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RESEARCH PAPER

Genocchi collocation method for accurate solution of nonlinear fractional differential equations with error analysis

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Abstract

In this study, we introduce an innovative fractional Genocchi collocation method for solving nonlinear fractional differential equations, which have significant applications in science and engineering. The fractional derivative is defined in the Caputo sense and by leveraging fractional-order Genocchi polynomials, we transform the nonlinear problem into a system of nonlinear algebraic equations. A novel technique is employed to solve this system, enabling the determination of unknown coefficients and ultimately the solution. We derive the error bound for our proposed method and validate its efficacy through several test problems. Our results demonstrate superior accuracy compared to existing techniques in the literature, suggesting the potential for extending this approach to tackle more complex problems of critical physical significance.

Keywords: Fractional-order modelling; collocation method; nonlinear phenomena; error bound

AMS 2020 Classification: 34A08; 65L60; 65L70

1 Introduction

During the last few years, fractional calculus (FC) has gained significant attention in the scientific and engineering communities due to its ability to provide more realistic simulations of real-life complex phenomena. FC is defined as the branch of mathematics that deals with derivatives and integrals of non-integer orders. Unlike traditional calculus, which focuses on integer-order derivatives and integrals, FC extends these operations to include fractional orders. The importance

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of the application of FC comes from the fact that it involves the derivative and integral of any order and is particularly useful in simulating models that exhibit memory effects or cannot be adequately described by classical approaches. With these important remarks and properties, researchers have been trying over the years to develop several definitions of FC [1]. One of the common definitions of FC is the Riemann-Liouville fractional derivative, which is defined as the integral of a function raised to a fractional power, followed by differentiation. On the other hand, another well-known important definition is the Caputo fractional derivative, which is defined as the integral of a function multiplied by a weight function, followed by differentiation [2]. The Caputo fractional derivative is one of the most important definitions in the field of FC due to many reasons. First, it can handle initial conditions more efficiently and can handle non-smooth functions and discontinuities. In addition, unlike other operators that require knowledge of the function's history at all times, the Caputo derivative only requires the function's values at the current time. This property makes this definition suitable for modeling real-world phenomena where the initial conditions are unknown or hard to obtain. Also, this definition provides the ability to handle non-smooth functions and discontinuities more effectively compared to other fractional operators. This makes it more versatile and applicable in a wider range of applications. Other definitions of fractional operators include the Grunwald-Letnikov fractional derivative, which is defined as a finite difference approximation of the fractional derivative, and the Atangana-Baleanu fractional derivative, which is defined using the Caputo fractional derivative and a non-singular kernel. Each of these definitions has its advantages and limitations and is suitable for specific applications. Choosing the appropriate fractional operator for a given problem requires careful consideration of the problem's nature and the desired properties of the solution.

In many real-life applications, differential equations are used to model physical processes, and the development of fractional calculus has led to a growing interest in fractional differential equations (FDEs). The study of FDEs has significant implications in various fields, including physics, engineering, and finance. For example, Kilbas et al. [3] were among the first to introduce the basics of fractional calculus and its application to differential equations. Podlubny [4] further expanded on the possible applications of fractional calculus to differential equations and was one of the earliest researchers to study FDEs. Agarwal et al. [5] investigated solutions to a class of semi-linear FDEs in the form of periodic solutions. In the field of biology, Rahman et al. [6] adapted the singular-type and nonsingular fractional-order derivatives for simulating the plant-pathogen-herbivore interactions model. Additionally, Ali et al. [7] employed the new sub-equation method to attain new traveling wave solutions of conformable time FDEs. Moreover, Uzun et al. [8] studied the forced oscillatory theory for higher-order fractional differential equations with a damping term via the ψ-Hilfer fractional derivative. In the field of biology, FDEs have been contributing to the understanding of the dynamics and spread of many viruses. For example, Atede et al. [9] investigated the solution of a COVID-19 model incorporating the effect of vaccination through a fractional model with verification using real data from Nigeria. Also, Anjam et al. [10] simulated the dynamics of a fractional pollution model in a system of three interconnecting lakes. These are some examples of the applications of FDEs in simulating real-life phenomena. For more details on the application of FDEs, the reader may refer to [11–17] and references therein.

In this paper, we introduce the Genocchi collocation method for solving the following form of fractional differential equation

$$u^{(\eta)}(x) = \sum_{m=0}^{r} \sigma_m u^{(m)}(x) + \mu\left(x, u(x)\right), \qquad a < x < b, \quad r-1 < \eta < r, \tag{1}$$

and boundary conditions

$$u^{(i)}(a) = \alpha_i, \qquad u^{(i)}(b) = \beta_i. \tag{2}$$

The study of fractional calculus has led to the development of various methods for solving fractional differential equations (FDEs) of the form $\mathcal{D}^{\eta}u(x) = f(x)$, where η is the fractional order of the derivative, $u(x)$ and $f(x)$ are continuous functions, and $\mathcal{D}^{\eta}$ denotes the fractional derivative operator. Many of these methods aim to find the most accurate approximation for the solution. For instance, Jajarmi et al. [18] developed a new iterative method to solve a class of non-linear fractional boundary value problems (BVPs), while Patnaik et al. [19] provided a fractional order nonlocal continuum model of an Euler-Bernoulli beam along with its analytic form and finite element solution. Isah et al. [20] suggested using a novel operational approach based on Genocchi polynomials to numerically solve nonlinear FDEs, while El-Gamel et al. [21] solved the Bagley-Torvik equation using Legendre basis functions. Abd-Elhameed et al. [23] created sixth-order Chebyshev polynomials for numerically solving linear and nonlinear forms of fractional order differential equations, and Zaky [24] created and examined a singularity-preserving spectral-collocation approach for the numerical solution of nonlinear tempered fractional differential equations. Chuanli Wang et al. [25] provided a Legendre spectral collocation method for Caputo fractional boundary value problems, while Ismail et al. [26] proposed a numerical technique using the Green function, which combines cosine and sine functions, to solve linear and nonlinear FDEs. Akguel and Yalcin [27] solved problems involving fourth-order fractional boundary values using the reproducing kernel Hilbert space approach, and Li et al. [28] provided a new reproducing kernel collocation technique for solving nonlocal fractional boundary value problems with nonsmooth solutions. Rehman et al. [29, 30] presented a numerical method based on the operational matrices of integration of the Haar wavelet to solve linear two-point and multi-point boundary value problems for FDEs, while Saeed et al. [31] used the Haar wavelet-quasilinearization approach to solve the nonlinear heat transfer equation. Pedas et al. [32, 33] presented spline and piecewise polynomial collocation techniques for numerical solutions of a class of boundary value problems for nonlinear Caputo fractional differential equations, respectively. Finally, Ur Rehman et al. [34] solved FDEs using Legendre wavelets and developed an operational matrix of fractional order integration to convert them into a system of algebraic equations. These methods contribute to the development of effective and efficient techniques for solving FDEs, which have significant applications in science and engineering.

The paper aims to investigate the solution of FDEs using the collocation technique accompanied by Genocchi polynomials. This technique offers several advantages and disadvantages that need to be considered when applying it. Firstly, one advantageous aspect of using the Genocchi collocation method is its simplicity and ease of implementation in selecting collocation points within the specified domain to approximate the solution of the model. Additionally, the flexibility of the proposed method in handling different forms of boundary conditions makes it suitable for simulating physical models with complex behavior. Furthermore, this method often leads to sparse linear systems, which can be efficiently solved using numerical techniques, thus reducing computational costs and improving efficiency. However, the choice of collocation points plays a crucial role in obtaining accurate results. Moreover, the method may encounter difficulties when dealing with problems involving irregular or complex geometries. To the best of the authors' knowledge, this is the first time FDEs have been solved using the Genocchi collocation technique. The novelty of the paper lies in the following points:

- A new design of a novel collocation approach based on Genocchi polynomials for simulating the model.

• The proposed algorithm is implemented to solve both linear and nonlinear fractional models of different complexities.
• An error analysis for the proposed algorithm is conducted to determine the error bound and estimate the residual error.
• The effectiveness of the method in solving these models suggests its potential application to other similar models.
• The proposed results obtained from the Genocchi collocation scheme are compared for each variant to verify the accuracy of the newly designed system.

The organization of the paper is as follows: In Section 2, some basic properties and definitions of fractional calculus are illustrated. Section 3 provides the properties of Genocchi polynomials, which are used in the subsequent sections to simulate the general model. Section 4 introduces a new approach to illustrate the main steps for solving the main model. Section 5 is devoted to investigating the error bound and residual error function of the proposed method through theorems. In Section 6, multiple examples are simulated to demonstrate the efficiency of our technique. The conclusion for the work is given in Section 7.

2 Basic definitions

In this section, we will introduce some important definitions using later in next sections for solving fractional boundary value problems, starting by the following definitions.

Definition 1 *[3] The Riemann-Liouville fractional integral of order η of $f(t)$ is given by*

$$I^{\eta} f(t) = \frac{1}{\Gamma(\eta)} \int_0^t (t-\tau)^{\eta-1} f(\tau) d\tau, \quad t > 0, \eta \in R^{+}, \tag{3}$$

where $\Gamma(\eta)$ is the well known gamma function.

Definition 2 *[3] The Riemann Liouville fractional derivative of order $\eta > 0$ is defined by*

$$\mathcal{D}_t^{\eta}(t) = \left(\frac{d}{dt}\right)^m I^{m-\eta} f(t), \quad (\eta > 0, m-1 < \eta < m).$$

Some properties of I^{η} are as following:

$$I^{\eta} I^{\varphi} f(t) = I^{\eta+\varphi} f(t), \quad \eta > 0, \varphi > 0, \tag{4}$$

$$I^{\eta} t^{\varphi} = \frac{\Gamma(\varphi+1)}{\Gamma(\eta+\varphi+1)} t^{\varphi+\eta}. \tag{5}$$

Definition 3 *[3] The Caputo fractional derivative D^{η} of a function $f(t)$ is defined as*

$$\mathcal{D}^{\eta} f(t) = \frac{1}{\Gamma(n-\eta)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\eta-n+1}} d\tau, \quad n-1 < \eta < n, n \in N. \tag{6}$$

Some properties of Caputo fractional derivatives are as follows:

$$\mathcal{D}^{\eta}t^{\varphi}=\begin{cases}0, & \varphi\in N\cup\{0\}\quad and\quad \varphi<\lceil\eta\rceil\\ \frac{\Gamma(\varphi+1)}{\Gamma(\eta+1-\varphi)}t^{\varphi-\eta}, & \varphi\in N\cup\{0\}\quad and\quad \varphi\geq\lceil\eta\rceil\\ & or\quad \varphi\notin N\quad and\quad \varphi>\lfloor\eta\rfloor\end{cases}, \tag{7}$$

where, $\lfloor\eta\rfloor$ denotes the largest integer less than or equal to η and $\lceil\eta\rceil$ is the smallest integer greater than or equal to η.

$$\mathcal{D}^{\eta}C=0,\qquad C=constant. \tag{8}$$

The operator D^{η} is a linear operator, since,

$$\mathcal{D}^{\eta}\left(Af(t)+Bg(t)\right)=AD^{\eta}f(t)+BD^{\eta}g(t), \tag{9}$$

where A and B are constants. The novelty of the paper lies in the fact that the use of the Genocchi polynomials has many advantages over other similar polynomials. The Genocchi polynomials have the advantage of providing accurate results with high accuracy of less basis. In addition, the computational cost of finding an accurate solution is less than the other methods in the literature.

3 Fundamental relations

In this section, we will illustrate the basic concepts of Genocchi polynomials and Genocchi operational matrix for integer and fractional derivatives that will be needed in later sections for solving this type of equation.

Genocchi polynomials and their properties

In this subsection, we will illustrate the basic concepts of Genocchi polynomials. The generating function of the Genocchi polynomials can take the following form [35–37]:

$$Q(x,t)=\frac{2te^{xt}}{e^{t}+1}=\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!},\qquad (|t|<\pi), \tag{10}$$

where $G_n(x)$ is the Genocchi polynomials of degree n and are defined on interval $[0,1]$ as

$$G_n(x)=\sum_{k=0}^{n}\binom{n}{k}G_k x^{n-k}, \tag{11}$$

where G_k is the Genocchi numbers and are defined by the generating function

$$Q(t)=\frac{2t}{e^{t}+1}=\sum_{n=0}^{\infty}G_n\frac{t^n}{n!},\qquad (|t|<\pi). \tag{12}$$

The first few Genocchi polynomials can be found in the form

$$G_1(x)=1,$$
$$G_2(x)=2x-1,$$
$$G_3(x)=3x^2-3x,$$

$$G_4(x) = 4x^3 - 6x^2 + 1,$$
$$G_5(x) = 5x^4 - 10x^3 + 5x.$$

These polynomials have many interesting properties and one of these important properties is the differential property. By differentiating both sides of Eq. (11) with respect to x, we get the following:

$$\frac{dG_n(x)}{dx} = nG_{n-1}(x), \qquad n \geq 1. \tag{13}$$

If we differentiate Eq. (11) k times, then we have

$$\frac{d^k G_n(x)}{dx^k} = \begin{cases} 0, & n \leq k \\ k!\binom{n}{k} G_{n-k}(x), & n > k \end{cases} \quad k, n \in N \cup \{0\}, \tag{14}$$

$$G_n(1) + G_n(0) = 0, \qquad n > 1. \tag{15}$$

In the next two subsections, we introduce the differentiation matrices for both integer and fractional derivatives of boundary value problems.

Genocchi operational matrix of integer derivative

First, we express the approximate solution in Eq. (11) in the following form

$$u_N(x) = \sum_{n=1}^{N} c_n G_n(x) = \mathbf{G}(x)\mathbf{C}, \tag{16}$$

where C are the unknown Genocchi coefficients and $G(x)$ are the Genocchi polynomials of the first kind, then they are given by

$$\mathbf{C}^t = \begin{bmatrix} c_1 & c_2 & ... & c_N \end{bmatrix}, \qquad \mathbf{G}(x) = \begin{bmatrix} G_1(x) & G_2(x) & ... & G_N(x) \end{bmatrix}.$$

The kth derivative of $u_N(x)$ can be expressed by

$$u_N^{(k)}(x) = \sum_{n=1}^{N} c_n G_n^{(k)}(x) = \mathbf{G}(x)\mathbf{M}^k\mathbf{C}, \qquad k = 1, 2, ... \tag{17}$$

where $\mathbf{M}$ is $N \times N$ operational matrix of derivative, and is given by

$$\mathbf{M} = \begin{bmatrix} 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Genocchi operational matrix of fractional derivative

We can find the fractional derivative of Genocchi polynomials in Eq. (11) from the following theorem.

Theorem 1 *[20] Let* $\mathbf{G}(x) = \begin{bmatrix} G_1(x) & G_2(x) & \dots & G_N(x) \end{bmatrix}$ *is the Genocchi vector and* $\eta > 0$. *Then the fractional derivative for any Genocchi polynomial can be found from*

$$D^{\eta} G_i(x) = \sum_{k=1}^{i} \frac{i! G_{i-k}}{(i-k)! k!} D^{\eta} x^k = \sum_{k=\lceil \eta \rceil}^{i} \frac{i! G_{i-k}}{(i-k)! \Gamma(k+1-\eta)} x^{k-\eta}, \tag{18}$$

where

$$D^{\eta} G_i(x) = 0, \qquad i \leq \lceil \eta \rceil,$$

and the matrix form of the fractional derivative will be in the form

$$\mathbf{H}(x) = \begin{bmatrix} 0 & 0 & \dots & H_{\lceil \eta \rceil + 1}(x) & \dots & H_N(x) \end{bmatrix}. \tag{19}$$

4 Method of solution

In this section, we solve the fractional differential boundary value problems with linear and nonlinear forms using Genocchi collocation method. First we approximate $u(x)$ as following

$$u_N(x) = \sum_{n=1}^{N} c_n G_n(x) = \mathbf{G}(x)\mathbf{C}, \tag{20}$$

and approximate the fractional derivative from Eq. (20) as

$$u_N^{(\eta)}(x) = \sum_{n=1}^{N} c_n G_n^{(\eta)}(x) = \mathbf{H}(x)\mathbf{C}. \tag{21}$$

Linear case

First, let $\mu(x, u(x)) = f(x)$ in Eq. (1), then

$$u^{(\eta)}(x) = \sum_{m=0}^{r} \sigma_m u^{(m)}(x) + f(x), \qquad 0 < x < 1, \quad r-1 < \eta < r, \tag{22}$$

after substituting equations (20), (21), and (17) in Eq. (22), we reach the following theorem.

Theorem 2 *If the assumed approximate solution of the fractional problem (22), and (2) are (20), (17), and (21), then the discrete Genocchi system for calculating the unknown coefficients is given by*

$$\sum_{n=1}^{N} c_n H_n(x_i) = \sum_{m=0}^{r} \sum_{n=1}^{N} \sigma_m c_n G_n^{(m)}(x_i) + f(x_i). \tag{23}$$

Proof By replacing each term in Eq. (22) with its approximation from equations (20), (17), and (21) and substituting collocation points given by the following equation

$$x_i = \frac{i-1}{N-1}, \qquad i = 1, 2, ..., N. \tag{24}$$

■

The matrix form of system (23) can be written by

$$\mathbf{\Psi C} = \mathbf{F}, \tag{25}$$

where

$$\mathbf{\Psi} = \mathbf{H} - \left(\sum_{m=0}^{r} \boldsymbol{\sigma}_m \mathbf{G} \mathbf{M}^m \right), \tag{26}$$

and

$$\boldsymbol{\sigma}_m = \begin{bmatrix} \sigma_m & 0 & ... & 0 \\ 0 & \sigma_m & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & \sigma_m \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & ... & H_{\lceil \eta \rceil + 1}(x_1) & ... & H_N(x_1) \\ 0 & 0 & ... & H_{\lceil \eta \rceil + 1}(x_2) & ... & H_N(x_2) \\ \vdots & \vdots & ... & \vdots & ... & \vdots \\ 0 & 0 & ... & H_{\lceil \eta \rceil + 1}(x_N) & ... & H_N(x_N) \end{bmatrix}.$$

The matrix forms of boundary conditions are given by

$$\mathbf{G}(0)\mathbf{M}^i\mathbf{C} = [\alpha_i], \qquad \mathbf{G}(1)\mathbf{M}^i\mathbf{C} = [\beta_i]. \tag{27}$$

After replacing r rows of the augmented matrix with boundary conditions, then the new augmented matrix takes the form

$$\bar{\mathbf{\Psi}}\mathbf{C} = \bar{\mathbf{F}}. \tag{28}$$

Finally, obtaining the unknown coefficients $\mathbf{C}$ by solving the resulting $N \times N$ system of linear algebraic equations.
In the next subsection, we will treat with nonlinear case of fractional boundary value problem.

Nonlinear case

By replacing $\mu(x, u(x)) = \sum_{m=1}^{r} \zeta_m u^m(x) + f(x)$, we reach the nonlinear form

$$u^{(\eta)}(x) = \sum_{m=0}^{r} \sigma_m u^{(m)}(x) + \sum_{m=1}^{r} \zeta_m u^m(x) + f(x), \qquad 0 < x < 1, \quad r-1 < \eta < r, \tag{29}$$

the nonlinear terms in Eq. (29) can be approximated according to the following theorem:

Theorem 3 [38] *The nonlinear term of the function $u^v(x_i), i = 1,2,...,N$ can be expressed as in the following matrix form*

$$\begin{aligned}\begin{bmatrix} u^m(x_1) \\ u^m(x_2) \\ \vdots \\ u^m(x_N) \end{bmatrix} &= \begin{bmatrix} u(x_1) & 0 & \dots & 0 \\ 0 & u(x_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u(x_N) \end{bmatrix}^{m-1} \begin{bmatrix} u(x_1) \\ u(x_2) \\ \vdots \\ u(x_N) \end{bmatrix} \\ &= (\bar{\mathbf{U}})^{m-1}\mathbf{U} \\ &= (\bar{\mathbf{G}}\bar{\mathbf{C}})^{m-1}\boldsymbol{GC},\end{aligned} \tag{30}$$

where

$$\bar{\mathbf{G}} = \begin{bmatrix} \boldsymbol{G}(x_1) & 0 & \dots & 0 \\ 0 & \boldsymbol{G}(x_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \boldsymbol{G}(x_N) \end{bmatrix}, \qquad \bar{\mathbf{C}} = \begin{bmatrix} \boldsymbol{C} & 0 & \dots & 0 \\ 0 & \boldsymbol{C} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \boldsymbol{C} \end{bmatrix}.$$

After substituting equations (20), (21), and (17) in Eq. (29), we reach the following theorem:

Theorem 4 *If the assumed approximate solution of the fractional problem (29), and (2) are (20), (17), and (21), then the discrete Genocchi system for calculating the unknown coefficients is given by*

$$\sum_{n=1}^{N} c_n H_n(x_i) = \sum_{m=0}^{r}\sum_{n=1}^{N} \sigma_m c_n G_n^{(m)}(x_i) + \sum_{m=1}^{r}\sum_{n=1}^{N} \zeta_m c_n G_n^m(x_i) + f(x_i). \tag{31}$$

Proof We begin by replacing each term in Eq. (29) with its approximation from equations (20), (17), and (21). Then, by substituting collocation points given by Eq. (24) into this system, we get the following matrix form:

$$\mathbf{\Psi C} = \mathrm{F}, \tag{32}$$

where

$$\mathbf{\Psi} = \mathbf{H} - \left(\sum_{m=0}^{r} \boldsymbol{\sigma}_m \mathbf{G}\mathbf{M}^m - \sum_{m=1}^{r} \zeta_m (\bar{\mathbf{G}}\bar{\mathbf{C}})^{m-1}\mathbf{G} \right), \tag{33}$$

and

$$\zeta_m = \begin{bmatrix} \zeta_m & 0 & \dots & 0 \\ 0 & \zeta_m & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \zeta_m \end{bmatrix},$$

after replacing r rows of augmented matrix with boundary conditions matrices from Eq. (27), then the new augmented matrix take the form

$$\bar{\Psi}\mathbf{C} = \bar{\mathrm{F}}. \tag{34}$$

Finally, obtaining the unknown coefficients **C** by solving the resulting $N \times N$ system of nonlinear algebraic equations by using the following algorithm:

Algorithm

• input (integer) N.
• input (double) tol.
• input (array) $C_{old} = \mathbf{C}_0$, (initial approximation, $\mathbf{C}_0$ with N dimension, can be chosen so that the boundary conditions are satisfied.)
• $\bar{\Psi}(\mathbf{C}_{old}).\mathbf{C}_{new} = \bar{\mathrm{F}}$ is a linear algebraic equation system. This system is solved and $\mathbf{C}_{new}$ is found.
• If $|\mathbf{C}_{old} - \mathbf{C}_{new}| < \mathtt{tol}$ then $\mathbf{C}_{new} = \mathbf{C}$. break (the program is finished).
• Else then $\mathbf{C}_{old} \leftarrow \mathbf{C}_{new}$.
• Go to the second stage. ■

5 Error bound

Error bound estimate

In this subsection, we will provide the error bound for the obtained solution of model 1. We provide the error bound for a special case of the model where the value of $\mu(x, u(x)) = \mathfrak{g}(x)$. Suppose that $\mathfrak{g}(x) \in C^{n+1}[0,1]$ and the space $\Xi = Span\{G_1(x), G_2(x), ..., G_N(x)\}$. Next, if the best approximation of $\mathfrak{g}(x)$ can be in the form $\mathbf{C}^T\mathbf{G}(x)$, then we reach the following theorem:

Theorem 5 *Suppose that $\mathfrak{g}(x) \in C^{n+1}[0,1]$ and define $\Xi = Span\{G_1(x), G_2(x), ..., G_N(x)\}$ where $\boldsymbol{C}^T\boldsymbol{G}(x)$ is the best approximation of the function $\mathfrak{g}(x)$ out of Ξ, then we have*

$$\|\mathfrak{g}(x) - \boldsymbol{C}^T\boldsymbol{G}(x)\| \leq \frac{\mathfrak{I}^{\frac{2m+3}{2}}\mathfrak{R}}{(m+1)!\sqrt{2m+3}}, \qquad x \in [x_i, x_{i+1}] \subseteq [0,1],$$

where $\mathfrak{R} = \max_{x\in[x_i,x_{i+1}]} |\mathfrak{g}^{(m+1)}(x)|$ and $\mathfrak{I} = x_{i+1} - x_i$.

Proof To prove this theorem. We first expand the function $u(x)$ in the following Taylor expansion form

$$u_1(x) = \mathfrak{g}(x_i) + \mathfrak{g}^{'}(x_i)(x - x_i) + \mathfrak{g}^{''}(x_i)\frac{(x - x_i)^2}{2!} + ... + \mathfrak{g}^{(n)}(x_i)\frac{(x - x_i)^n}{n!}. \tag{35}$$

Then, for the previous form of Taylor expansion, if we apply the modulus for both sides of Eq. (35), we can deduce in the following compact form

$$|\mathfrak{g}(x) - u_1(x)| \leq |\mathfrak{g}^{(n+1)}(\aleph_x)|\frac{(x - x_i)^{n+1}}{(n+1)!},$$

where

$$\aleph_x \in [x_i, x_{i+1}].$$

With the assumption that $\mathbf{C}^T\mathbf{G}(t)$ is the best approximation of the function $\mathfrak{g}(x)$ out of the space Ξ and that $u_1(t) \in \Xi$, then we have

$$\|\mathfrak{g}(x) - \mathbf{C}^T\mathbf{G}(x)\|_2^2 \leq \|\mathfrak{g}(x) - u_1(x)\|_2^2 = \int_{x_i}^{x_{i+1}} |\mathfrak{g}(h) - u_1(h)|^2 dh$$
$$\leq \int_{x_i}^{x_{i+1}} \|\mathfrak{g}(x)^{(m+1)}(\aleph_x)\|^2 \frac{(h-x_i)^{m+1}}{(m+1)!} dh \leq \frac{\mathfrak{I}^{2m+3}\mathfrak{R}^2}{((m+1)!)^2(2m+3)}.$$

Then, finally taking the square root for both sides, we conclude that

$$\|\mathfrak{g}(x) - \mathbf{C}^T\mathbf{G}(x)\| \leq \frac{\mathfrak{I}^{\frac{2m+3}{2}}\mathfrak{R}}{(m+1)!\sqrt{2m+3}}.$$

■

This theorem provides a local error bound for the proposed main equation of $\mathcal{O}(\mathfrak{I}^{\frac{2m+3}{2}})$.

Residual error function

In this subsection, We can easily check the accuracy of the suggested method in terms of the residual error function. Since the truncated Genocchi series in Eq. (16) is considered as an approximate solution of Eq. (1), then by substituting the approximate solution $u_N(x)$ and its derivatives into Eq. (1), the resulting equation must be satisfied, and when substituting the collocation points defined as

$$x = x_i \in [0,1], \qquad i = 1, 2, ..., N,$$

the residual error function for the approximate solution can be calculated in the form

$$|\,\mathfrak{R}_N(x_i)\,| = |\,u^{(\eta)}(x) - \sum_{m=0}^{r} \sigma_m u^{(m)}(x) - \mu(x, u(x))\,| \cong 0, \tag{36}$$

or

$$\mathfrak{R}_N(x_i) \leq 10^{-\tau i},$$

where $\mathfrak{R}_N(x_i)$ are the residual error function defined at the collocation points x_i and τi is any positive integer. If $\max 10^{\tau i} = 10^{\tau}$ (τ is any positive integer) can be prescribed which can be considered as the tolerance for the obtained error, then the value of the number of iterations N is increased until the residual error $\mathfrak{R}_N(x_i)$ at each of the points become smaller than the prescribed tolerance 10^{τ} which shall prove that the method converge to the desired solution as the residual error approaches zero. Also, we can calculate the error function at each of the collocation points to prove the efficiency of the proposed technique which can be described as

$$\mathfrak{R}_N(x_i) = u^{(\eta)}(x) - \sum_{m=0}^{r} \sigma_m u^{(m)}(x) - \mu(x, u(x)).$$

Then, if $u_N(x) \to 0$, as N has sufficiently enough value, then the residual error decreases and this proves that the proposed method converges correctly.

6 Numerical simulation

In this section, we present 7 examples [20, 21, 25, 33, 34, 39, 40] for linear and nonlinear forms of fractional problems using Genocchi collocation method. The error measurements for verifying the results in the later examples can be used in the following form

$$\mathbf{e}_N(x) = |(u(x) - u_N(x))|,$$

and the maximum absolute error is given by

$$\|\mathbf{e}_N(x)\|_\infty = max\|u(x) - u_N(x)\|.$$

In addition, the L_2 norm can be defined in the following form:

$$\|\mathbf{e}_N(x)\|_2 = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathbf{e}_N(x))^2}.$$

Example 1 *[21] Consider the following linear fractional BVP*

$$u'' + u^{(3/2)} + u = x + 1, \qquad 0 < x < 1,$$

with boundary conditions

$$u(0) = 1, \quad u(1) = 2,$$

and exact solution $u = x + 1$. We provide the details for obtaining the approximate solution for $N = 6$ as follows, let the approximate solution in the form

$$u(x) = c_1G_1(x) + c_2G_2(x) + \cdots + c_6G_6(x),$$

then

$$M^2 = \begin{bmatrix} 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using collocation points $x_i = \frac{i-1}{5}$, $\quad i = 1, 2, \cdots, 6$, then we have

$$G = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & -3 \\ 1 & -0.6 & -0.48 & 0.792 & 0.928 & -2.42208 \\ 1 & -0.2 & -0.72 & 0.296 & 1.488 & -0.92256 \\ 1 & 0.2 & -0.72 & -0.296 & 1.488 & 0.92256 \\ 1 & 0.6 & -0.48 & -0.792 & 0.928 & 2.42208 \\ 1 & 1 & 0 & -1 & 0 & 3 \end{bmatrix}_{(6\times 6)},$$

$$\boldsymbol{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.02776 & -4.44071 & -3.39109 & 13.42249 \\ 0 & 0 & 4.28190 & -3.99644 & -7.76451 & 12.95335 \\ 0 & 0 & 5.24423 & -2.09770 & -10.90800 & 6.37099 \\ 0 & 0 & 6.05552 & 0.80740 & -11.62659 & -3.38417 \\ 0 & 0 & 6.77027 & 4.51352 & -9.02703 & -12.57337 \end{bmatrix}_{(6\times 6)},$$

and the augmented matrix becomes as

$$[\boldsymbol{\Psi}, \mathbf{F}] = \begin{bmatrix} 1 & -1 & 6 & -11 & 0 & 27 & , & 1 \\ 1 & -0.6 & 8.54776 & -10.84871 & -12.06309 & 34.76041 & , & 1.2 \\ 1 & -0.2 & 9.56190 & -6.10044 & -20.67651 & 20.91079 & , & 1.4 \\ 1 & 0.2 & 10.52423 & 0.00631 & -23.82000 & -1.58645 & , & 1.6 \\ 1 & 0.6 & 11.57552 & 7.21540 & -20.29860 & -24.72209 & , & 1.8 \\ 1 & 1 & 12.77028 & 15.51352 & -9.02703 & -39.573377 & , & 2 \end{bmatrix}.$$

Next, the augmented matrix for the boundary conditions according to Eq. (27) can take the forms

$$[\psi_1, \alpha_0] = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & -3 & , & 1 \end{bmatrix},$$

$$[\psi_2, \beta_0] = \begin{bmatrix} 1 & 1 & 0 & -1 & 0 & 3 & , & 2 \end{bmatrix}.$$

Replacing the first and last rows with the previous representation of the boundary conditions, the new augmented matrix takes the form

$$[\bar{\boldsymbol{\Psi}}, \bar{\mathbf{F}}] = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & -3 & , & 1 \\ 1 & -0.6 & 8.54776 & -10.84871 & -12.06309 & 34.76041 & , & 1.2 \\ 1 & -0.2 & 9.56190 & -6.10044 & -20.67651 & 20.91079 & , & 1.4 \\ 1 & 0.2 & 10.52423 & 0.00631 & -23.82000 & -1.58645 & , & 1.6 \\ 1 & 0.6 & 11.57552 & 7.21540 & -20.29860 & -24.72209 & , & 1.8 \\ 1 & 1 & 0 & -1 & 0 & 3 & , & 2 \end{bmatrix}.$$

Then, by solving the above linear system the Genocchi coefficients can be found as

$$\boldsymbol{C} = \begin{bmatrix} 1.5000 \\ 0.5000 \\ -3.1258E-17 \\ -2.6724E-16 \\ -1.3235E-17 \\ -8.2262E-17 \end{bmatrix},$$

and the approximate solution is

$$u_6(x) = 1 + x + 0.2757E - 15x^2 - 0.9366E - 15x^3 + 0.1168E - 14x^4 - 0.4936E - 15x^5.$$

By using Genocchi collocation method for solving this form of fractional boundary value problem at $N = 6$ *having the exact solution* $u = x + 1$, *we reach that the approximate solution is equal to the exact solution with running time* 5.079 *seconds. For* $N = 14$ *the absolute error and the residual error are represented in*

Table 1. From this table, it can be noted that the method provides accurate results using a few numbers of Genocchi bases. In addition, a comparison between exact and approximate solutions is presented in Figure 1.

Table 1. Absolute and residual error for Example 1 at $N = 14$.

x	$\lvert \mathbf{e}_N(x) \rvert$	$\lvert \mathfrak{R}_N \rvert$
0.0	1.5543E-15	9.1807E-15
0.1	1.3323E-15	9.6109E-16
0.2	8.8818E-16	5.0143E-16
0.3	4.4409E-16	3.0309E-16
0.4	0.0000	3.1559E-16
0.5	2.2204E-16	1.4750E-16
0.6	4.4409E-16	1.2567E-16
0.7	8.8818E-16	1.1833E-16
0.8	1.1102E-15	6.3519E-17
0.9	1.5543E-15	1.3599E-16
1.0	1.7764E-15	1.6221E-15

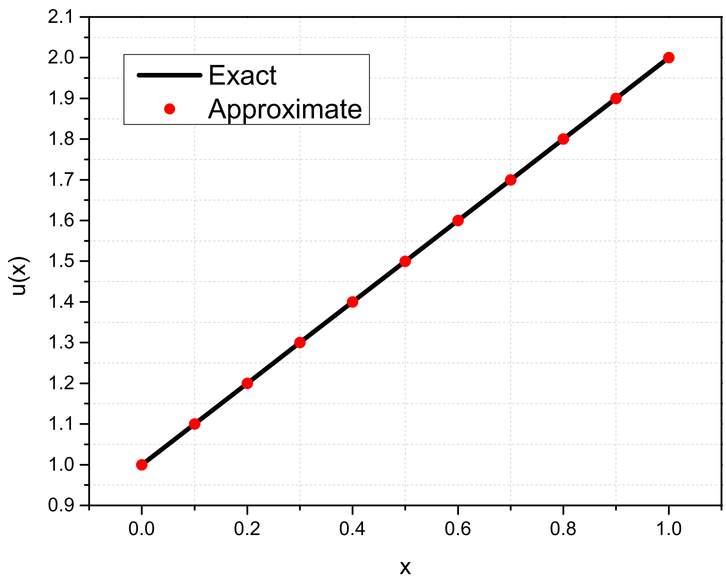

Figure 1. Comparison between exact and Genocchi solution for Example 1.

Example 2 *[21, 22] Consider the linear fractional IVP taken the form*

$$u'' + u^{(3/2)} + u = 7x + \frac{8}{\sqrt{\pi}}x^{3/2} + x^3 + 1, \qquad 0 < x < 1,$$

with initial conditions

$$u(0) = 1, \quad u'(0) = 1,$$

and exact solution

$$u = x^3 + x + 1.$$

Comparing the approximate solution obtained by Genocchi collocation method and shifted Legendre collocation method [21] at N = 15 with the exact solution in Table 2 and the Genocchi solution and shifted Legendre solution are represented in Figure 3. The absolute error for Genocchi solution when N = 15 is appearing in Table 3 and compared to the results reported by using the Lucas Wavelet Scheme in [22]. Based on these results, it can be seen that the proposed method provides better accuracy. In addition, it can be noticed from Figure 2, which appears the exact and Genocchi approximate solution that our method is very accurate.

Table 2. Exact and approximate solution for Example 2.

x	Exact	Approximate	Shifted Legendre [21]
0.10	1.101000	1.101000	1.101000
0.25	1.265625	1.265625	1.265625
0.50	1.625000	1.625000	1.625000
0.75	2.171875	2.171875	2.171875
1.0	3.000000	3.000000	3.000002

Table 3. Absolute error for Example 2.

x	$\lvert \mathbf{e}_N(x) \rvert$	Lucas Wavelet [22]
0.0	2.2204E-16	×
0.1	4.4409E-16	1.99E-15
0.2	4.4409E-16	×
0.3	6.6613E-16	×
0.4	6.6613E-16	×
0.5	6.6613E-16	4.90E-14
0.6	6.6613E-16	×
0.7	4.4409E-16	×
0.8	0.0000	×
0.9	0.0000	×
1.0	4.4409E-16	1.96E-13

Example 3 *[39] Consider another form of linear fractional IVP*

$$u^{(\eta)} + u = (x^2 + 2x^{2-\eta}/\Gamma(3-\eta)) + (x^3 + 6x^{3-\eta}/\Gamma(4-\eta)), \quad 0 < x < 1,$$

with initial condition

$$u(0) = 0,$$

the exact solution

$$u = x^3 + x^2.$$

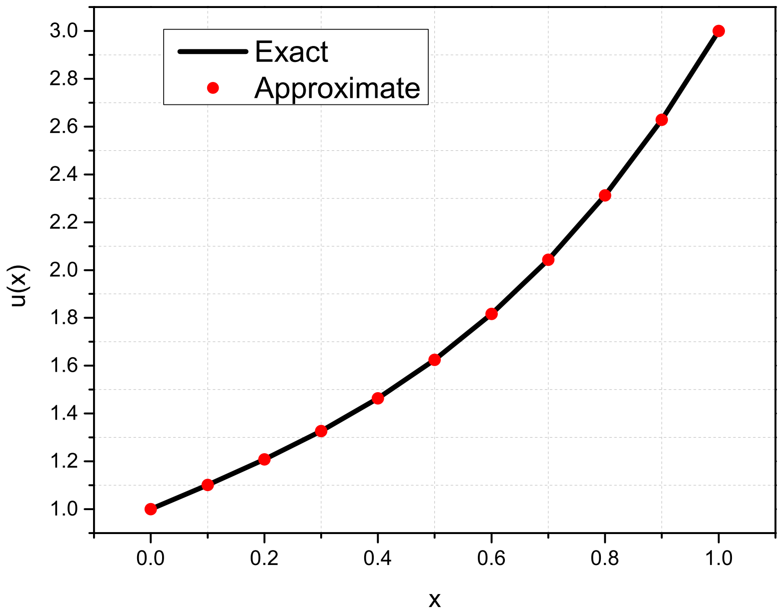

Figure 2. Comparison between exact and Genocchi solution for Example 2.

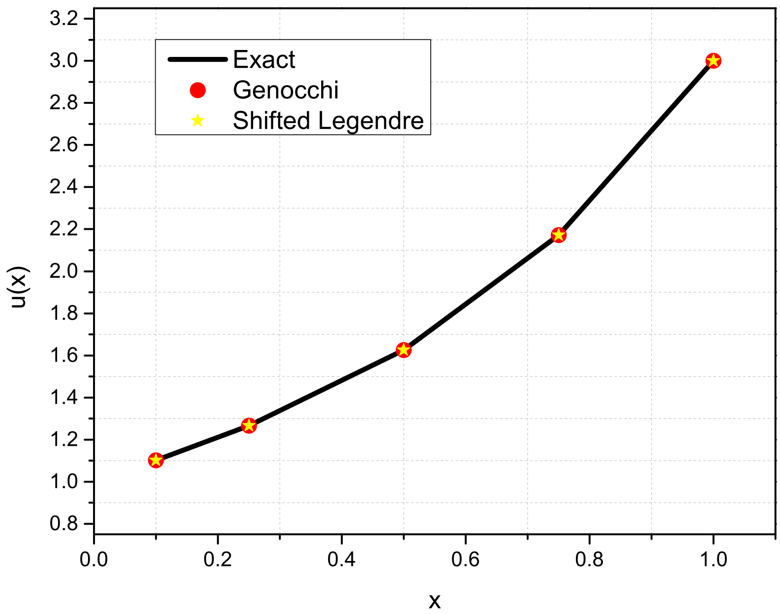

Figure 3. Comparison between Genocchi solution and Shifted Legendre for Example 2.

Taking the value of $\eta = 1/2$*, we reach the absolute error for* $N = 6$ *using Genocchi collocation method tabulated in Table 4. In addition, the running time for simulating the results is found to be 5.651 seconds with an error norm of* $\|e_6(x)\|_2 = 3.0978E-15$*. The value of the acquired norm reveals the ability of the method to provide accurate solutions. In addition, the behavior of exact and approximate Genocchi solution is in Figure 4.*

Table 4. Absolute error for Example 3.

x	$\|\mathbf{e}_N(x)\|$
0.0	1.8111E-15
0.1	1.9082E-16
0.2	1.5127E-15
0.3	2.9143E-15
0.4	3.8580E-15
0.5	4.3299E-15
0.6	4.4409E-15
0.7	4.2188E-15
0.8	3.7748E-15
0.9	2.8866E-15
1.0	1.7764E-15

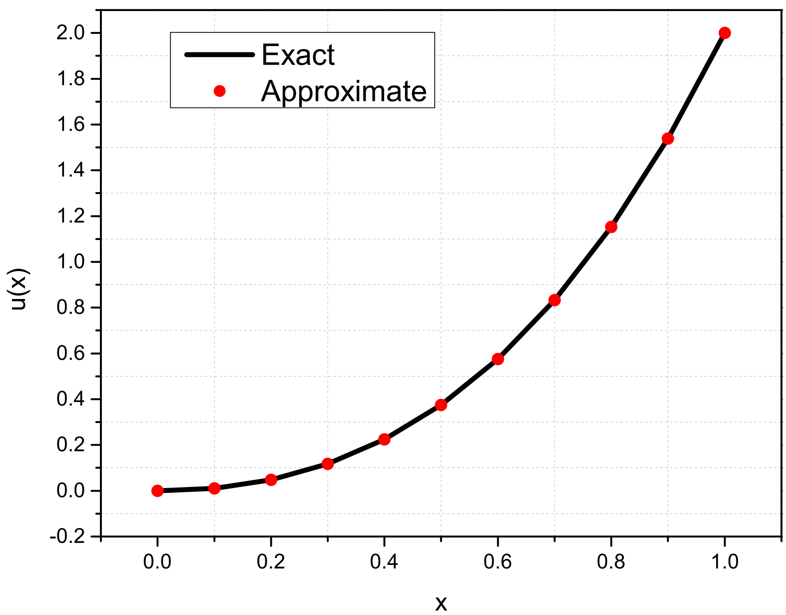

Figure 4. Comparison between exact and Genocchi solution for Example 3.

Example 4 *[20] Consider the following nonlinear fractional BVP*

$$u'' + \Gamma(\frac{4}{5})x^{\frac{6}{5}}u^{(\frac{6}{5})} + \frac{11}{9}\Gamma(\frac{5}{6})x^{\frac{1}{6}}u^{(\frac{1}{6})} - (u')^2 = 2 + \frac{1}{10}x^2, \quad 0 < x < 1,$$

with boundary conditions

$$u(0) = 1, \quad u(1) = 2,$$

and exact solution

$$u = x^2 + 1.$$

Seeing from Table 5 which represents the absolute error obtained by Genocchi collocation method for $N = 6$ with a running time 10.912 seconds, our method is very accurate for solving this type of fractional BVPs.

Besides that comparison between exact and approximate Genocchi solution is shown in Figure 5.

Table 5. Absolute error for Example 4.

x	$\|\mathbf{e}_N(x)\|$
0.0	0.0000
0.1	3.1752E-14
0.2	5.6177E-14
0.3	6.6391E-14
0.4	4.5519E-14
0.5	1.9762E-14
0.6	1.3101E-13
0.7	2.6557E-13
0.8	3.6660E-13
0.9	3.3085E-13
1.0	0.0000

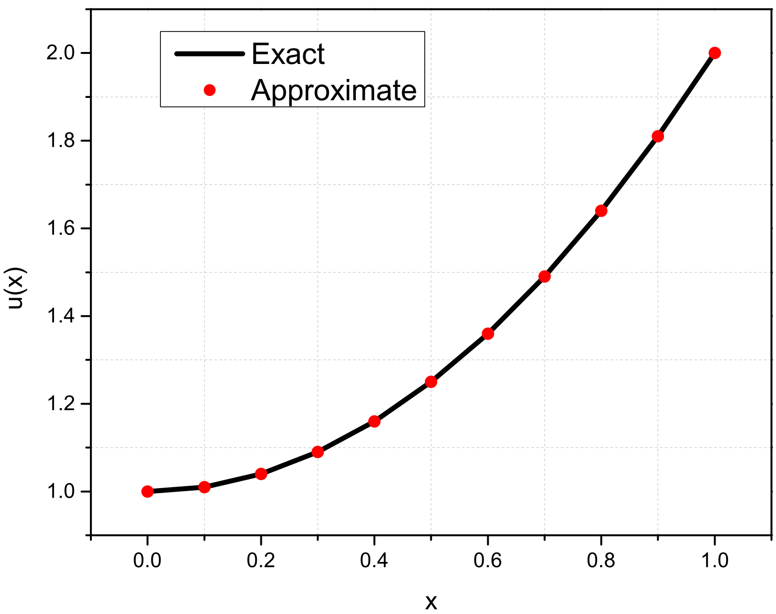

Figure 5. Comparison between exact and Genocchi solution for Example 4.

Example 5 *[33] Consider the following nonlinear fractional BVP*

$$u^{(\frac{3}{2})} - u^3 = \frac{\Gamma(2.9)}{\Gamma(1.4)} x^{0.4} - (x^{1.9} - 1)^3,$$

with boundary conditions

$$u(0) = -1, \quad u(1) = 0,$$

the exact solution is

$$u = x^{1.9} - 1.$$

Representing the absolute error obtained by Genocchi collocation method with N = 10 in Table 6 and the comparison between maximum absolute error obtained by Genocchi collocation method and spline collocation method [33] for different values of N in Table 7. In addition, the exact and approximate Genocchi solutions are shown in Figure 6.

Table 6. Absolute error for Example 5.

x	$\lvert \mathbf{e}_N(x) \rvert$
0.0	6.6613E-16
0.1	2.3761E-4
0.2	2.9613E-4
0.3	3.0385E-4
0.4	2.9104E-4
0.5	2.6203E-4
0.6	2.1928E-4
0.7	1.6267E-4
0.8	9.4943E-5
0.9	1.2709E-5
1.0	6.3838E-16

Table 7. Comparison between maximum absolute error for Example 5.

N	$\lVert \mathbf{e}_N(x) \rVert$	Spline collocation [33]
4	1.8688E-03	1.24E-3
8	4.6148E-04	3.57E-4

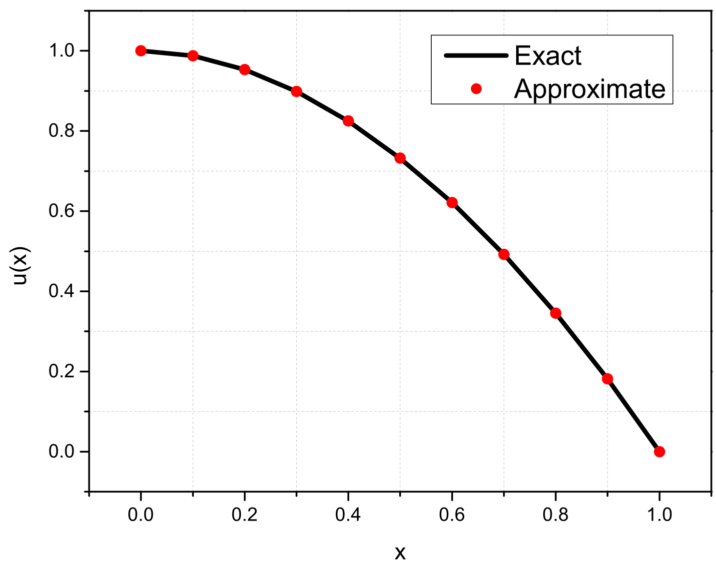

Figure 6. Comparison of exact and Genocchi solutions at $\eta = 3/2$ for Example 5.

Example 6 *[40] Consider the following nonlinear fractional BVP*

$$u^{(\frac{3}{2})} + e^{-2\pi}u^2 = \frac{105\sqrt{\pi}}{32}x^2 + e^{-2\pi}x^7, \quad 0 < x < 1,$$

with boundary conditions

$$u(0) = 0, \quad u(1) = 1,$$

the exact solution

$$u = x^{7/2}.$$

Table 8 represents the comparison between the absolute error obtained by Genocchi collocation method $N = 10$, and Legendre wavelet method [34]. In addition, it is found that the error measure of the $\|e_6(x)\|_2 = 8.0268E-06$ and the behavior of exact and approximate solutions is graphed in Figure 7.

Table 8. Comparison of absolute error for Example 6.

x	$\lvert \mathbf{e}_N(x)\rvert$	Legendre wavelet [34]
0.0	5.7246E-17	x
0.1	1.0507E-5	9.6996E-5
0.2	1.3141E-5	9.3927E-4
0.3	1.2742E-5	1.5087E-3
0.4	1.1182E-5	3.3989E-4
0.5	8.7996E-6	2.4163E-3
0.6	5.9264E-6	3.1023E-4
0.7	2.5945E-6	1.4799E-3
0.8	9.2167E-7	6.3407E-4
0.9	5.2079E-6	4.6701E-3
1.0	1.1102E-16	x

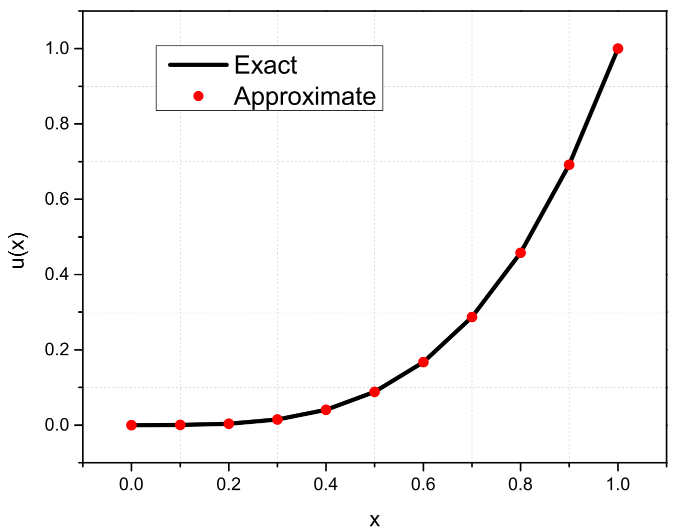

Figure 7. Comparison between exact and Genocchi solution at $\eta = 3/2$ for Example 6.

Example 7 *[25] Consider the following nonlinear fractional BVP*

$$u^{(5/4)} - u^2 = -\frac{\Gamma(128/17)}{\Gamma(128/17-\eta)} x^{111/17-\eta} - (x - x^{111/17})^2, \quad 0 < x < 1,$$

with boundary condition

$$u(0) = 0, \quad u(1) = 0,$$

the exact solution

$$u = x - x^{111/17}.$$

A comparison between exact and approximate Genocchi solution is represented in Figure 8, and the absolute error for N $= 10$ *obtained by Genocchi collocation method is represented in Table 9.*

Table 9. Absolute error for Example 7.

x	$\lvert \mathbf{e}_N(x) \rvert$
0.0	1.9559E-16
0.1	1.4825E-07
0.2	3.2983E-08
0.3	1.0581E-07
0.4	2.6299E-07
0.5	4.4011E-07
0.6	6.3645E-07
0.7	8.6430E-07
0.8	1.1047E-06
0.9	1.4344E-06
1.0	1.9559E-16

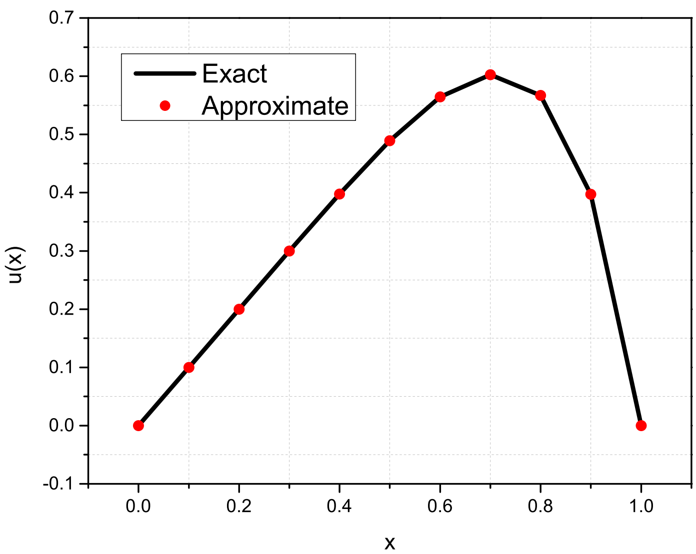

Figure 8. Comparison between exact and Genocchi solution at $\eta = 5/4$ for Example 7.

7 Conclusion

In this paper, we have examined the application of the Genocchi collocation technique for solving a general form of linear and nonlinear fractional models. The models of fractional order have great applications in science and engineering. Some basic definitions for the fractional order derivative are introduced and utilized for treating the fractional term in the main model. Then, the collocation technique is adapted for converting the model into a system of nonlinear algebraic equations which is then solved using a novel technique to find the values of the unknown coefficients, and hence, the solution is found. The error bound for the proposed technique is provided ensuring that the proposed technique has a local bound of $\mathcal{O}(\mathfrak{I}^{\frac{2m+3}{2}})$. The accuracy of the proposed technique is tested for several examples of different forms and the results are compared to other forms the literature provides the effectiveness of the technique in providing more accurate results with less computational cost. Thus, the method proved to be an effective technique for simulating similar models and has other important applications.

Declarations

List of abbreviations

Not applicable.

Ethical approval

The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

Consent for publication

Not applicable.

Conflicts of interest

The authors confirm that there is no competing interest in this study.

Data availability statement

Data availability is not applicable to this article as no new data were created or analyzed in this study.

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Author's contributions

M.G.: Conceptualization, Methodology, Writing - Original Draft, Writing - Review & Editing, Project administration, Supervision. N.S.: Methodology, Software, Validation, Formal analysis, Data Curation, Writing - Original Draft, Visualization. W.A.: Conceptualization, Methodology, Software, Validation, Formal analysis, Data Curation, Writing - Original Draft, Writing - Review & Editing, Supervision. All authors have read and agreed to the published version of the manuscript.

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