$P(u, u_x, u_t, u_{xx}, u_{xxx}, u_{tt}, u_{tx}, \ldots) = 0, \tag{2}$$

where, the function $u(x, y, t)$ is of unknown nature, while P represents a polynomial that involves $u(x, y, t)$ as well as its derivatives.

Step 1: The subsequent transformation of the traveling wave is characterized by the

$$u(x, y, t) = u(\xi), \xi = k(x + y - ct), \tag{3}$$

where k, c are nonzero constants and can be computed at a later stage.

By substituting Equation (3) into Equation (2), we obtain the subsequent nonlinear ordinary differential equation (NODE).

$$N(u, u', u'', u''',, \ldots) = 0. \tag{4}$$

Step 2: According to the MEFM, it is assumed that the desired solution can be described as follows:

$$u(\xi) = \frac{\sum_{i=0}^{n} A_i\left[\exp\left(-\Omega(\xi)\right)\right]^i}{\sum_{j=0}^{m} B_j\left[\exp\left(-\Omega(\xi)\right)\right]^j} = \frac{A_0 + A_1 e^{-\Omega} + \cdots + A_m e^{-n\Omega}}{B_0 + B_1 e^{-\Omega} + \cdots + B_n e^{-m\Omega}}, \tag{5}$$

where A_i $(0 \leq i \leq m)$ and B_j $(0 \leq j \leq n)$.

The balance principle is employed to derive positive integer values for the variables m and n

$$\Omega' = e^{-\Omega(\xi)} + \mu e^{\Omega(\xi)} + \lambda. \tag{6}$$

Equation (6) exhibits the subsequent families of solutions as described in [12].

Family 1: For $\mu \neq 0, \lambda^2 - 4\mu > 0$, then we have the solution

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + EE) - \frac{\lambda}{2\mu}\right)\right). \tag{7}$$

Family 2: For $\mu \neq 0, \lambda^2 - 4\mu < 0$, then we get the solution

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 + 4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + EE) - \frac{\lambda}{2\mu}\right)\right). \tag{8}$$

Family 3: For $\mu = 0, \lambda \neq 0, \lambda^2 - 4\mu > 0$, then we have the solution

$$\Omega(\xi) = -\ln\left(\frac{\lambda}{e^{\lambda(\xi + EE)} - 1}\right). \tag{9}$$

Family 4: For $\mu \neq 0, \lambda \neq 0, \lambda^2 - 4\mu = 0$, then we get the solution

$$\Omega(\xi) = \ln\left(-\frac{2\lambda(\xi + EE) + 4}{\lambda^2(\xi + EE)}\right). \tag{10}$$

Family 5: For $\mu = 0, \lambda = 0, \lambda^2 - 4\mu = 0$, then we have the solution

$$\Omega(\xi) = \ln(\xi + EE), \tag{11}$$

where, EE is a integral constant.

Step 3: Substituting Equation (5) and its derivatives in Equation (4), we acquire the algebraic equation system. The system has been solved using the Mathematica software package, resulting in the acquisition of solutions for the CTFZE.

3. APPLICATION

In this part, we will employ the MEFM to acquire solutions for the CTFZE. Let us contemplate the subsequent traveling wave transform:

$$u(x, y, t) = u(\xi), \xi = \left(kx + ry - c\frac{t^\alpha}{\alpha}\right). \tag{12}$$

The subsequent NODE is derived as

$$krc^2\left(\frac{u''}{u}\right)'' - rk^3\left(\frac{u''}{u}\right)'' - 2ck(u^2)'' = 0. \tag{13}$$

When the balancing procedure is implemented on Equation (12), it results in the establishment of the subsequent connection.

$$n = m + 1.$$

By selecting $m = 1$, then we get $n = 2$. Hence, Equation (5) is acquired for m and n values in the following.

$$u(\xi) = \frac{A_0 + A_1 e^{-\Omega} + A_2 e^{-2\Omega}}{B_0 + B_1 e^{-\Omega}}. \tag{14}$$

By rearranging Equation (14) according to the corresponding term in Equation (13), a set of algebraic equations is obtained. This set consists of the coefficients of the exponential function $e^{-\Omega(\xi)}$.

The Mathematica software tool yielded the following coefficients that are deemed appropriate.

Case:

$$A_0 = -\frac{\sqrt{r}\sqrt{S}\lambda B_0}{\sqrt{2}(-2S + k^3 r(\lambda^2 - 4\mu))^{1/4}(kr(\lambda^2 - 4\mu))^{1/4}},$$

$$A_1 = -\frac{\sqrt{r}\sqrt{S}(2B_0 + \lambda B_1)}{\sqrt{2}(-2S + k^3 r(\lambda^2 - 4\mu))^{1/4}(kr(\lambda^2 - 4\mu))^{1/4}},$$

$$A_2 = -\frac{\sqrt{2}\sqrt{r}\sqrt{S}B_1}{(-2S + k^3 r(\lambda^2 - 4\mu))^{1/4}(kr(\lambda^2 - 4\mu))^{1/4}},$$
$$c = -\frac{\sqrt{-2S + k^3 r(\lambda^2 - 4\mu)}}{\sqrt{kr(\lambda^2 - 4\mu)}}.$$

By substituting the given coefficients into Equation (13), the resulting solutions are as follows:

Family 1: For $\mu \neq 0, \lambda^2 - 4\mu > 0$, then we obtain the solution,

$$u_1(x, y, t) = \left(-\frac{\sqrt{r}\sqrt{S}\left(\varpi + \lambda\sqrt{\varpi}\mathrm{Tanh}\left[\frac{1}{2}(\zeta)\sqrt{\varpi}\right]\right)}{\sqrt{2}(-2S + k^3 r\varpi)^{1/4}(kr(\varpi))^{1/4}\left(\lambda + \sqrt{\varpi}\mathrm{Tanh}\left[\frac{1}{2}(\zeta)\sqrt{\varpi}\right]\right)}\right). \tag{15}$$

where, $\zeta = \mathrm{EE} + kx + ry - \frac{ct^{\alpha}}{\alpha}, \varpi = \lambda^2 - 4\mu.$

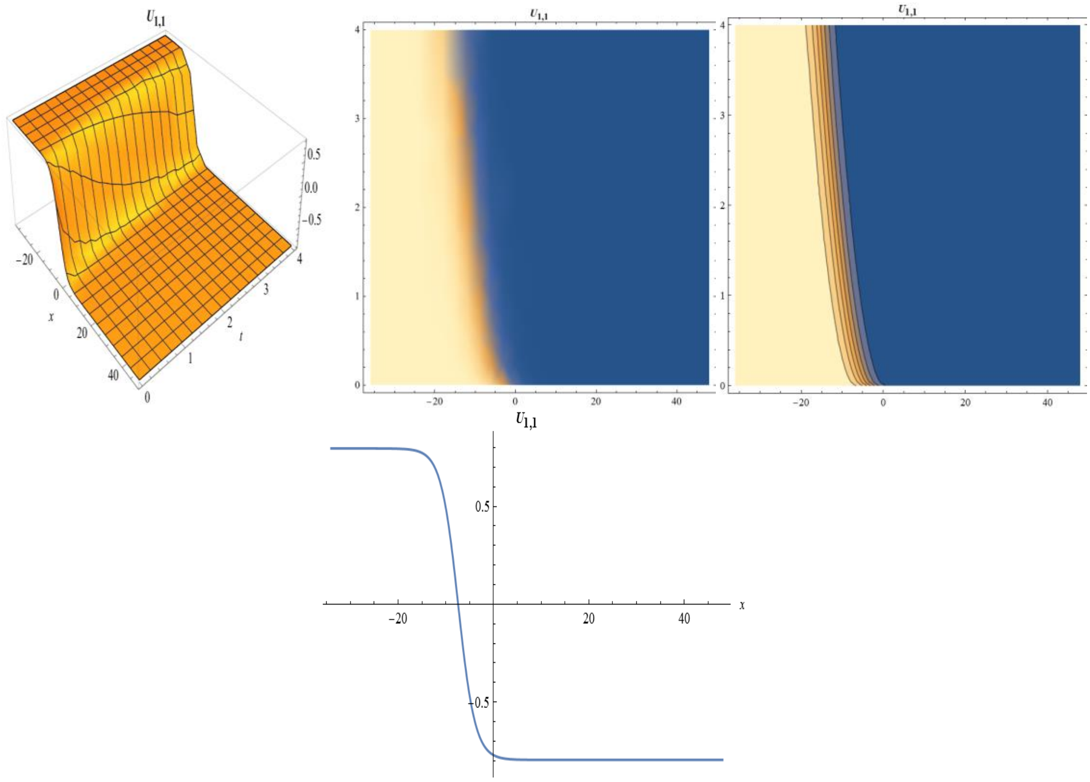


Figure 1: The 2D, 3D, density, contour graphs of Eq. (15) for $k = 0.25, \lambda = 3, \mu = 1, B_0 = 1.2, B_1 = 0.55, r = -0.75, S = 0.24, A_0 = -1.2812, A_1 = -1.44135, A_2 = 0.391477, c = -0.757958, \alpha = 0.5, EE = 0.82, y = 1.2, t = 0.5$.

Family 2: When $\mu \neq 0, \lambda^2 - 4\mu < 0$, then we have the solution,

$$u_2(x, y, t) = \left(\frac{\sqrt{r}\sqrt{S}\left(-\varpi + \lambda\sqrt{-\varpi}\mathrm{Tan}\left[\frac{1}{2}\zeta\sqrt{-\varpi}\right]\right)}{\sqrt{2}(-2S + k^3 r(\varpi))^{1/4}(kr(\varpi))^{1/4}\left(\lambda - \sqrt{-\varpi}\mathrm{Tan}\left[\frac{1}{2}\zeta\sqrt{-\varpi}\right]\right)}\right), \tag{16}$$

where, $\zeta = \mathrm{EE} + kx + ry - \frac{ct^{\alpha}}{\alpha}, \varpi = \lambda^2 - 4\mu$.

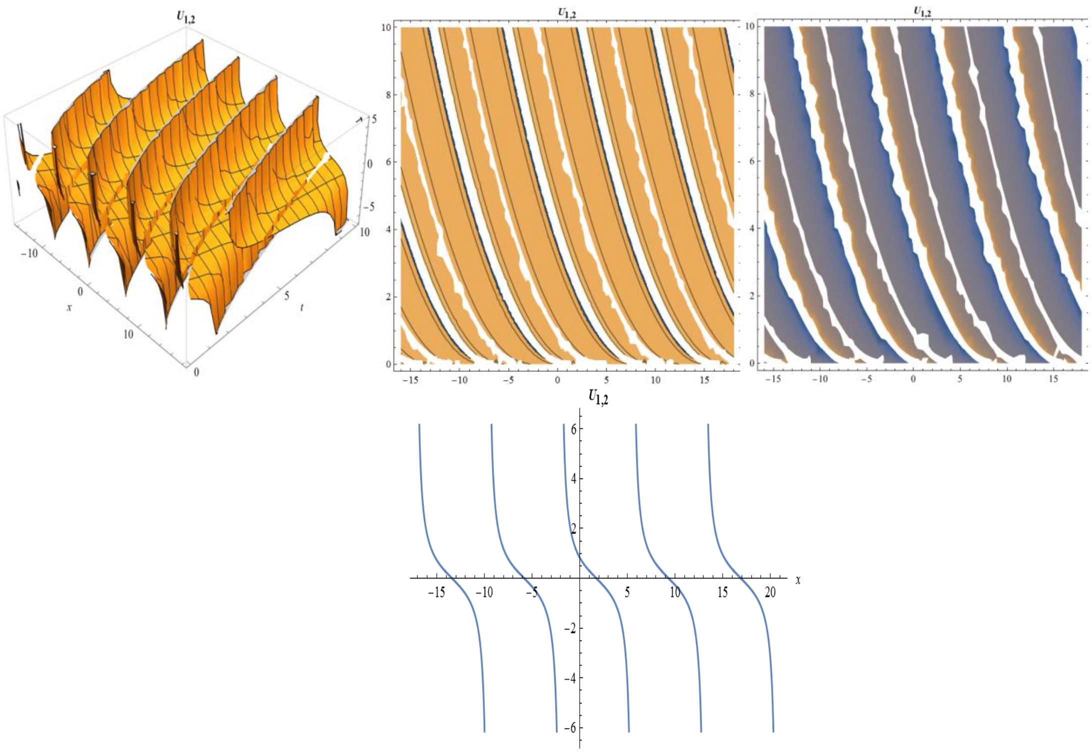
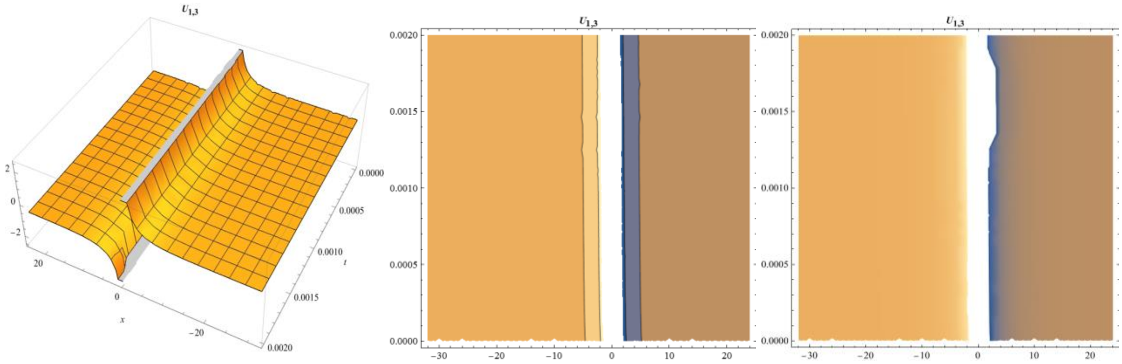


Figure 2: The 2D, 3D, density, contour graphs of Eq. (15) for $k = 0.25, \lambda = 1, \mu = 3, B_0 = 1.2, B_1 = 0.55, r = -0.75, S = -0.24, A_0 = 0.340068, A_1 = 0.836002, A_2 = 0.311729, c = -0.543348, \alpha = 0.5, EE = 0.82, y = 1.2, t = 0.5$.

Family 3: For $\mu = 0, \lambda \neq 0, \lambda^2 - 4\mu < 0$, the solution of Eq. (1) is obtained as,

$$u_3(x,y,t) = \left(-\frac{\sqrt{r}\sqrt{S}\lambda \mathrm{Coth}\left[\frac{1}{2}\zeta\lambda\right]}{\sqrt{2}(kr\lambda^2)^{1/4}(-2S + k^3 r\lambda^2)^{1/4}} \right), \tag{17}$$

where, $\zeta = \mathrm{EE} + kx + ry - \frac{ct^{\alpha}}{\alpha}$.



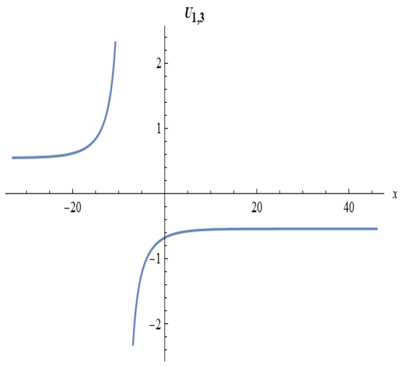


Figure 3: The 2D, 3D, density, contour graphs of Eq. (15) for $k = 0.25, \lambda = 1, \mu = 0, B_0 = 1.2, B_1 = 0.55, r = -0.75, S = 0.24, A_0 = -0.653316, A_1 = -1.60607, A_2 = -0.598873, c = -1.61941, \alpha = 0.5, EE = 0.82, y = 1.2, t = 0.5$.

4. CONCLUSION

In this article, traveling wave solutions of CTFZE are effectively obtained using MEFM. The outcomes indicate that the MEFM is a highly efficient mathematical approach for solving NPDEs. The solutions acquired were verified using the Mathematica program, and visual representations in the form of two-dimensional and three-dimensional graphs, as well as density and contour plots, were generated using proper parameters.

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CONFLICT OF INTEREST

The authors stated that there are no conflicts of interest regarding the publication of this article.

AUTHORSHIP CONTRIBUTIONS

Autors' contributions are equal.

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