Teaching Multiplication Based on Realistic Mathematics Education to Students at Risk of Mathematics Learning Difficulties[1]

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Abstract

This study aimed to develop, implement, and evaluate a Realistic Mathematics Education (RME)-supported instructional design for primary school students with Mathematics Learning Difficulties (MLD) risk who had difficulty in multiplication at the third-grade level. In this study, a design-based research method was utilized since it was aimed to apply and assess the RME-supported instructional design to primary school students with MLD risk. The study selected the participants from primary school students with mathematics learning difficulties who had difficulty in number concepts and operations with natural numbers at the third-grade level. Participant students were determined by using the criterion sampling method. As data collection tools, the Numbers and Operations Mathematics Achievement Test (NOMAT), and Learning Disabilities Symptom Screening Test, curriculum-based assessment, interview, and researcher diary were employed. The content analysis method was integrated into the study to analyze the data obtained from qualitative data collection tools. Descriptive statistics methods were used to analyze the data obtained from quantitative data collection tools. The data obtained from the research is presented under three themes: findings on analysis, design, and development, findings on implementation, and findings on evaluation. The results of the research show that teaching with RME-supported instructional design is effective in students' multiplication instruction and contribute to their permanent learning.

Keywords: At Risk of Mathematics Learning Difficulties, Primary School, Realistic Mathematics Education, Multiplication Teaching

Introduction

International reports indicate that, on average, one in every two children in the world does not meet the minimum proficiency level in mathematical skills (United Nations [UN], 2018). In recent years, particularly in developing countries, students who do not meet minimum proficiency levels in mathematical skills have increased (The World Bank [WB], 2018). In Turkey, the percentage of students who do not meet minimum proficiency levels in mathematical skills exceeds the Organisation for

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Economic Co-operation and Development (OECD) averages (Ministry of National Education [MEB], 2019). When the state of mathematics in Turkey was examined in the Programme for International Student Assessment (PISA) 2018, approximately 37% of the students performed at or below the first level. Students who do not meet the minimum proficiency levels in mathematical skills are at risk of experiencing math learning difficulties, which can have negative consequences for students.

Students with difficulty acquiring fundamental mathematical skills and low mathematics performance are defined as students at risk of MLD (Hellstrand et al., 2019). The prevalence of students with MLD varies between 4% and 8%, and these students have difficulties acquiring fundamental mathematical skills (Hellstrand et al., 2019). Students who perform at or below the twenty-fifth percentile in standardized mathematics tests are identified as at risk of MLD (Geary, 2011). Studies defining students with MLD, including students at risk of MLD, generally used cut-off scores ranging from 25% to 50% (Bryant et al., 2021; Geary, 2013). On the other hand, longitudinal studies have reported that, on average, 30% of students in primary school have MLD (Zhang et al., 2020).

In the mathematics education program implemented in Turkey, multiplication follows addition and subtraction instruction (MEB, 2018). Similar teaching processes are adopted in the mathematics curricula of European countries, the United States, and Israel (Rotem & Henik, 2020). In Turkey, instruction on multiplication begins in the second grade of elementary school. In multiplication instruction for second-grade students, the concept of multiplication as repeated addition is initially taught. Then, the commutative property of multiplication and the effects of one and zero in multiplication are emphasized (MEB, 2018). In the third grade, multiplication instruction covers multiplication tables, multiplying two-digit numbers by two-digit numbers, and multiplying three-digit numbers by one-digit numbers (MEB, 2018). Additionally, students encounter multiplication involving place value and digit value in the third grade, which introduces them to more complex multiplication concepts.

When students at risk of MLD are not supported with instructional interventions appropriate to their characteristics, they lag behind their peers academically (Bender, 2016; Zhang et al., 2020). Therefore, students at risk of MLD should be supported with effective instruction considering their developmental levels and individual characteristics (Van de Walle et al., 2018). For effective mathematics teaching to students at risk of MLD, it is necessary to develop intervention programs appropriate to students' current learning levels and individual characteristics, to make use of various teaching strategies, and to support teaching with instructional materials (Gersten et al., 2009). In addition, there is a need for instructional designs that allow students to learn at their pace and in which students construct knowledge themselves (Gersten et al., 2009; Re et al., 2014). Such designs can ensure that students at risk of MLD receive education without lagging behind their typically developing peers.

The real-life problems used in the RME approach can help the students at risk of MLD integrate their school-based learning into real-life situations, providing a valuable contribution to their education. In the RME approach, starting teaching with real-life problems increases students' academic achievement and makes teaching more enjoyable (Laurens et al., 2017). RME supports teachers in helping students improve their maths skills (Hasibuan et al., 2018). Because RME provides students with the tools for solving real-life problems, critical thinking, and discussion (Juandi et al., 2022). In the RME approach, teachers guide students and enable them to realize their mathematics learning (Altun, 2015). In addition,

thanks to the interaction built among students during the teaching process, students are enabled to develop different solutions to the problem (Widjaja & Heck, 2003).

The intervention studies generally focused on teaching mathematics topics such as the number concept (Aunio et al., 2021; Clarke et al., 2019), basic four operations (Koponen et al., 2018), problem-solving (Lucangeli et al., 2019), and fractions (Wang et al., 2019). In intervention studies for teaching mathematics topics, the effectiveness of instructional interventions such as explicit instruction (Aunio et al., 2021), strategy instruction (Rodriguez et al., 2019), computer-assisted instruction (Hellstrand et al., 2019), and self-regulated instruction (Ennis & Losinski, 2019; Wang et al., 2019) were tested. Since each student with MLD has different characteristics, more studies using different instructional strategies are needed (Monei & Pedro, 2017). It can be stated that students with MLD were preferred as the participant group in the studies. Studies on teaching mathematics to students with MLD risk have been limited (Jitendra et al., 2013).

In the literature, there is no study on the effect of the RME approach on the mathematics teaching process of students with MLD risk. In this study, the instructional design prepared based on RME was implemented and evaluated for students with MLD risk. This design study is important in terms of being an original design that can be used in teaching mathematics to students with MLD risk. This study is important as it is the first to test the effectiveness of the RME approach in teaching mathematics to students with MLD risk. The RME-supported instructional design is thought to support students in the subjects in which students with MLD risk have difficulty. In addition, RME-supported instructional design is expected to guide teachers and researchers in supporting the difficulties of students with MLD risk in multiplication.

Aim and Research Questions

This study aimed to develop, implement, and evaluate an RME-supported instructional design for primary school students with MLD risk who had difficulty in multiplication at the third-grade level. In line with this purpose, the research questions are given below. For primary school students with MLD risk who have difficulty in multiplication at the third-grade level,

1. What kind of instructional design can be developed?
2. How does the developed RME-supported instructional design affect student learning?
3. What are the evaluation results of the implemented RME-supported instructional design?

Method

In this study, a design-based research method was utilized. Design-based research is a study in which a new product is designed, the designed product is implemented, and its effectiveness and efficiency are evaluated (Richey & Klein, 2014). In this study, a design-based research method was employed since it was aimed to apply and assess the RME-supported instructional design to primary school students with MLD risk. In addition, design-based research was preferred in this study to revise the instruction by student characteristics and provide an opportunity for cyclical implementation. Design-based research is generally designed by considering the stages of analysis, design, development, implementation, and evaluation. In this context, the research was shaped in four stages.

In the first stage, the student's individual learning needs were identified by determining the subjects in which students with MLD risk had difficulty in multiplication. Expert (one researcher each from the fields of special education and classroom education) and teacher (n = 24) opinions were taken, and the Numbers and Operations Mathematics Achievement Test (NOMAT) (Author & Author, 2022) was used to determine the learning needs. In the second stage, an RME-supported instructional design was developed. RME-supported instructional design was developed using the ADDIE instructional design model to support the difficulties experienced by students in teaching multiplication to students with MLD risk. The suitability of the developed instructional design for students with MLD risk was evaluated by taking expert opinion (one expert from the fields of mathematics education and curriculum design). In the third stage, the developed RME-supported instructional design was applied to students with MLD risk in two design cycles. At this stage, the effectiveness and efficiency of the instructional design were evaluated through interviews (teacher, student, and parent), NOMAT, and researcher diary. In the fourth stage, the evaluation determined the effect of the RME-supported instructional design on the teaching of multiplication. The assessment used interviews (teacher, student, and parent) and follow-up tests (NOMAT).

Participants

The study selected the participants from primary school students with mathematics learning difficulties who had difficulty in number concepts and operations with natural numbers at the third-grade level. The participating students were determined by using the criterion sampling method. It was taken into consideration that the students did not have difficulty in reading and writing skills or had no disability or disease. In addition, criteria such as having difficulty in the concept of numbers and operations with natural numbers were considered. Participating students were identified after obtaining permission from Trabzon University Social and Human Sciences Research and Ethics Committee (81614018-000-E.525) and Bayburt Provincial Directorate of National Education (E-13616634-605.01-19029017). More than one assessment tool was used to determine the students participating in the first and second implementation cycles. The sample numbers (n) expressed in determining the participating students are given in parentheses as the first and second implementations. Firstly, NOMAT was administered to third-grade students (n = 63; n = 68) in three primary schools randomly selected from a province in Northeast Anatolia. The test scores were ranked from the highest to the lowest, and the students in the bottom 25% (n = 16; n = 17) were determined. Secondly, face-to-face interviews were conducted with the classroom teachers of the students in the bottom 25%. After the interviews, the students who had good reading and comprehension skills developed writing skills but had difficulties in operations with numbers and natural numbers and were still attending school (n = 8; n = 10) were identified. In the third stage, classroom teachers while asked to fill in the Learning Disability Symptom Screening Test for students limited by the test and interview, respectively. These lists were completed separately for each student by the classroom teachers with the support of the families and evaluated by the researchers. In the last stage, the classroom teachers and the families of the students were interviewed again, and the students who did not have difficulty in reading, reading comprehension, and writing skills but had difficulty in number concepts and operations with natural numbers (n = 5; n = 6) were determined. Participant students were selected from these students to participate in the implementation cycles (n = 3, n = 2).

Table 1

Demographic characteristics of participating students

	Code name	Gender	Age	Formal diagnosis	Health problem	Special situation
First application cycle	Ege	Male	9	None	None	Low economic income
	İrem	Female	9	None	None	-
	Ceyda	Female	9	None	None	Lives separately from her father
Second application cycle	Deniz	Male	10	None	None	Grade repetition
	Emre	Male	9	None	None	Hyperactivity

When Table 1 is analyzed, the first implementation cycle was carried out with two female and one male student, and the second was carried out with two male students. The participants were students in the same age group attending the third grade. Only one student was ten years old because he repeated the first grade.

Research Process

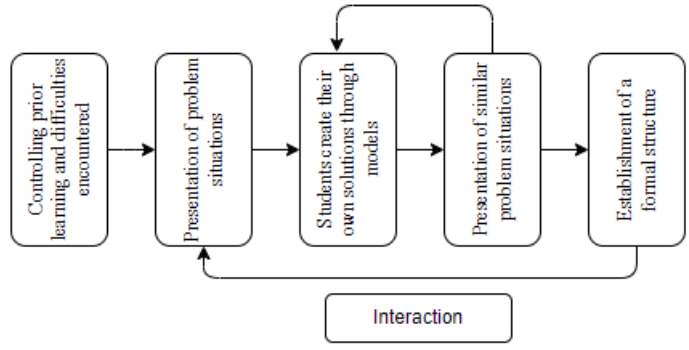
Analysis. Within the scope of the needs analysis, the individual learning needs of students with MLD risk and the difficulties they experienced in multiplication were determined. In this direction, firstly, a literature review was conducted to determine the learning needs of the students and the difficulties they experienced in multiplication. Secondly, face-to-face interviews were conducted with 26 classroom teachers. In the interviews, data was collected about topics such as the subjects the students had difficulties with, what kind of teaching was provided, and what kind of activities and materials were used in the teaching process. Thirdly, expert opinions were obtained from two expert lecturers, one of whom has studies on learning disabilities in the field of special education and one of whom has studies on mathematics learning disabilities in the field of primary school mathematics education. In addition, the assessment tools used to identify the participating students were used in the analysis phase.

Design. The ADDIE instructional design model was used to develop the RME-supported instructional design. For students with MLD risk, critical topics for teaching multiplication were determined. Due to the spiral structure of mathematics, second-grade level topics were also determined, considering that students with MLD risk might have had difficulties in previous grades. In the second grade level, multiplication (with single-digit numbers), and in the third grade, creating the multiplication table and multiplication (with two-digit and three-digit numbers) were determined.

Lesson plans were developed by considering RME-supported instructional design for teaching the determined topics to students with MLD risk. The stages of the lesson plan developed by considering the RME approach are presented below (See Figure 1).

Figure 1

Stages of an RME-supported lesson plan



The RME-supported lesson plan starts by checking the prior learning of students at risk of MLD. At this stage, the topics that students at risk of MLD have difficulty in multiplication are identified using curriculum-based assessment and NOMAT. Then, real-life problems are presented for the topics students have difficulty with. Students read the problem and express what they understand from the problem, and the lesson starts. In the next stage, students create and present their own products and structures through models related to the problem situation. In this step, students are guided by the teacher, and hints are provided when necessary. Students use real objects presented to them while creating their products and structures (informal solutions). Students who create informal solutions share their answers with their friends at the next stage (interaction), creating a discussion environment. The shared solutions are written on the board, and students' opinions are taken. In this way, it is ensured that students see different solutions along with their solutions. The interaction phase continues from the beginning to the end of the lesson in relation to the other phases. In the next stage, different problem situations similar to the real-life problem which is laid at the beginning of the lesson are presented. The same steps are followed, and the lesson is repeated. It is ensured that the students reinforce the lesson through similar problem situations. The student is expected to create a formal structure in the last stage. The student expresses the solution with numbers and symbols based on informal solutions. At the end of the course, the knowledge and skills that the student is supposed to acquire are measured and evaluated with curriculum-based assessment. The student who reaches the required achievement passes to the next lesson.

Development. In the development step, the suitability of the designed instructional model for students with mathematics learning difficulties was evaluated by taking expert opinions.

Implementation. The developed RME-supported instructional design was implemented in two cycles for students at risk of MLD. The first implementation cycle was implemented in a detached primary school in June 2021 in the central district of a province in Northeastern Anatolia. The selection of the school was influenced by its adequacy in terms of infrastructure and equipment and the positive attitude of the school administration. The first implementation cycle was carried out by the researcher in the support education room of the school with three students at risk of MLD and lasted for two weeks. The implementation was planned as 30-minute lesson sessions three days a week. The applications were implemented by the first researcher in the form of small-group teaching. The applications were carried

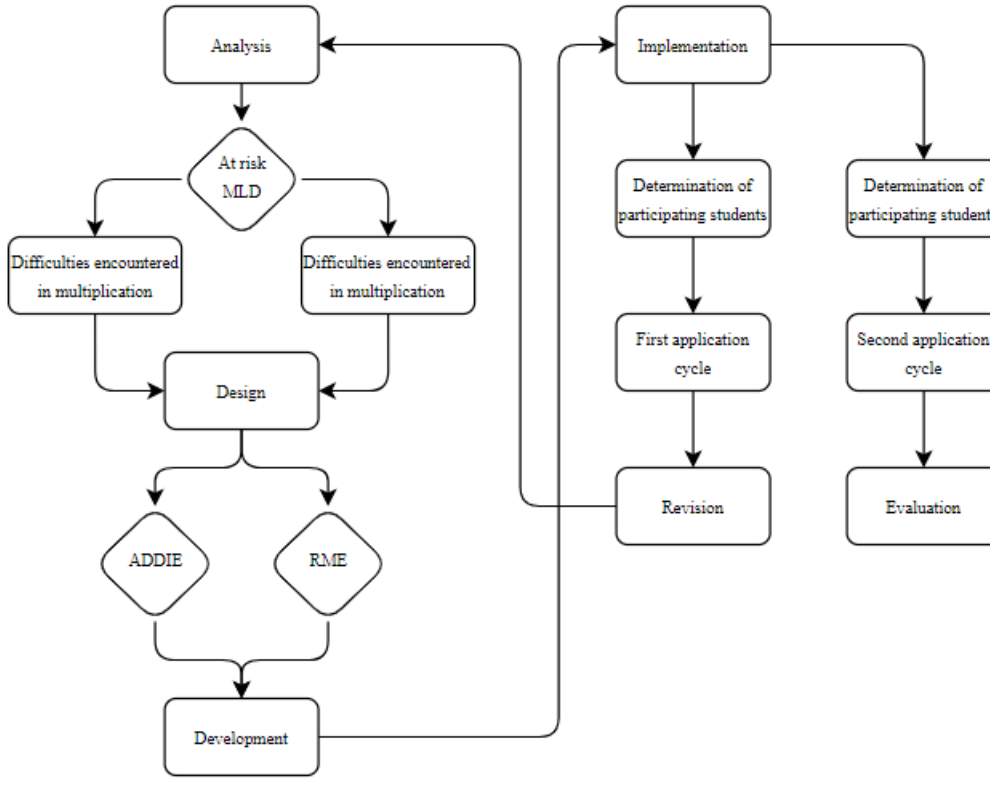
out simultaneously when the students had their maths lessons. Before the first application cycle, the subjects which the participants had difficulties with and their current situations were determined by applying NOMAT to them. After the application, NOMAT was used as a post-test, and the students were evaluated.

The second implementation cycle was implemented in a detached primary school in November 2021 in the central district of a province in Northeast Anatolia. The second implementation cycle was carried out by the researcher in the support education room of the school with two students at risk of MLD and lasted for two weeks. The implementation was planned as 30-minute lesson sessions three days a week. The applications were implemented by the first researcher in the form of small-group teaching. The applications were conducted after the lessons at school. Before the second implementation cycle, NOMAT was administered to the participants and the subjects whose difficulties and their current situations were determined. After the application, NOMAT was used as a post-test, and the students were evaluated.

Evaluation. For evaluating the RME-supported instructional design, semi-structured interviews were conducted with the students, their class teachers, and their parents. In addition, NOMAT was applied to students at risk of MLD as a follow-up test 5 weeks after the end of the intervention. The research process is summarised in Figure 2 below.

Figure 2

Flowchart for the research process



Instruments

Quantitative and qualitative data collection tools were used together in the data collection process to develop, implement, and evaluate an RME-supported instructional design for primary school students with MLD risk who had difficulty in multiplication at the third-grade level.

Numbers and Operations Mathematics Achievement Test (NOMAT)

The test was developed to identify students with MLD and MLD risk (Author & Author, 2022). In addition, it was also used to measure the achievement in the sub-learning areas of natural numbers and operations with natural numbers at the third-grade level of primary school. An item pool consisting of 47 questions was created by considering the subjects that students with MLD and MLD risk had difficulty in the sub-learning area of numbers and operations with numbers. As a result of the expert opinions received, a pilot application was made with 34 items. Accordingly, the test was applied to 171 people. Within the scope of item analysis, item difficulty, and discrimination indices were calculated for each test question. As a result of the analysis, the final test consisting of 21 questions was formed. The KR-20 reliability coefficient of the test was calculated as 0.93, and the Spearman-Brown value for the two halves was calculated as 0.86.

Learning Disability Symptom Screening Test (LDSST)

The test was adapted into Turkish by Korkmazlar (1993), taking into account the questionnaire used in the diagnosis and education of students with learning disabilities at The Developmental Centre (London). Later, the test was finalized by the Attention Deficit Hyperactivity and Specific Learning Disabilities Association by making additions to the test. The test consisted of 88 items in total in the sub-factors of academic achievement, arithmetic skills, reading and writing skills, organization skills, study habits, orientation skills, motor skills, sequencing skills, verbal expression skills, social-emotional behaviors, mobility, attention skills, visual, auditory and tactile perception. The items in the test are graded and scored as never (0), sometimes (1), often (2) and always (3). A student with an average score of 30 and above on the test is classified as a student with MLD risk. In addition, the Cronbach alpha value for the reliability of the test was calculated as 0.97. Finally, it was reported that all items in the test showed significant differences according to the lower and upper quartile groups.

Curriculum-Based Assessment (CBA)

In this study, curriculum-based assessment was used to determine the current knowledge of students with MLD risk in multiplication and to decide on the instruction to be provided.

Interview

Within the scope of the research questions, semi-structured interviews were conducted with various participant groups (experts, teachers, parents, and students) at different stages of the study. While preparing the questions asked in the interview, two faculty members from the primary school mathematics education field were consulted. A pilot interview was also conducted. The interviews with the experts were conducted online, while the other interviews were conducted face-to-face.

Various expert opinions were consulted at different stages of the study. In the expert interview conducted at the analysis stage, questions such as the characteristics of students with MLD risk, the difficulties experienced by students in multiplication, what kind of instructional design can be developed, how multiplication can be taught, and what the examples of activities and materials that can be used were asked. In the design and development phase, the suitability of the RME-supported instructional design in terms of content and design was evaluated. While coding the experts, the abbreviation EO was used, and a sequence number was given. For example, EO1 represents the first person whose expert opinion was taken.

Interviews were conducted with 26 classroom teachers at different stages of the study. Firstly, in the analysis phase, questions such as what the characteristics of students with MLD risk are, what difficulties they experience in multiplication, and what are the student characteristics were asked in the interview with the teachers. In the evaluation phase, the RME-supported instructional design applied to students with MLD risk was evaluated in line with the opinions of two classroom teachers. The evaluation revealed the contribution, usefulness, and applicability of the RME-supported instructional design to teach multiplication topics. While coding the teachers, the abbreviation TO was used, and a sequence number was given. For example, TO1 represents the first person whose teacher's opinion was taken.

Researcher Diary

The first researcher kept a researcher diary from the beginning to the end of the research process. The researcher's diary significantly contributed to the detailed explanation and reporting of the research. The researcher's diary was recorded online and backed up daily. The researcher's diary was written with the support of the notes kept by the first researcher and the recorded video footage. The researchers monitored the video recordings, identified the missed points, and made the necessary arrangements. While reporting the research process, the RME-supported teaching process was reflected naturally with direct quotations from the researcher's diary.

Data Analyses

In this study, qualitative and quantitative data collection tools were used together. The content analysis method was used to analyze the data obtained from qualitative data collection tools. In this sense, similar codes were brought together within the scope of categories and themes and organized and interpreted in a way the reader can understand. Firstly, the data obtained were transferred to a Word document and coded by the first researcher. Then, the second researcher reviewed the codes, and the inter-coder reliability between the two researchers was calculated. The inter-coder reliability score was calculated using the Miles and Huberman (1994) formula obtained by dividing the total number of agreements by the total number of opinions and multiplying by 100. Accordingly, the inter-coder reliability coefficient was calculated as 0.93. Descriptive statistics methods were used to analyze the data obtained from quantitative data collection tools.

Validity and Reliability

In this study, several measures were taken to ensure validity and reliability. Data triangulation was provided by using different data collection tools, and after the interviews, the recorded data were summarised to the participant, and participant confirmation was obtained. In the research, the opinions of two experts who have studied in the field of primary school mathematics education and one who has studied mathematics learning disabilities were taken at certain intervals. Data triangulation, participant confirmation, and expert opinion provided credibility and contributed to internal validity. The implementation process of the RME-supported instructional design was revealed in detail with the researcher's diary, video recordings, and photographs taken from student products. In reporting the research, the research process was described by including direct quotations from the participant's views. Transferability was ensured by presenting the teaching and participants in detail and external validity was contributed. Data triangulation and examination of the process and results by another researcher provided consistency and contributed to reliability.

Findings

The data obtained from the research are presented under three themes: findings on analysis, design, and development, findings on implementation, and findings on evaluation.

Findings on Analysis, Design, and Development

Under this theme, the findings are presented regarding what kind of instructional design can be developed for primary school students with MLD risk who have difficulty in multiplication at the third-

grade level. In this direction, firstly, the opinions of 24 classroom teachers and two experts were obtained, and the difficulties experienced by the students in multiplication were revealed. Secondly, the difficulties experienced by students with MLD risk in multiplication were revealed with the support of the assessment tools used to identify participating students. Student difficulties obtained from teacher and expert opinions are presented in Table 2.

Table 2

Difficulties experienced by students with MLD risk in multiplication

Evaluation Tool	Codes
Teacher and expert opinions	Multiplication
	Multiplying two-digit two-digit numbers
	Multiplying two-digit numbers with three-digit numbers
NOMAT	Multiplication
	Creating a multiplication table
	Solving problems requiring multiplication and division
LDSST	Difficulty with basic arithmetic operations (addition, subtraction, multiplication and division)
	Slow processing speed
	Staying below the class level in learning the multiplication table
	Difficulty in learning basic four operation symbols (+, -, x, ÷)

When Table 2 is analyzed, it is seen that students with MLD risk have slower processing speed, have difficulty in multiplication, and make processing errors compared to their peers. In addition, it can be stated that students have difficulty in learning the multiplication table and comprehending the repeated addition meaning of multiplication.

The data obtained from classroom teachers and experts, regarding the characteristics which the instructional design developed for students with MLD risk should have, are presented in Table 3 under two categories.

Table 3

Characteristics That The Instructional Design Developed for Students with Mld Risk Should Have

Category	Teacher opinions	f	Expert opinions	f
Teaching materials	Should attract students' attention and interest	11	Interesting materials should be prepared	1
	Must be appropriate to the level of the students	13	Individual characteristics of students should be taken into account	2
	Must be related to daily life	5	Must be close and related to daily life	1

Teaching Strategies	Teaching strategies should be supported with examples from real life	9	Differences in teaching methods should be provided	1
	An individual programme should be implemented	9	Individual training should be offered	1
	It should be shaped concretely (tools, materials)	13	Different teaching materials should be used	2

When Table 3 is analyzed, it is stated that the instructional materials in the instructional design developed for students with MLD risk should consist of materials appropriate to the student's level, take into account the students' differences, and attract the student's interest. While developing the instructional design, it was emphasized that the instruction should be shaped concretely, supported with real-life examples, and planned with an individual program.

Findings Related to Implementation

Under this theme, it was revealed how the RME-supported instructional design developed for primary school students with MLD risk who had difficulty in multiplication at the third-grade level impacted student learning. The findings regarding implementing the RME-supported instructional design were revealed in line with the data obtained from the researcher's diary NOMAT pre-test and post-test scores. The findings obtained from the implementation cycles were presented by considering the stages of the RME-supported instructional design. The findings obtained from the first implementation cycle are presented in Table 4.

Table 4

Findings from the First Implementation Cycle

Phase	Findings
Checking prior learning	Within the scope of the CBA, it was determined that the participant students had difficulty in multiplication, creating the multiplication table, "*multiplying a two-digit natural number by a two-digit natural number at most, and multiplying a three-digit natural number by a one-digit natural number at most*". All three participant students had difficulty in both second and third-grade subjects.
Presentation of the problem situation	The problem situation presented to the students for teaching multiplication did not have the expected effect on the students. This situation was expressed in the researcher's diary as follows: "*It was observed that students with MLD risk did not understand the problem situation presented and could not create their own solutions.*"
Students create their own informal solutions through models	Students with MLD risk used egg parcels, unit cubes, ping pong balls, mangala, ice cream sticks, and base blocks as real objects while creating their products and structures for multiplication. Students with MLD risk formed their solutions to the problem by drawing shapes. This situation

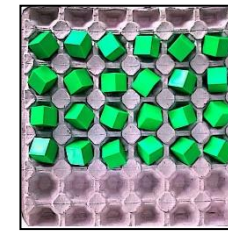
	was described in the researcher's diary: *"For the problem presented, students were asked to produce solutions to the problem by drawing a shape or using a model. İrem drew a circle, made eggs, and stated that rhythmic counting would be done."*
Interaction	Thanks to the interaction with students with MLD risk, it was observed that students created different solutions for the problem situations. This situation was expressed in the researcher's diary as follows: "*Since a chicken lays 6 eggs every day, how many eggs does a chicken lay four days later?"* The students formed their answers by repeated addition and rhythmic counting. It can be stated that students with MLD risk made sense of the changing property of multiplication. Teaching about the change property of multiplication was reflected in the researcher's diary as follows: "*Ege, one of the students, suggested to her friend that if you cannot count four times eight, you can count eight times four."* This statement can be interpreted as an indicator that the students both made sense of the property of change and were able to create different solutions for the problem situation.
Creation of formal solutions	It was observed that all three participant students were able to construct the operation using numbers and symbols. The results obtained from the CBA at the end of the lesson indicate that the students have gained a formal structure.

The findings obtained from the second implementation cycle are presented in Table 5.

Table 5

Findings from the second implementation cycle

Phase	Findings
Checking prior learning	Within the scope of CBA and NOMAT, it was determined that the participant students had difficulty in multiplication, creating the multiplication table, "*multiplying two-digit natural numbers by a two-digit natural number at most, and multiplying a three-digit natural number by a one-digit natural number at most*". Two participant students had difficulties in both second and third-grade subjects. The following statements were made in the researcher's diary regarding the difficulties experienced by students with MLD risk: "*They had difficulty understanding the digit shift operation in the second multiplier when multiplying two-digit numbers by two-digit numbers, and even if they did, they did it by heart."*
Presentation of the problem situation	It was observed that the RME-supported instructional design supported the problem-posing activities of students with MLD risk. This situation was expressed in the researcher's diary as "*the presentation of the problem situation in the classroom environment in a way close to real life attracted the students' attention, and they were more willingly involved in the lesson."*

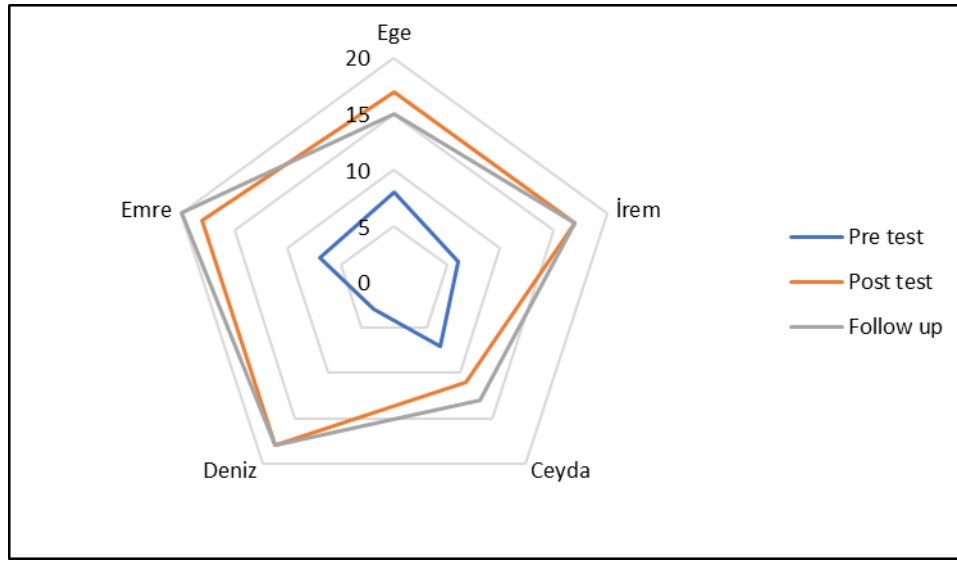
Students create their own informal solutions through models	Students with MLD risk used egg parcels, unit cubes, ping pong balls, mangala, ice cream sticks, and base blocks as real objects while creating their products and structures for multiplication. In addition, students made their products and structures through figures, tables, and diagrams. Deniz, one of the participant students, said, *"On the farm, a hen named Yellow Bird lays 6 eggs daily. After 4 days, how many eggs does she lay in total?"* A sample visual created using real objects for the question is presented below. 
Interaction	Thanks to the interaction with students with MLD risk, it was observed that the students created different solutions for the problem situations. This situation was expressed in the researcher's diary: "*In the multiplication of 29 and 4, Emre reached the solution by counting nine times four instead of four times nine when multiplying four and nine*". The interaction established with the students in the RME-supported instructional design contributes to the students with MLD risk to create different informal solutions for problem situations. This situation was expressed in the researcher's diary: "*Two rows of eggs were arranged on the egg parcel with three eggs in each row. When asked how many eggs there are here, the students answered by counting one by one, counting 2 and 3 rhythmically*." Interaction with students at risk of MLD contributed to students' use of mathematical concepts and expressions. This situation was discussed in the researcher's diary: "V*erbal expressions such as five times six equals thirty, five times six equals thirty, five times six equals thirty were emphasized and followed throughout the multiplication teaching."*
Creation of formal solutions	It was observed that both of the two participant students formed the operation by using symbols. This situation was reflected in the researcher's diary: "*It was observed that the students used mathematical expressions and concepts through the multiplication table, expressed them with symbols, and made sense of the unit element, absorber element, and the property of change."*

Findings for Evaluation

The findings regarding the evaluation of the RME-supported instructional design applied to primary school students with MLD risk who had difficulty in multiplication at the third-grade level were presented in line with the data obtained from the NOMAT follow-up test and interviews with the students' classroom teachers. Firstly, the findings obtained from the NOMAT follow-up test applied to the participant students 5 weeks after the RME-supported instruction were shared. The NOMAT pre-test and post-test findings of the participant students were also shared to compare the results more effectively. The number of correct scores obtained by the students participating in the first and second application cycles from the pre-test, post-test, and follow-up tests are presented below.

Figure 3

NOMAT Pre-Test, Post-Test and Follow-Up Test Results of The Participant Students



When the figure is analyzed, it is seen that RME-supported instructional design effectively taught multiplication topics to students with MLD risk. All students showed progress after RME-supported instruction. In addition, it can be stated that the follow-up test results conducted 5 weeks after the instruction are similar to the post-test results.

To evaluate the RME-supported instruction, classroom teachers were asked to implement the lesson plan for teaching multiplication. The findings obtained from the interviews with the classroom teachers (n=2) after the implementation are presented. Classroom teachers stated that the present problem situations facilitated students' understanding because they were based on real-life situations. The teacher interviewed expressed this situation: "*The selection of problem situations from real life increased students' interest in the lesson."*

Classroom teachers stated that the real objects used in multiplication attracted students' attention and contributed to their understanding of the lesson. The first teacher stated the effect of real objects on teaching as "*the use of real objects suitable for the subjects attracted students' attention."* The second teacher stated that *"the use real objects of enabled students to learn by doing and experiencing*."

Discussion

In this study, the expert and teacher opinions findings revealed that students had difficulty in multiplication and creating multiplication tables. In addition, it was found that students with MLD risk had slower processing speeds than their peers and had difficulty understanding and using multiplication symbols. The fact that each student with MLD risk exhibits different characteristics can be the reason for other difficulties. In previous studies, it has been reported that students at risk of MLD have difficulties in mathematical expressions and concepts, confuse symbols, and make processing errors (Geary & Hoard, 2005). Considering that multiplication topics form the basis for subsequent

mathematics learning (Van de Walle et al., 2018), it can be ensured that the topics in which students with MLD risk have difficulty are identified and supported with appropriate methods and strategies.

In the findings obtained from the expert and teacher opinions on developing an instructional design for students at risk of MLD, it has been determined that different teaching materials, different methods, and strategies that are interesting and have traces from daily life should be used in mathematics teaching and teaching should be supported with examples from everyday life. When the studies on this subject are examined, it is stated that different intervention programs should be developed for students with MLD risk (Fletcher et al., 2018; Deruaz et al., 2020). In addition, it is stated that the developed intervention programs should be suitable for student needs (Van de Walle et al., 2018), should be supported by different methods and strategies, should support teaching with concrete materials, and should include repetition and exercises in teaching (Gersten et al., 2009; Re et al., 2014; Witzel & Little, 2016).

In the findings obtained from the researcher diary for determining the effectiveness of the RME-supported instructional design in teaching multiplication to students with MLD risk, it has been determined that the presented real-life problems supported the difficulties faced by students with MLD risk in multiplication. Real-life problems associated with daily life can be used instead of verbal problems in teaching multiplication (Van de Walle et al., 2018; Witzel & Little, 2016). Based on the real-life problems presented to students, students with MLD risk can create informal solutions using different methods and techniques. In this way, students may have the opportunity to experience other contexts related to the subject.

In this study, it has been determined that the use of real objects in teaching multiplication within the scope of RME-supported instructional design contributed to the students with MLD risk creating their own solutions. In the teaching of mathematical concepts and skills, using different types of models and drawings creates meaningful learning in students, while also providing support for generalization and abstraction (Treffers, 1991).

In the follow-up test findings for the evaluation of the RME-supported instructional design model applied for teaching multiplication to primary school students with MLD risk, it has been determined that the number of correct answers of the students with MLD risk in the follow-up test was similar to the number of correct answers in the post-test. This shows that teaching with RME-supported instructional design contributed to students' permanent learning of multiplication. Koponen et al. (2018) found that students who participated in the multi-component strategy instruction intervention significantly improved their computational fluency and maintained their computational fluency during the five-month follow-up period.

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