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Improving Low Mathematics Achievers' Number Sense via Number Line Training with Board Games

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Mehmet Hayri Sarı*

Sinan Olkun**

Abstract

This study investigated the effect of a number line training via lineer board games with specific contents, namely numerical and figural, on students' number line estimation skills, arithmetic performance, and mathematics achievement. A total of 30 first graders (15 in experimental and 15 in control group) participated in the study. There were 2 speech and language disorder (SLD) students, one in each, in the control and experimental groups. Number line training with a board game, Sahibingo® was implemented in the experimental group, while a non-numerical board game, Animalbingo, was used in the control group. Training lasted for 30 minutes a day, one day a week, and 4 weeks (2 hours in total). Results indicated that students, including the SLD, in the experimental group improved their number line estimation skills in 0-10 and 0-20 number range, but not in the 0-100 range. Improvements did not reflect in arithmetic performance and mathematics achievement scores. Although there are some improvements in small range number line estimations possibly because of the familiarity there is no improvement in the larger range and in terms of mathematics achievement scores, possibly because of the shorter period of training. It can be concluded that low mathematics achievers' number sense can be improved via lineer board games with numerical content. Future research may investigate the effect of longer periods and may include other students with special needs.

Keywords: Number sense, number line training, board games, mathematics achievement, low mathematics achievers.

^{*}Corresponding Author: Assoc. Prof. Dr., Nevşehir Hacı Bektaş Veli University, Faculty of Education, Department of Elementary Education, Nevşehir, Turkey. E-mail: mhsari@nevsehir.edu.tr, https://orcid.org/ 0000-0002-7159-2635

^{**}Prof. Dr., Ankara University, Faculty of Educational Science, Department of Elementary Education, Ankara, Turkey. E-mail: olkun@ankara.edu.tr, https://orcid.org/0000-0003-3764-2528

Matematikte Düşük Başarılı Öğrencilerin Sayı Hissinin Masa Oyunları ve Sayı Doğrusu Eğitimi Yoluyla Geliştirilmesi

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Mehmet Hayri Sarı* Sinan Olkun**

Öz

Bu calısmada, sayısal ve sekilsel olmak üzere belirli iceriklere sahip masa ovunları aracılığıyla verilen savı doğrusu eğitiminin öğrencilerin savı doğrusunda tahmin, aritmetik performansı ve matematik başarısı üzerindeki etkisi araştırılmıştır. Çalışmaya toplam 30 birinci sınıf öğrencisi (15 deney ve 15 kontrol grubu) katılmıştır. Deney ve kontrol gruplarında birer tane olmak üzere 2 konusma ve dil bozukluğu (KDB) öğrencisi bulunmaktadır. Deney grubunda doğrusal bir masa oyunu olan Sahibingo® ile sayı doğrusu eğitimi uygulanırken, kontrol grubunda sayısal olmayan bir masa oyunu olan Animalbingo kullanılmıştır. Eğitim günde 30 dakika, haftada bir gün ve 4 hafta (toplam 2 saat) sürmüştür. Sonuçlar, KDB de dahil olmak üzere deney grubundaki öğrencilerin sayı doğrusu tahmin becerilerinin 0-10 ve 0-20 sayı aralığında gelistiğini, ancak 0-100 aralığında gelismediğini göstermiştir. İyilesmeler aritmetik performansa ve matematik başarı puanlarına yansımamıştır. Küçük aralıktaki sayı doğrusu tahminlerinde muhtemelen aşınalık nedeniyle bazı gelişmeler olmasına rağmen, daha büyük aralıkta ve matematik başarı puanları açısından, muhtemelen eğitim süresinin daha kısa olması nedeniyle, herhangi bir gelişme olmamıştır. Düşük matematik başarısına sahip öğrencilerin sayı duyularının sayısal içerikli doğrusal masa oyunları ile geliştirilebileceği sonucuna varılabilir. Gelecekteki araştırmalar daha uzun sürelerin etkisini araştırabilir ve özel ihtiyaçları olan diğer öğrencileri de kapsayabilir.

Anahtar Sözcükler: Masa oyunları, sayı hissi, düşük matematik başarısı, sayı doğrusu eğitimi, matematik başarısı.

^{*}Sorumlu Yazar: Doç. Dr., Nevşehir Hacı Bektaş Veli Üniversitesi, Eğitim Fakültesi, Temel Eğitim Bölümü, Sınıf Eğitimi Anabilim Dalı, Ankara, Türkiye. E-posta: mhsri@nevsehir.edu.tr, https://orcid.org/0000-0002-7159-2635

^{**}Prof. Dr., Ankara Üniversitesi, Eğitim Bilimleri Fakültesi, Temel Eğitimi Bölümü, Sınıf Eğitimi Anabilim Dalı, Ankara, Türkiye. E-posta: olkun@ankara.edu.tr, https://orcid.org/0000-0003-3764-2528

Introduction

In the last 40 years, researchers in the fields of developmental psychology, neuroscience and education have focused on determining the mechanisms underlying mathematical development and increasing mathematics achievement. Early and a significant portion of school mathematics is merely a quantification process. Therefore, how children's understanding of numerical quantities changes over time with birth and how this change is related to mathematical performance has been a key question in studies of numerical cognition (cf. Wynn, 1992a; Wynn, 1992b; Xu et al., 2005; Xue & Spelke, 2000). In fact, mathematical competence relies heavily on number sense, the ability to understand magnitude information (Obersteiner & Tumpek, 2016) from various representations.

What Constitutes Number Sense?

Research shows that human beings have a cognition that has a separate core system for representing numbers. The core number system consists of two subsystems, namely the small or Exact Number System (ENS) and the large or Approximate Number System (ANS) (Feigenson et al., 2004; Spelke & Kinzler, 2007). However, it is argued that unlike animals, in humans there is a separate subsystem that connects the quantity and the symbol it represents. This system is called Access to Symbols (ATS) (Rousselle & Noel, 2007). It is stated that altogether form a module, a basic core system that enables people to learn mathematics (Butterworth, 2005).

It can be said that there are contradictory claims regarding the role of the core system in mathematics performance. While some researchers emphasize that precision in ANS has a strong relationship with mathematics achievement (Halberda et al., 2008; Libertus et al., 2011; Starr et al., 2013; Wilson & Dehaene, 2007), others see precision in the ENS as an important measure of mathematical success (Landerl et al., 2004). Still others claim that access to symbols is more important to mathematics achievement (Rousselle & Noel, 2007; Rubinsten & Henik, 2005).

Claims that the ANS within the core system is the source of developmental variation in math performance is a subject of significant ongoing debate (cf. Coolen et al., 2022; Passolunghi et al., 2014). ANS has to do with an evolutionarily conserved intuitive number sensitivity that is innate—genetically transmitted—that develops with age (Halberda et al., 2012). ANS forms the basis for the acquisition and development of symbolic number systems for counting, arithmetic, and further algebraic operations (Dehaene, 1997). It also allows quantities to be represented on an internal, mental number line and approximate discrimination of large amounts (Dehaene, 2009). Typical ANS tasks include the approximation of symbolic and non-symbolic quantities and comparison of non-symbolic quantities (Landerl et al., 2004; Wilson & Dehaene, 2007). Among these tasks, the approximate value of numerical quantities consists of placing a series of numbers on a given mental number line (MNL) (Siegler & Booth, 2005).

While different theoretical approaches to number line performance have been proposed, one point of agreement is that it is important to understand, at least in part, number line estimation performance because it is reliably linked to formal math skills in both children with normal development and children with mathematical learning disabilities (Zax et al., 2019). For example, in a meta-analysis study conducted by Schneider et al. (2018), the correlation value (r) between number line estimation and mathematical achievement was found to be .443 when the average effect size of a total of 263 studies with 10,576 participants aged between 4 and 14 were taken into account. In addition, they revealed that individual differences in number line estimation accuracy were highly correlated with individual differences in counting, arithmetic, and standard mathematics achievement tests (Schneider et al., 2018).

Individuals with dyscalculia have problems with various numerical comprehension skills such as, determining quantities, arithmetic, number words, counting, numbers and spatially representing numbers (Kucian & von-Aster, 2015). Past research has paid particular attention to training such basic numerical skills as writing numbers, ordering numbers by magnitude, comparing numerical quantities, or estimating the relative position of a number on an empty number line. Since these skills form the basis of number sense and the initial steps for complex skills, they are a strong predictor of future mathematical success (Whyte & Bull, 2008). In this context, the ineffectiveness of traditional educational interventions on students with weak number sense sensitivity and mathematics difficulties has led researchers to different teaching methods (Shalev et al., 2005).

Although there is research that does not support the strong claim that number line knowledge plays a central role in the development of mathematical skills (e.g. Coolen et al., 2022; Friso-Van Den Bos et al., 2018; LeFevre et al., 2013; Muldoon et al., 2013) the abundance of evidence suggests that number line estimation performance is not only related to actual numerical performance, but is also a predictor of future mathematical improvement (Link et al., 2014). Therefore, more research is needed on whether improving number line performance improves children's math performance.

Practices to Develop Number Line Estimation Performance

The question "does improving children's number line performance reflect on their math achievement?" has been asked by many researchers (e.g. Laski & Siegler, 2007). To answer this question, Siegler and his colleagues designed simple board games based on the game Chutes and Ladders, which is often played in daily life (Siegler & Booth, 2004; Siegler & Ramani, 2008; Siegler & Ramani, 2009). Studies have shown that these simple board games help the development of children's number sense (cf. Ramani & Siegler, 2008; Siegler & Ramani, 2008; Siegler & Ramani, 2009). In these studies, it has been shown that it is possible to improve the representation of number size spatially in children with both normal development and developmental dyscalculia by training. It has been observed that playing with these board games not only improves the precision of the mental number line, but also supports other numerical skills that are not taught directly during these games, such as counting skills, number naming and number comparison. (Laski & Siegler, 2014; Ramani & Siegler, 2008). In other board games prepared with similar approaches, children's estimation performance on number lines improved too (e.g. Whyte & Bull, 2008).

Considering the positive effect of playing with board games on children's numerical development and mathematics achievement, researchers have made attempts on technology-supported education to ensure the development of number sense (e.g. Fischer et al., 2011; Fischer et al., 2015; Kucian et al., 2011; Link et al., 2013; Wilson et al., 2006). In particular, it has been given importance by researchers because of the flexibility of digital technology to be designed to adapt to the skills of each child and to provide intensive education in a stimulating environment (Kullik, 2004). In addition, technology use in education and training has drawn new paths for researchers, giving important educational results at both behavioral and neural levels (Moeller et al., 2015).

It is seen that the studies to develop the sense of number, supported by technology, help in the development of children's number perception. It has been revealed that technology-supported trainings, especially aimed at improving the number representation of children with dyscalculia, have a positive effect on the mathematical competence of these children (Aragón-Mendizábal et al., 2017; Fischer et al., 2011; Fischer et al., 2015; Käser et al., 2013; Kohn et al., 2020; Kucian et al., 2011; Link et al., 2013; Mera et al., 2022; Obersteiner et al., 2013; Ramani et al., 2017; Sari & Olkun, 2020; Wilson et al., 2006). For example, in a study by Ramani et al. (2017), playing both the number game and the working memory game for 10 sessions improved children's numerical knowledge, especially in understanding the magnitude of numbers. Children with lower pre-existing numerical magnitude knowledge improved the most. Similarly, in Kohn et al. (2020)'s study, 67 dyscalculic children from 2nd to 5th grade played the Calcularis game at least 42 training sessions, 20 minutes long. After the game, there was a significant decrease in the children's Total Absolute Errors (TAE) and a significant increase in linearity. These improvements have been shown to be stable after a 3-month interval post-training.

There is also evidence that games designed to improve number sense do not improve in the short run (Hellstrand et al., 2020). In the research conducted by Hellstrand and colleagues (2020), the Number Race game was utilized as an intervention for mathematically low achiever children in the first grade. Other low-achieving and typical-achieving groups were trained in their classrooms. The short-term improvement was not statistically significant in the mathematical performance of low-performing children who played number race game in 15-minute sessions, 3-4 days a week, for a period of four-weeks.

In summary, the findings of both technology-based games and board games designed to improve the number sense support the claim that training on spatial representations of numbers improves the proper placement of numbers on an empty number line, additionally; it also improves other numerical competencies (reading numbers, number comparison, arithmetic operations, etc.). Although the educational interventions carried out give an idea to the researchers about the development of number sense, it seems important to resolve some of the issues in the research with the current study in order to generalize the results to a large extent and to use these games in education. One of these limitations is the fact that educational interventions carried out so far were no true experimental designs, usually missing the control group (cf. Wilson et al, 2006; Whyte & Bull, 2008; Siegler & Ramani, 2008; Ramani & Siegler, 2008; Käser et al., 2013; Laski & Siegler, 2014). Secondly, none of these interventions were conducted in regular classrooms which has both typical students and students with combined learning difficulties. In this study, in contrast to the number line game played in the experimental group, a non-numerical game was played on the number line model in the control group. In both of the control and experimental groups there were typical and atypical students. The current study aimed at investigating whether the lineer games played in both groups have a significant effect on children's mathematical success, arithmetic performance and number line estimation skills.

Method

Research Design

The present study utilized a pretest-posttest, controlled experimental design (Cohen et al., 2018). Participants were randomly assigned to one of the two groups. An intervention was applied to both groups and pre-test and post-tests were administered to measure the change in the groups (Cohen et al., 2018). The design is given in Table 1.

Table 1. Experimental process design

Experimental group	RO ₁	v	O2	
Control group	RO ₃	А	O_4	

O= refers to the process of measurement, X= Exposure of a group to an experimental event

Participants

The study recruited 30 children, 15 children in experimental group (N=15), and 15 children in control group (N=15). All children (14 girls, 16 boys; age $M_{control}$ = 78.15 month, age $M_{experimental}$ = 77.57 month) were included in the analysis. Participants' families gave written consent to participate in both groups (experimental and control group).

The standard mathematics achievement test developed by Fidan, (2013) and the arithmetic performance test (Devos, 1992) were used to determine the participants. The standard mathematics achievement test and the arithmetic performance test were applied to 138 first-year students, and the students who were in the lower 35% of both tests constituted the research study group. After the application and evaluation of both tests, a total of 32 students were determined as the participants of the research. These students were randomly assigned to the control and experimental groups. One student from the experimental group and one student from the control group left because they did not want to continue the experimental process. As a result, there were 15 first-year students in each group. In addition, one student in the experimental group has a speech and language disorder (SLD)¹, while one of the two students in the control group has a speech and language disorder (SLD), while the other student has attention deficit and hyperactivity disorder (ADHD)².

Pre- and posttest measures

Three tests were administered to obtain information about children's knowledge of numbers, estimation, and arithmetic skills. The detailed features of the three tests, namely *Mathematics achievement test (MAT)*, *Arithmetic performance test (APT), and Number Line Estimation Test (NLE)*, are explained in Fidan (2013), Olkun et al. (2013), and Olkun and Sarı (2016) respectively. The order of the presentation to each child was the same. The KR-20 reliability coefficient of the MAT was .96, while the current reliability coefficient of the test has been calculated as .91. The original KR-20

¹ The student was already diagnosed SLD, by an expert, speech and language therapist, and the family and the school was informed.

² The student was already diagnosed ADHD by a psychiatrist.

reliability coefficient of the test wad .95 under time constraint (Olkun et al., 2013). The current study found the KR-20 coefficient as .94. The reliability measures of the NLE were, .75, .66, and, .72 for 0-10, 0-20, and 0-100 number lines respectively.

Intervention

In the research, different games were developed for the control and experimental groups. These are the Sahibingo[©] games played in the experimental group and the Animalbingo games played in the control group.

Game 1: Composed of the Turkish abbreviation of number sense bingo, Sahibingo is a game that aims to improve the approximate number system (ANS) in human cognition (Figure 1a). There is a visual design in the game that will support the perception of important concepts such as the size of the number, its distance from other numbers, its location and order. In other words, the cards used in the Sahibingo game are linearly equally spaced as on the mental number line and are given in order of magnitude from left to right (Figure 1a). Thus, the child learns the relative magnitude of any number together with its positional relationship with other numbers. It allows the child to perceive the number not just as a symbol, but as a concept with magnitude. The game has been developed for number lines between 0-10 and 0-100. Playing cards and checkers in the range of 0-100 are in red. Each player gets one of the playing cards as in Figure 1a. Each player draws the game pieces in a bag in turn. The number on the drawn checker (Figure 1b) is spoken aloud, and the player who has the number on the checker takes that checker and puts it on the number on his own card. Whichever player completes the numbers on his/her cards wins the game. The Sahibingo game was designed by the researchers based on the relevant literature on numerical cognition.



a) Sahibingo playing cards

b) Sahibingo game checkers

Figure 1. Sample Images from Sahibingo Game

Game 2: The Animalbingo game was designed for the control group as opposed to the Sahibingo game in the experimental group. A linearly designed playing card has animal figures on it, in place of numbers. These animal figures are closed by matching the children using the checkers in a bag. Each player in turn draws a checker from the checkers in the bag (Figure 2b), and the animal figure in the drawn checker is on whose card that player takes that checker and puts it on the animal figure in the card. The player who finds all the animal figures on his card wins the game. This game in the control group was designed as a non-numerical version of the game in the experimental group. Playing cards are in two different colors, blue and red (see Figure 2a) and each color card consists of 6 different playing cards in itself. The Animalbingo game was designed by the researchers to liken the game used in the experimental group except the numerical aspects to best reveal the effect of relevant content (numerical in the experimental group and nonnumerical in the control group).



a) Animalbingo playing cards



b) Animalbingo game checkers

Figure 2. Sample images from Animalbingo Game

Procedure

The experimental process was carried out in the primary school, where the children were educated. The games designed for both the control and experimental group were played during the time deemed appropriate by the parents and the classroom teacher. All participants were first informed about the nature and structure of the experiment. Parents of the participants signed the informed consent form.

The experimental group played the games every week on Thursday, and the control group students played the games every week on Friday. Students in both control and experimental groups played the games in groups of three. In an empty classroom at the school, the students sat at the round tables in threes and played 6 different game cards. The experimental process lasted for 30 minutes per week for 4 weeks for each group. After the experimental process was completed, the number line test was immediately applied to the students. However, the mathematics achievement test and arithmetic performance test were applied to the students in both the control and experimental groups 4 weeks later.

Ethical Procedures

The Research Ethics Committee of the Nevşehir HBV University (date and number = 27.03.2023, 2023.03.85) authorized the recruitment, tasks, and overall procedure. The research was conducted on a voluntary basis and no reward or money was paid to the participants

Results

Table 2 represents the comparisons of the Mathematics Achievement (MAT) and Arithmetic Performance (APT) of Control and Experimental Groups. Results of Independent Samples t-test Analysis of Pre-test – Post-test Difference Scores in Mathematics Achievement Test showed no significant difference [t(28)=1.345; p>.05]. In other words, the games played in both the control and experimental groups did not have a significant effect on increasing students' mathematics achievement. Similarly, when the arithmetic performances of the students in the control and experimental groups are

compared in terms of group variable, there is no significant difference (t(28)=,702; p>.05). It can be said that the games played in both the control and experimental groups did not have a significant effect on increasing the students' arithmetic performances.

Table 2. Arithmetic Performance and mathematics achievement of experimental and control groups. results of independent samples t-test analysis of pre-test – post-test difference scores

Variable	Group	Ν	\overline{X}	SD	DF	t	p*
Mathematics	Exp.	15	3,53	2,61	20	1,345	190
Achievement	Control	15	2,33	2,26	20		,109
Arithmetic Performance	Exp.	15	4,87	3,89	28	702	/88
	Control	15	3,73	4,89	20	,702	,400

Independent Samples t-test Analysis of Pre-test - Post-test Difference Scores of Control and Experimental Groups for MNL-10, MNL-20 and MNL-100 are depicted in Table 3. Results showed significant differences between the control and experimental group students' Total Absolute Errors (TAE) in MNL-10 (t(28)= -6.437; p< .05), in favor of experimental group. In other words, the experimental group significantly improved their estimation skills in the number line range of 0-10 (X = -2.15) as opposed to the control group (X=0.33). Similarly, when the pre-test - post-test mean score difference of the total absolute errors in MNL-20 is compared, we see that there is a significant group. That means, the experimental group significantly improved the control group (X=0.67). Finally, the comparison of the pre-test – post-test mean score difference of the total absolute errors in MNL-10 group (X=0.67). Finally, the comparison of the pre-test – post-test mean score difference of the total absolute errors in MNL-10 group (X=0.67). Finally, the comparison of the pre-test – post-test mean score difference of the total absolute errors in MNL-100 yielded no significant difference between the control and experimental groups (t(28)= .811; p>.05). That means, the games played in the experimental group did not have a significant effect on the improvement in estimation skills in the number line range of 0-100.

Table 3. Independent samples t-test analysis of pre-test - post-test difference scores of experimental and control groups for MNL-10, MNL-20 and MNL-100

Variable	Group	Ν	\overline{X}	SD	DF	t	p*
MNL-10	Exp.	15	-2,15	1,33	- 28	6 127	001
	Control	15	,33	,678	20	-0,437	,001
MNL-20	Exp.	15	-1,37	1,14	- 28	2 717	001
	Control	15	,67	,972	28	-3,/1/	,001
MNL-100	Exp.	15	-,01	,593	- 28	011	424
	Control	15	,25	1,11	- 28	-,011	,424



Figure 3. The comparison of the mathematics achievement test mean scores of the experimental group, control group and students with special education needs

Looking at Figure 3, in which the scores obtained from the experimental design are depicted in terms of students with special education needs, it can be said that Student 1 improved his/her mathematics achievement score as much as other groups, while Student 2 and Student 3 in the control group did not change at all.

As seen in Figure 4, the improvement in Student 1's APT score from pre-test to post test is wider than that of Student 2 and Student 3, who are located in control group



Figure 4. The comparison of the arithmetic performance test mean scores of the experimental group, control group and students with special education needs

As depicted in Figure 5, the improvement in Student 1's MNL10 TAE score from pre-test to post test is wider than that of Student 2 and Student 3, who are located in control group. Student 1 improved his/her estimation skill on MNL10 as much as the students in the experimental group.



Figure 5. The comparison of the MNL-10 TAE difference scores of the experimental group, control group and students with special education needs

As shown in Figure 6, the improvement in Student 1's MNL20 TAE score from pre-test to post test is much wider than that of Student 2 and Student 3, who are located in control group. Student 1 improved his/her estimation skill on MNL20 much more than the students in the experimental group too.



Figure 6. The comparison of the MNL20 TAE difference scores of the experimental group, control group and students with special education needs

As indicated in Figure 7, the improvement in Student 1's MNL100 TAE score from pre-test to post test is much wider than that of Student 2 and Student 3, who are located in control group. Student 1 improved his/her estimation skill on MNL100 much more than the students in the experimental group too.



Figure 7. The comparison of the MNL100 TAE difference scores of the experimental group, control group and students with special education needs

Discussion

The summary of the results indicates that the games played in both the control and experimental groups did not have a significant effect on increasing neither the arithmetic performances nor the mathematics achievement of the students. On the other hand, the games played in the experimental group significantly improved the estimation skills in the number line range of 0-10 and 0-20, however they did not have a significant effect in the number line range of 0-100. In terms of students with special education needs, Student 1 in the experimental group improved his/her mathematics achievement score as much as other students, while Student 2 and Student 3 in the control group did not change at all. The improvement in Student 1's arithmetic performance score from pre-test to post test is wider than that of Student 2 and Student 3. The improvements in Student 1's MNL-10, MNL-20 and MNL-100 TAE scores from pre-test to posttest are much wider than that of Student 2 and Student 3, who are located in control group. Student 1 improved his/her estimation skill on MNL100 much more than the students in the experimental group too.

It seems that it is possible to improve number line estimation skills in a short-term training (30 minutes per week, for 4 weeks for each group=2 hours total) with relevant games such as Sahibingo[®]. On the other hand, such a short-term training do not seem to be enough for an improvement in neither the arithmetic performance nor the mathematics achievement scores. Students with special education needs (SLD) also benefited from the experimental games, however; students (1 SLD, and 1 ADHD) who played the control games did not improve in any of the measured numerical skills.

As in the current study, children's showing a significant decrease in total absolute error averages and a significant increase in linearity after the number line game designed to develop the approximate number system is consistent with the findings in the literature (e.g. Elofsson et al., 2016; Kohn et al., 2020; Kucian et al., 2011; Ramani & Siegler, 2008; Sarı & Olkun, 2020; Sarı & Olkun, 2023; Siegler & Ramani, 2008; 2009; Wilson et al., 2006). Siegler and his colleagues, showed in their research that linearly designed board games improved the number line estimations of young children (dyscalculic and typically developing) even after a short-term training phase (Siegler & Ramani, 2008; 2009; Ramani & Ramani, 2008; Siegler, 2008). These results have been replicated with both digital and board games at different times and studies (e.g. Elofsson et al., 2016; Kohn et al., 2020; Kucian et al., 2011; Sarı & Olkun, 2020; Sarı & Olkun, 2023; Wilson et al., 2006). There are also opposing findings that number line games do not show significant improvements in children's ANS performance (e.g. Wilson et al., 2009). Wilson and his colleagues retested the "Number Race" game, which they had previously used and contributed to the development of number line performance, with a reading game in the control group. After a total of two hours of play time with 5 to 6 year-old children from low socio-economic status, the children showed improvement in various numerical skills, but there was no improvement in ANS performance. The reasons for these opposing findings may include sample differences (homogeneity and heterogeneity), duration of playing the games, content and scope of the games (directly aiming to improve number line performance, including basic numerical competencies such as magnitude comparison and quick counting) along with number line training in the game content.

Remarkable improvements in both typically developing and SLD students occurred in number line estimation skills only in the number range of 0-10 and 0-20, but not in 0-100 range. This finding may be explained with the fact that the participants of this study (first graders) are more familiar with numbers from zero the 20 but less with 20 to 100. According to Gallagher-Mitchell et al. (2018), the scale of the number line presented to children and adults varies according to age. Younger children are presented with smaller scales (e.g., 0-10, 0-20, or 0-100), and older children and adults are presented with larger scales (e.g., 0-1000). This reflects the expected extent of their number familiarity. Therefore, one of the factors affecting accuracy in number line estimation performances is familiarity with numbers (Ebersbach et al., 2008; Ebersbach et al., 2015; LeFevre et al., 2013; Sullivan & Barner, 2014). As children become familiar with a particular numerical range (e.g. 0-10, 0-20, 0-100), their predictions improve within that range. However, this situation may differ for unfamiliar number line ranges (e.g. 101-1000) (Sullivan & Barner, 2014).

Another important finding of the research is that improvements did not reflect in arithmetic performance and mathematics achievement scores. We have different hypotheses on the possible reasons for the lack of improvement in either the arithmetic performance or mathematics achievement scores in the current study. The first of these, in fact, no improvement in mathematics achievement and arithmetic performance was not a surprise because of the fact that the games used in both the experimental and control group did not include any curricular demands contained in the tests to measure them, namely MAT and APT. Relevant games to the curricular demands of arithmetic performance and mathematics achievement may produce results that are more positive. The game used in the experimental group is more relevant to different aspects of number sense, such as the relative magnitude and neighborhood of numbers, symbolic number reading etc. Although most studies have shown that precision of ANS acuity is a predictor of future mathematics achievement (Decarli et al., 2023; Halberda et.al., 2008; Libertus et. al., 2011; Mazzocco et al., 2011; Starr et al., 2013), and even though it is accepted that symbolic arithmetic is built on more primitive quantitative representations (Starr et al., 2013), there is also evidence that games designed to develop number sense do not improve mathematical performance (e.g., Hellstrand et al., 2020; Räsänen et al., 2009). For example, in the study conducted by Hellstrand and colleagues (2020), no statistically significant improvement was seen in the mathematical performance of low-achieving children in the experimental group after training with the Number Race game. Therefore, opposing findings in studies may also be due to methodological differences. For example, in our research, a cut-off score of 35% was used to identify low-achieving children. In addition, the experimental and control groups had a heterogeneous structure and came from a middle-level socio-economic families. Siegler and Ramani's (2009) study revealed that playing numerical board games helped children from low socioeconomic backgrounds, developed a better concept of numerical magnitude, which later helped them make more precise predictions on the number line. It would be beneficial to take these situations into consideration in future studies.

Our second hypothesis in the current study is that, although there was an increase in the arithmetic performance of the groups after number line training, why no significant difference was observed between the experimental and control groups. There is evidence in the literature showing that the approximate number system and exact number systems of individuals with dyscalculia do not function properly (Landerl et al., 2004). It is claimed that subitizing forms the basis of the development of the concept of number (Schindler et al., 2020) and is an important requirement for arithmetic learning (Libertus, 2019; Schindler et al., 2020). For this reason, there is an ongoing debate about whether arithmetic operations are performed with the ENS or the ANS. However, there is evidence that both systems are used (Cohen & Dehaene, 2000). For example, while students who have internalized number facts and can operate with any two numbers give a single answer as expected (such as 7+6=13), students with dyscalculia can give more than one answer (such as 7+6=12 or 14). This is an indication that the ANS as well as the ENS is involved. As a result of internalization over time, it is possible to recall the correct answer (Sarı & Olkun, 2020). Another reason why no significant difference was observed between the experimental and control groups on arithmetic performance may be that children with mathematics difficulties depend too much on working memory resources (cf. Nelwan et.al., 2022). Because in our research, a timed (1 minute for each of four operations) arithmetic performance test was used. As a matter of fact, during the application of the arithmetic performance test by the researchers, it was observed that the children in the experimental and control groups made finger counting. Nelwan (2022) also claims that individuals with mathematics learning disabilities follow a different developmental model and use their ANS abilities less during arithmetic operations, and therefore may be more dependent on working memory resources during mathematical tasks (Nelwan et al., 2022). It would be useful to test these claims in future studies.

Another possibility of the non-improvement might be the shorter period of training. Although it has been shown that short-term number line training shows improvements in young children's ability to estimate the number line, compare magnitudes, count, recognize numbers, and learn new arithmetic problems (cf. Siegler & Ramani, 2008; 2009; Ramani & Siegler, 2008). There are also studies (e.g., Hellstrand et al., 2020; Sarı & Olkun, 2020) in which short-term number line training does not have a significant effect on mathematics performance. A possible reason for the contrasting findings due to differences in playing time may be individual differences. The experimental process of our study was conducted with heterogeneous groups. Considering that mathematics learning disability is a heterogeneous disorder resulting from individual differences in development or function at the neuroanatomical, neuropsychological, behavioral and interactional levels (Kaufmann et al., 2013; Skagerlund & Träff, 2016), each child may need different educational periods. Relatively longer period of training may cause improvement in estimations skills as well as arithmetic skills and math achievement in general. Future research may investigate the effect of longer periods, more specific arithmetic games, and may include other students with special needs.

The findings of this study also showed that it is possible to design games to improve different aspects of numbers, such as representing numbers as analog magnitudes, based on individual students' specific needs. The games can be used individually, in small groups, as well as in the classroom. Since games are more engaging for the students than traditional instruction, they are especially relevant for routine and boring tasks such as learning number facts. In addition, it will be beneficial to structure the experimental processes by considering the hypotheses put forward in our study in future research.

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