Journal of Research in Economics • İktisat Araştırmaları Dergisi • Cilt: 7 / Issue • Sayı: 2 October • Ekim 2023 • ss/pp. 110-130
ISSN: 2636-8307 • DOI: http://dx.doi.org/10.29228/JORE.27

ARAŞTIRMA MAKALESİ / RESEARCH ARTICLE

THE UNEXPECTED SHORTFALL: AN ALTERNATIVE RISK MEASURE*

Ekrem KILIÇ**

Abstract

The international prudential regulation standard – the Basel standards – introduces a substantial change to its market risk framework. The change is part of a comprehensive revision of the standard to address the weaknesses discovered during the global financial crisis (GFC) of 2008. One of the key changes is the replacement of Value-at-Risk (VaR) with Expected Shortfall (ES) as the primary risk measure in the framework. By incorporating the tail events, ES partially answers the concerns raised about the VaR during the GFC. However, ES as well lacks a mechanism to extrapolate the historical shocks. This paper proposes an alternative measure – unexpected shortfall (US) – which aims to serve as a better safety barrier for financial institutions. Based on the evidence from 3 conventional currency pairs (EUR/USD, USD/TRY, EUR/TRY) and 1 cryptocurrency pair (BTC/USD), the new measure displayed violations in a reasonably close range of the expected values and backtest analyses suggested that the incurred excessive losses for US are less than both VaR and ES.

Keywords: Market Risk, Capital Adequacy, Value at Risk, Expected Shortfall, Basel IV, FRTB
Jel Classification: G170, G180, G280

1. Introduction

The global financial crisis (GFC) of 2008 has become one of the most significant turning points in the financial markets. The collapse of the US housing market has been followed by the bankruptcy of some major US banks such as Lehman Brothers and Bear Sterns. After such big failures in the market, the crisis has quickly spread globally. Although the first wave of the crisis has eased out by 2010, the global recession after the crisis has been prolonged for almost a decade.

Aftermath of the GFC, the failures in the financial system initiated a series of new regulations and rules. The Basel Committee on Banking Supervision (BCBS) also responded by introducing some new capital requirements as well as starting a comprehensive review of the existing ones. In 2016, after eight years from the crisis, an extensive review on minimum capital requirements for market risk – better known as the Fundamental Review of Trading Book (FRTB) – has been concluded with a

* This paper was adapted from a part of Ekrem Kılıç's PhD thesis "Post-crisis Changes in Capital Adequacy Regulations for Market Risk: the Impact of Basel IV" to be submitted at Marmara University, Department of Economics.

** Wolters Kluwer Financial Services (UK) Ltd, E-mail: ekrem.kilic@wolterskluwer.com, ORCID: 0000-0002-0045-426X

Submission Date: 25.12.2022
Revision Date: 07.07.2023
Acceptance Date: 11.07.2023

revised framework. Among others, one of the major changes of the new framework is the replacement of the Value-at-Risk (VaR) based internal model approach with Expected Shortfall (ES).

Starting from 1990s, VaR emerged as being the standard risk measure for risk practitioners and eventually it become the main risk measure of Basel framework with 1996 amendments. Although the technique initially introduced for market risk, it has been applied to all types of risks. However, especially during the GFC, the perception of VaR has been dramatically changed and VaR started to be considered as an inadequate or even misleading measure of risk. One source criticism to VaR was about its insensitivity to tail events. VaR as a measure gives a false sense of security as if in the case of a financial shock, the loss would be as much as VaR itself. However, VaR rather represents a threshold, or a truncation point of the tail, so the loss that exceeds the VaR may be quite larger than the VaR value itself. This is more likely especially if the financial portfolio contains derivative or highly leveraged financial products. The seconds popular criticism to VaR, also known as Black Swan theory, suggested that using the historical price movements to forecast the future losses is deemed to fail in case there is an unprecedent shock occurs. In its original form as pioneered by Taleb (2009), the Black Swan theory may imply that it is simply not possible to measure such risks. However, at least it indicates that a better risk measure should not rely solely on the historical changes, in some way it should also extrapolate and potentially amplify the historical losses.

ES addresses the first criticism by simply replacing the notion of worst loss at a certain significance level, with the value of the losses which exceeds the VaR. In this way, ES is sensitive to the extreme values in the tail. However, similar to VaR, ES as well doesn't extrapolate the price movements, instead aims to represent what the historical return distribution implies as the potential loss.

This study proposes a new risk measure aims to extrapolate the risk by modelling directly the violations and their magnitude. The second section of the study explains the VaR and ES methodologies used as well as the new measure, unexpected shortfall. The third section delivers an empirical comparison of different risk measures. Finally, the last section summarises the findings of this study.

2. Methodology

VaR is a measure of the worst loss on a financial portfolio of assets due to changes in the risk factors over a given time horizon and at a confidence level. From a statistical point of view, VaR can be derived using the inverse of the cumulative distribution function (CDF) of the returns.

$$V_t^{\alpha} = -\Phi^{-1}(\alpha|\mathfrak{I}_t)$$

where V_t^{α} is VaR at time and at a significance level α, $\Phi^{-1}(.)$ is the inverse of the conditional cumulative distribution function of the returns, $\mathfrak{I}_t$ is the information set available at time *t*.

ES, on the other hand, is the expected value of the losses exceeding VaR and can be formulated as follows;

$$ES_t^{\alpha} = -\mathbb{E}(r_{t+1} | r_{t+1} < -V_t^{\alpha})$$

where ES_t^{α} is ES at time *t* and at a significance level α, $\mathbb{E}(.)$ is the expected value operator and r_{t+1} is the portfolio return at *t*+1.

2.1. Parametric Approach

The first group of VaR/ES models assumes the profit/loss distribution of the portfolio follows a parametric distribution. If we assume the portfolio/loss distribution follows a normal distribution, the estimation of VaR simply requires fitting the normal distribution to the historical data. The normal distribution is defined by mean and the standard deviation;

$$r_t \sim N(\mu_t, \sigma_t)$$

where μ_t is the mean and σ_t is the standard deviation. Therefore, to fit the normal distribution to a data set, one needs to estimate these two parameters. Referring to the market efficiency hypothesis, it is also quite common to assume $\mu = 0$; in this case, VaR calculation would only require the estimation of standard deviation.

After fitting the normal distribution, VaR can be calculated as;

$$V_t^{\alpha} = z_{\alpha} \cdot \sigma_t \cdot \mathbb{V}_t$$

where z_{α} is the threshold value for significance level α calculated using the inverse CDF of normal distribution and $\mathbb{V}_t$ is the present value of the financial assets at time .

Since standard deviation or the volatility is the key element of the parametric approach, a difference in the volatility estimation between VaR calculations imply a different parametric VaR model. This study reports the results of the parametric VaR uses moving average (MA), exponentially moving average (EWMA) and generalised autoregressive heteroscedacity (GARCH) (Bollerslev, 1986) volatility estimators.

Under the normal distribution assumption ($N(0, \sigma_t)$), similar to VaR, ES can also be calculated analytically as;

$$ES_t^{\alpha} = -\frac{\sigma_t}{1-\alpha} \cdot \phi(\Phi^{-1}(\alpha)) \cdot \mathbb{V}_t$$

where $\phi(.)$ probability density function and $\Phi^{-1}(.)$ is the inverse cumulative density function.

2.2. Simulation-based Approaches

The second class of VaR/ES calculation approaches is the simulation-based approach which relies on scenario creation and evaluation for generating the profit/loss distribution. Therefore, the general algorithm for all methods under this category shares the following common steps;

i. Generate the market scenario with the corresponding fx rates, yield curves etc.
ii. Calculate the value change of the portfolio under each scenario
iii. Calculate the desired percentile of the value changes (profit/loss) for VaR
iv. Calculate the mean of the losses exceeding VaR for ES

Then the differences between alternative simulation-based approaches arise in the way how a certain step above is implemented.

2.2.1. Historical Simulation

The main assumption of the historical simulation is that the best representation of the possible market scenarios that may occur in the next time horizon is the past movements of the risk factors.

For calculating under the historical simulation method, first, the past risk factor movements need to be calculated. Then the series of the past returns can be defined as;

$$\{x_{i,n}\} = \left\{\ln\left(\frac{p_{i,t-n}}{p_{i,t-k-n}}\right)\right\}_{n=0}^{h-1}$$

where $\{x_{i,n}\}$ is the return series of the i-th risk factor, $p_{i,t}$ is the market value of the i-th risk factor (e.g. FX rate) at time *t, k* is the time horizon for the returns and *h* is the number of scenarios. Then the scenario series;

$$\{s_{i,n}\} = \{p_{i,T}e^{x_{i,n}}\}_{n=0}^{h-1}$$

where $\{s_{i,n}\}$ is the series of scenarios for the i-th risk factor projecting *k*-days ahead and $p_{i,T}$ is the most recent value of the i-th risk factor. The next step is to ccalculate implied profit loss for each scenario. Then, the profit/loss series can be defined as;

$$\{R_n\}_{n=0}^{h-1} = \{V(\boldsymbol{s}_n) - \mathbb{V}_t\}_{n=0}^{h-1}$$

where $\{R_n\}$ is the series of profit or losses projecting *k*-days ahead, $\boldsymbol{s}_n$ series of vectors contain scenario values of each risk factor, *v*(.) valuation function and $\mathbb{V}_t$ is the current value of the portfolio.

Finally, VaR, then, is the desired percentile of the profit/loss series;

$$V_t^{\alpha} = -\mathbb{P}(\{R_n\}, \alpha)$$

where $\mathbb{P}(.)$ is the percentile function and α is the significance level. ES, under the historical simulation method, can be calculated by taking the average of the losses that exceed;

$$ES_t^{\alpha} = -\frac{\sum_{n=0}^{h} R_n I_{V_t^{\alpha}}(R_n)}{\sum_{n=0}^{h} 1_{V_t^{\alpha}}(R_n)}$$

where $I_{V_t^{\alpha}}(.)$ Is defined as follows;

$$I_{V_t^{\alpha}}(r) := \begin{cases} 1, & r \leq -V_t^{\alpha} \\ 0, & r > -V_t^{\alpha} \end{cases}$$

Although the historical simulation is a relatively straightforward way to calculate VaR, one of the main disadvantages is having few observations in the profit/loss distribution tails. For instance, if is estimated using 252 scenarios (with 1-year data) and under 99% significance level; 1% tail of the distribution will be represented by the worst 3 scenarios. Therefore, if the dataset covers a relatively low volatile period, then it will tend to underestimate the risk. Similarly, if the dataset is of a high volatility period, would overestimate the risk. In addition, the historical simulation implicitly treats all the scenarios equally likely to occur in the next time horizon; therefore, if there is a significant volatility increase in the market, the historical simulation is slow to respond.

2.2.2. Filtered Historical Simulation

Hull and White (1998) introduced a volatility adjustment scheme for the realisation of the daily changes in the risk factors. Following a similar notion, Barone-Adesi et al. (1999) developed a more general form of the same adjustment mechanism for n-day changes and called this method as Filtered Historical Simulation (FHS).

In line with the conditional heteroscedasticity models, under the FHS method, the changes in the risk factors are standardised using the volatility estimate of that day;

$$e_t = \frac{x_t}{\sigma_t}$$

where e_t is the standardised return, x_t is the change in the risk factor at time *t* and σ_t is the volatility estimate at time *t*. Then, the adjusted changes can be derived as;

$$x_t^* = \sigma_T e_t$$

where x_t^* is the volatility adjusted return derived using the return at time t and σ_T is the current estimate of volatility. Finally, the adjusted returns are translated into scenarios;

$$\{s_{i,n}\} = \{p_{i,T} e^{x^*_{i,n}}\}_{n=0}^{h-1}$$

The rest of the calculation follows the same steps as the standard historical simulation method described in the previous subsection.

2.2.3. Weighted Historical Simulation

Another method that attempts to improve the performance of the historical simulation is the weighted historical simulation (WHS) developed by Boudoukh et al. (1998). The WHS method follows the same steps for generating scenarios and deriving the profit/loss distribution as the standard historical simulation. However, in the calculation of the percentile, opposed to the regular percentile calculation where implicitly all the scenarios are treated as equally likely, WHS implements a time-dependent weighting scheme.

WHS uses a decaying weighting for the scenarios based on the date of the realised returns used in the derivation of that particular scenario. The weight of each scenario in WHS is defined as;

$$w_t = \frac{1-\lambda}{1-\lambda^T}\lambda^{T-t}; \qquad for\ t = 1 \dots T$$

where λ is the decay factor. Boudoukh et al. (1998) compare results of WHS using two values for the decay factor; 0.97 and 0.99. In this study, the results are calculated by setting the decay factor to 0.99.

2.2.4. Bootstrapped Historical Simulation

Bootstrapping is a technique in statistics for increasing the sample size by replacing the existing observations and re-sampling. The use of bootstrap techniques in VaR was relatively less popular, mainly due to the fact that re-shuffling of the observations makes better sense when VaR is calculated for a time horizon longer than 1-day. Zenti and Pallotta (2000) have implemented bootstrapping for multi-day time horizons in one of the early studies. Since the new Basel IV increases the calculation time horizon for VaR/ES to 10-days; the use of the Bootstrapped Historical Simulation (BHS) might become more popular.

Under the BHS method, the daily returns constitute the pool of samples. Then, for creating an n-day scenario, n daily returns are randomly drawn from the pool; and converted to n-day returns (in the case of logarithmic returns simply by summation).

2.2.5. Monte Carlo Simulation

The Monte Carlo simulation method for VaR/ES as well follows the same notion and incorporates stochastic processes for modelling the movement of the risk factors. Apart from the way how the scenarios are generated, the Monte Carlo simulation for VaR/ES follows the same steps as the historical simulation.

There are several different stochastic processes developed for different classes of risk factors. However, for the asset prices such as commodities, stocks and FX rates, the most commonly used model is the geometric Brownian motion (GBM) [1] which is shown in the following equation;

$$\hat{x}_{t+\delta} = \left(\mu - \frac{1}{2}\sigma^2\right)\delta + \sigma\varepsilon\sqrt{\delta}$$

where $\hat{x}_{t+\delta}$ is the simulated return from t to $t+\delta$, δ , is the time increment with $0 < \delta < 1$, μ is the average return of the risk factor, σ is the volatility of the risk factor, and ε is random with $\varepsilon \sim N(0,1)$.

Then a scenario price that is produced by the process can be shown as;

$$s_{t+\delta} = s_t e^{\hat{x}_{t+\delta}}$$

By repeating the same process $1/\delta$ times, the scenario process for the next day (or next time interval) can be produced. Therefore, after $1/\delta$ iterations, one scenario can be produced for VaR/ES calculation; however, by repeating the whole experiment many times, thousands of scenarios can be generated. In this study, a business day is represented by 100 iterations, therefore δ is set to 1/100.

2.3. New Measure: Unexpected Shortfall

ES delivers a better risk buffer than VaR by taking to account the possibility of extreme events in the tail of the profit loss distribution. However, ES as well limited to the observed historical data and does not extrapolate the magnitude of the potential losses. As per its definition, ES deals with the expected value of the losses exceeding VaR, therefore does not aim to project the unexpected.

The shortfall, by definition, is observed on the violation days. Therefore, the realised shortfalls can be used for modelling the distribution of the shortfall conditionally on a VaR estimate. However, considering the heteroscedasticity of VaR and ES, instead of the nominal shortfall, it would be preferable to define the shortfall as a magnitude relative to VaR. Then the shortfall magnitude can be defined as;

1 Glasserman (2004) can be seen for extensive review of stochastic processes in finance and particularly for the derivation of GBM.

$$SM_t = 1 - \frac{R_t}{V_t^{\alpha}} \qquad for\ R_t < -V_t^{\alpha}$$

By definition *SM*, will take values from zero to positive infinity. Intuitively, it is expected that *SM* follows a left-skewed size distribution. In the context of credit risk, CreditRisk+ [2] framework uses gamma distribution to model the severity multiplier of the losses (Gordy, 2002) which is conceptually very similar to SM . The actuarial models as well use gamma distribution for modelling the size of the losses (Kleiber & Kotz, 2003). If we assume *SM* follows a gamma distribution, then we can calculate a VaR like unexpected shortfall magnitude (USM) at a certain significance level.

$$USM_t = -\Gamma^{-1}(k, \theta, \gamma)$$

where k and θ are the shape and scale parameters of the gamma distribution and γ is the significance level of *USM*. The unexpected shortfall magnitude indicates the highest shortfall magnitude under the chosen significance level. The final step is to convert this magnitude of shortfall to the nominal loss amount, then the unexpected loss can be shown as;

$$US_t = (1 + USM_t) \cdot V_t^{\alpha}$$

The current BCBS framework requires the banks to calculate the VaR at a 1% significance level. Basel IV, on the other hand, mandates an ES calculation under 2.5% significance level. As far as US is concerned, there are 2 confidence levels involved; the first one for the underlying VaR, and the second is the significance level of the measure itself.

In this study, 2 sets of confidence levels are considered. The first set uses Basel IV's 2.5% significance level assumption for shortfall observations and combines it with 1% for US. The second set uses, 5% and 1% respectively.

3. Model Validation and Comparison

3.1. Backtesting

Since there are many methodologies to calculate VaR, validation of VaR is essential and an integral part of the capital adequacy framework. The process to measure the performance of VaR estimates and validate their statistical significance is called backtesting. The procedure involves the comparison between VaR estimate and the realised profit/loss.

Backtesting methodologies check if the violation series – derived from VaR and the realised profit/loss series – fits into the expected statistical properties of such series. A violation occurs when the realised loss is larger than VaR calculated for a certain day. Then, the violation series can be defined as;

2 CreditRisk+ is a well-known Credit VaR methodology introduced by Credit Suisse.

$$\{I_t\}_{t=0}^{H} = I_t = \begin{cases} 1, & R_t \leq -V_t^{\alpha} \\ 0, & R_t > -V_t^{\alpha} \end{cases}$$

where I_t is an indicator function, R_t is the realised return at time *t*, *H* is the number of days used in backtesting and V_t^{α} is VaR calculated for time *t*. Then the expected value of the I_t is equal to α, the significance level of VaR.

Kupiec (1995) introduced a statistical test focusing on the distance between the observed density of the violations and the assumed density. Christoffersen (1998) improved the idea by introducing a likelihood ratio test of conditional coverage which would be applied to the smaller samples. The likelihood ratio of both unconditional coverage (UC) and conditional coverage (CC) follows a Chi distribution asymptotically with degrees of freedom 1 and 2 respectively.

3.2. Reality Check

Basel IV mandates the use of VaR for validating ES-derived capital adequacy numbers, but the horizon of the calculation is not consistent with the backtest either. In the literature, unfortunately, this aspect seems to be neglected and no study investigated backtesting VaR or ES results with a longer horizon (i.e 10-day). Considering, that the capital requirement aims to be a buffer for the potential losses, in this study, the results are compared against both the realised return after 10-days and the maximum loss born in the same time interval.

4. Empirical Analysis

4.1. Data

Sample data consists of 3 pairs of FX rates and 1 crypto currency price from January 2014 to March 2022. The FX rates comprise 1 hard currency pair (EUR/USD) and 2 emerging market pairs (USD/TRY & EUR/TRY). On the other hand, the cryptocurrency analysed is BTC (BTC/USD). All the data is obtained from Yahoo Finance.

Table 1 shows the descriptive statistics of all 4 data sets used. The sample data represents different volatility profiles. Both the standard deviation and the range of the returns indicates that EUR/USD is less volatile while TRY rates shows higher volatility and BTC is the most volatile. This allows comparing the performance of the models under both high and low volatility. TRY pairs exhibit positive skewness and therefore more extreme values on the positive tail of the distribution.

Table 1: Descriptive Statistics of Daily Logarithmic Returns

	EUR/USD	USD/TRY	EUR/TRY	BTC/USD
Mean	-0.0089%	0.0973%	0.0871%	0.2439%
Median	-0.0088%	0.0365%	0.0386%	0.2001%
St. Dev.	0.4875%	1.2596%	1.2539%	3.8827%
Min	-2.7752%	-18.8638%	-17.8132%	-37.1695%
Max	2.8545%	22.7990%	20.9004%	25.2472%
Skewness	(0.0370)	1.7485	1.2498	(0.1357)
Excess Kurtosis	3.1957	83.2896	66.1116	7.2491
Count	2,149	2,149	2,150	2,753

The descriptive statistics of 10-daily logarithmic returns are shown in Table 2. In general, 10-daily returns exhibit the same features as the daily returns. However, the mean and the medians seem to diverge from zero for all series except EUR/USD. As it would be expected, another difference is that the standard deviations for all return series are higher.

Table 2: Descriptive Statistics of 10-Daily Logarithmic Returns

	EUR/USD	USD/TRY	EUR/TRY	BTC/USD
Mean	-0.0886%	0.9774%	0.8749%	2.5204%
Median	-0.0883%	0.5685%	0.5045%	1.4584%
St. Dev.	1.4552%	4.0406%	3.9717%	12.8479%
Min	-7.1710%	-21.9982%	-21.9372%	-45.0320%
Max	5.7399%	42.4045%	38.4444%	82.3086%
Skewness	(0.1098)	2.4040	2.0679	0.5393
Excess Kurtosis	1.4720	19.8108	16.0880	1.8901
Count	2,140	2,140	2,141	2,744

4.2. Backstest Result

The results in this section are calculated using 1512 daily VaR calculations at 99% significance level and 1-day liquidity horizon. Therefore, the expected value of the number of violations is 15.12.

Table 3 displays the likelihood ratio test results for EUR/USD. Considering a 10% significance level for the likelihood ratio test, 6 out of 11 models can pass the test for both unconditional coverage and conditional coverage tests. The historical simulation model can pass the likelihood ratio test only for conditional coverage.

Table 3: LR Tests of EUR/USD VaR 99% with 1-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	27	7.64	0.57%	8.66	1.31%
FHS GARCH	26	6.51	1.07%	7.45	2.41%
FHS MA	24	4.47	3.45%	5.28	7.15%
HIS	23	3.58	5.86%	4.32	11.54%
MC EWMA	21	2.06	15.12%	2.68	26.18%
MC GARCH	25	5.45	1.96%	6.32	4.24%
MC MA	21	2.06	15.12%	2.68	26.18%
PAR EWMA	19	0.93	33.49%	1.44	48.69%
PAR GARCH	22	2.77	9.59%	3.45	17.80%
PAR MA	21	2.06	15.12%	2.68	26.18%
WHS	10	1.99	15.85%	2.14	34.38%

The likelihood ratio test results for USD/TRY are shown in Table 4. Majority of the models pass the LR tests for USD/TRY. The historical simulation model passes the unconditional coverage test, but fails the conditional coverage. It is also interesting to note that FHS MA models fails due to having more violations than expected, but WHS contrarily fails as it has too few violations.

Table 4: LR Tests of USD/TRY VaR 99% with 1-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	19	0.93	33.49%	1.44	48.69%
FHS GARCH	17	0.23	63.38%	0.64	72.74%
FHS MA	24	4.47	3.45%	5.23	7.33%
HIS	21	2.06	15.12%	6.66	3.58%
MC EWMA	11	1.25	26.31%	1.43	48.95%
MC GARCH	19	0.93	33.49%	1.44	48.69%
MC MA	16	0.05	82.17%	0.41	81.28%
PAR EWMA	10	1.99	15.85%	2.14	34.38%
PAR GARCH	17	0.23	63.38%	0.64	72.74%
PAR MA	16	0.05	82.17%	0.41	81.28%
WHS	6	7.20	0.73%	7.26	2.65%

The results for EUR/TRY data display more violations, and therefore less models pass the unconditional coverage test. However, 9 out of 11 models pass the conditional coverage test. This is largely because the violation numbers do not diverge largely from the expected values and independence criteria is satisfied.

Table 5: LR Tests of EUR/TRY VaR 99% with 1-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	23	3.58	5.86%	4.32	11.54%
FHS GARCH	14	0.09	76.94%	0.37	83.26%
FHS MA	22	2.77	9.59%	3.45	17.80%
HIS	25	5.45	1.96%	17.57	0.02%
MC EWMA	9	2.93	8.71%	3.05	21.80%
MC GARCH	17	0.23	63.38%	2.02	36.42%
MC MA	21	2.06	15.12%	2.68	26.18%
PAR EWMA	9	2.93	8.71%	3.05	21.80%
PAR GARCH	17	0.23	63.38%	2.02	36.42%
PAR MA	19	0.93	33.49%	1.44	48.69%
WHS	3	14.63	0.01%	14.65	0.07%

The results for BTC/USD are displayed in Table 6. None of the VaR models can pass both of the likelihood ratio tests. Only WHS pass the conditional coverage test, however the rest of the models yielded quite large numbers of violations.

Table 6: LR Tests of BTC/USD VaR 99% with 1-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	33	15.97	0.01%	20.34	0.00%
FHS GARCH	25	5.45	1.96%	6.10	4.73%
FHS MA	32	14.41	0.01%	19.09	0.01%
HIS	31	12.92	0.03%	14.95	0.06%
MC EWMA	39	26.53	0.00%	26.58	0.00%
MC GARCH	45	39.00	0.00%	39.16	0.00%
MC MA	44	36.80	0.00%	38.67	0.00%
PAR EWMA	35	19.26	0.00%	19.35	0.01%
PAR GARCH	34	17.58	0.00%	17.70	0.01%
PAR MA	36	20.99	0.00%	22.22	0.00%
WHS	8	4.09	4.32%	4.18	12.34%

In the second part of this section, the likelihood ratio test results for VaR under 97.5% significance level and with a 10-day liquidity horizon are displayed. Considering there are 1512 data points, the expected value of the number of violations is 37.8 instances.

EUR/USD results exhibit dramatical differences from the 1-day VaR results calculated at 99%. Only FHS GARCH, MC EWMA and BHS models pass the unconditional coverage test and all models fail the conditional coverage test.

Table 7: LR Tests of EUR/USD VaR 97.5% with 10-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	50	3.67	5.53%	146.04	0.00%
FHS GARCH	40	0.13	71.96%	21.33	0.00%
FHS MA	50	3.67	5.53%	173.75	0.00%
HIS	55	7.05	0.79%	237.06	0.00%
MC EWMA	31	1.34	24.79%	67.02	0.00%
MC GARCH	27	3.51	6.10%	14.42	0.07%
MC MA	106	85.40	0.00%	317.96	0.00%
PAR EWMA	28	2.86	9.09%	66.72	0.00%
PAR GARCH	25	5.04	2.48%	17.16	0.02%
PAR MA	108	89.76	0.00%	330.62	0.00%
WHS	23	6.89	0.86%	93.61	0.00%
BHS	35	0.22	64.05%	137.91	0.00%

USD/TRY results are also parallel to EUR/USD. FHS GARCH and BHS models pass the unconditional coverage test. However, none of the models passes the conditional coverage test.

Table 8: LR Tests of USD/TRY VaR 97.5% with 10-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	49	3.12	7.75%	131.21	0.00%
FHS GARCH	37	0.02	89.48%	34.99	0.00%
FHS MA	49	3.12	7.75%	148.76	0.00%
HIS	56	7.85	0.51%	212.53	0.00%
MC EWMA	2	60.71	0.00%	60.71	0.00%
MC GARCH	18	13.16	0.03%	24.37	0.00%
MC MA	56	7.85	0.51%	157.13	0.00%
PAR EWMA	1	67.25	0.00%	67.25	0.00%
PAR GARCH	15	18.22	0.00%	47.28	0.00%
PAR MA	55	7.05	0.79%	150.96	0.00%
WHS	22	7.95	0.48%	77.37	0.00%
BHS	35	0.22	64.05%	148.61	0.00%

The models performed slightly better with EUR/TRY data. Half of the models pass the unconditional coverage test and produced violation numbers reasonably close to the expected value. However, again all the models failed in the conditional coverage test.

Table 9: LR Tests of EUR/TRY VaR 97.5% with 10-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	41	0.27	60.30%	117.36	0.00%
FHS GARCH	39	0.04	84.41%	38.57	0.00%
FHS MA	37	0.02	89.48%	111.09	0.00%
HIS	52	4.91	2.68%	206.67	0.00%
MC EWMA	1	67.25	0.00%	67.25	0.00%
MC GARCH	15	18.22	0.00%	39.07	0.00%
MC MA	47	2.13	14.40%	174.13	0.00%
PAR EWMA	1	67.25	0.00%	67.25	0.00%
PAR GARCH	14	20.17	0.00%	42.28	0.00%
PAR MA	43	0.70	40.18%	168.65	0.00%
WHS	26	4.24	3.96%	91.64	0.00%
BHS	37	0.02	89.48%	120.48	0.00%

BTC/USD data confirms the same message. This time WHS passes the unconditional coverage test, but none of the models pass the conditional coverage.

Table 10: LR Tests of BTC/USD VaR 97.5% with 10-Day Liquidity Horizon

VaR Method	Violations	UC Test	UC Prob	CC Test	CC Prob
FHS EWMA	56	7.85	0.51%	132.56	0.00%
FHS GARCH	92	57.28	0.00%	144.97	0.00%
FHS MA	62	13.36	0.03%	130.29	0.00%
HIS	69	21.31	0.00%	268.04	0.00%
MC EWMA	28	2.86	9.09%	75.39	0.00%
MC GARCH	57	8.68	0.32%	48.00	0.00%
MC MA	114	103.30	0.00%	353.58	0.00%
PAR EWMA	15	18.22	0.00%	47.28	0.00%
PAR GARCH	53	5.58	1.81%	39.28	0.00%
PAR MA	91	55.44	0.00%	293.72	0.00%
WHS	43	0.70	40.18%	95.40	0.00%
BHS	58	9.54	0.20%	187.28	0.00%

4.3. Reality Check Results

In this section, VaR and ES results are compared using the realised shortfall. The results display the accumulated difference between the risk measure and the realised loss when the loss exceeds the amount predicted by the measure. Since the required base liquidity horizon is 10 days, the losses are derived using 10-day returns.

If the bank would have used the risk measure as a direct indicator of the capital buffer without any adjustments, how much loss would have been accumulated during the analysis period (last 1250 observation).

Table 11: Cumulative Realised Shortfall with 10-Day Returns

		EUR/USD	USD/TRY	EUR/TRY	BTC/USD
FHS EWMA	VaR @99	-5.51%	-11.16%	-9.79%	-79.91%
	ES @97.5	-5.15%	-10.67%	-10.65%	-88.53%
FHS GARCH	VaR @99	-4.44%	-14.62%	-16.49%	-231.10%
	ES @97.5	-4.78%	-14.45%	-16.02%	-191.70%
FHS MA	VaR @99	-5.16%	-16.80%	-18.12%	-102.53%
	ES @97.5	-5.60%	-15.92%	-18.19%	-101.96%
HS	VaR @99	-21.82%	-92.07%	-89.47%	-245.37%
	ES @97.5	-21.14%	-90.30%	-90.87%	-245.96%
MC EWMA	VaR @99	-1.27%	0.00%	0.00%	-44.64%
	ES @97.5	-0.85%	0.00%	0.00%	-41.51%
MC GARCH	VaR @99	-3.44%	-7.89%	-8.23%	-146.93%
	ES @97.5	-3.38%	-7.16%	-7.09%	-140.71%
MC MA	VaR @99	-17.95%	-19.47%	-31.79%	-288.80%
	ES @97.5	-16.89%	-17.65%	-30.06%	-277.03%
PAR EWMA	VaR @99	-0.64%	0.00%	0.00%	-15.38%
	ES @97.5	-0.62%	0.00%	0.00%	-14.51%
PAR GARCH	VaR @99	-2.86%	-5.86%	-7.13%	-113.00%
	ES @97.5	-2.76%	-5.68%	-6.93%	-110.73%
PAR MA	VaR @99	-15.52%	-15.95%	-27.55%	-180.74%
	ES @97.5	-15.16%	-15.52%	-26.96%	-176.00%
WHS	VaR @99	0.00%	-8.03%	-8.62%	-59.85%
	ES @97.5	-2.09%	-37.54%	-26.84%	-109.23%
BHS	VaR @99	-13.95%	-38.21%	-27.69%	-211.16%
	ES @97.5	-13.75%	-37.05%	-26.07%	-212.03%
FHS EWMA	VaR @99	-5.51%	-11.16%	-9.79%	-79.91%
	ES @97.5	-5.15%	-10.67%	-10.65%	-88.53%

VaR@99, VaR at 99% significance level; ES@97.5, ES at 97.5% significance level.

Although the BCBS requires banks to calculate the risk measure for a liquidity horizon of 10-days, the 10-day returns don't reflect the worst loss during each 10-day period. Table 12 reports the cumulative shortfalls calculated using the maximum loss. The results indicate that the potential losses during the liquidity horizon could be quite larger than the simple return value.

Table 12: Cumulative Realised Shortfall with 10-Day Min-Max Returns

		EUR/USD	USD/TRY	EUR/TRY	BTC/USD
FHS EWMA	VaR @99	-8.81%	-176.46%	-243.12%	-361.21%
	ES @97.5	-8.45%	-165.55%	-240.36%	-354.50%
FHS GARCH	VaR @99	-26.05%	-89.64%	-84.16%	-511.34%
	ES @97.5	-25.98%	-73.80%	-82.29%	-406.15%
FHS MA	VaR @99	-21.93%	-313.71%	-250.28%	-1013.44%
	ES @97.5	-21.33%	-288.70%	-250.90%	-988.50%
HS	VaR @99	-19.48%	-652.86%	-584.25%	-1056.47%
	ES @97.5	-18.89%	-621.74%	-568.03%	-1071.18%
MC EWMA	VaR @99	-1.94%	-18.97%	-21.09%	-79.89%
	ES @97.5	-1.75%	-18.24%	-19.49%	-74.02%
MC GARCH	VaR @99	-16.89%	-38.93%	-38.54%	-285.09%
	ES @97.5	-16.12%	-37.76%	-37.18%	-273.47%
MC MA	VaR @99	-40.60%	-301.39%	-255.95%	-1483.90%
	ES @97.5	-37.75%	-290.08%	-249.37%	-1448.46%
PAR EWMA	VaR @99	-1.35%	-9.47%	-9.90%	-22.44%
	ES @97.5	-1.27%	-9.00%	-9.43%	-21.36%
PAR GARCH	VaR @99	-14.71%	-35.56%	-34.28%	-225.41%
	ES @97.5	-14.33%	-34.93%	-33.56%	-221.50%
PAR MA	VaR @99	-36.89%	-271.47%	-228.15%	-1148.59%
	ES @97.5	-36.12%	-267.21%	-223.80%	-1131.80%
WHS	VaR @99	-16.39%	-563.72%	-524.59%	-886.77%
	ES @97.5	-15.34%	-598.10%	-545.05%	-1013.94%
BHS	VaR @99	-15.37%	-475.13%	-383.86%	-1201.36%
	ES @97.5	-15.18%	-467.11%	-375.13%	-1160.55%
FHS EWMA	VaR @99	-8.81%	-176.46%	-243.12%	-361.21%
	ES @97.5	-8.45%	-165.55%	-240.36%	-354.50%

Note: VaR@99, VaR at 99% significance level; ES@97.5, ES at 97.5% significance level.

4.4. Unexpected Shortfall Results

In this section, the unexpected shortfall results are reported. The results consist of the outcome of 2 significance level pairs.

Table 13 shows the US results for EUR/USD. Shortfall column displays the total of the realised losses exceeding the US for the entire analysis period of 1250 days. The number of US violations, similarly, shows the number of days where the realised loss was larger than the estimated US. Since, the US results are calculated at 99% significance level, it is expected 1 loss will exceed US out of 100 VaR violations. Therefore, when the VaR exhibit less violation, the expected number of the US violations will also be smaller. The expected number of US violations for the US measure which uses 97.5% VaR estimates is 0.3125. On the other hand, the US measure which uses 95% VaR has twice as many expected US violations, 0.625. In the light of these expected values, PAR MA result seems to produce too many US violations. The other models produce less than or equal to 2 US violations.

Table 13: US Results for EUR/USD

Model	US Sig Level	Shortfall	# of US Violation
FHS EWMA	US@97.5/99	0.00%	0
	US@95/99	-0.04%	1
FHS GARCH	US@97.5/99	0.00%	0
	US@95/99	-0.21%	1
FHS MA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
HS	US@97.5/99	-0.28%	1
	US@95/99	-1.28%	2
MC EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
MC GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
MC MA	US@97.5/99	-0.07%	2
	US@95/99	-0.47%	2
PAR EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR MA	US@97.5/99	-0.02%	1
	US@95/99	-0.41%	5
WHS	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
BHS	US@97.5/99	0.00%	0
	US@95/99	-0.08%	1

Note: US@97.5/99, US using VaR results at 97.5% significance level and calculated at 99% significance level; US@95/99, US using VaR results at 95% significance level and calculated at 99% significance level.

Both USD/TRY and EUR/TRY results for US produced the US violations less than or equal to 2 violations. The results also exhibit the expected feature of having less US violations for US@97.5/99 results.

Table 14: US Results for USD/TRY

Model	US Sig Level	Shortfall	# of US Violation
FHS EWMA	US@97.5/99	-0.12%	1
	US@95/99	-0.56%	2
FHS GARCH	US@97.5/99	0.00%	0
	US@95/99	-0.31%	1
FHS MA	US@97.5/99	0.00%	0
	US@95/99	-0.40%	1
HS	US@97.5/99	0.00%	0
	US@95/99	-0.99%	1
MC EWMA	US@97.5/99	0.00%	0

	US@95/99	0.00%	0
MC GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
MC MA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR MA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
WHS	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
BHS	US@97.5/99	0.00%	0
	US@95/99	0.00%	0

Note: US@97.5/99, US using VaR results at 97.5% significance level and calculated at 99% significance level; US@95/99, US using VaR results at 95% significance level and calculated at 99% significance level.

Table 15: US Results for EUR/TRY

Model	US Sig Level	Shortfall	# of US Violation
FHS EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
FHS GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
FHS MA	US@97.5/99	0.00%	0
	US@95/99	-0.21%	2
HS	US@97.5/99	-0.01%	1
	US@95/99	-1.50%	2
MC EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
MC GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
MC MA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR GARCH	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR MA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
WHS	US@97.5/99	0.00%	0
	US@95/99	-1.16%	2
BHS	US@97.5/99	0.00%	0
	US@95/99	0.00%	0

Note: US@97.5/99, US using VaR results at 97.5% significance level and calculated at 99% significance level; US@95/99, US using VaR results at 95% significance level and calculated at 99% significance level.

The US measure produces promising results for BTC/USD as well. Again, all the violations are within the expected range. One major difference, however, is in the aggregate loss amount. Although, BTC/USD exhibits few US violations as the other dataset, the magnitude of the losses is much larger.

Table 16: US Results for BTC/USD

Model	US Sig Level	Shortfall	# of US Violation
FHS EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
FHS GARCH	US@97.5/99	-10.51%	1
	US@95/99	-11.09%	1
FHS MA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
HS	US@97.5/99	-7.88%	1
	US@95/99	-8.84%	2
MC EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
MC GARCH	US@97.5/99	-9.26%	1
	US@95/99	-12.40%	1
MC MA	US@97.5/99	-0.99%	1
	US@95/99	-3.68%	2
PAR EWMA	US@97.5/99	0.00%	0
	US@95/99	0.00%	0
PAR GARCH	US@97.5/99	-6.81%	1
	US@95/99	-10.38%	1
PAR MA	US@97.5/99	-0.35%	1
	US@95/99	-1.53%	1
WHS	US@97.5/99	-2.92%	1
	US@95/99	-3.78%	1
BHS	US@97.5/99	0.00%	0
	US@95/99	-0.69%	1

Note: US@97.5/99, US using VaR results at 97.5% significance level and calculated at 99% significance level; US@95/99, US using VaR results at 95% significance level and calculated at 99% significance level.

5. Conclusion

The Basel IV reforms replace VaR with ES as the primary risk measure for the internal model approach. In this study, a capital measure unexpected shortfall is proposed as an alternative. The replacement of VaR with the ES addresses the problems related to the fat tails. However, similar to VaR itself, ES as well forecasts the risks as much as it has been observed in the past. In addition, the capital requirement amount is expected to provide a cushion for financial institutions. Neither VaR nor ES is directly translated into a capital requirement. Because, by definition, both measures are expected to fail a number of times that is not acceptable for a safety cushion. Therefore, BCBS uses an arbitrary mechanism to transform the VaR results into capital requirement amounts. This calculation makes use of an average of the last 60 days of VaR, stressed VaR and some multipliers. Basel IV as well defines an arbitrary process for deriving the capital requirement from the ES results.

The proposed risk measure, the unexpected shortfall, addresses both of these issues. First, it allows extrapolating the violation information gathered from the VaR and forecasts the magnitude of upcoming shortfalls. Secondly, it delivers a measure suitable for the use of capital requirements without arbitrary transformations. In this study, the results for 99% US based on 95% VaR and 99% US based on 97.5% are displayed. The expected violation for these US estimates is 0.025% and 0.05% respectively. In a real capital requirement context, more conservative significance levels might be selected to reduce the possibility of a US violation near impossible. However, for the sake of demonstrating the approach, less conservative significance levels are selected, to allow observing US violations.

The results for US are very promising. Almost all the models generated US violations in a reasonably close range of the expected values. Although the underlying VaR analysis displayed a large number of violations for a 10-day liquidity horizon, the US measure in a way fixed the inaccuracy of the model and delivered a reasonable capital cushion. Similarly, the US measure seems to work with BTC/USD data as good as the other datasets. However, the magnitude of the US violation for BTC/USD is relatively higher than the others.

The findings in this study suggest that the replacement of VaR by ES may partially address the problems faced in the GFC. However, the new capital framework doesn't bring completely a new way of measuring the risk. In fact, the findings indicate that the results produced by both measures are quite parallel in many cases. This thesis also proposes an alternative way of approaching the problem; rather than modelling only the profit loss distribution, modelling the distribution of shortfall magnitude brings a new dimension.

References

Barone-Adesi, G., Giannopoulos, K., & Vosper, L. (1999). VaR without correlations for portfolios of derivative securities. *Journal of Futures Markets*, 583-602.

Basel Committee on Banking Supervision. (1996). *Amendment to the capital accord to incorporate market risks*. Bank for International Settlements.

Basel Committee on Banking Supervision. (2016). Minimum capital requirements for market risk. Bank for International Settlements.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, 121-131.

Boudoukh, J., Richardson, M., & Whitelaw, R. (1998). The best of both worlds. *Risk*, 64-67.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International economic review*, 841-862.

Glasserman, P. (2004). *Monte Carlo methods in financial engineering*. New York: Springer.

Gordy, M. B. (2002). Saddlepoint approximation of CreditRisk+. *Journal of banking & finance*, 26(7), 1335-1353.

Hull, J., & White, A. (1998). Incorporating Volatility Updating Into The Historical Simulation Method for Value At Risk. *Journal of Risk*, 5-19.

J.P. Morgan. (1996). *RiskMetrics – Technical Document* (4th Edition).

Kleiber, C., & Kotz, S. (2003). *Statistical size distributions in economics and actuarial sciences*. John Wiley & Sons.

Kupiec, P. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *The Journal of Derivatives*, 3(2).

Taleb, N. (2009). *The Black Swan*. Random House.

Zenti, R., & Pallotta, M. (2000). Risk analysis for asset managers: Historical simulation, the bootstrap approach and value at risk calculation. Available at SSRN 251669.