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Research Article

Determining secondary school mathematics teachers' errors and misconceptions in geometry[1]

Selin Uygun[2] and Esra Altıntaş[3]*

Sait İlbeyi İmam Hatip Secondary School, Kars, Turkiye

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Abstract

The aim of this study is to determine the mistakes and misconceptions experienced by the teachers on the basis of the definitions and examples of geometry subjects in the error and misconception detection test prepared for elementary school mathematics teachers in secondary school geometry subjects. Errors and misconceptions experienced by teachers will open the door to mistakes and misconceptions that students will experience during the lesson. For this reason, by identifying the errors and misconceptions experienced by primary school mathematics teachers in geometry subjects, producing solutions will ensure the prevention of misconceptions. The study group of the research consists of 20 primary school mathematics teachers working in secondary schools in the center of Kars in the 2021-2022 academic year. Since the teachers who could be reached while forming the study group were included in the research, the appropriate sampling method was used. In the research, the "Error and Misconception Identification Test", which was developed by the researcher and determined by taking expert opinion, was applied. In the results of the research, it was determined that primary school mathematics teachers mostly experienced errors and misconceptions about basic geometric concepts, quadrilaterals and prisms. It was determined that the teachers did not experience the misconception that they only made mistakes in some questions.

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Introduction

Although geometry is an important branch of mathematics, it is necessary for students to see and understand some facts in the environment they live in (Doyuran, 2014). Geometry has a long history of being closely connected to the world (Jones, 2000). Geometry should be taught at an early age in terms of containing concrete shapes and facilitating mathematics teaching (Berkant and Çadırlı, 2019). It is observed that students have difficulties and difficulties in the geometry subjects taught within the scope of mathematics course, and as a result, they develop a negative attitude towards geometry. The role of the teacher in the classroom is of great importance in eliminating these negative attitudes and prejudices experienced by students. For this reason, educational environments should be supported with rich content regarding geometry, which is given to students in the early stages, and action should be taken according to the students' thinking levels (Pusey, 2003).

The fact that concepts learned in the previous class can be used again in the next class in geometry teaching emphasizes the importance of the graduality principle of geometry. Therefore, according to Kiriş (2008), it should be

[1] This study was produced from first author' master thesis.

[2] Mathematics Teacher, Sait İlbeyi İmam Hatip Secondary School, Kars, Türkiye, E-mail: selin.uygun17@hotmail.com, ORCID No: 0000-0002-6040-4256

[3] Corresponding author: Assoc.Prof., Aydin Adnan Menderes University, Faculty of Education, Math Education Department, Aydin, Turkiye. E-mail:altintas.esra1982@gmail.com ORCID: 0000-0003-3311-7179

taken into consideration that each concept learned is related to the previous concept. According to Ayyıldız and Altun (2013), if these concepts are not learned correctly, the foundations of knowledge cannot be formed and connections between events cannot be established. In this context, "misconceptions" occur as a result of individuals misattributing the connections between events. According to Osoje (2015), misconception is expressed as an individual's misunderstandings and misinterpretations based on misunderstandings.

Using the expressions “misconception” and “error” together causes these two expressions to be confused with each other. Error occurs due to misconception. This means that a student who has a misconception may, as a result, exhibit incorrect approaches to some subjects (İncikabı and Kılıç, 2013). According to TDK (2011), error: wrong; It means an unintentional and unknowing mistake, flaw, mistake, error. Borasi (1987) states that errors accepted in mathematics education can be a powerful tool for diagnosing learning and that it may be possible to directly improve learning. In other words, focusing on errors provides a deep understanding of mathematical concepts. Error is defined as incorrect use of mathematical concepts and inaccuracies in operations and calculations (Erbaş et al., 2010). In this way, misconception appears as a comprehensive expression that includes error. The reason for this is that misconceptions usually manifest themselves in the mistakes made by students (Erdem and Gürbüz, 2017).

When we look at the studies conducted, we see that these studies were conducted with teacher candidates and students. In the study conducted by Köprücü (2020), 13 studies with transportation permits between 2000 and 2020 were examined. According to the data obtained, it was determined that not much work has been done on misconceptions in geometry. In the study conducted by Paksu et al. (2012), it was observed that teacher candidates' knowledge about the concept of dimension was insufficient, and they focused on different criteria such as the number of corners, number of edges, number of diagonals, and number of visible faces when deciding on the number of dimensions. In his research, Usta (2018) found that prospective teachers could not suggest solutions to detect student errors in converting volume measurement units to liquid measurement units. It was observed that the students could not make the association between volume measurements and liquid measurements. In the study conducted by Şengün and Yılmaz (2021), it was determined that there was difficulty in explaining the bisector and bisector, there were difficulties in using the ruler and protractor, and the related concepts were confused with the concept of height. In the study conducted by Erdoğan and Dur (2014), it was observed that pre-service mathematics teachers' knowledge of quadrilaterals and prototype images that they learned at the primary-secondary school level was dominant. Türnüklü and Ergin (2016) found in their research that students were far from academic definitions and mostly tried to describe the surfaces of prisms. It was observed that the students expressed the expression "it is a three-dimensional object" in different ways for the prism, and the expressions "all three-dimensional objects are prisms" caused overgeneralization. In the study conducted by Kartal and Çınar (2017), prospective teachers were more successful in questions that required knowing the mathematical definition of polygon. Although prospective teachers answered correctly whether the shapes given to them were polygons or not, the majority of them still could not define polygons completely and correctly. In Yazıcı (2019) study, prospective teachers ignored the fact that the concept of point was undefined and stated that the "pen tip" used to explain the concept of point was a point, and regarding the concept of line, the candidates expressed the line as a straight line and frequently referred to the concept of line. It was determined that they confused the expressions of truth with each other and perceived the example of truth as real truth. Çakmak et al. (2014) concluded in their study that teacher candidates mostly had difficulty in determining and defining the critical features of three-dimensional objects. As a result of their study by Bozkurt and Koç (2012), it was seen that the majority of teacher candidates could not define prism. According to the results obtained from the definitions of the concept of prism, it was seen that they were not sufficient in using mathematical language and expressing the concept that was wanted to be defined. Ulusoy (2022) stated in his study that students defined parallelism and vertical line segments by using three types of reasoning. It has been concluded that concept images shaped by prototypes and formal examples are effective in these definitions. In his study, Fischbein (1993) concluded that the concept of

dimension was not developed in students and most of them thought of a point as a round object. According to this result, it was concluded that figural representations prevent the definition of concepts and create misconceptions. According to their results, Gutieerez and Jaime (1999) stated that prospective teachers had a weak concept image regarding the height in a triangle. They also concluded that although prospective teachers had formal definitions of quadrilaterals, their prototype images affected their formal concepts. Tall and Winner (1981) emphasized in their study that students tend to use concept images instead of using previous concepts in the concept learning process, and that examples should be enriched during concept learning. In his study, Blanco (2001) included his findings regarding errors in teaching and learning the basic concepts of geometry. According to the results he obtained, he stated that the students wrote the height definition of the triangle correctly, but had difficulties in drawing it. Cunningham and Roberts (2010) stated that teachers provide inadequate definitions and prototype examples when they encounter a concept they are not familiar with. Skordoulis et al. (2009), in their study on prospective mathematics teachers' understanding of the concept of size, examined the candidates' correct knowledge of the dimensions of geometric shapes and their geometric dimension measurements.

Due to the cumulative structure of mathematics and the cumulative structure of geometry, which is a branch of mathematics, errors and misconceptions occur in the learning and teaching of some concepts. The structure of geometric concepts used by the teacher in the classroom contributes to the development of students' geometric thinking skills (Erdoğan, 2006). Teachers should provide the infrastructure for the formation of conceptual knowledge instead of memorized knowledge in the classroom. Students who learn the concepts can also use the concepts they have learned on other subjects to be learned. Students who cannot achieve conceptual learning as desired have misconceptions about geometric concepts and have difficulty understanding geometric concepts. In this research, primary school mathematics teachers are asked questions about defining basic geometric concepts, how to teach a given geometric concept, drawing geometric shapes, perception of geometric dimensions, and the errors and misconceptions that occur are determined. The mistakes and misconceptions experienced by teachers will open the door to mistakes and misconceptions that students will experience during the lesson. Therefore, the most important thing should be to identify the mistakes and misconceptions that primary school mathematics teachers experience in geometry and to produce solutions. When the literature is examined, it will be possible to say that the studies were conducted with teacher candidates and students. When the thesis studies on errors and misconceptions in geometry subjects are examined, we can say that there was an intensity between 2007 and 2014 and that these studies were carried out with students (Gülkılık, 2008; Kiriş, 2008; Ay, 2014; Doyuran, 2014). For this reason, in this research, a study was conducted with primary school mathematics teachers and it is important in terms of determining the mistakes and misconceptions that teachers experience in geometry subjects. It is thought that conducting the study with primary school mathematics teachers who have spent a certain amount of time in the profession and spend time with students in the classroom environment will contribute to the literature. In addition, it is thought that determining the mistakes and misconceptions experienced by primary school mathematics teachers in geometry subjects is also important in terms of structuring the new concepts that students will learn in geometry lessons.

Problem of Study

The aim of this research is to determine the errors and misconceptions experienced by teachers based on their definitions and examples on geometry subjects in the error and misconception determination test prepared for primary school mathematics teachers in secondary school geometry subjects. For this purpose, the problem was "What are the mistakes and misconceptions that primary school mathematics teachers experience in geometry?" It was determined as. In line with the research problem the sub-problems are as follows:

- What kind of mistakes do primary school mathematics teachers have in secondary school geometry subjects?
- What are the misconceptions of primary school mathematics teachers about secondary school geometry?

Method

Research Model

The aim of the study was to identify the mistakes and misconceptions experienced by primary school mathematics teachers in secondary school geometry subjects. Therefore, this study is a case study model, one of the qualitative research methods. Case studies, also known as case studies, are accepted as a method in which one or more events, environments, situations or groups are examined in depth (Büyüköztürk et al., 2020). This study, which was conducted to identify the errors and misconceptions experienced by primary school mathematics teachers in geometry subjects, categorize them, explain them with their reasons and make suggestions, constitutes an example of the case study model, one of the qualitative research methods.

Study Group

The study group of the research consists of 20 primary school mathematics teachers working in secondary schools in Kars center and districts in the 2021-2022 academic year. Since the teachers who could be reached while determining the study group were included in the study, the appropriate sampling method, one of the non-random sampling methods, was used. The appropriate sampling method is defined as selecting the study group from easily accessible and applicable respondents (Büyüköztürk et al., 2020).

Data Collection and Analysis

The analysis of the data collected in this research was done with content analysis, one of the qualitative analysis types. Content analysis is a systematic technique in which some words of the study are summarized into smaller categories by coding within the framework of certain rules (Büyüköztürk et al., 2020). With content analysis, researchers make interpretations of the message in the text by determining and analyzing the meanings and relationships of words and concepts (Büyüköztürk et al., 2020). Inferences were made by determining the errors and misconceptions experienced by primary school mathematics teachers in secondary school geometry subjects, examining the definitions given, and subjecting them to content analysis along with the answers obtained from the teachers' opinions.

By conducting a literature review, criteria were created to identify errors and misconceptions. Deficiencies in expressions and incorrect use of words and terms determined after the teachers' definitions and interviews were treated as errors. The definitions and explanations made by the teachers and the examples they gave were examined together. Confusion of concepts on topics that are related to each other, inability to associate them with examples, having incorrect information, different answers that are not related to the question, prototype definitions, answers that create a concept image, and answers that appear to have concepts that do not match scientific concepts are examined in the misconception category.

Procedure

The test has been prepared in accordance with the objectives determined to address all achievements in secondary school geometry subjects. While determining the outcomes of the questions in the "Errors and Misconceptions Identification Test", care was taken to include topics that addressed geometry subjects at the secondary school level.

Table 1. Questions and learning outcomes in the error and misconception identification test

1.	a) Define the concept of "point". Explain by giving an example. b) Define the concept of "Dimension". Explain by giving an example. b1) Define the concepts of two and three dimensions. Explain by giving an example.	*M.5.2.1.2. It expresses the position of a point relative to another point using direction and units.*
2.	Can you define the concepts of line, line segment and ray? Express with examples.	*M.5.2.1.1. Explains line, line segment, ray and shows it with symbols.*
3.	Define the concept of steepness. Explain by giving an example.	*M.5.2.1.5. Draws a perpendicular to a line from a point on or outside it.*
4.	Describe the types of triangles. Express with examples.	*M.5.2.1.5. Draws a perpendicular to a line from a point on or outside it.*
5.	Define the concept of quadrilateral. Explain by drawing a figure.	*M.5.2.2.3. Determines and draws the basic elements of rectangle, parallelogram, rhombus and trapezoid*
6.	a) Can you prove the sum of the interior angles of a triangle? b Can you prove the area of a triangle?	*M.5.2.2.4. Determines the sum of the measures of the interior angles of triangles and quadrilaterals and finds the angle that is not given.* *M.6.3.2.1. Creates the area relationship of the triangle and solves related problems.*
7.	a) Explain the difference between a circle and a circle. Express with an example. b) Explain the concepts of circle and perimeter-area in a circle.	*M.6.3.3.3. Solve problems that require calculating the length of a circle given its diameter or Radius* *M.7.3.3.3. Calculates the area of the circle and circle segment.*
8.	How would you describe the transition from liquid measurements to volume measurements? Express with examples	*M.6.3.5.2. Relates liquid measurement units to volume measurement units.*
9.	a) How do you express that the angle goes to infinity? Express with examples. b) Express the stages of drawing the bisector of an angle. Explain in sentences by drawing figures c) Express the steps of drawing a triangle. Explain in sentences by drawing figures.	*M.6.3.1.1. She knows that the angle is formed by two rays with the same starting point and represents it with a symbol.* *M.7.3.1.1. Determines the bisector by dividing an angle into two equal angles.* *M.8.3.1.4. Draws a triangle given the dimensions of a sufficient number of elements.*
10.	 How do you show the sum of interior angles in polygons without using mathematical relations?	*M.7.3.2.2. Determines the diagonals, interior and exterior angles of polygons; Calculates the sum of the measurements of the interior angles and exterior angles.*
11.	a) Is a cube a prism? Explain with reasons. b) How do you name prisms? Explain with example.	*M.8.3.4.1. Recognizes right prisms, determines their basic elements, constructs them and draws their expansion.*
12.	a) Describe the difference between congruence and similarity in triangles. Express with an example. b) While explaining the concepts of congruence and similarity in triangles, what are the gains you convey to the students regarding the subject of ratio and proportion? Express it with explanation	*M.8.3.3.1. Relates congruence and similarity, determines the side and angle relationships of congruent and similar shapes..*

The researcher included achievements that address secondary school geometry subjects and include subjects at all grade levels. The content validity of the prepared error and misconception detection test was ensured by taking the

opinions of two faculty members who are experts in the field of geometry. Then, a pilot application was conducted and an error and misconception detection test was applied to eight graduate students. In practice, it was observed that students answered the test within 45 minutes. Therefore, the application time of the test was determined as 45 minutes. After the pilot application, the error and misconception detection test was applied to twenty primary school mathematics teachers and data was collected.

Results

In this part of the research, the findings and comments obtained from the error and misconception test prepared regarding secondary school geometry subjects are included. In addition, the findings and comments of the interview held after the analysis are also included in this section.

Table 2. Define the concept of *point* frequency and percentage values regarding the distribution of teacher answers and errors-misconceptions regarding the question "explain with example"

Answers		**f**	**%**
Academically accepted answer	Point; is a non-existent, dimensionless, abstract geometric term.	5	23,80
Misconceptions	The mark left by the pen on the paper	15	71,42
	Multiplication sign	1	4,76
	Total	21	100

When the answers given by primary school mathematics teachers to the question about defining the concept of "dot" and giving examples are examined in Table 2, it is seen that 23.80% gave the academically correct answer and 76.18% had various misconceptions. According to the interviews conducted with primary school mathematics teachers after the analysis, it was seen that the teachers did not make any mistakes in this question. After analyzing the answers given by the teachers who had misconceptions, an interview was conducted about the concept of "dot". When the answers given and the data obtained after the interview were brought together, it was determined that primary school mathematics teachers used the definition of "the mark left by the pen on the paper" in their lessons for the concept of "point". This definition for the concept of "point" shows that teachers have a prototype or concept image. According to the answer to this question, which includes the concept image rather than the concept definition, it is seen that the majority of teachers have misconceptions about the concept of "point". Some answers obtained from primary school mathematics teachers are given below;

> "The mark left by the pen on the paper".... (S1).
> "The shape that appears when we put the tip of the pen on the paper, the shape that appears when we touch it".....S3).
> "A dot is the mark left by a pen on a surface. Used instead of multiplication sign in mathematics"....(S11).
> "A geometric concept that has no width, length or height is called"....(S14).

After the interview with primary school mathematics teachers who were thought to have misconceptions, one of the teachers stated, "I explain the concept of point to the student in this way to concretize it." Another teacher answered: "I use this definition in my lessons, and to elaborate, I state that the dot is the smallest building block." Another teacher said, "In mathematics class, students use "." instead of "x" as the multiplication sign. They use the sign. He responded to the interview by saying, "I can use this when explaining the concept of point." Another teacher who had a misconception stated that he defined a point in his lessons as "the mark left by the pen on the paper".

Conclusion and Discussion

This research covers the examination of the mistakes and misconceptions experienced by primary school mathematics teachers regarding secondary school geometry subjects. Suggestions regarding the results are also included under this heading. “What are the mistakes and misconceptions that primary school mathematics teachers experience in geometry?” problem and "What kind of mistakes do primary school mathematics teachers have in secondary school geometry?" and “What kind of misconceptions do primary school mathematics teachers have about secondary school geometry?” The results of the sub-problems are given in this section.

When the results of the first sub-problem are examined, the primary school mathematics teachers' deficiencies in expression, verbal errors, not being able to read the question correctly, using different expressions than the desired answer in the question, questions left blank, lack of information, giving relations, including information other than what is requested in the question are evaluated under the error heading. It was observed that primary school mathematics teachers had difficulty in conceptually defining the answers they gave to the question about the cube being a prism, they contained incomplete expressions and they started from specialized situations. The answers of the teachers who included these statements were evaluated under the heading of errors. Lack of expression and use of mathematical language in the conceptual definition of prisms (Bozkurt and Koç, 2012); Using some specialized situations and thinking that only square and rectangular prisms have the property of being prisms (Türnüklü and Ergin, 2016) have shown that teachers made mistakes.

When we look at the findings regarding the second sub-problem, the answers that primary school mathematics teachers gave personalized and bookish definitions, far from academic definitions, and especially expressions containing prototypes and concept images, were examined under the title of misconception. Through the interviews, it was clarified that the answers given by the teachers who had misconceptions were misconceptions.

According to the results obtained from the findings, it was seen that primary school mathematics teachers mentioned the prototype structure in their answers regarding the definition of the concept of "point". It was observed that 71.42% of the teachers could not make a conceptual definition of the point with the answer "the mark left by the pen on the paper". As a result of the interviews, it was determined that this definition was used in the lessons. In the activity about comparing the size and weight of points in two different drawings given by Fischbein (1993), students stated that the point formed by the line formed by crossing many lines is larger. According to the findings, Fischbein's (1993) study will be exemplary considering that teachers' failure to academically define the "point" in lessons may lead to size-related problems in students. It has been stated that the majority of primary school mathematics teachers ignore that the concept of point is undefined and assume it to be a "pen tip" (Doyuran, 2014; Yazıcı, 2019).

In defining the concepts of "line, line segment, ray", it was observed that primary school mathematics teachers included the expressions length and straight line for the concepts of "line" and "ray". Yazıcı (2019) stated in his study that prospective teachers frequently used the expression "straight line" regarding the concept of "right". It would be possible to say that teachers mostly make mistakes in the concepts of "line" (Ubuz, 1999; Yazıcı, 2019) and "ray". According to the findings, it was observed that teachers' possession of prototype structures related to the concept of "line segment" led them to misconceptions. Considering the statement mentioned by Yazıcı (2019) that teacher candidates are at least mistaken in the concept of "line segment", it can be seen that there is a difference from the study in this regard. It has been observed that teachers experience misconceptions due to the prototype structures they have (Gutieerez and Jaime, 1999; Doyuran, 2014; Ulusoy, 2022).

When we look at the studies conducted on geometry misconceptions, it was seen that polygons and quadrilaterals were the most common topics in the subject distribution between 2000 and 2020 (Köprücü, 2020). In this study, it was observed that the teachers were successful in providing the desired explanations about the interior angles of polygons and there were no teachers who had misconceptions. When a similar study was conducted by Kartal and Çınar (2017), it was seen that the participants were successful in knowing the definition of polygons. It would be possible to say that teachers have concept images regarding the concept of "Quadray". Teachers who cannot reach the

correct definition academically have difficulty in giving conceptual definitions to students in their lessons. The reasons behind the non-existence of the concept of "quadrangle" extend to basic geometric concepts. Considering that teachers make definitions such as combinations of points and adding line segments end to end, this supports this (Erdoğan and Dur, 2014; Yurtyapan and Karataş, 2020).

Within the scope of the angle concept, it has been determined that teachers teach in their lessons by dividing the "angle into two equal parts" for drawing the "bisector" and they confirmed this data during the interviews. Blanco (2001) states in his study that there are difficulties in understanding the auxiliary elements in the triangle. In the study conducted by Şengün and Yılmaz (2021), it was determined that students had difficulty in explaining the concept of bisector.

It has been stated that teachers use prototype expressions in teaching the concept of "Prism" and include these expressions in their lessons. Teachers are mistaken in thinking that the side faces of the prism concept consist only of rectangles. This is where teachers made mistakes in the definition of "cube". Because teachers accept the side faces only as rectangles and also think that the condition of being a prism depends on this. Çakmak et al., (2014) stated that there are difficulties in determining the critical properties of three-dimensional objects. In this study, it was seen that explanations were made by ignoring some critical features about the "cube". Considering that the concept of "cube" is a special prism, it has also been observed that teachers are inadequate in defining prism properties. This result is similar to the study conducted by Bozkurt and Koç (2012).

Looking at the results, it can be seen that primary school mathematics teachers mostly make mistakes and misconceptions in basic geometric concepts, quadrilaterals and prisms. According to this result, it turns out that teachers' misconceptions about basic geometric concepts affect their subsequent learning. The reason for this situation is that geometric concepts (point, line segment, line, ray, angle, plane, etc.) are the basis of the subjects of quadrilaterals and prisms. At the same time, another result obtained from this research showed that teachers constantly use concept images and prototype structures in their lessons. Tall and Winner (1981) state that students use concept images instead of using concepts in their concept learning processes. In other words, it is thought that the reasons why students use classical, bookish definitions may be due to teachers using prototype expressions of concepts in their teaching processes. For this reason, it may be inevitable that the misconceptions that teachers will experience due to their concept images will also create problems in students' learning of concepts. It has been stated in many studies that prototype expressions are frequently used in teaching concepts in geometry subjects (Tall and Winner, 1981; Mason, 1989; Gutieerez and Jaime, 1999; Cunningham and Roberts, 2010; Erdoğan and Dur, 2014; Doyuran, 2014; Ulusoy, 2022).

Recommendations

- In this research, the mistakes and misconceptions experienced by primary school mathematics teachers in secondary school geometry subjects were investigated and questions were prepared to cover all subjects. As a result, important misconceptions have been reached on some subjects rather than all subjects, and these issues need to be examined in depth.
- It is seen that the misconceptions experienced by primary school mathematics teachers on basic geometric concepts are reflected in the teaching of other subjects and misconceptions occur in these subjects as well. For this reason, it is thought that conducting studies on basic geometric concepts may be effective.
- In this research, errors and misconceptions were identified, but no analysis was made for these errors and misconceptions. For this reason, it is important to conduct a comprehensive study on secondary school geometry subjects together with teachers and students in order to provide information on how to resolve the errors and misconceptions that will be obtained.

- According to the errors and misconceptions obtained from the research, the prototype structures that teachers have should be taken into consideration. Accordingly, attention should be paid to the contents of the textbooks used by teachers in their lessons and expressions containing concept images should be reviewed.

Limitations of Study

It is limited to the 2021-2022 academic year. This research is limited to official public secondary schools in Kars province.This research is limited to primary school mathematics teachers. The secondary school mathematics program is limited to all subjects in the field of geometry.

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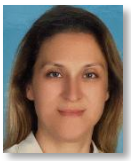
Biodata of the Authors

Selin Uygun graduated from Sakarya University, Department of Primary Mathematics Teaching in 2016. He completed his master's degree in Mathematics Education at Kafkas University Institute of Science and Technology in 2023. He has been teaching in Kars since 2016. Institution: Kars Ministry of National Education, Kars, Turkey. Email: selin.uygun17@hotmail.com ORCID: 0000-0002-6040-4256



Esra Altıntaş completed his undergraduate education at Ege University, Faculty of Science, Department of Theoretical Mathematics. Her master's and doctorate education in Marmara University, Institute of Educational Sciences, Department of Mathematics Education. She worked as a teaching and research assistant at the Faculty of Education at Kafkas University between 2005-2007. Between 2007-2015, she worked as a research assistant at Marmara University, Faculty of Education, math education program, and as an assistant at Kafkas University between 2015-2018. She received the title of associate professor in 2018 and worked in the math education program of the faculty of education at Kafkas University until 2022. Since 2022, he has been working in the mathematics education program of Aydın Adnan Menderes University, Faculty of Education. Her areas of expertise are math education, math education for gifted students, and differentiated instruction

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