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Rate of Convergence for Modified Srivastava–Gupta Type Operators

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Abstract. In this paper, we give a generalization of Srivastava-Gupta operators introduced by Srivastava and Gupta and estimate the rate of convergence for these operators for the real valued continuous bounded functions on the positive semi-axis

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1. Introduction

In [6], Srivastava and Gupta defined very important linear positive operators (Later, called the Srivastava-Gupta operators), because they contain some of the well known operators such as Phillips operators, Baskakov – Durrmeyer type operators as a special case.The authors obtained on the rate of convergence of these operators by means of the decomposition technique for functions of bounded variation. They show that the new operators preserve only the constant functions. Recently, these operators and their generalizations were investigated by many authors (see, [1,2,4–13]). For examples, in [13], Yadav proposed a modified form of these operators so as to preserve linear functions, In [6] İspir and Yuksel introduced the Bezier variant of the Srivastava - Gupta operators and estimate the rate of convergence of these operators for functions of bounded variation.

In this paper, we give a generalization of the Srivastava–Gupta operators as follows:

$$G_{n,c}(f;x,\alpha_n,\beta_n) = G^*_{n,c}(f;x) = \alpha_n \sum_{k=1}^{\infty} P_{n,k}(x,c) \int\limits_0^{\infty} P_{n+c,k-1}(t,c) f(\frac{\beta_n - c}{\beta_n} t) dt + P_{n,0}(x,c) f(0) \tag{1.1}$$

where

$$P_{n,k}(x,c) = \frac{(-x)^k}{k!} \Phi^{(k)}_{n,c}(x)$$

with

$$\Phi_{n,c}(x) = \begin{cases} e^{-\alpha_n x} & , \ c = 0 \\ (1+cx)^{-\frac{\alpha_n}{c}} & , \ c \in \mathbb{N} \end{cases}$$

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and α_n, β_n are the unbounded sequences of positive numbers satisfying the following properties:

$$\lim_{n\to\infty}\alpha_n = \lim_{n\to\infty}\beta_n = \infty,\ \lim_{n\to\infty}\frac{\alpha_n}{\beta_n} = 1,\ \lim_{n\to\infty}\frac{\alpha_n}{\alpha_{n+c}} = 1.$$

Note that more properties of the function $\Phi_{n,c}(x)$ are included in [11]. In case $\alpha_n = \beta_n = n$ in (1.1), we immediately obtain the modified Srivastava–Gupta operators

$$G_{n,c}(f;x) = G^*_{n,c}(f;x) = n\sum_{k=1}^{\infty}P_{n,k}(x,c)\int_0^{\infty}P_{n+c,k-1}(t,c)f(\frac{n-c}{n}t)dt + P_{n,0}(x,c)f(0) \tag{1.2}$$

studied R. Yadav (see [13]). The author obtained the moments of the operators defined by (1.2) for the special cases in terms of the confluent hypergeometric series and hypergeometric series and studied their statistical convergence, asymptotic formula and error estimate in terms of higher order of modulus of continuity.

In the present paper, we investigate approximation properties of the operators $G_{n,c}(f;x,\alpha_n,\beta_n)$ defined by (1.1) and establish the rate of convergence for the real valued continuous bounded functions on the interval $[0,\infty)$.

2. Auxiliary Results

In this section, we will give some lemmas required for proving the main result. Now, we consider the two cases as given in (1.1). First, we shall assume that $c = 0$ and $x \in [0,\infty)$. Then, it easily follows that (1.1) gives

$$G^*_{n,0}(f;x) = \alpha_n\sum_{k=1}^{\infty}\frac{(\alpha_n x)^k}{k!}e^{-\alpha_n x}\int_0^{\infty}\frac{(\alpha_n t)^{k-1}}{(k-1)!}e^{-\alpha_n t}f(t)dt + e^{-\alpha_n x}f(0) \tag{2.1}$$

where $P_{n,k}(x,0) = \frac{(-x)^k}{k!}\Phi^{(k)}_{n,0}(x)$ and $\Phi_{n,0}(x) = e^{-\alpha_n x}$.

Lemma 2.1. *For $r \geq 1$, we have*

$$G^*_{n,0}(t^r;x) = \frac{\Gamma(r+1)}{{\alpha_n}^{r-1}}xe^{-\alpha_n x}F^1_1(r+1,2,\alpha_n x) \tag{2.2}$$

where F^1_1 is the confluent hypergeometric series defined as $F^1_1(a,b;x) = \sum_{k=0}^{\infty}\frac{(a)_k}{(b)_k}\frac{x^k}{k!}$ and $(a)_k$ is the Pochhammer symbol defined as $(a)_k = a(a+1)(a+2)...(a+k-1)$.

Proof. If we take $f(t) = t^r$ in (2.1), then we have

$$\begin{aligned} G^*_{n,0}(t^r;x) &= \alpha_n\sum_{k=1}^{\infty}\frac{(\alpha_n x)^k}{k!}e^{-\alpha_n x}\int_0^{\infty}\frac{(\alpha_n t)^{k-1}}{(k-1)!}e^{-\alpha_n t}t^r dt \\ &= \alpha_n\sum_{k=1}^{\infty}\frac{(\alpha_n x)^k}{k!(k-1)!}e^{-\alpha_n x}\int_0^{\infty}{\alpha_n}^{k-1}e^{-\alpha_n t}t^{k+r-1}dt. \end{aligned}$$

Consider the integral $\int_0^{\infty}{\alpha_n}^{k-1}e^{-\alpha_n t}t^{k+r-1}dt$. We easily get that

$$\begin{aligned} \int_0^{\infty}{\alpha_n}^{k-1}e^{-\alpha_n t}t^{k+r-1}dt &= \int_0^{\infty}e^{-u}{\alpha_n}^{k-1}\frac{u^{k+r-1}}{{\alpha_n}^{k+r-1}}\frac{1}{\alpha_n}du \\ &= \frac{1}{{\alpha_n}^{r+1}}\int_0^{\infty}e^{-u}u^{k+r-1}du \\ &= \frac{1}{{\alpha_n}^{r+1}}\Gamma(k+r) \end{aligned}$$

here we use the substitution $\alpha_n t = u$. And so, we get

$$\begin{aligned}
G^*_{n,0}(t^r;x) &= \alpha_n e^{-\alpha_n x}\sum_{k=1}^{\infty}\frac{(\alpha_n x)^k}{k!(k-1)!}\frac{\Gamma(k+r)}{{\alpha_n}^{r+1}}\\
&= \alpha_n e^{-\alpha_n x}\sum_{k=0}^{\infty}\frac{(\alpha_n x)^{k+1}}{(k+1)!k!}\frac{\Gamma(k+r+1)}{{\alpha_n}^{r+1}}.\\
&= \alpha_n e^{-\alpha_n x}\sum_{k=0}^{\infty}\frac{(\alpha_n x)^{k+1}}{(k+1)!k!}\frac{(r+1)_k\Gamma(r+1)}{{\alpha_n}^{r+1}}\\
&= \alpha_n^2 x e^{-\alpha_n x}\frac{\Gamma(r+1)}{{\alpha_n}^{r+1}}\sum_{k=0}^{\infty}(r+1)_k\frac{(\alpha_n x)^k}{(2)_k k!}\\
&= \frac{\Gamma(r+1)}{{\alpha_n}^{r-1}}x e^{-\alpha_n x}\sum_{k=0}^{\infty}\frac{(r+1)_k}{(2)_k k!}(\alpha_n x)^k\\
&= \frac{\Gamma(r+1)}{{\alpha_n}^{r-1}}x e^{-\alpha_n x}F^1_1(r+1,2,\alpha_n x)
\end{aligned}$$

here we used following equalities

$$\begin{aligned}
(k+1)! &= (2)_k = 2.3.4...(2+k-1) = 2.3...(k+1),\\
(r+1)_k &= (r+1)(r+2)...(r+1+k-1) = (r+1)(r+2)...(r+k),\\
(r+1)_k\Gamma(r+1) &= (r+1)(r+2)...(r+k)\Gamma(r+1).
\end{aligned}$$

Now, taking into account that $F^1_1(a,b,\alpha_n x) = e^{\alpha_n x}F^1_1(b-a,b,-\alpha_n x) = x\frac{\Gamma(r+1)}{{\alpha_n}^{r-1}}F^1_1(1-r,2,-\alpha_n x)$, we establish the desired result and completes the proof of the lemma. □

Remark 2.2. From Eq. (2.2), we have

$$G^*_{n,0}(1;x) = 1, G^*_{n,0}(t;x) = x,\ G^*_{n,0}(t^2;x) = x^2 + \frac{2}{\alpha_n}.$$

Now, for the case $c = 1$ in (1.1), one have

$$G^*_{n,1}(f;x) = \alpha_n\sum_{k=1}^{\infty}P_{n,k}(x,1)\int_0^{\infty}P_{n+1,k-1}(t,1)f(\frac{\beta_n-1}{\beta_n}t)dt + P_{n,0}(x,1)f(0)$$

where $P_{n,k}(x,1) = \frac{(-x)^k}{k!}\Phi^{(k)}_{n,1}(x)$ and $\Phi_{n,1}(x) = (1+x)^{-\alpha_n}$.

Lemma 2.3. *For $r \geq 1$, one has*

$$G^*_{n,1}(t^r;x) = \frac{\alpha_n^2(\beta_n-1)^r\Gamma(\alpha_{n+1}-r-1)\Gamma(r+1)}{\beta_n^r\Gamma(\alpha_{n+1})(1+x)^{\alpha_n+1}}xF^2_1(\alpha_n+1,r+1,2;\frac{x}{1+x}) \tag{2.3}$$

where F^2_1 is the hypergeometric series defined by $F^2_1(a,b,c;x) = \sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(c)_k}\frac{x^k}{k!}$.

Proof. Putting $f(t) = t^r$ in (1.1), we have

$$\begin{aligned}
G^*_{n,1}(t^r;x) &= \alpha_n\sum_{k=1}^{\infty}\frac{(\alpha_n)_k}{k!}\frac{x^k}{(1+x)^{\alpha_n+k}}\int_0^{\infty}\frac{(\alpha_{n+1})_{k-1}}{(k-1)!}\frac{t^{k-1}}{(1+t)^{\alpha_{n+1}+k-1}}(\frac{\beta_n-1}{\beta_n}t)^r dt\\
&= \alpha_n\frac{(\beta_n-1)^r}{\beta_n^r}\sum_{k=1}^{\infty}\frac{(\alpha_n)_k}{k!}\frac{x^k}{(1+x)^{\alpha_n+k}}\frac{(\alpha_{n+1})_{k-1}}{(k-1)!}\beta(k+r,\alpha_{n+1}-r-1)\\
&= \frac{\alpha_n(\beta_n-1)^r\Gamma(\alpha_{n+1}-r-1)}{\beta_n^r}\sum_{k=0}^{\infty}\frac{(\alpha_n)_{k+1}}{(k+1)!}\frac{x^{k+1}}{(1+x)^{\alpha_n+k+1}}\frac{(\alpha_{n+1})_k}{k!}\frac{\Gamma(k+r+1)}{\Gamma(k+\alpha_{n+1})}
\end{aligned}$$

using the identities

$$\Gamma(k+r+1) = (r+1)_k\Gamma(r+1),$$

$$\begin{aligned}(\alpha_n)_{k+1} &= \alpha_n(\alpha_n+1)...(\alpha_n+k), \alpha_n(\alpha_n+1)_k \\ &= \alpha_n(\alpha_n+1)...(\alpha_n+k), \alpha_n(\alpha_n+1)_k \\ &= (\alpha_n)_{k+1},\end{aligned}$$

we get

$$\begin{aligned}G^*_{n,1}(t^r;x) &= \frac{\alpha_n(\beta_n-1)^r\Gamma(\alpha_{n+1}-r-1)\Gamma(r+1)}{\beta_n^r}\sum_{k=0}^{\infty}\frac{(\alpha_n)_{k+1}}{(k+1)!}\frac{(\alpha_{n+1})_k}{k!}\frac{(r+1)_k}{\Gamma(k+\alpha_{n+1})}\frac{x^{k+1}}{(1+x)^{\alpha_n+k+1}} \\ &= \frac{\alpha_n^2(\beta_n-1)^r\Gamma(\alpha_{n+1}-r-1)\Gamma(r+1)}{\beta_n^r\Gamma(\alpha_{n+1})(1+x)^{\alpha_n+1}}xF_1^2(\alpha_n+1,r+1,2;\frac{x}{1+x}),\end{aligned}$$

and the proof of the lemma is complete. □

Remark 2.4. From Eq. (2.3), we have

$$\begin{aligned}G^*_{n,1}(1;x) &= \frac{\alpha_n}{\alpha_{n+1}-1}+\left(1-\frac{\alpha_n}{\alpha_{n+1}-1}\right)(1+x)^{-\alpha_n}, \\ G^*_{n,1}(t;x) &= \frac{\alpha_n^2(\beta_n-1)}{\beta_n(\alpha_{n+1}-1)(\alpha_{n+1}-2)}x, \\ G^*_{n,1}(t^2;x) &= \frac{\alpha_n^2(\beta_n-1)^2}{\beta_n^2(\alpha_{n+1}-1)(\alpha_{n+1}-2)(\alpha_{n+1}-3)}\left[(\alpha_n+1)x^2+2x\right].\end{aligned}$$

Note that the conclusion now follows immediately from Remark 2.2 and Remark 2.4:

$$G^*_{n,c}(1;x) = \frac{\alpha_n}{\alpha_{n+c}-c}+(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{1}{(1+x)^{\alpha_n}}, \tag{2.4}$$

$$G^*_{n,c}(t;x) = \frac{\alpha_n^2(\beta_n-c)}{\beta_n(\alpha_{n+c}-c)(\alpha_{n+c}-2c)}x, \tag{2.5}$$

$$G^*_{n,1}(t^2;x) = \frac{\alpha_n^2(\beta_n-c)^2}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}\left[(\alpha_n+c)x^2+2x\right]. \tag{2.6}$$

A simple computation from (2.4, 2.5, 2.6) also shows that

$$G^*_{n,c}(t-x;x) = \frac{\alpha_n^2(\beta_n-c)-\alpha_n.\beta_n(\alpha_{n+c}-2c)}{\beta_n(\alpha_{n+c}-c)(\alpha_{n+c}-2c)}x-(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{x}{(1+x)^{\alpha_n}},$$

$$\begin{aligned}G^*_{n,c}((t-x)^2;x) = &\frac{\alpha_n^2(\beta_n-c)^2(\alpha_n+c)-2\alpha_n^2\beta_n(\beta_n-c)(\alpha_{n+c}-3c)+\alpha_n\beta_n^2(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x^2 \\ &+\frac{2\alpha_n^2(\beta_n-c)^2}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x+(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{x^2}{(1+x)^{\alpha_n}}.\end{aligned}$$

3. Rate of Convergence

In this section, we estimate the rate of convergence of the operator (1.1) by using second order modulus of continuity of $f \in C_B[0,\infty)$.

Let $C_B[0,\infty)$ be the class on real valued continuous bounded functions f with the norm $\|f\| = \sup_{x\in[0,\infty)}|f(x)|$. Also we denote by $C_B^2[0,\infty) = \{g \in C_B[0,\infty) : g', g'' \in C_B[0,\infty)\}$. For $f \in C_B[0,\infty)$ and $\delta > 0$, the second order modulus of continuity and the Peetre's K-functional are defined as

$$\omega_2(f;\delta) = \sup_{0<h\leq\delta}\sup_{x\in[0,\infty)}|f(x+2h)-2f(x+h)+f(x)|$$

and

$$K_2(f;\delta) = \inf_{g\in C_B^2[0,\infty)} \left\{\|f-g\| + \delta \|g\|_{C_B^2[0,\infty)}\right\}$$

where $\|g\|_{C_B^2[0,\infty)} = \|g\| + \|g'\| + \|g''\|$, respectively.

Theorem 3.1. *Let $f \in C_B[0,\infty)$.Then*

$$\left|G_{n,c}^*(f;x) - f(x)\right| \leq C' \omega_2(f;\sqrt{\delta_{n,x}})$$

where C' is a positive constant and

$$\begin{aligned}\delta_{n,x} &= \frac{\alpha_n^2(\beta_n-c)^2(\alpha_n+c) - 2\alpha_n^2\beta_n(\beta_n-c)(\alpha_{n+c}-3c) + \alpha_n\beta_n^2(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x^2 \\ &+ \frac{2\alpha_n^2(\beta_n-c)^2}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x,\end{aligned}$$

holds for all $x \in [0,\infty)$.

Proof. Let $x \in [0,\infty)$ and $g \in C_B^2[0,\infty)$. Using Taylor's expansion, we see that

$$g(t) = g(x) + g'(x)(t-x) + \int_x^t (t-u)g''(u)du \tag{3.1}$$

for all $t \in [0,\infty)$. If we apply the operator $G_{n,c}^*$ defined by (1.1) in (3.1), taking into account (from remark 2) that

$$G_{n,c}^*(g;x) - \left(\frac{\alpha_n}{\alpha_{n+c}-c} + (1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{1}{(1+x)^{\alpha_n}}\right)g(x) = g'(x)G_{n,c}^*((t-x);x) + G_{n,c}^*(\int_x^t (t-u)g''(u)du\,;x)$$

holds. Also an easily verification shows that

$$\left|\int_x^t (t-u)g''(u)du\right| \leq \frac{1}{2}\left\|g''\right\|(t-x)^2.$$

So, using (2.4, 2.5, 2.6) we have

$$\begin{aligned}&\left|G_{n,c}^*(g;x) - \left(\frac{\alpha_n}{\alpha_{n+c}-c} + (1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{1}{(1+x)^{\alpha_n}}\right)g(x)\right| \\ \leq\ & \left\|g'\right\|\left[\frac{\alpha_n(\alpha_n\beta_n - c\alpha_n - \beta_n\alpha_{n+c} + 2c\beta_n)}{\beta_n(\alpha_{n+c}-c)(\alpha_{n+c}-2c)}x - (1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{x}{(1+x)^{\alpha_n}}\right] \\ &+\frac{1}{2}\left\|g''\right\|\left\{\frac{\alpha_n^2(\beta_n-c)^2(\alpha_n+c) - 2\alpha_n^2\beta_n(\beta_n-c)(\alpha_{n+c}-3c) + \alpha_n\beta_n^2(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x^2\right. \\ &\left.+\frac{2\alpha_n^2(\beta_n-c)^2}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x + (1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{x^2}{(1+x)^{\alpha_n}}\right\}.\end{aligned} \tag{3.2}$$

On the other hand, from (2.4) we have

$$\begin{aligned}G_{n,c}^*(f;x) &= \alpha_n\sum_{k=1}^{\infty}P_{n,k}(x,c)\int_0^{\infty}P_{n+c,k-1}(t,c)f(\frac{\beta_n-c}{\beta_n}t)dt + P_{n,0}(x,c)f(0), \\ \left|G_{n,c}^*(f;x)\right| &\leq \|f\|\end{aligned} \tag{3.3}$$

and so we get

$$\begin{aligned}\left|G^*_{n,c}(f;x)-f(x)\right| &\leq |f(x)-g(x)|\left|G^*_{n,c}(1;x)\right|+\left|f(x)-\left(\frac{\alpha_n}{\alpha_{n+c}-c}+(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{1}{(1+x)^{\alpha_n}}\right)g(x)\right|\\ &+\left|G^*_{n,c}(g;x)-\left(\frac{\alpha_n}{\alpha_{n+c}-c}+(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{1}{(1+x)^{\alpha_n}}\right)g(x)\right|\\ &\leq 3\|f-g\|+\left\|g'\right\|\left|\frac{\alpha_n(\alpha_n\beta_n-c\alpha_n-\beta_n\alpha_{n+c}+2c\beta_n)}{\beta_n(\alpha_{n+c}-c)(\alpha_{n+c}-2c)}x-(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{x}{(1+x)^{\alpha_n}}\right|\\ &+\frac{1}{2}\left\|g''\right\|\left|\frac{\alpha_n^2(\beta_n-c)^2(\alpha_n+c)-2\alpha_n^2\beta_n(\beta_n-c)(\alpha_{n+c}-3c)+\alpha_n\beta_n^2(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x^2\right.\\ &\left.+\frac{2\alpha_n^2(\beta_n-c)^2}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x+(1-\frac{\alpha_n}{\alpha_{n+c}-c})\frac{x^2}{(1+x)^{\alpha_n}}\right|.\end{aligned}$$

Using (3.2) and (3.3)

$$\left|G^*_{n,c}(f;x)-f(x)\right|\leq 3\|f-g\|+\delta_{n,x}\left(\left\|g'\right\|+\left\|g''\right\|\right)$$

where

$$\begin{aligned}\delta_{n,x} &= \frac{\alpha_n^2(\beta_n-c)^2(\alpha_n+c)-2\alpha_n^2\beta_n(\beta_n-c)(\alpha_{n+c}-3c)+\alpha_n\beta_n^2(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x^2\\ &+\frac{2\alpha_n^2(\beta_n-c)^2}{\beta_n^2(\alpha_{n+c}-c)(\alpha_{n+c}-2c)(\alpha_{n+c}-3c)}x,\end{aligned}$$

holds for all sufficiently large n. Finally, taking infimum over all $g\in C_B^2[0,\infty)$, considering the inequality $K_2(f,\delta)\leq C\omega_2(f;\sqrt{\delta})$ (see [3]), and the proof is complete. □

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