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Research Article

Risk Assessment for Breast Cancer with Integrated Group Decision-Making Method

Murat Kirişci *
İstanbul University-Cerrahpaşa, Department of Biostatistics and Medical Informatics
İstanbul, Türkiye

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Abstract: The most prevalent invasive malignancy in women is breast cancer. The second most common cause of cancer deaths in women, behind lung cancer, is breast cancer. It begins with developing a tiny tumor or mass and spreads from breast cells, primarily in the milk ducts (ductal carcinoma) or glands (lobular carcinoma). Every woman needs to be aware of her risk of developing breast cancer to be proactive about risk reduction measures and for better care of the disease, even though the causes of breast cancer are not fully known. Numerous variables that can either raise or decrease the likelihood of acquiring breast cancer have been identified by independent investigations. By looking at these risk factors, it is feasible to determine a woman's estimated risk of acquiring a malignant breast illness. Fermatean fuzzy sets can adequately describe the uncertain data for determining breast cancer risk. The cumulative prospect theory is used to build the traditional Tomada de Decisão Iterativa Multicritério (TODIM) approach, which can be used to reflect the psychological behavior of the decision-maker. The Fermatean fuzzy cumulative prospect theory-TODIM approach is proposed in this paper to handle the problem of group decision-making. Using the entropy weight method with Fermatean fuzzy sets to obtain attribute weight information simultaneously improves rationality. This article applies the mentioned method to the risk assessment of breast cancer. It illustrates the risk assessment model based on the proposed method, concentrating on hot topics in contemporary culture.

Keywords: Breast cancer, Fermatean fuzzy environment, cumulative prospect theory, TODIM, group decision-making.

1. Introduction

Women of all ages are susceptible to breast cancer, which is a highly diverse disease. The breast comprises many tissues, including dense and fatty tissue that includes milk glands, lobes, and lobules. Breast cancer occurs when breast cells multiply uncontrollably, leading to tumor formation.

*Correspondence: murat.kirisci@iuc.edu.tr
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Breast cancer is said to be metastatic if it spreads to other organs. There are two basic varieties of breast cancer: non-invasive breast cancer, which stays in the lobular part of the breasts, and invasive breast cancer, which spreads to nearby tissues or distant organs. In 2020, there were 2.3 million new instances of breast cancer in women and 685.000 deaths globally, according to the WHO data. For more targeted medicine and treatment, early prognosis prediction is therefore essential. Breast cancer is difficult to anticipate and treat since it is a complicated disease with a wide range of clinical outcomes. For instance, medical personnel face challenges with manual interpretation due to the high dimensionality of multimodal data. Therefore, the creation of computational algorithms becomes essential for precisely forecasting the prognosis of breast cancer. The importance of these methods in clinical decision-making is highlighted by the fact that these algorithms can help doctors choose the best course of action for their patients.

Artificial intelligence, cognitive science, psychology, philosophy, and other academic disciplines are all interested in how people reason and form opinions in practical situations. Various mathematical and statistical models usually describe these processes; decision-making (DM) becomes essential. Behavior management chooses which behavior patterns an individual or organization should use to accomplish a specific goal. Research indicates that while many decisions in daily life can be made without conscious thought, more thought and effort are needed to make complex and important decisions. In discrete situations with well-defined and limited options, multi-attribute decision-making, or MCDM, is employed. MCDM problems have a limited number of possible solutions. MCDM techniques are frequently used in decision-making processes like ranking, comparing, and selecting options. These methods are typically chosen because they enable quick DM without requiring intricate mathematical computations or sophisticated package software. The MCDM approach can only accomplish one objective. The goal is to solve the choice dilemma most cost-effectively and beneficially possible.

When there are several alternative outcomes of a particular event, but their likelihood is unknown, this is known as uncertainty. As a result, the DM must comprehend uncertainty. It takes time and effort to comprehend the likelihood that events will occur in reality. Consequently, there is uncertainty at every stage of the DM process. A strong basis for logical reasoning with vague and imperfect data is provided by fuzzy logic theory. Thanks to fuzzy logic theory, computers can understand human language and apply human knowledge. At this point, it starts employing symbols instead of numerical expressions. Fuzzy sets (FS) are symbolic expressions of this type. FSs are known to include choice variables, such as probability states.

1.1. Research Motivation

Quantifying the degrees of membership ($\mathcal{M}$) and non-membership ($\mathcal{N}$) in a single numerical number is only partially justified or technically sound in human cognitive and decision-making activities. When information must be provided as intervals rather than single-valued numbers, interval numbers may be used. The decision-maker can more easily convey his or her preference for $\mathcal{M}$ and $\mathcal{N}$ using intervals. Because of a lack of information, decision-makers may find it challenging to express their thoughts accurately with an exact number in specific real-world DM challenges. In Intuitionistic fuzzy set (IFS) theory, the $\mathcal{N}$ is defined in addition to the $\mathcal{M}$, whereas FS theory is designed only to reveal the $\mathcal{M}$ defined in the range. Pythagorean fuzzy sets (PFS), which Yager proposed [34] and in certain instances developed as an extension of IFSs since IFSs cannot adequately convey uncertainty PFSs employ the notion that the sum of the squares of $\mathcal{M}$ and $\mathcal{N}$ are less than or equal to 1 for circumstances when decision-making is impossible when $\mathcal{M}$ and $\mathcal{N}$ are added together. The Fermatean fuzzy set FFS is ascribed to Senapati and Yager [34]. The property is "the sum of the cubes of $\mathcal{M}$ and $\mathcal{N}$ are less than or equal to 1"attained by the $\mathcal{M}$ and $\mathcal{N}$ in the FFS.

The TODIM method ranks the alternatives in the MCDM problem. The TODIM method is a method used to make decisions under risky conditions. The form of the value function in the method is similar to the loss and gain function of the prospect theory. This function reflects the behavioral characteristics of decision-makers, such as risk aversion, and shows the degree of dominance of alternatives over each other. The global value function combines the gains and losses according to all decision criteria and allows the ranking of alternatives. The TODIM technique is a method that allows the use of qualitative data expressed with linguistic variables along with quantitative data.

The main advantage of the TODIM method compared to other behavioral decision techniques is that it considers the behavioral characteristics of DMs with limited rationality. The method includes gains and losses relative to the reference point in case of uncertainty, thus making DMs more sensitive to losses. In the case of decision-making based on full rationality, DMs only aim to maximize utility. In contrast, in the TODIM technique, DMs maximize total utility by considering losses. Therefore, the TODIM method can be considered a behavioral decision-making method based on partial rationality.

The primary finding of cumulative prospect theory (CPT) (and its precursor, prospect theory) is that people typically consider potential outcomes about a specific reference point, which is frequently the status quo rather than the ultimate status. We refer to this phenomenon as the framing effect. Additionally, their risk attitudes toward gains (i.e., outcomes above the reference point) and losses (i.e., outcomes below the reference point) differ, and they are typically more concerned with prospective losses than with potential profits (loss aversion). Lastly, people undervalue "average"events while overvalue extraordinary ones. Prospect Theory, which holds that people outweigh unexpected events regardless of their relative outcomes, is contrasted with the last statement.

1.2. Literature

Globally, breast cancer (BC) is the most common invasive cancer in women. After lung cancer, breast cancer is the second leading cause of death for women. It starts with the formation of a small tumor or mass and is brought on by breast cells, particularly those in the glands (lobular carcinoma) or milk ducts (ductal carcinoma) [38]. The border of a benign (non-cancerous) tumor is smooth and distinct. There may be irregularly bordered or hypothesized cancerous (malignant) lumps [31]. Even if the exact causes of BC are unknown, every woman should be aware of her risk of contracting the illness so that she can take proactive steps to reduce her risk and treat it successfully. Numerous factors that either increase or decrease the risk of getting breast cancer have been identified by independent studies [39–42]. It is possible to estimate a woman's predicted probability of developing malignant breast disease by evaluating these risk factors.

An annual breast cancer screening utilizing digital mammography is recommended for all women over the age of 40 to spot worrisome lesions early. Digital mammograms do not accurately indicate the post-screening risk of developing a malignant breast disease in those diagnosed with normal or benign findings, even though they are believed to be effective in detecting suspicious breast masses and lesions and grading the findings on a zero to six scale by BI-RADS [43] guidelines. Certain women should be treated separately by patients with lower risk factors since they may have genetic predispositions or other BC risk factors that put them in the high-risk category. In order to help women become more BC-conscious, it is imperative to create an integrated BC risk assessment model that incorporates the results of the initial screening study with the individual's demographic risk factors. This would make it possible for high-risk women to ask their doctors for sensible guidance on the best follow-up plan, increasing the likelihood that malignant tumors will be discovered early.

Zadeh's [35] concept of an FS highlighted the ambiguity and absurdity of a $\mathcal{M}$. Atanassov [2] then discovers the IFS, which could provide more detailed evaluation information by linking an item to a component's $\mathcal{N}$. However, because of their significant limitations in giving preference information, IFSs are designed to make it difficult for judgment specialists to make the proper assessments. Along with the $\mathcal{M}$, IFS also specifies the $\mathcal{N}$. IFS theory states that $\mathcal{M}$ and $\mathcal{N}$ fall within the $[0,1]$ range.

Imprecision must be taken into consideration in any DM procedure. Numerous methods and instruments have been put forth to address the unclear environment of collective DM. One of the most recent methods for dealing with uncertainty is FFS [23]. These sets provide a wider range of applications than the FS [35] extensions, the IFS [2], and the PFS [33]. Recently, FFs have inspired many studies [1, 5, 6, 10, 14, 24–26].

Problems in the real world are often very complicated. Complexity can be attributed to ambiguities, randomness, or limited understanding brought on by a lack of data or poor quality of information. Most tasks require identifying the variables in the problem statement using linguistic language. More precise forecasts and advantageous solutions will result from an understanding of decision-makers knowledge of confusing facts. Zadeh's [35] FS idea is a key component of fuzzy modeling, a mathematical approach that describes uncertainty in human systems. It is indisputable that FS theory cannot individually determine the satisfaction or dissatisfaction of human judgment, even though the study of partial membership required a significant divergence from conventional reasoning. Atanassov [2] created the intuitionistic fuzzy set (IFS) theory to overcome this limitation. Numerous academics in a range of optimization-related domains have since used IFSs. IFS is not designed to handle scenarios when the sum of $\mathcal{M}$ and $\mathcal{N}$ for some alternatives is more than one. To get around this restriction, Yager [33], [34] developed Pythagorean fuzzy sets (PFSs), which loosen it up so that the only condition at each option evaluation is that $\mathcal{M}$ and $\mathcal{N}$ sum of squares is less than 1. Senapati and Yager [23, 24] developed the FFS concept in response to the limitations imposed by PFSs. FFS theory was proposed by Senapati and Yager [23, 24] in response to the limitations imposed by IFSs and PFSs. The $\mathcal{M}$ and $\mathcal{M}$ cubic sum in an FFS must be less than or equal to 1. In addition, FFS-related applications are depicted in [5, 6, 8, 18, 24, 25, 27].

The $\mathcal{M}$'s ambiguity and vagueness were illustrated using [35]'s concept of an FS. Atanassov's intuitionistic FS (IFS) [2] links an element's $\mathcal{N}$ to an item, providing a more comprehensive explanation of assessment data. Yager [33, 34] developed the Pythagorean FS(PFS) idea to broaden the range of $\mathcal{M}$ and $\mathcal{N}$ so that $\mathcal{M}^2+\mathcal{N}^2 \leq 1$ in response to the IFS vulnerability previously described. Because of this, PFS offers professionals more evaluation opportunities to express their opinions on various objectives. The complexity of the DM framework increases the difficulties specialists have in producing reliable evaluation data. The development of IFS and PFS has addressed the ambiguity and vagueness caused by the complex subjectivity of human cognition. The FFS was the first to expand the scope of information assertions by adding the cubic sum of $\mathcal{M}$ and $\mathcal{N}$. Therefore, FFS manages ambiguous choice situations more efficiently and practically than IFS and PFS. Senapati and Yager started the FFS [23]. The fundamental characteristics of FFS were initially provided by Senapati and Yager [24, 25].

Garg et al. [5] have established general aggregation operators, based on Yager's t-norm and t-conorm, to cumulate the FF data in decision-making environments. In [17], a hybrid MCDM based on IVFF was proposed for risk analysis related to autonomous vehicle driving systems. Kirisci [8] defined new correlation coefficients based on the Fermatean hesitant fuzzy elements and interval-valued Fermatean hesitant fuzzy elements. The least common multiple expansion was used in the newly defined correlation coefficients. In [12], a three-way method for computing the correlation coefficients between FFSs has been given using the notions of variance and covariance. New distance and cosine similarity measures amongst FFSs have been defined [10]. A method was established to construct similarity measures between FFSs based on the cosine similarity and Euclidean distance measures. In [11], a new correlation coefficient and weighted correlation coefficient formularization have been proposed to evaluate the affair between two FFSs. In [14], an extended version of the ELECTRE-I model called the FF ELECTRE-I method for multi-criteria group decision-making with FF human assessments has been presented. Kirisci [15] defined the Fermatean hesitant fuzzy set and gave aggregation operations based on the Fermatean hesitant fuzzy set. The interval-valued Fermatean fuzzy linguistic Kernel Principal Component Analysis model has been given in [16]. The definition of FF soft sets and some properties were introduced [9]. Furthermore, the Fermatean fuzzy soft entropy and the formulas for standard distance measures, such as Hamming and Euclidean distance, were defined [9]. A new model for group decision-making methods in which experts' preferences can be expressed as incomplete FF-preference relations has been presented [27]. A multi-criteria decision-making strategy to evaluate the risk probabilities of autonomous vehicle driving systems by combining the AHP technique with interval-valued FFSs has been proposed in [28]. First, the interval-valued IFS was described in [3]. It represented the $\mathcal{M}$ and $\mathcal{N}$ by the closed subinterval of the interval [0,1]. The interval-valued PFS (IVPFS), whose $\mathcal{M}$ and $\mathcal{N}$ are represented by an interval number, was further proposed by [29]. Several operations and relations of IVPFS are also examined. Jeevaraj defined the IVFFS [6].

Gomes and Lima [7] provide the traditional TODIM approach for the first time due to the complexity of the decision environment. The TODIM approach is always used to evaluate some MADM conditions while considering the DMs confidence level to deliver a more equitable solution under risk. As a result, while the TODIM technique is an excellent MADM method, it has limitations; it does not have to provide an adequate mechanism for generating attribute weights and needs a comprehensive approach. As a result, Tian et al. [20] improved on the traditional TODIM technique. They used it with the CPT to change the weighting of attributes to make more reasonable decisions in practice. On the other hand, the risk assessment of science and technology projects could be considered a classic MAGDM issue. Some research is similar. Tüysüz and Kahraman [21] found the fuzzy analytic hierarchy process (AHP) to help analyze the risk of an information technology project. Kumar et al. [22] studied the risks of software projects and developed a new blended MCDM technique based on fuzzy DEMATEL, FMCDM, and TODIM knowledge. Suresh and Dillibabu [32] were also looking for a better model for software project evaluation and developed a framework for fuzzy DEMATEL, ANFIS MCDM, and IF-TODIM. Zhao et al. [36] proposed a CPT-TODIM method based on intuitionistic fuzzy sets for the MAGDM problem and used the CRITIC method to obtain the weight information of the attributes. In [37], the new CPT-TODIM approach based on PFSs has been implemented. Liao et al. [19] introduced the extended TODIM with CPT for probabilistic hesitant fuzzy multiple attributes group decision-making.

1.3. Necessity

The FFSs could effectively depict the imprecise or vague information of risk assessment issues of breast cancer. In light of this, the primary goal of this work is to offer a technique for evaluating breast cancer risk. To consider the limited rationality of physicians' thinking, we expand this unique TODIM method based on the CPT to the FFSs in this paper. We also use FFSs to transmit experts' appraisals of each alternative for each attribute. This combination has potential applications in related circumstances, which can strengthen and resupply the research. As a result, applying this research topic to MCDM for risk evaluation issues is intriguing.

Since the information description of breast cancer pre-diagnosis lays a solid foundation for later disease diagnosis, the current paper mainly focuses on the imprecision and incompleteness that existed in the problem modeling procedure.

A selectable method is required to reflect the psychological behaviors of physicians, and due to this requirement, the classical TODIM method based on cumulative prospect theory (CPT-TODIM) will be created.

To increase rationality, the weight information of the attributes will be obtained.

1.4. Originality

The literature has identified numerous hazards that can either increase or lower the chance of developing breast cancer. It is possible to evaluate a woman's likelihood of developing a malignant breast disease by examining these risk factors. Bridging the gap between FFSs and CPT-TODIM and investigating efficient models and ways with the aid of FFS CPT-TODIM in deficient information systems is essential, given that FFSs are expected to be a fundamental tool for breast cancer risks. This serves as the main driving force for the research in the paper.

1.5. Contribution

The following significant contributions can be eventually specified:

(1) FFS CPT-TODIM processes uncertain information in modeling breast cancer risks.

(2) The FFS CPT-TODIM method can more comprehensively address the bounded rationality of physicians regarding breast cancer risks.

(3) A comprehensive FF MCDM approach is constructed via FF CPT-TODIM. By using FFSs, physicians' evaluation of each alternative for each attribute can be captured more robustly.

2. Preliminaries

U, the initial universe set, will be used throughout the article.

For $\alpha_P : U \to [0,1]$ and $\beta_P : U \to [0,1]$, the FFS P is shown by $P = \{(u, \alpha_P(u), \beta_P(u)) : u \in U\}$, where the inequality $0 \leq \alpha_P^3(u) + \beta_P^3(u) \leq 1$ [23] is valid.

It is defined as $\gamma_P(u) = \sqrt[3]{1 - (\alpha_P^3(u) + \beta_P^3(u))}$ degree of indeterminacy of u to P.

Take three FFSs $P = \{\alpha_P, \beta_P\}$, $P_1 = \{\alpha_{P_1}, \beta_{P_1}\}$ and $P_2 = \{\alpha_{P_2}, \beta_{P_2}\}$. Then, some operations as follows [23]:

i. $P_1 \cap P_2 = \min\{\alpha_{P_1}, \alpha_{P_2}\}, \max\{\beta_{P_1}, \beta_{P_2}\}$,

ii. $P_1 \cup P_2 = \max \alpha_{P_1}, \alpha_{P_2}, \min \beta_{P_1}, \beta_{P_2}$,

iii. $P^t = \beta_P, \alpha_P$,

iv. $P_1 \boxplus P_2 = \left(\sqrt[3]{\alpha_{P_1}^3 + \alpha_{P_2}^3 - \alpha_{P_1}^3 \alpha_{P_2}^3}, \beta_{P_1}\beta_{P_2}\right)$,

v. $P_1 \boxtimes P_2 = \left(\alpha_{P_1}^3 \alpha_{P_2}^3, \sqrt[3]{\beta_{P_1}^3 + \beta_{P_2}^3 - \beta_{P_1}^3 \beta_{P_2}^3}\right)$,

vi. $\alpha P = \left(\sqrt[3]{1 - (1 - \alpha_P^3)^\lambda}, \alpha_P^\alpha\right), \quad \lambda > 0$,

vii. $P^\lambda = \left(\alpha_{P_1}^3, \sqrt[3]{1 - (1 - \beta_P^3)^\lambda}\right), \quad \lambda > 0$.

Let $P = \{\alpha_P, \beta_P\}$ be an FFN, then the score and accuracy functions of P are defined as:

$$SC(P) = \frac{1 + \alpha_P^3 - \beta_P^3}{2},$$

$$AC(P) = \alpha_P^3 + \beta_P^3$$

where $SC(P) \in [-1, 1]$ and $AC(P) \in [0, 1]$.

For any two FFNs $P_1 = \{\alpha_{P_1}, \beta_{P_1}\}$ and $P_2 = \{\alpha_{P_2}, \beta_{P_2}\}$,

(K1) If $SC(P_1) < SC(P_2)$, then $P_1 < P_2$,

(K2) If $SC(P_1) = SC(P_2)$, then

(A) If $AC(P_1) < AC(P_2)$, then $P_1 < P_2$,

(B) If $AC(P_1) = AC(P_2)$, then $P_1 \sim P_2$.

Let $P_i = \{\alpha_{P_i}, \beta_{P_i}\}$, $(i = 1, \cdots, n)$ be a collection of FFNs and $\omega = (\omega_1, \cdots, \omega_n)^T$ be the weight vector of P_i. Then, the Fermatean fuzzy weighted average (FFWA) operator is a mapping $FFWA : P^n \longrightarrow P$, where

$$FFWA(P_1, \cdots, P_n) = \left(\sum_{i=1}^{n} \omega_i \alpha_i, \sum_{i=1}^{n} \omega_i \beta_i \right).$$

3. CPT-TODIM based on FFSs

First, we will give the TODIM method based on CPT [20]. As follows, there is a decision matrix C, in which the schemes and attributes are provided by decision-makers:

$$C = (c_{ij})_{m\times n} \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots & & \\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & & \\ c_{m1} & c_{m2} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix}. \tag{1}$$

The weighting vector of attributes is represented by $\varpi = (\varpi_1, \cdots, \varpi_n)^T$, which satisfies $\varpi_j \geq 0$ and $\sum_{j=1}^{n} \varpi_j = 1$.

Step 1: Figure out the modified weights $\Theta^*_{ikj}(\varpi_j)$ based on the original weighting vector of attributes and the weighting function by

$$\Theta_{ikj}(\varpi_j) = \begin{cases} \frac{((\varpi_j)^{\lambda}}{(\varpi_j)^{\lambda} + (1-\varpi_j)^{\lambda})^{1/\lambda}} & , c_{ij} \geq c_{kj} \\ \frac{((\varpi_j)^{\mu}}{(\varpi_j)^{\mu} + (1-\varpi_j)^{\mu})^{1/\mu}} & , c_{ij} < c_{kj}, \end{cases} \tag{2}$$

$$\Theta^*_{ikj}(\varpi_j) = \frac{\Theta_{ikj}(\varpi_j)}{\max\{\Theta_{ikj}(\varpi_l) : l \in n\}} \quad j \in n, \quad \forall (i,k), \tag{3}$$

where λ, μ are the parameters, which are used to describe the curvature of the weighting function.

Step 2: The relative predominance $\Delta_j(S_m, S_k)$ of scheme S_m compared with S_k in the attribute A_j can be computed by

$$\Delta_j(S_m, S_k) = \begin{cases} \frac{\Theta^*_{ikj}(\varpi_j).(c_{ij} - c_{kj})^{\zeta}}{\sum_{j=1}^{n} \Theta^*_{ikj}(\varpi_j)} & , c_{ij} > c_{kj} \\ 0 & , c_{ij} = c_{kj} \\ -\delta \cdot \frac{(\sum_{j=1}^{n} \Theta^*_{ikj}(\varpi_j)).(c_{ij} - c_{kj})^{\eta}}{\Theta^*_{ikj}(\varpi_j)} & , c_{ij} < c_{kj}, \end{cases} \tag{4}$$

where δ, ζ, η are the parameters.

Step 3: Equation (5) is applied to calculate the overall predominance $\Upsilon(S_m)$ of scheme S_m:

$$\Upsilon(S_m) = \sum_{k=1}^{m} \sum_{j=1}^{n} \Delta_j(S_m, S_k). \tag{5}$$

Step 4: Acquire the standard overall predominance

$$\Omega(S_m) = \frac{\Upsilon(S_m) - \min_m(\Upsilon(S_m))}{\max_m(\Upsilon(S_m)) - \min_m(\Upsilon(S_m))}. \tag{6}$$

Step 5: Rank the standard overall predominance $\Omega(S_m)$ to get the best scheme that has the biggest $\Omega(S_m)$ value.

Based on the above knowledge, a new model will be established to answer the multiple attribute group decision-making issue with Fermatean fuzzy information. There are three collections of information, which are named the set of alternatives $S = \{S_1, \cdots, S_m\}$, the set of attributes $A = \{A_1, \cdots, A_n\}$ and the set of decision makers $J = \{J_1, \cdots, J_p\}$. Through building the relation between the alternative and attribute, we can get the Fermatean fuzzy decision matrix $R^z = (r_{ij}^z)_{m\times n} = (\alpha_{ij}^z, \beta_{ij}^z)_{m\times n}$ provided by the decision maker J_z, where r_{ij}^z as well as α_{ij}^z respectively, indicate the membership degree and the non-membership degree about the alternative S_m keeping in line with the attribute A_j and satisfy $\alpha_{ij}^z, \beta_{ij}^z \in [0,1]$ and $(\alpha_{ij}^z)^3, (\beta_{ij}^z)^3 \leq 1$. Furthermore, the weighting vector of attribute $\varpi = (\varpi_1, \cdots, \varpi_n)^T$ and $\varrho = (\varrho_1, \cdots, \varrho_n)^T$ is the weighting vector of decision makers.

Algortihm of CPT-TODIM based on FFSs:

Stage 1: Process and Integrate the Information from Independent Decision Makers

1. Take advantage of Equation (7) to ensure the unification of all of the attributes:

$$\begin{aligned} M^z &= (m_{ij}^z)_{m\times n}, \\ m_{ij}^z &= (\phi_{ij}^z, \psi_{ij}^z) = \begin{cases} a_{ij}^z = (\alpha_{ij}^z, \beta_{ij}^z) & , A_j \, \textit{is a positive attribute} \\ (m_{ij}^z)^c = (\beta_{ij}^z, \alpha_{ij}^z) & , A_j \, \textit{is a negative attribute.} \end{cases} \end{aligned} \tag{7}$$

2. The Fermatean fuzzy power weighted averaging (FFPWA) operator makes the integration of Fermatean fuzzy decision matrices deriving from independent decision-makers come true. The specific process of calculation refers to Equations (8) - (11):

$$d(m_{ij}^z, m_{ij}^t) = \frac{\sqrt[3]{(\phi_{ij}^z - \phi_{ij}^t)^3 + (\psi_{ij}^z - \psi_{ij}^t)^3}}{\sqrt{2}} \tag{8}$$

$$sup(m_{ij}^z, m_{ij}^t) = 1 - d(m_{ij}^z, m_{ij}^t) \tag{9}$$

$$X(m_{ij}^z) = \sum_{t=1, t\neq z}^{s} \varphi_t sup(m_{ij}^z, m_{ij}^t), \quad z = 1, 2, \cdots, s \tag{10}$$

$$\begin{aligned} g_{ij} &= FFPWA_{\varphi}(m_{ij}^1, \cdots, m_{ij}^s) = \frac{\boxplus_{z=1}^{s} \left(\varphi_z(1 + X(m_{ij}^z)m_{ij}^z\right)}{\sum_{z=1}^{s} \varphi_z(X(m_{ij}^z))} \\ &= \left(\sqrt[3]{1 - \prod_{z=1}^{s} \left(1 - (\phi_{ij}^z)^3\right)^{\varphi_z(1+X(m_{ij}^z))/\sum_{z=1}^{s} \varphi_z(X(m_{ij}^z))}}, \right. \\ & \left. \prod_{z=1}^{s} \left((\psi_{ij}^z)^3\right)^{\varphi_z(1+X(m_{ij}^z))} \sum_{z=1}^{s} \varphi_z(X(m_{ij}^z))\right) \end{aligned} \tag{11}$$

Stage 2: Acquire the Attribute Weights based on Existing Information

3. To get the original weighting vector of attributes ϖ, all related equations are sequentially listed: For $j, h = 1, 2, \cdots, n$,

$$\Lambda_{jh} = \frac{\sum_{i=1}^{m} \left(SC(g_{ij}) - \frac{1}{m}\sum_{i=1}^{m} SC(g_{ij})\right) . \left(SC(g_{ih}) - \frac{1}{m}\sum_{i=1}^{m} SC(g_{ih})\right)}{\sqrt{\sum_{i=1}^{m} \left(SC(g_{ij}) - \frac{1}{m}\sum_{i=1}^{m} SC(g_{ij})\right)^2 . \left(SC(g_{ih}) - \frac{1}{m}\sum_{i=1}^{m} SC(g_{ih})\right)}}, \tag{12}$$

$$\Gamma_j = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} \left(SC(g_{ij}) - \frac{1}{m} \sum_{i=1}^{m} SC(g_{ij})\right)^2}, \tag{13}$$

$$\varpi_j = \frac{\Gamma_j . \sum_{h=1}^{n} (1 - \Lambda_{jh})}{\sum_{j=1}^{n}} \left(\Gamma_j . \sum_{h=1}^{n} (1 - \Lambda_{jh})\right). \tag{14}$$

4. Utilize the weighting function shown in Equations (15) and (16) to calculate the modified weights:

$$\Theta_{ikj}(\varpi_j) = \begin{cases} \frac{(\varpi_j)^{\lambda}}{((\varpi_j)^{\lambda} + (1-\varpi_j)^{\lambda})^{1/\lambda}} & , g_{ij} \geq g_{kj}, \\ \frac{(\varpi_j)^{\mu}}{((\varpi_j)^{\mu} + (1-\varpi_j)^{\mu})^{1/\mu}} & , g_{ij} < g_{kj}, \end{cases} \tag{15}$$

$$\Theta_{ikj}^{*}(\varpi_j) = \frac{\Theta_{ikj}(\varpi_j)}{\max\{\Theta_{ikj}(\varpi_l) : l \in n\}} \quad j \in n, \quad \forall (i,k), \tag{16}$$

where λ, μ are the parameters, which are used to describe the curvature of the weighting function.

Stage 3: Carry Through Pairwise Comparison for Any Alternative and Acquire the Eventual Standard of Ordering

5. Determine the relative predominance of alternative S_m compared with S_k underneath the attribute A_j:

$$d_{ikj} = \frac{\sqrt[3]{(\delta_{ij} - \vartheta_{ij})^3 + (\delta_{ij} - \vartheta_{ij})^3}}{\sqrt{2}}, \tag{17}$$

$$\Delta_j(S_m, S_k) = \begin{cases} \frac{\Theta^*_{ikj}(\varpi_j).(d_{ikj})^{\zeta}}{\sum_{j=1}^n \Theta^*_{ikj}(\varpi_j)} & , g_{ij} > g_{kj} \\ 0 & , g_{ij} = g_{kj} \\ -\delta \cdot \frac{\sum_{j=1}^n \Theta^*_{ikj}(\varpi_{ij})}{\Theta^*_{ikj}(\varpi_j)} \cdot (d_{ikj})^{\eta} & , c_{ij} < c_{kj}, \end{cases} \tag{18}$$

where δ, ζ, η are the parameters.

6. Determine the overall predominance $\Upsilon(S_m)$ and the standard overall predominance $\Omega(S_m)$ of the alternative S_m over all others in according to Equations (19) and (20):

$$\Upsilon(S_m) = \sum_{k=1}^{m} \sum_{j=1}^{n} \Delta_j(S_m, S_k), \tag{19}$$

$$\Omega(S_m) = \frac{\Upsilon(S_m) - \min_m(\Upsilon(S_m))}{\max_m(\Upsilon(S_m)) - \min_m(\Upsilon(S_m))}. \tag{20}$$

7. Rank the standard overall predominance $\Upsilon(S_m)$ and the bigger value of the standard overall predominance means the more excellent alternative.

4. Risk Analysis of Breast Cancer

Everyone wants to know how to lower their breast cancer risk. Although doctors do not know what causes breast cancer, they know there are factors linked to a higher-than-average risk of developing the disease. Some factors associated with increased breast cancer risk — being a woman, age, and genetics, for example — cannot be changed. Other factors — lack of exercise, smoking cigarettes, and eating certain foods — can be altered by lifestyle choices.

By choosing the healthiest lifestyle options, one can empower oneself and keep the risk of breast cancer as low as possible. If a factor cannot be changed (such as your genetics), you can learn about protective steps to help keep your risk as low as possible. We will classify a given individual's BC risk level into three different grades: S_1-Normal, S_2-Benign, and S_3-Malignant. The 14 main influencing personal risk factors related to the three main risk factors affecting BC

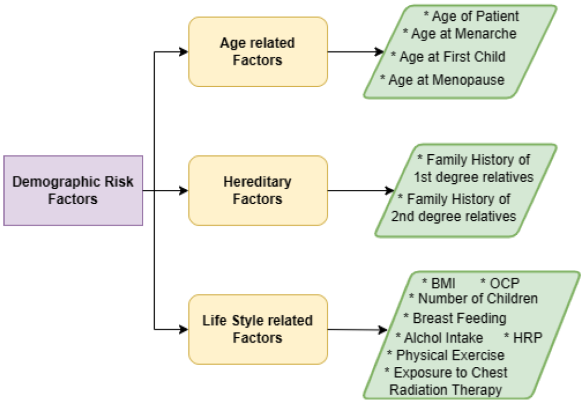

Figure 1: Demographic risk factors of breast cancer [30]

Table 1: Group decision matrix

	A_{11}	A_{12}	A_{13}	A_{14}	B_{11}	B_{22}	C_{31}	⋯
S_1	(0.6321, 0.4610)	(0.6357, 0.2744)	(0.3310, 0.5863)	(0.6485, 0.3762)	(0.8945, 0.2376)	(0.8637, 0.2551)	(0.7581, 0.3548)	⋯
S_2	(0.8577, 0.1646)	(0.7378, 0.2819)	(0.6974, 0.2119)	(0.6994, 0.3712)	(0.6513, 0.2487)	(0.3514, 0.6879)	(0.1542, 0.5897)	⋯
S_3	(0.4620, 0.3766)	(0.4925, 0.4526)	(0.5971, 0.2637)	(0.7038, 0.3100)	(0.6347, 0.4465)	(0.6405, 0.3542)	(0.5330, 0.4112)	⋯

	C_{35}	C_{36}	C_{37}	C_{38}
S_1	(0.6485, 0.2998)	(0.8214, 0.1258)	(0.2471, 0.7902)	(0.2416, 0.6548)
S_2	(0.6669, 0.3125)	(0.3126, 0.4121)	(0.3146, 0.3251)	(0.4011, 0.6175)
S_3	(0.7301, 0.2694)	(0.3082, 0.6812)	(0.3677, 0.5807)	(0.2103, 0.8899)

are shown in Figure 1 [30].

BC risk assessment could be regarded as a classical MCDM issue. Based on the above steps, in the following, we intend to apply the proposed FF-CPT TODIM method in this paper to the risk assessment of BC.

1. Uniform the positive and negative attributes by applying Equation (7) and concentrate a group decision matrix Q by utilizing Equations (8)-(11). The final results are shown in Table 1.

2. Take advantage of Equations (12)-(14) to obtain the original weighting vector of attributes.

3. Utilize the weighting function shown in Equations (15) and (16) to calculate the modified weights (Take $\lambda = 0.61$ and $\mu = 0.69$).

4. Determine the relative predominance of alternative S_m compared with S_k underneath the attribute A_j according to Equations (17) and (18) (Take $\delta = 0.91$, $\zeta = 0.88$, and $\eta = 2.25$). The original risk weights are denoted by Table 2. Modified weights tables for 14 risk criteria are not shown in the study as they would take up too much space.

5. Calculate the overall predominance and the standard overall predominance of the alternative S_m over all others according to Equations (19) and (20). The results are: $\Upsilon(S_1) = -16.6326$,

Table 2: The original risks weight

	A_{11}	A_{12}	A_{13}	A_{14}	B_{11}	B_{22}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{36}	C_{37}	C_{38}
ϖ	0.078	0.062	0.084	0.065	0.075	0.096	0.063	0.087	0.070	0.067	0.065	0.056	0.074	0.058

$\Upsilon(S_2) = -0.1402,\ \Upsilon(S_3) = -13.3205,\ \Omega(S_1) = 0.2843,\ \Omega(S_2) = 0.3927,\ \Omega(S_3) = 0.4585.$

6. Rank the standard overall predominance $\Omega(S_3)$ has biggest value that means the S_3 emerges as the most risky case.

5. Discussion

5.1. Comparative Analysis:

In this section, the proposed method is compared with the previously given IF CPT-TODIM [36], PF CPT-TODIM [37] and PHFS CPT-TODIM [19] methods. Table 3 presents the findings of a comparison between the suggested approach and established techniques. It was also seen from the rankings that the results obtained by the new method overlapped with the methods given, especially by PFS and PHFS. However, it was seen that S_1 was the third option in all methods.

Table 3: Ranking comparison

Method	S_1	S_2	S_3
IF CPT-TODIM [37]	3	1	2
PF CPT-TODIM [36]	3	2	1
PHFS CPT-TODIM [19]	3	2	1
Proposed Method	3	2	1

5.2. Superiority of Suggested Method

The FS, IFS, and PFS are combined to create the FFS. Total squares that are equal to or less than one, as well as member and nonmember satisfaction levels, are used to calculate PFS. The decision-maker rarely gives the $\mathcal{M}$ and $\mathcal{N}$ a specific attribute that would make the squares total greater than 1. As a result, the PFS cannot deal with this situation effectively. FFS, which can handle inconsistent and partially unknown data -both prevalent in real-world scenarios- is one of the most complete methods for getting over this restriction.

The results of the suggested strategy overlap with those of the available methods, according to the current and sensitivity assessments. The primary benefit of the suggested method over readily available DM solutions is that it incorporates extra data and tackles data uncertainty by accounting for aspects like $\mathcal{M}$ and $\mathcal{N}$ of criteria. The item's information may be examined more precisely and impartially. In the DM process, it is also a valuable tool for handling imprecise and erroneous data. As a result, the predicted information loss occurs since the reasoning for giving one parameter a score value has no bearing on the other values.

Conversely, there is no discernible loss of information with our suggested method. The intended methodology has an advantage over current approaches in that it can identify the degree of similarity and differentiation across data, avoiding incorrectly motivated judgments. The DM process can be aided by combining unclear and inaccurate information.

5.3. Limitations

This study still raises several issues. First, risk and uncertainty are not the same thing. This study primarily concerns the consequences of risk selection rather than promoting ambiguity. Given the complexity of evaluating women's potential for BC, risk aversion is crucial to uncertainty avoidance. Prospect theory was used in this work to operationalize risk choice. However, in order to identify possible hazards with BC, a more comprehensive evaluation could be necessary. Future research should concentrate on combining general risk choice criteria with particular BC risk markers.

Beyond the advantages of the suggested FF-based technique, its incapacity to fully assess the available possibilities limits its use in particular DM situations. When there are several criteria and options, creating FFSs is simpler. In order to overcome these constraints, we hope further to investigate the following topics in our upcoming work:

- The scope of the application can be expanded to include scenarios that can be obtained with different data.
- Extending the scope of outranking-based interval rough set theory methods -such as VIKOR, ELECTRE, DEMATEL, ANP, FMEA, BWM, and others- is another long-term objective.
- We aim to determine how various MCDM methods can be applied to the FF values.

Despite identifying and listing risks and sub-risks to BC, this article may need to locate and include further risks. Subjective weighting values were taken into consideration while applying the assessments. The results are, therefore, predicated on subjective weighting data.

6. Conclusion

The TODIM approach is always used to examine some MCDM circumstances by considering the DMs confidence level to provide a more reasonable option under risk. As a result, the TODIM method is an ideal MCDM method. However, it has limits. It is not required to give a suitable method for determining attribute weights, nor is it required to present a comprehensive methodology. FFS has emerged as a powerful extension of the FS that enables several degrees of truth connected with each preference information to express ambiguity and vagueness effectively. This article examines the difficulties of MCDM with FFSs. We propose the FF-CPT-TODIM technique, which exceptionally illustrates the actual state of mind for decision-makers based on the corresponding knowledge of FFSs and the classical TODIM method. In addition, an example of breast cancer risk assessment is shown to validate the applicability of the FF-CPT-TODIM method in handling MCDM problems.

As a result of the evaluation made by following the steps of the algorithm, it was seen that S_3-Malignant came first among the risks related to breast cancer. The parameter values may change the calculated result in the fourth section and there is no doubt that we need to select the perfect parameters to address the problem we are studying. The responsibility of this paper is not to analyze the parameters but to establish a brilliant PF-CPT-TODIM method for MCDM issues. In future studies, breast cancer risks can be evaluated using different risk analysis methods. Again, risk assessment can be done using different sets.

Declaration of Ethical Standards

The author declares that the materials and methods used in her study do not require ethical committee and/or legal special permission.

Conflicts of Interest

The author declares no conflict of interest.

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