Osmangazi Journal of Educational Research Volume 10 (Special Issue) 2023

100th Anniversary of the Republic of Türkiye

RESEARCH Open Access

Suggested Citation: Bağdat, O. & Bağdat, A. (2023). Should I learn division algorithm?: An investigation of elementary school students' solution strategies on realistic division with remainder (DWR) problems. *Osmangazi Journal of Educational Research, 10*(Special Issue), 273-292.

Submitted: 09/10/2023 **Revised:** 24/10/2023 **Accepted:** 24/10/2023 **DOI:** 10.59409/ojer.1373059

Should I Learn Division Algorithm?: An Investigation of Elementary School Students' Solution Strategies on Realistic Division with Remainder Problems***

***Osman Bağdat **Ayşe Bağdat**

Abstract. The division with remainder (DWR) problems offer significant potential for students to make sense of the division operation. The purpose of the study is to investigate elementary school students' solution strategies for DWR problems. In particular, this study aims to compare the problem-solving strategies in DWR problems employed by second-grade students, who are versed in multiplication, but have not been introduced to division; with those of third and fourth-grade students who are familiar with division but not have yet to engage with the interpretation of remainders. This qualitative research obtained data from 144 students in second, third, and fourth-grades in a public primary school. A total of six different DWR problems were presented to the students, including types as remainder divisible, remainder not divisible and remainder as a whole. The findings indicated that the strategies used by students in solving DWR problems differed. While second-grade students prefer strategies such as repetitive addition, repetitive subtraction, grouping, verbal explanation and using models, there is a noticeable tendency to use the division algorithm by fourth-grade students. However, it was noticed that students were unable to interpret the remainder in a meaningful way, especially from the third-grade, when they began to learn the division algorithm. According to the study, it is suggested that rather than moving immediately to the division algorithm, teachers should spend more time helping students understand division through contextual problems and representations.

Keywords. Realistic problems, context, division algorithm, division with remainder (DWR) problems, student strategies.

***** (**Responsible Author**) Asst. Prof. Dr., Anadolu University, Faculty of Education, Eskişehir, Türkiye
e-mail: osmanbagdat@anadolu.edu.tr

****** Middle School Mathematics Teacher, Ministry of National Education, Eskişehir, Türkiye
e-mail: ayseozatabagdat@gmail.com

******* This research was presented as a paper at the IX International Eurasian Educational Research Congress

Division is the most difficult and complex arithmetical operation to teach and learn among the four operations in terms of semantic structures (Kouba, 1989) comprehending both its meaning (Steffe, 1988), operation technique (Bağdat, 2020; Doruk & Doruk, 2019) and requiring preliminary knowledge such as multiplication (Kaasila, Pehkonen, & Hellinen, 2010). There are many reasons for its difficulty and complexity: (1) the lack of knowledge about the partitioning and measurement meanings of division is one of the primary reasons for students' difficulties with division (Doğan-Coşkun & Ev-Çimen, 2019a), (2) insufficient understanding of the functions of the divisor, dividend, quotient, and remainder in division as well as their relationships with one another (Correa, Nunes, & Bryant, 1988; Squire & Bryant, 2002; Doğan-Coşkun & Ev-Çimen, 2019a), (3) requiring good estimation skills and (4) inadequate consideration of the meanings of quotative and partitive division (Anghileri, Beishuizen, & van Putten, 2002). According to earlier research, teaching division should not only concentrate on rehearsing the division algorithm but also incorporate contextual issues that allow students to examine the various interpretations of division, the connections between the division algorithm phases, and the meaning of the quotient and remainder (Van de Walle, 2007). This is where Division with Remainder problems (DWR) differentiates itself as a noteworthy option for the sense-making division operation.

Division with Remainder Problems

Mathematics problems require students make sense of the context, comprehend the problem, consciously select problem-solving strategies, apply selected strategies, and consider whether the outcome is sensible (Polya, 2004). However, while solving problems, students usually focus on some phrases in the problem text, apply computational procedures, do not think about the context and consequences of the problem, and provide arithmetically correct, but senseless answers (İncikabı, Ayanoglu, & Uysal, 2020; Li & Silver, 2000; Rodríguez, Lago, Hernández, Jiménez, Guerrero, & Caballero, 2009; Silver, 1988; Silver, Mukhopadhyay, & Gabriele, 1992; Verschaffel, 2000). DWR problems can be defined as problems that involve a daily life context and are aimed at interpreting the meaning of the remainder obtained in a division problem. A number of researchers have an interest in DWR problems because they are regarded as non-routine problems (Rodríguez, et al. 2009; Arıkan & Ünal, 2016) and offer a tremendous potential to help students learn division by sense-making (Li & Silver, 2000; Silver, Shapiro, & Deutsch, 1993). Consider, for instance, a problem that asks how many buses are required for a 350-person trip where the students would travel in 40-person buses. Students would divide 350 by 40 in this problem, finding that the remaining would be 30 and the result would be eight. Because the quotient is eight, some of the students would think that the result is eight. In the upper classes, students may even divide the remaining 30 by eight and find the result 8.75. However, in this case, students should note that the correct response is nine, given that the remaining 30 passengers must board a different bus. While the quotient in conventional division problems provides the answer, in this case, the quotient obtained is insufficient for the solution, and the student needs to interpret the remainder. In DWR problems, the remainder can be interpreted differently depending on the context. In this respect, DWR problems can be classified into three different categories: 1) remainder divisible, 2) remainder not divisible and 3) remainder as a whole. For instance, if you were to divide five liters of water into four bottles, you would divide the five into four and then divide the remaining one so that each container would hold 1.25 liters of water. If an individual desires to share five balloons among four people, he would find one by ignoring the last balloon because he is unable to distribute it after giving each of them a balloon. If a group of five individuals decides to take a taxi that accommodates four passengers, they must divide the group by four and then call another taxi to accommodate the fifth passenger. Thus, by adding the remaining one to the quotient, he will find the result two. As noticed, in all three problems, the number five is divided by four, but different answers are obtained by interpreting the remainder according to the context.

Since 90s', many different groups of participants', ranging from young students (Arıkan & Ünal, 2016; Cai & Cifarelli, 2004; Cai & Silver 1995; Cooper & Harries, 2005 Guerrero & Riveira, 2001; Li & Silver, 2000; Silver, et. al, 1993) to teachers' (Chen, van Dooren, Chen & Verschaffel, 2010; Doğan-Coşkun & Ev-Çimen, 2019b) performance were examined on DWR problems. Li and Silver (2000) asked DWR problems to third-grade students who had not yet learned the division algorithm. Although third-grade students had not yet learned the formal division algorithm, their various strategies such as repetitive addition or subtraction enabled them to interpret the remainder successfully. Guerrero and Riveira (2001) investigated the difference between third-grade students before and after they were taught the division algorithm by asking them numerical computation and DWR problems. The results showed that learning the division algorithm improved students' performance in division with small integers, but not in solving DWR problems. Some of the students who learned the division algorithm thought that the quotient obtained by using the division operation in DRW problems would always lead to the correct answer, thus they obtained incorrect results. Even though they learned the division algorithm, the majority of students continued to use alternative approaches and had comparable results.

Contrary to popular belief, studies conducted with middle school students indicated that students experienced considerable difficulties (e.g., Silver et. al, 1993; Cooper & Harries, 2005). Cai and Silver (1995), who examined the performance of Chinese and American middle school students on DWR problems, indicated that although students showed high achievement in performing operations and only one out of four students from both countries were able to interpret the remainder correctly. Cai and Cifarelli (2004) discovered that Chinese and US students, similarly, performed better on DWR problems in a study concentrate more on computational skills rather than comprehension. These findings demonstrate that even after learning the division method, students still struggle to understand the remainder in DWR problems. This study intends to look into primary school children's approach in DWR problems. Considering this significance, the purpose of this study is to compare the problem-solving strategies in DWR problems employed by second-grade students, who are versed in multiplication, but have not been introduced to division; with those of third and fourth-grade students who are familiar with division but have yet to engage with the interpretation of remainders.

Methodology

The qualitative design was used in this study's data gathering, analysis, and interpretation procedures. In qualitative research, students' perspectives and interpretations are revealed in a flexible manner through the use of data gathering methods like observation, interviews, and document analysis (Yıldırım & Şimşek, 2011). Consequently, the goal of the current study was to compare the strategies used by students in the second, third, and fourth-grades when solving DWR problems.

Participants and Data Collection Process

This study was carried out with primary school students studying in Eskişehir province of Türkiye. A total of 144 students, including 54 second-grade, 51 third-grade and 39 fourth-grade students in six different classes, participated in the study. The data were obtained from the open-ended forms including a total of six DWR problems. These problems consisted of three different categories: 1) remainder divisible, 2) remainder not divisible and 3) remainder as a whole. Table 1 shows the problems posed to the students.

Table 1.

DWR Problems

Remainder divisible problems	
1	If Murat and Selin share 23 cookies, how many cookies will each of them have?
2	When we divide 26 litres of milk equally into 4 bottles, how many litres of milk will fill each bottle?
Remainder not divisible	
3	There are 35 scouts in a scout group. How many tents are required for the scouts who will stay in tents for 4 people each?
4	Hilal reads a book of 18 pages a day. How many days will Hilal finish a 80-page book?
Remainder as a whole	
5	Two brothers will share the 13 balloons their father bought them. How many balloons will each of them get?
6	13 pencils will be divided equally among 4 people. How many pencils will each person get?

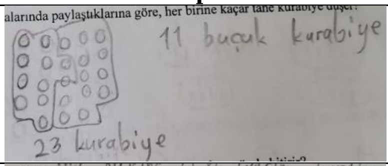
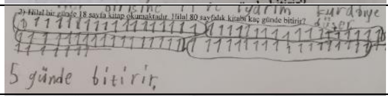
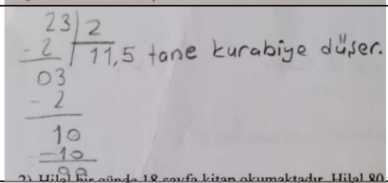
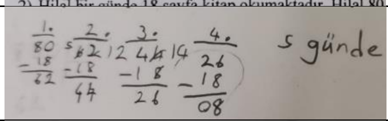
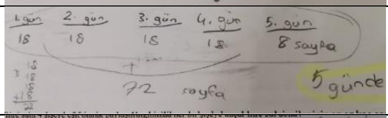
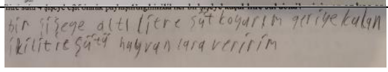
In the first and second problems in Table 1, students are expected to obtain a decimal number by dividing the remaining. For instance, the student should be capable to share the cookies as 11.5 in the first problem. In the third and fourth problems, since one remaining value cannot be ignored, the quotient value should be increased by one. For example, in the third problem, eight tents won't be sufficient; in order to accommodate the remaining three people, one more tent would be required, hence the correct response is 8+1 = 9. In the fifth and sixth problems, students are expected to ignore the remainder by thinking that the balloon and the pencil cannot be shared in half. In the study, these six problems were mixed and the students were asked to explain their solutions in detail and not to use erasers. Particular attention was paid to the fact that second-grade students had learned multiplication but not division during the data collection phase. Students in the third and fourth-grades learned division, but they didn't formally learn how to find the decimal values needed for division. On the other hand, it is assumed that students might develop a variety of strategies based on their experiences in daily life. The purpose of this study was to compare and determine the extent to which the strategies employed by students who learned division differ from those who didn't learn division algorithm.

Data Analysis

This study utilized content analysis method which provides to bring together similar data within the framework of certain concepts and themes and to interpret them by organizing them in a way that the reader can understand (Yıldırım & Şimşek, 2011). In the current study, strategies employed by students were examined to discern both their similarities and differences, leading to the establishment of pertinent categories. These strategies were also analysed in terms of the accuracy of the solution. As a result, the findings were presented with frequency tables in terms of solution strategies and accuracy of solutions. The sample student strategies that arose from the data analysis are shown in Table 2.

Table 2.

Sample Student Strategies Arose From Content Analysis

Strategy	Sample solution	Explanation
Modelling		Correct response to the first problem: 11.5 cookies for each person.
Grouping		Correct response to the fourth problem: Hilal completes a 80-page book in 5 days
Division algorithm		Correct response to the first problem: 11.5 cookies for each person
Repetitive substraction		Correct response to the fourth problem: Hilal completes a 80-page book in 5 days
Repetitive addition		Correct response to the fourth problem: Hilal completes an 80-page book in 5 days
Verbal explanation		Incorrect response to the fourth problem: I fill each bottle with six litres of milk and give the remainder to the animals.

The data were analyzed by two experts in the field. These experts reached a consensus regarding the accuracy of the solutions. To establish a shared understanding of the strategies employed in the solutions, they initially analyzed several problems collaboratively, aiming to develop a consistent terminology for strategies. As a matter of fact, the following categories emerged as a result of the analysis as division algorithm, verbal responses, model utilization, repetitive additition and subtraction. In the following problems, they carried out this analysis process separately and reached a consensus over 90%. Here, especially in the problem of dividing the balloons to ignore the remainder, they discussed whether it would be a mistake or a strategy for the students to divide balloons 6 to 7 for example, and they agreed that it was an erroneous answer.

Findings

The findings presented under three different headings in order to interpret three different problem types of DWR problems.

Findings of the Remainder Divisible Problems

Table 3 shows the findings of the first problem:

Table 3.

Findings of the First Problem

If Murat and Selin share 23 cookies, how many cookies will each of them have?		
Grade	**Correct**	**Incorrect**
2	29 (%54)	25 (%46)
	➨ 18 students sketched a model. ➨ 10 students provided a verbal answer and found the answer 11 and a half. ➨ 1 student used a division algorithm.	➨ 3 students sketched a model and divided 11 to 12. ➨ 2 students sketched a model and divided 12 to 12. ➨ 2 students found 46 by dividing each of the 23 pieces by 2. ➨ 1 student divided it as 10, 10, 3. ➨ 1 student divided it as 10, 13. ➨ 16 students provided blank, incomplete or irrelevant solutions.
3	1 (%2)	51 (%98)
	➨ Provided a verbal answer	➨ 18 students divided the number 23 by 2 using the division algorithm and did not make any explanation. ➨ 26 students divided the number 23 by 2 using the division algorithm and found 11 cookies. ➨ 7 students provided blank, incomplete or irrelevant solutions.
4	3 (%8)	36 (%92)
	➨ The division algorithm was used.	➨ 7 students divided the number 23 by 2 using the division algorithm and left it halfway and did not make an explanation. ➨ 26 students divided the number 23 by 2 using the division algorithm and found 11 cookies. ➨ 1 student sketched a model and found the answer 11 cookies. ➨ 2 students provided blank, incomplete or irrelevant solutions.

As seen in Table 3, 18 (54%) of second-graders solved the problem correctly by sketching models. The responses of two second-graders who sketched models are displayed in Figure 1.

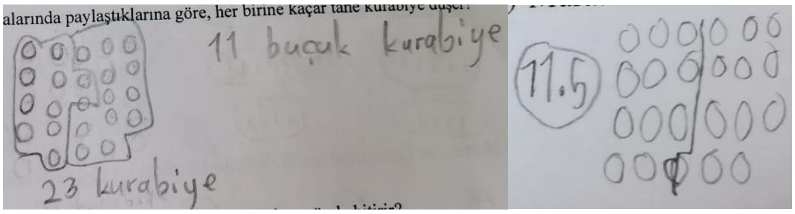


Figure 1. Sample Second-Graders' Response Illustrating the Correct Solution to the First Problem via Modelling.

In the second-grade, 10 students provided verbal responses and obtained the result 11.5, while one student reached the correct response by using the division algorithm. In the third and fourth-grades, students did not prefer to use models and very few students (one student in the third-grade and three students in the fourth-grade) solved the problem correctly. While 25 (46%) second-grader, 51 (98%) third-graders and 36 (92%) fourth-graders reached incorrect responses. In the second-grade, students reached incorrect responses by inaccurate reasoning without using the division algorithm, third and fourth-graders mostly used division algorithm and found the result 11 by ignoring the remainder. Figure 2 shows the solution of a fourth-grader who reached the correct response using the division algorithm.

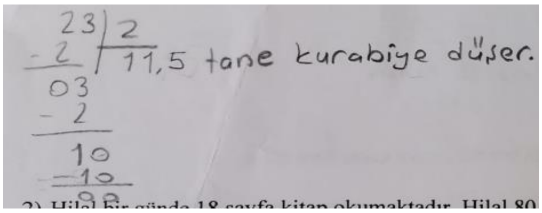


Figure 2. Sample Fourth-Grader's Response Illustrating the Correct Solution to the First Problem via Division Algorithm.

Table 4 represents the findings of the second problem:

Table 4.

Findings of the Second Problem

When we divide 26 litres of milk equally into 4 bottles, how many litres of milk will fill each bottle?		
Grade	**Correct**	**Incorrect**
2	5 (%9)	49 (%91)
	➨ 2 students sketched a model. ➨ 2 students provided verbal answers. ➨ 1 student used a division algorithm.	➨ 8 students filled 6 litres of milk for each bottle and ignore 2 litres of milk. ➨ 3 students provided incorrect repetitive subtraction. ➨ 1 student provided incorrect repetitive addition. ➨ 2 students used a division algorithm. ➨ 35 students provided blank, incomplete or irrelevant solutions.
3	0 (%0)	51 (%100)
		➨ 26 students divided 26 by 4 with the division algorithm and did not continue. ➨ 20 students found the answer 6 by dividing 26 by 4 with the division algorithm. ➨ 1 student found the answer 6 by dividing 26 by 4 with the division algorithm and completed it to 7. ➨ 4 students provided blank, incomplete or irrelevant solutions.
4	6 (%15)	33(%85)
	➨ 5 students found the answer 6.5 using the division algorithm. ➨ 1 student found the answer 6.5 using a model.	➨ 10 students divided 26 by 4 with the division algorithm and did not continue. ➨ 21 students found the answer 6 by dividing 26 by 4 with the division algorithm. ➨ 2 students divides 26 by 4 with the division algorithm and finds the answer 6 and completes it to 7.

Table 4 shows us that the majority of the second, third and fourth-graders provided incorrect solutions. Two of second-graders who provided correct responses explained the solution by sketching a model, two of them provided verbal answer and one of them used division algorithm. None of the third-graders provide correct solution and the majority of fourth-graders (85%) provided the incorrect solutions. Among the fourth-graders, one of the six students reached the correct response by using a model, and five of them reached the response of 6.5 by using the division algorithm. Figure 3 below illustrates the correct responses of the students using a model.

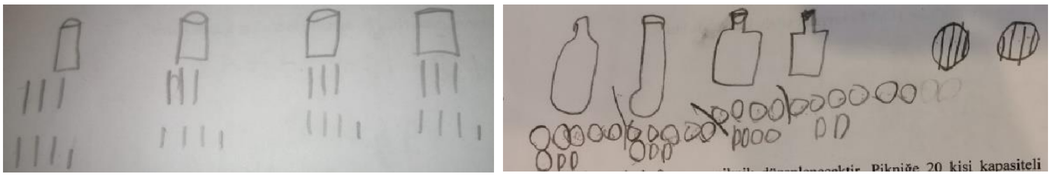


Figure 3. Sample Student Responses Illustrating the Correct Solution to the Second Problem via Modelling.

Figure 4 displays the correct response of a student who formulated a verbal explanation consistent with the solution to the problem of dividing 26 litres of milk among 4 bottles. In this solution, the student addressed the problem by disregarding the remainder.

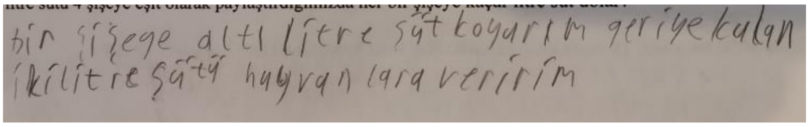


Figure 4. Sample Student Responses Illustrating the Correct Solution to the First Problem via Verbal Explanation.

Table 5 presents the findings of the third problem.

Table 5.

Findings of the Third Problem

Hilal reads a book of 18 pages a day. How many days will Hilal finish an 80-page book?		
Grade	**Correct**	**Incorrect**
2	19 (%35)	35 (%65)
	➨ 7 students provided repetitive subtraction. ➨ 7 students provided repetitive addition. ➨ 3 students provided verbal answers. ➨ 2 students developed a group formation strategy.	➨ 10 students provided repetitive subtraction as 80-18-18-18-18-18, but could not reach the correct response. ➨ 3 students provided repetitive addition but could not reach the correct response. ➨ 22 students provided blank, incomplete or irrelevant solutions.
3	12 (%23)	40 (%77)
	➨ 9 students provided repetitive addition. ➨ 1 student provided repetitive addition. ➨ 2 students reached the answer 18x5 by multiplication.	➨ 4 students used a division algorithm. ➨ 5 students provided repetitive addition but could not reach the correct response. ➨ 1 student provided repetitive subtraction but could not reach the correct response.

		➨ 30 students provided blank, incomplete or irrelevant solutions.
4	7 (%18)	33 (%82)
	➨ 5 students found 4+1 = 5 days using the division algorithm. ➨ 1 student using the division algorithm $4\frac{8}{18}$ day. ➨ 1 student divided 80 by 18 using the division algorithm and found 4.5 days.	➨ 28 students used division algorithm, divided 80 by 18 and could not reach correct response. ➨ 4 students provided an incorrect operation. ➨ 1 student provided repetitive addition but could not reach the correct response.

Table 5 shows the findings of the third problem. While the majority of the second, third and fourth-graders reached incorrect response, the most successful group was the second-graders (35%). In second-grade, the students mostly reached the correct solution by repetitive addition or repetitive subtraction strategy. In addition, three students provided the correct response verbally and two students responded correctly by grouping strategy. Figure 5-6 illustrate the solution examples of second-graders who provided different solutions.

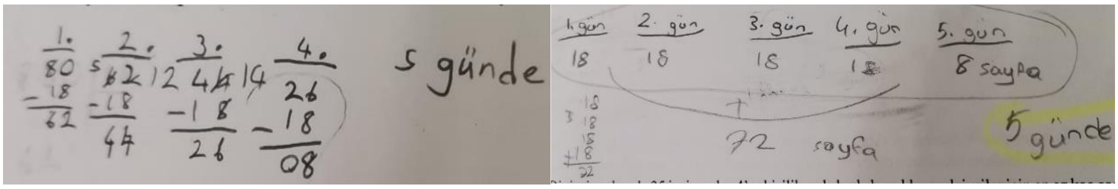


Figure 5. Sample Second-Graders' Responses Illustrating the Correct Solution to the Third Problem via Repetitive Addition and Repetitive Subtraction Strategies.

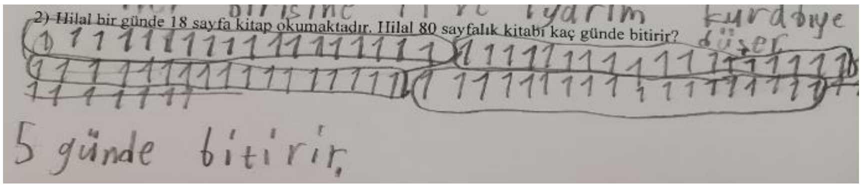


Figure 6. Sample Second-Grader's Response Illustrating the Correct Solution to the Third Problem via Grouping Strategy.

The third-graders' responses show us that there were more students (77%) who reached incorrect result than the second-graders. It was observed that most of the third-graders preferred repetitive addition while solving the problem. In comparison to students in third-grade, more fourth-graders (82%) provided incorrect responses. Four of fourth-graders who reached correct solution used division algorithm while the other two students made an effort to illustrate the day using fractional expressions.

Table 6 shows the findings of the fourth problem:

Table 6.

Findings of the Fourth Problem

There are 35 scouts in a scout group. How many tents are required for the scouts who will stay in tents for 4 people each?		
Grade	Correct	Incorrect
2	25 (%35)	29 (%65)
	➨ 6 students used repetitive subtraction strategy. ➨ 7 students used repetitive addition strategy. ➨ 12 students sketched a model.	➨ 4 students sketched a model. ➨ 5 students used repetitive addition strategy. ➨ 3 students used repetitive subtraction strategy ➨ 17 students provided blank, incomplete or irrelevant solutions.
3	5(%10)	46(%90)
	➨ 5 students found the quotient 8 with the division algorithm and added 1 to the result.	➨ 23 students divided 35 by 4 and found the answer 8 with the standard division algorithm. ➨ 16 students divided 35 by 4 with standard division algorithm but could not reach the correct response. ➨ 7 students provided incorrect, blank or irrelevant responses.
4	7 (%18)	32 (%82)
	➨ 7 students found the quotient 8 with the division algorithm and added 1 to the result.	➨ 18 students divided 35 by 4 and found the result 8 with the standard division algorithm. ➨ 11 students divided 35 by 4 with standard division algorithm but could not reach the correct response. ➨ 1 student left it blank. ➨ 1 student found 8¾ tents with the division algorithm. ➨ 1 student found 8.15 with the division algorithm.

Table 6 demonstrates that students in the second-grade scored the best performance (%25). Almost half of the students who solved the problem correctly used repetitive subtraction or repetitive addition, and half of them reached the correct answer by sketcing a model. 90% of third-graders reached incorrect answers. All of the students who reached correct answers found the quotient eight by using the division algorithm and obtained nine by adding one. 82% of the fourth-graders provided incorrect solutions to the problem. The students who reached at the solution correctly, on the other hand, employed the division algorithm. Figure 7-9 show examples of various types of student solutions.

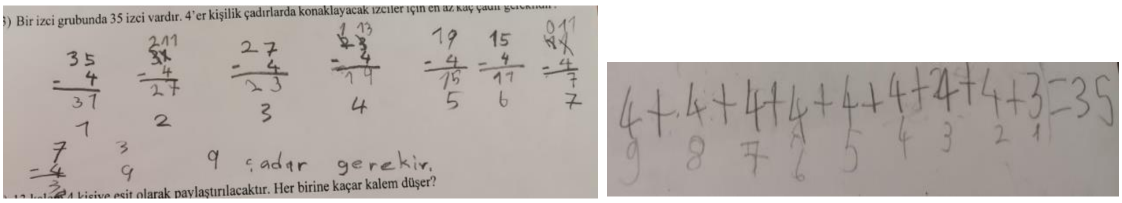


Figure 7. Sample Second-Graders' Responses Illustrating the Correct Solution to the Fourth Problem via Repetitive Subtraction and Repetitive Addition Strategies.

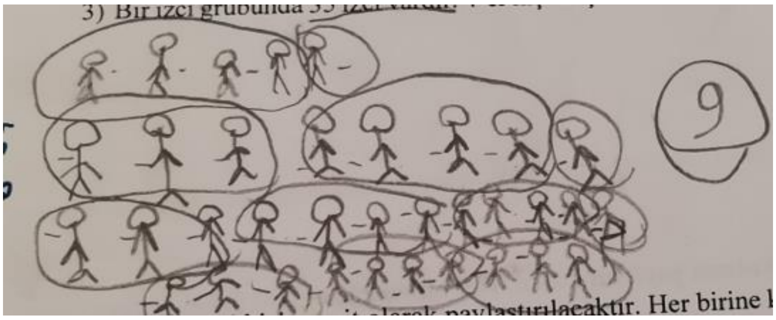


Figure 8. Sample Second-Grader's Response Illustrating the Correct Solution to the Fourth Problem via Grouping Strategy.

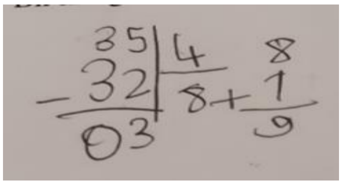


Figure 9. Sample Third-Grader's Response Illustrating the Correct Solution to the Fourth Problem via Division Algorithm.

Table 7 shows the findings of the fifth problem:

Table 7.

Findings of the Fifth Problem

13 pencils will be divided equally among 4 people. How many pencils will each person get?		
Grade	**Correct**	**Incorrect**
2	16 (%30)	38 (%70)
	➨ 2 students reached the solution by using repetitive subtraction strategy. ➨ 3 students provided verbal solutions. ➨ 11 students reached the solution by drawing a model.	➨ 7 students divided 3 whole pencils and 1 quarter pencil. ➨ 4 students divided 3 whole pencils and 1 half pencil. ➨ 1 student reached the incorrect solution by repetitive subtraction. ➨ 2 students reached the incorrect solution by repetitive addition. ➨ 24 students obtained blank, incomplete or irrelevant solutions.
3	46 (%90)	5 (%10)
	➨ 46 students reached the solution using the division algorithm.	➨ 5 students provided an operation error.
4	35 (%90)	4 (%10)
	➨ 35 students reached the solution using the division algorithm.	➨ 2 student found 3.25. ➨ 1 student found 3.5. ➨ 1 irrelevant answer.

Table 7 presents the results for the fifth problem. From the data, it is evident that 70% of the second graders' arrived at an incorrect solution. Among those who responded correctly, the majority

(16 students) utilized models for their solutions, while three students employed verbal explanations, and two students relied on repetitive subtraction strategies. Responses of third and fourth-graders reveal a significant rise in the proportion of correct answers. 90% of third-graders and 90% of fourth-graders responded the problem correctly using the division algorithm. Figure 10-12 show the sample student solutions.

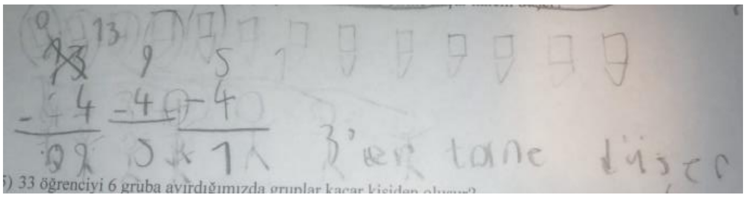


Figure 10. Sample Second-Grader's Response Illustrating the Correct Solution to the Fifth Problem via Repetitive Subtraction Strategy.

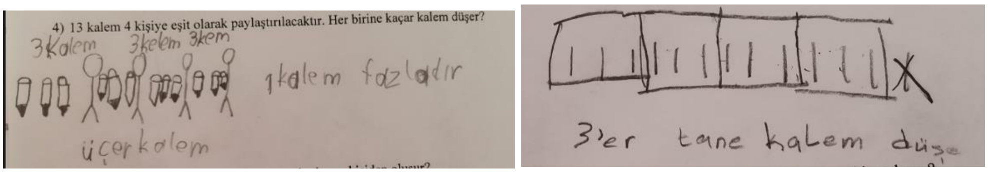


Figure 11. Sample Second-Graders' Responses Illustrating the Correct Solution to the Fifth Problem via Modelling.

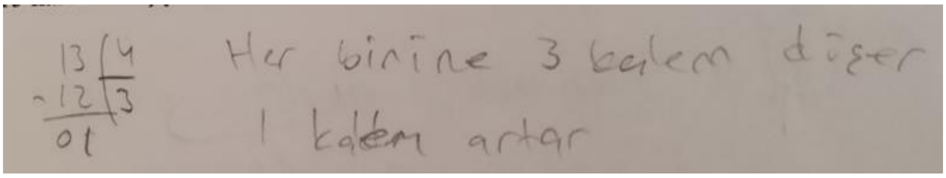


Figure 12. Sample Fourth-Grader's Response Illustrating the Correct Solution to the Fifth Problem via Using the Division Algorithm.

In this problem, since the remainder did not affect the solution in any way, it was sufficient to obtain a solution using the division algorithm. It is thought that this situation caused an increase in the success of third and fourth-graders.

Table 8 shows the findings of the sixth problem.

Table 8.

Findings of the Sixth Problem

Two brothers have to share the 13 balloons their father bought them. How many balloons will each em get?		
Grade	**Correct**	**Incorrect**
2	10 (%19)	44 (%81)
	➨ 10 students sketched a model for dividing by 6 and ignoring the remainder.	➨ 16 students found 6 and a half. ➨ 2 students divided 6 by 7. ➨ 2 students divided 7 by 7.

		➨ 24 students provided blank, incomplete or irrelevant solutions.
3	44 (%86)	7 (%14)
	➨ 44 students used division algorithm and found the answer 6.	➨ 1 student divided 13 balloons into 2 and found 6+1=7. ➨ 6 students provided blank, incomplete or irrelevant solutions.
4	32 (%82)	7(%18)
	➨ 31 students used division algorithm and found the answer 6. ➨ 1 student said that 6 balloons fall for two siblings and the remaining 1 balloon is popped.	➨ 3 students found 6 and a half. ➨ 4 students provided blank, incomplete or incorrect solutions.

Table 8 shows the findings of the sixth problem. In contrast to the preceding problems, second graders were the group that struggled with this problem the most, 81% of second-graders responded the balloon problem incorrectly. All of the students who responded correctly preferred to sketch models. On the other hand, third and fourth-graders had a great increase in the number of correct responses compared to second-graders. 86% of third-graders and 82% of fourth-graders responded the problem correctly and almost all of the students reached the correct solution by using the division algorithm. Similar to the fifth problem, it was sufficient to obtain a solution by using the division algorithm since the remainder did not affect the result in any way. Figure 13 shows the solution of the second-grader who provided a model, and Figure 14 illustrates the solution of the third-grader who provided an incorrect response.

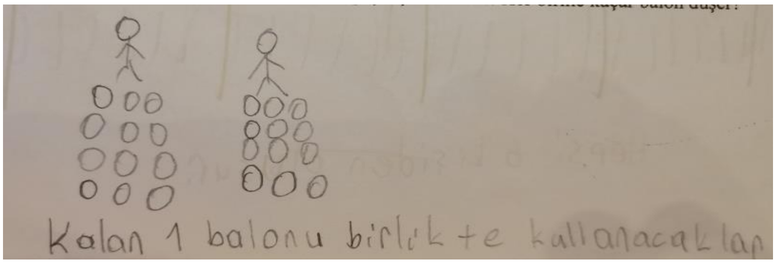


Figure 13. Sample Secon-Grader's Response Illustrating the Correct Solution to the Sixth Problem via Modelling.

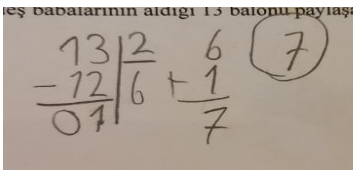


Figure 14. Sample Third-Grader's Response Illustrating the Incorrect Solution to the Sixth Problem via Using the Division Algorithm.

Conclusion and Discussion

The purpose of the study is examining the solution strategies of primary school students in DWR problems. In particular, this study aims to compare the problem-solving strategies in DWR problems employed by second-grade students, who are versed in multiplication, but have not been introduced to division; with those of third and fourth-grade students who are familiar with division but have yet to engage with the interpretation of remainders. The results indicated that the methods used by primary school second, third and fourth-grade students while solving DWR problems differed. While second-grade students prefer to use strategies such as repetitive addition, repetitive subtraction, grouping and using models, it is noticeable that there is a tendency to use the division algorithm towards the fourth-grade. Robinson et al. (2006) reported similar results with upper-grades, showing that strategies shifted in step with age. Students in fourth-grade employed the repetitive addition strategy, whereas those in fifth through seventh-grade utilized the multiplication strategy more frequently. On the other hand, it was noticed that especially from the third-grade, when the division algorithm began to be learned, students could not interpret the remainder in a meaningful way, many of them left the operations with remainders by applying the division algorithm and obtained incorrect results. Although the accuracy of the results varied according to the grades and problems, it was observed that, contrary to expectations, there was no significant increase in the number of correct solutions as students progressed to higher-grades, and even the success decreased significantly, especially at the third-grade level. Silver et al. (1993) found that students who performed repetitive subtraction or repetitive subtraction were highly successful in interpreting the remainder problems. In the current study, it was observed that second-grade students preferred similar strategies and achieved higher success, perhaps these strategies allowed students to reason about the problem. Guerrero and Riveira (2001) asked numerical calculation and DWR problems to third-grade students before and after teaching the division algorithm, and found that the most successful students were the students who used alternative approaches such as repetitive addition and repetitive subtraction. Li and Silver (2000), in their study in which they asked third-grade students who had not yet learnt the division algorithm to solve DWR problems, discovered that students similarly achieved high levels of success by using alternative approaches. This study does not claim that students should continue to solve problems in traditional ways, but it points to the problems that arise when the division algorithm is launched in a meaningful way. On the other hand, it was observed that students' solutions were algorithmic (division algorithm) towards the upper-grades, and the diversity of solutions and using representations decreased, as in the study conducted with sixth-grade students by

Incikabı et al. (2020). It can be expected that students use the division algorithm they have learned instead of making long repetitive subtraction or modelling this subtraction, but the use of representation is also a way for students to express their reasoning, so in this study, it was seen that students abandoned reasoning strategies very early in problems. Li and Silver (2000) stated in their study that it is necessary to observe how the depth of students' strategies in problems changes with the school mathematics launched.

While second-grade students were more successful in remainder divisible and remainder not divisible problems, third and fourth-grade students were more successful in remainder as a whole problems. However, it was seen that the success of third and fourth-grade students in remainder as a whole problem was due to the nature of the problem. In these problems, it can be claimed that the remainder has no impact on the solution, therefore, students would reach the correct answer even if they do not interpret the remainder. As a matter of fact, the students reached the quotient they obtained as a response and found the correct result. In other words, since these types of problems can be solved only by using the division algorithm without sufficient reasoning, it can be said that third and fourth-grade students showed higher success in these problems unlike other problems. On the other hand, it is seen that the reasoning used by second-grade students in Problems 5 and 6, in which they were unsuccessful compared to other problems, was not meaningless. For example, in the problem where two brothers should share 13 balloons (perhaps due to an error caused by the fact that equal sharing was not mentioned in the way the problem was asked), the students divided the balloons 7 by 8, many of them left the problem blank and the majority of them found the answer 6.5 with an erroneous reasoning. These results indicate that second-grade students mostly provide solutions based on reasoning, while third and fourth-grade students provide standard solutions using the division algorithm. In a study conducted by Li and Silver (2000) with students who did not learn division, they stated that students prefer the way that makes the most sense to them. In addition, while second-grade students focused more on the context, it was observed that third and fourth-grade students focused on the necessary operation without focusing on the context. Cooper and Harries (2005), Graeber, Tirosh and Glover (1989) and Kaasila et al. (2009) emphasised similar points in their study by stating that students do not think and reason sufficiently on realistic contexts. Finally, while second-grade students were more successful in the first problem about dividing the remainder, they had difficulty in solving the second problem. This situation can be explained by the difference in context or the fact that the divisor was two in the first problem while the divisor was four in the second problem may have made the problem difficult. In the fourth-grade, on the contrary, it was observed that the success increased in favour of the second problem. The growth of the divisor in division caused the second-graders to have more difficulty in reasoning.

Recommendations

Based on the findings of this study, several recommendations emerge. Firstly, educators should ensure a gradual transition to the division algorithm in problem-solving, and alongside this, discussions can be facilitated around student-invented strategies related to the division algorithm. Moreover, the use of multiple representations in problem-solving should be promoted. The study suggests a pedagogical shift towards an approach centered on reasoning and problem-solving skills, emphasizing "using division for problem-solving" rather than focusing solely on specific solution methods for "solving problems that require division." To foster this mindset, the significance of employing realistic contexts and fostering a deep understanding of problems is underscored (Cooper & Harries, 2005; Verschaffel et al., 2000). Consistent with the principles of realistic mathematics education (Gravemeijer, 1994), arithmetic operations should be introduced and mastered within the framework of solving realistic problems, rather than being treated as isolated subjects.

About Authors

First Author: Osman Bağdat is a member of Anadolu University, Faculty of Education, Türkiye. He is currently working at the Mathematics Education Department. His research interest includes mathematics education.

Second Author: Ayşe Bağdat is middle school mathematics teacher at Ministry of National Education. She got her master's degree from Eskişehir Osmangazi University in Eskişehir, Türkiye and continues her doctoral education in Hacettepe University, Department of Mathematics Education. She mainly works in fields on mathematics education.

Conflict of Interest

It has been reported by the authors that there is no conflict of interests..

Funding

No funding was received.

Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration.

ORCID

Respectively write the ORCIDs of the authors, as in the example below.

Osman Bağdat https://orcid.org/0000-0002-4007-7518

Ayşe Bağdat https://orcid.org/0003-0022-9393

References

Anghileri, J., Beishuizen, M., & van Putten, K. (2002). From informal strategies to structured procedures: Mind the gap!. *Educational Studies in Mathematics, 49*(2), 149–170.

Arıkan, E. E., & Ünal, H. (2016). An examination of preservice mathematics teachers' realistic approaches with division with remainder (DWR) problems. *International Journal of Social and Educational Sciences, 3*(5), 1-12.

Bağdat, A., & Ev-Çimen, E. (2023). An investigation of sixth-grade students' skills of posing problems regarding the order of operations via multiple-choice tests. *Turkish Studies-Educational Sciences*, *18*(2), 533-551. https://dx.doi.org/10.7827/TurkishStudies.64586

Cai, J. & Cifarelli, V. (2004). Thinking mathematically by Chinese learners: A cross-national comparative perspective. In L. Fan, N.-Y. Wong, J. Cai & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders*. Singapore, Singapore: World Scientific.

Cai, J., & Silver, E. A. (1995). Solution processes and interpretations of solutions in solving a division-with-remainder story problem: Do Chinese and US students have similar difficulties?. *Journal for Research in Mathematics Education, 26*(5), 491-497.

Chen, L., Van Dooren, W., Chen, Q., & Verschaffel, L. (2011). An investigation on Chinese teachers' realistic problem posing and problem-solving ability and beliefs. *International Journal of Science and Mathematics Education, 9,* 919-948.

Cooper, B., & Harries, T. (2005). Making sense of realistic word problems: Portraying working class 'failure' on a division with remainder problem. *International Journal of Research & Method in Education, 28*(2), 147-169.

Correa, J., Nunes, T., & Bryant, P. (1998). Young children's understanding of division: The relationship between division terms in a noncomputational task. *Journal of Educational Psychology, 90,* 321-329.

Doğan-Coşkun, S. D., & Ev-Çimen, E. (2019a). Pre-service elementary teachers' problem-solving abilities for problems focusing on the partition and comparison meanings of the division operation. *Eskişehir Osmangazi University Journal of Social Sciences, 20,* 1-21.

Dogan-Coşkun, S., & Ev-Çimen, E. (2019b). Pre-service elementary teachers' difficulties in solving realistic division problems. *Acta Didactica Napocensia, 12*(2), 183-194.

Doruk, M., & Doruk, G. (2019). Analysis of the problems posed by the fifth-grade students related to multiplication and division. *Van Yüzüncü Yıl University Journal of Education, 16*(1), 1338-1369.

Ev-Çimen, E., & Tat, T. (2018). Investigation of eight grade students' problem posing abilities for interpretation of the remainder in division problems. *Journal of Research in Education and Teaching*, *7*(4), 1-11.

Graeber, A., Tirosh, T., & Glover, R. (1989). Preservice teachers' misconceptions in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education, 20(1)*, 95–102.

Gravemeijer, K. (1994). *Developing realistic mathematics education.* Utrecht: Cd-ß Press.

Guerrero, L., & Rivera, A. (2001). Does the acquisition of mathematical knowledge make students better problem solvers? An examination of third-graders' solution of division-with-remainder (DWR) problems. In *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Snowbird, Utah.

İncikabı, L., Ayanoglu, P., & Uysal, R. (2020). Sixth-grade students' procedural and conceptual understandings of division operation in a real-life context. *International Electronic Journal of Elementary Education, 13*(1), 35-45.

Kaasila, R., Pehkonen, E., & Hellinen, A. (2010). Finnish pre-service teachers' and upper secondary students' understanding of division and reasoning strategies used. *Educational Studies in Mathematics, 73,* 247-261.

Kouba, V. L. (1989). Children's solution strategies for equivalent set multiplication and division word problems. *Journal for Research in Mathematics Education, 20*(2), 147-158.

Li, Y., & Silver, E. A. (2000). Can younger students succeed where older students fail? An examination of third graders' solutions of a division-with-remainder (DWR) problem. *The Journal of Mathematical Behaviour, 19*(2), 233-246.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method* (Vol. 85). Princeton University Press.

Robinson, K. M., Arbuthnott, K. D., Rose, D., McCarron, M. C., Globa, C. A., & Phonexay, S. D. (2006). Stability and change in children's division strategies. *Journal of Experimental Child Psychology, 93*(3), 224-238.

Rodríguez, P., Lago, M. O., Hernández, M. L., Jiménez, L., Guerrero, S., & Caballero, S. (2009). How do secondary students approach different types of division with remainder situations? Some evidence from Spain. *European Journal of Psychology of Education, 24*, 529-543.

Silver, E. A. (1988). Solving story problems involving division with remainders: The importance of semantic processing and referential mapping. In:M. J. Behr, C. B. Lacampagne, & M. M. Wheeler (Eds.), *Proceedings of the tenth annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 127-133). De Kalb, IL: Northern Illinois University.

Silver, E. A., Mukhopadhyay, S., & Gabriele, A. J. (1992). Referential mapping and the solution of division story problems involving remainders. *Focus on Learning Problem in Mathematics, 14* (3), 29-39.

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: an examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education, 24*(2), 117-135.

Steffe, L. P. (1988). Children's construction of number sequences and multiplying schemes. *Number Concepts and Operations in the Middle Grades, 2,* 119-140.

Squire, S., & Bryant, P. (2002). The influence of sharing on children's initial concept of division. *Journal of Experimental Child Psychology, 81*(1), 1-43.

Van de Walle J. A. (2007). *Elementary and middle school mathematics* (6th ed.). Boston, MA: Pearson.

Verschaffel, L., Greer, B. & De Corte, E. (2000) Making sense of word problems. Lisse: Swets & Zeitlinger.

Yıldırım, A., & Şimşek, H. (2011*). Qualitative research methods in social sciences.* Ankara: Seçkin Publishing.