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Cultural Interactions of Medieval Societies Hidden in The Symmetry of Ornaments

Bezemelerin Simetrilerinde Saklı Olan Ortaçağ Kültür İlişkileri

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Abstract

We trace the culture on the mosaics of seven medieval civilizations by taking the symmetry of the ornaments found on the mosaics as characteristic evidence for the cultural group in which the artwork is created. We classify the ornaments into 17 wallpaper groups according to their symmetry. Applying the cosine law to the frequency of each symmetry group we calculate the pairwise correlation of the cultural interactions. Finally, a hierarchical cluster analysis is performed on the correlations to find cultural groups that are closely related to each other. The results show the formation of two clusters: The Islamic cluster includes the Arabs and the Seljuk dynasties. The Hellenistic group inspired other Anatolian civilizations and even influenced the Umayyads in Andalusia. These results are consistent with the calculations based on multidimensional scaling. Our work demonstrates the possibility of applying symmetry analysis and statistical methods to the visual arts to reveal valuable details such as the hidden influences in art practice.

Keywords: *Ornaments, symmetry, wallpaper groups, correlation, hierarchical cluster analysis.*

Öz

Mozaiklerde bulunan bezemelerin simetrilerini kullanarak, yedi Ortaçağ uygarlığının kültürel izlerini gözlemliyoruz. Simetrileri, sanat eserinin yaratıldığı kültürün bir özelliği olarak değerlendiriyoruz. Öncelikle iki boyutlu periyodik bezemeleri simetrilerine göre 17 Wallpaper grubuna ayırıyor ve her uygarlık için simerti gruplarının kullanım sıklığını sayısal olarak belirliyoruz. Bu değerlere kosinüs yöntemini uygulayarak, her bir topluluk çifti için bir tane olmak üzere, uygarlıklar arasındaki ilişkileri gösteren toplam 21 korelasyon sayısı elde ediyoruz. Bu korelasyon sayılarına uygulanan hiyerarşik kümeleme tekniği, uygarlıkların birbirlerine yakınlaştıkları iki ana kümeyi ortaya çıkarıyor: Birincisi Araplar ve Selçukluların yer aldığı İslam kümesi, ikincisi ise etkisini Anadolu'da olduğu kadar ilginç olarak Endülüs'te de belli eden Helen-Roma grubu. Uyguladığımız çok boyutlu ölçekleme yöntemi sonuçları desteklemektedir. Böylece matematiğin değişik dallarından esinlenen simetri analizi ve istatistiksel yöntemler yardımıyla, farklı uygarlıkların sanat eserlerindeki simetrileri incelemiş ve uygarlıklar arasındaki sanatsal etkileşmeleri bulma yolunda yenilikçi bir adım atmış oluyoruz.

Anahtar Kelimeler: *Bezemeler, simetri, wallpaper grupları, korelasyon, hiyerarşik kümeleme, çok boyutlu ölçekleme.*

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1. Introduction

We study the traces of faith and culture on mosaics in a non-classical but scientific way. We have chosen medieval peoples of the Middle East, mostly either Christian or Moslem. We focus on the two-dimensional periodic ornaments found in most mosaic artwork of these groups. Symmetry is used as evidence for the characteristic nature of the artwork. We analyze the symmetry of the ornaments in terms of mathematical groups and obtain an objective quantity of the particular mosaic work.

This idea was introduced by György Pólya (Pólya 1924: 278-282), who postulated that, analogous to the symmetry properties of atoms on an ordered crystal surface, periodic ornaments can be identified by their symmetry and arranged into 17 crystallographic groups, the *wallpaper groups*. Edith Müller applied this concept to ornaments in the Alhambra Palace in Granada in her dissertation in mathematics at the University of Zurich (Müller 1944). In her work, Müller proved that the wallpaper group takes into account all possible symmetries that a craftsman can create. After half a century, Washburn and Crow introduced the idea that the symmetry distribution of all patterns in the 17 wallpaper groups is characteristic of the culture in which the patterns are created (Washburn - Crow 1999). Thus, the pattern in an artisan-made mosaic is associated with a unique quantitative measure.

The assignment of two-dimensional periodic ornaments to the corresponding wallpaper groups later became a standard method (Makowicky 2016; Bonner 2017), which we applied to different cultural groups to compare their performances (Erbudak 2020a: 95-101). We anticipate that this comparison will contain invaluable information about the interactions of the civilizations under consideration. Since the collected information about the symmetry of the mosaics is in quantitative form, we are able to apply statistical recipes to draw rigorous conclusions. This is our only contribution to this topic.

2. Methods

2.1. Wallpaper Group

In crystallography (Hammond 2015), the two-dimensional ordered surface phase is uniquely described by the point-group operations that hold an object, which can be an atom or a molecule, in our case a motif, in place and apply rotations and reflections around one or more mirrors. The rotations are expressed in numbers n that divide 2π and, when applied, leave the motive unchanged. Allowed numbers are $n = 1, 2, 3, 4, 6$. Larger numbers lead to overlaps, 5 leads to voids. Together with the reflections, these operations allow 10 possibilities, called the *crystal class*.

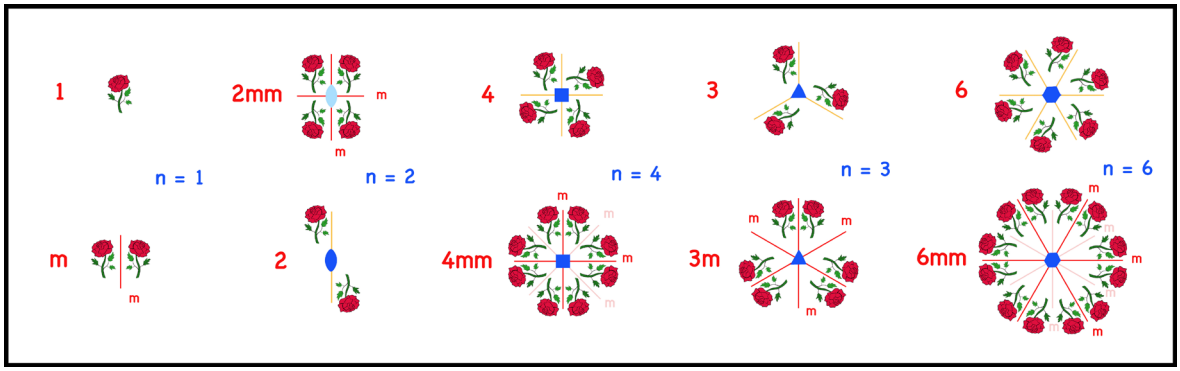


Figure 1
Point-group operations applied to a motif. Note that some mirror symmetry is created by rotation, as in the case of *4mm* and *6mm*. For *2mm*, rotational symmetry is caused by the double mirror reflection.

Figure 1 shows a rose as a motif and illustrates the possible crystal classes produced by point-group operations, i.e., mirror reflection and rotation by *n*.

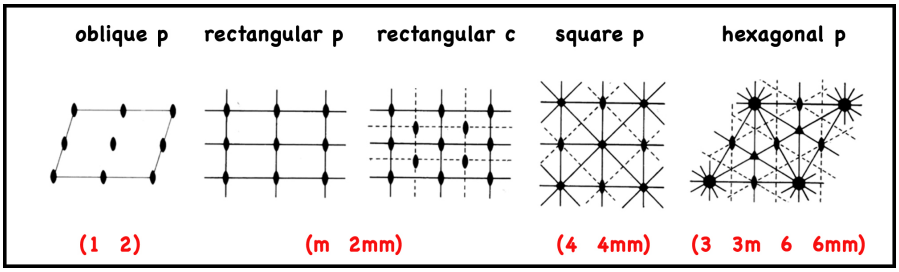


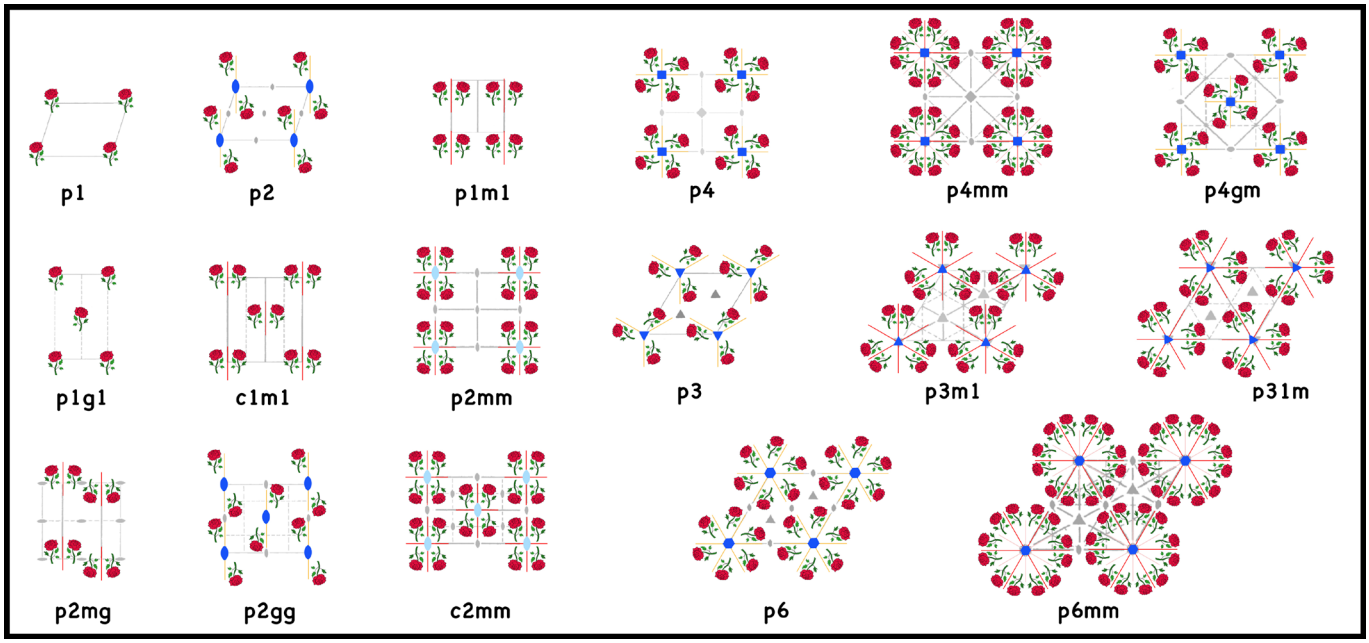
Figure 2
Crystal systems, oblique, rectangular, square, and hexagonal. The oblique system is suitable for crystal classes *1* and *2*, the rectangular systems *p* or *c* for crystal classes m and *2mm,* the square lattice for *4* and *4mm*, and the hexagonal lattice for crystal classes *3, 3m, 6* and *6mm*.

To obtain an extended pattern in two dimensions, as we have in a lattice or grid, the stationary point group must be embedded in a translation operator with a suitable symmetry. Figure 2 shows that the translation group consists of 5 *crystal systems:* the oblique, the rectangular *p* and the rectangular *c*, the square, and the hexagonal. The letter *c* represents a centered cell with a motive in the middle of the lattice. Otherwise, it is a primitive cell denoted by *p*.

When crystal classes *1* and *2* are embedded in the oblique lattice, we obtain *p1* and *p2*. The crystal classes *m* and *2mm* can be used with the rectangular system *p* or *c* and give *pm, cm, pg* and *p2mm, p2mg (= p2gm), p2gg, c2mm*. The character *g* denotes the glide reflection resulting from the combination of point-group and translational symmetry. The square lattice is used for crystal classes *4* and *4mm*, resulting in the symmetries *p4* and *p4mm, p4gm*. Finally, crystal classes *3, 3m, 6* and *6mm* are embedded in the hexagonal lattice, leading to symmetries *p3, p3m1, p31m* and *p6, p6mm*. Thus, the combination of the point-group and the translational symmetry results in 17 groups, called the *wallpaper group*, consisting of *p1, p1m1, p1g1, c1m1, p211, p2mm, p2mg, p2gg, c2mm, p3, p3m1, p31m, p4, p4mm, p4gm, p6, p6mm*. They can be used to describe any two-dimensional pattern.

Figure 3 presents the 17 wallpaper groups with the corresponding symmetry operations, rotation and reflection. A single rose is the elementary starting point.

Figure 3
17 wallpaper groups, as described in the text. The group designations are given below each group member.



An application of these classifications to periodic ornaments is shown for each rotational symmetry *n* in Figure 4. The identification of the rotational symmetry is straightforward. Other symmetry properties are found by identifying mirror and glide reflections. Recently, the wallpaper classification was applied to the artworks of the Seljuk dynasty of Rum (Erbudak 2020b: 177-198) and compared with the classical arrangement based on local characteristics of the patterns (Schneider 1980).

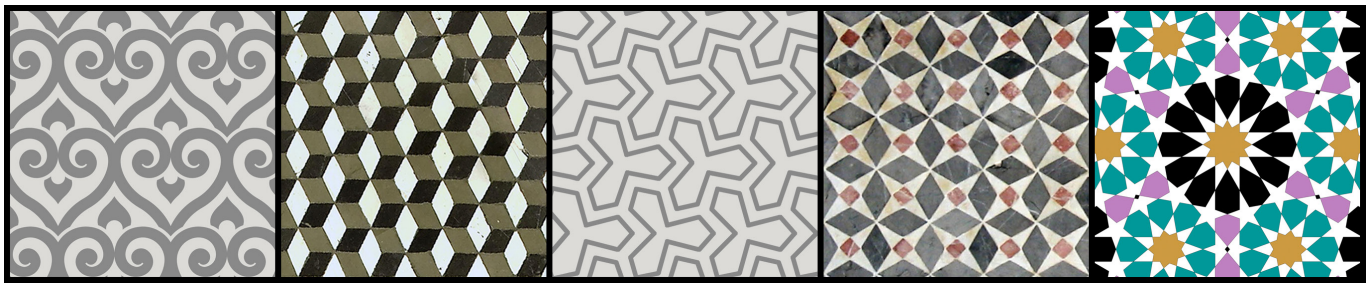


Figure 4
Ornaments with different rotational symmetries. From left-to-right: Armenian *c1m1*, Eastern Roman *p211*, Rum Seljuk *p31m*, Hellenic *p4mm*, and Andalusian *p6mm*.

2.2. Medieval Civilizations

We consider the Hellenic colonies first established in the coastal regions of Asia Minor around 650 BC (Brinton et al. 1963). Later, in the *Classical Period*, the settlements flourished and gained power. In the *Hellenistic Period*, several cities are founded in the eastern regions, mainly in the course of the conquest by Alexander the Great and later by his generals.

Figure 5
The Middle East stretches from Persia in the east to the Anatolian peninsula. This region was home to the Armenians, the Hellenic and Roman Civilizations, the Arabs, and the Seljuks. The expansion of the Eastern Roman Empire is highlighted. In the west, Andalusia was founded on the Iberian peninsula by the Berber Groups, and the Umayyads established a glorious kingdom.

The Romans conquered most of the Hellenic foundations in the 2nd century BC. The most important foundation was that of the Eastern Roman (Byzantine) Empire with its capital at Constantinople (Vasiliev 1952; Kaldellis 2019). At its height, the Empire stretched from North Africa to the Black Sea and included Italy, the Balkans, Greece and the Mediterranean islands.

Another civilization we will consider is the Armenian one (Payaslian 2007). The historical Armenians moved westward under the threat by the Moslem neighbors from the east. In Cilicia, they came into contact with Hellenistic culture.

In the 7th century, Islam emerged on the Arabian Peninsula (Brinton et al. 1963). It was the Arabs who spread the religion from the Atlantic coast to India in less than a century. Their successful dynasty was the Umayyads, with Damascus as their capital, who were succeeded in 750 by the Abbasids from Bagdat. The

Umayyads moved across North Africa to the Iberian Peninsula and established a kingdom in Cordoba. It was a stronghold of science, medicine and philosophy for almost eight centuries.

The last group we looked at was the Seljuks (Başan 2010). They came from the prairies in the southeast of what is now Mongolia. Around the year 1000, they moved to Afghanistan and then to Persia, where they established an empire. They were successful warrior people, called Turan by the Persians. In the late 11th century, they founded the Seljuk dynasty of Rum in Asia Minor with Konya as its capital (Mecit 2014). Their architectural marvels still inspire the travellers today.

3. Classification of Ornaments

We have analyzed the planar ornaments of the above civilizations and classified them into 17 wallpaper groups. The use of each symmetry group is expressed as a percentage, and the information specific to each cultural group can be considered as a vector with 17 components. We repeated the classification process for each cultural group.

For the ornaments of the Hellenistic culture, we considered 61 artifacts from three sites: the terrace houses of Ephesus (Tabanlı 2007), the archeological site of Zeugma (Zeugma), and Antiochia ad Cragum (Dodd 2020). We have already described these Greco-Roman sources and their influence on other Anatolian cultures (Erbudak - Onat 2020). A wealth of ornaments was found in the Basilica di San Marco in Venice (Vio 2012); 114 of them are included in this work as representative of the Eastern Roman civilization. A comprehensive collection of Armenian ornaments was published by Kyurkchyan (Kyurkchyan - Khatcherian 2010; Kyurkchyan 2016). We have analyzed 123 of them. Arabic ornaments are frequently published under Islamic Art (Bourgoin 1879; Critchlow 1976; Makovicky - Makovicky 1977: 58-66; Abas - Salman 1995; Bonner 2017). Here, we sorted out 225 artifacts. The data for the Umayyads of Andalusia, consisting of 103 ornaments, come from the archive of Brian Wichmann (Wichmann 2020). The Great Seljuks founded a dynasty in Persia. Therefore, it is difficult to clearly distinguish between the Persians and Seljuks in their artistic achievements. The 125 ornaments used here are from various sources (Hutt - Harrow 1977; Hattstein - Delius 2000; Makovicky 2016; Bonner 2017). We have extracted 364 periodic ornaments from Schneider's work on the Seljuks of Rum (Schneider 1980). The results are presented in Table 1. The first column represents the wallpaper group. The other columns show the number and resulting percentage use of the wallpapers in each cultural group.

	Hellenistic	E Roman	Armenian	Arabs	Umayyads	Seljuk Emp	R Seljuks
p1	6/9.8	16/14.0	1/0.8	0/0	1/1.0	1/0.8	1/0.3
p1m1	1/1.6	11/9.7	6/4.9	0/0	0/0	0/0	0/0
p1g1	0/0	1/0.9	0/0	0/0	0/0	1/0.8	0/0
c1m1	1/1.6	5/4.4	3/2.4	0/0	0/0	0/0	2/0.6
p211	0/0	3/2.6	0/0	8/3.6	1/1.0	3/2.4	4/1.1
p2mm	4/6.6	11/9.7	7/5.7	13/5.8	2/1.9	8/6.4	32/8.8
p2mg	1/1.6	6/5.3	1/0.8	1/0.4	0/0	1/0.8	3/0.8
p2gg	0/0	1/0.9	1/0.8	0/0	0/0	4/3.2	2/0.6
c2mm	4/6.6	4/3.5	2/1.6	29/12.9	2/1.9	14/11.2	31/8.5
p3	0/0	0/0	1/0.8	0/0	1/1	0/0	3/0.8
p3m1	0/0	0/0	0/0	0/0	0/0	0/0	0/0
p31m	0/0	0/0	3/2.4	1/0.4	0/0	2/1.6	7/1.9
p4	1/1.6	2/1.8	15/12.2	1/4.4	28/27.2	12/9.6	30/8.2
p4mm	38/62.4	40/35.1	67/54.5	65/28.9	53/51.5	32/25.6	101/27.8
p4gm	4/6.6	13/11.4	6/4.9	12/5.3	6/5.8	16/12.8	19/5.2
p6	0/0	1/0.9	2/1.6	10/4.4	3/2.9	10/8.0	29/8.0
p6mm	1/1.6	0/0	8/6.6	76/33.8	6/5.8	21/16.8	100/27.5
# / %	61/100	114/100	123/100	225/100	103/100	125/100	364/100

Table 1
Occurrence of each symmetry group in the artifacts of seven cultures, in numbers (#) and percentages (%). Rotational symmetries are grouped together. The Eastern Roman Empire is known as Byzantium. Umayyads are located in Andalusia, and Seljuks of Rum in addition to the Seljuk Empire are Moslem peoples emigrated from Asia.

We note that some groups place great emphasis on ornaments with fourfold symmetry. Others frequently use artwork with sixfold symmetry. Threefold symmetry is rarely used. Hellenistic patterns mostly feature the *4mm* group, while Seljuks prefer the double-mirror symmetry, *viz*., the holohedral group.

4. Correlation of the Results

Beyond interpreting the results presented in Table 1, we can compare the creation of artworks of the cultural groups by analyzing their correlations pairwise. A quantitative comparison can be made by using different metrics such as *Jaccard* or *Pierson* correlation. Here we apply the *cosine similarity* pairwise to the occurrence of the symmetry groups of all cultural groups (Ye 2011: 91-97). This metric is used to evaluate the correlation of partially overlapping systems. We have already used this idea to calculate the overlap of two vectors based on their dot product (Erbudak 2020a: 95-101). For seven groups, we obtain here 21 similarity values s_{ij}. However, it is common to work with dissimilarity values, $d_{ij} = 1 - s_{ij}$. For simplicity, the resulting dissimilarity values are each multiplied by 1000 and listed in Table 2.

D	Hellenic	E Roman	Armenian	Arabs	Umayyads	Seljuk Emp	R Seljuks
Hellenic	0						
E Roman	79	0					
Armenian	38	127	0				
Arabs	337	408	285	0			
Umayyads	119	218	42	326	0		
Seljuk Emp	243	263	189	89	191	0	
R Seljuks	296	353	223	21	248	54	0

Table 2
21 dissimilarity values *D*, based on the occurrence of wallpaper groups in the artwork of 7 medieval civilizations (see Table 1 for abbreviations). Similarity is highest for the least dissimilar value. $D = 0$ is the self-similarity.

Correlated pairs have a large similarity value *S* and thus small dissimilarity value *D*. In the table, we see that Seljuks of Rum and Moslem Arabs are closely related ($D = 21$). Similarly, Armenians and the Hellenic groups show a relatively close correlation ($D = 38$). There is some dissimilarity between the Eastern Roman and two Seljuk groups.

4.1. Hierarchical Cluster Analysis

The matrix presented in Table 2 contains all the information about the similarity relations between the 7 pairs of cultures. It is almost impossible to obtain an intuitive picture of the overall similarity relationship between them, since this task requires the simultaneous consideration of 21 dissimilarity values (Bijnen 1973). Hierarchical Cluster Analysis (HCA) is used to investigate whether similar objects in a group form a cluster. Based on the similarity of the objects, HCA generates a tree-like structure called *dendrogram*. It is built iteratively by connecting similar objects with vertical lines called *nodes*. The nodes are located at dissimilarity values of the connected objects. The horizontal lines are called *branches*. After each link, a new matrix is computed using the average values of the linked objects, i.e., by aggregating the dissimilarity values of the connected objects. In the aggregation, we have used the averaging method compared to minimum or maximum (complete) clustering schemes, since there is already a considerable amount of overlap between different cultural practices. We also calculated a similar dendrogram for a smaller set of cultural groups (Erbudak - Onat 2020) using the minimum and maximum linkage methods. Averaging resulted in better defined clusters. This process is repeated until a single object remains (Sokal - Michener 1958: 1409-1438).

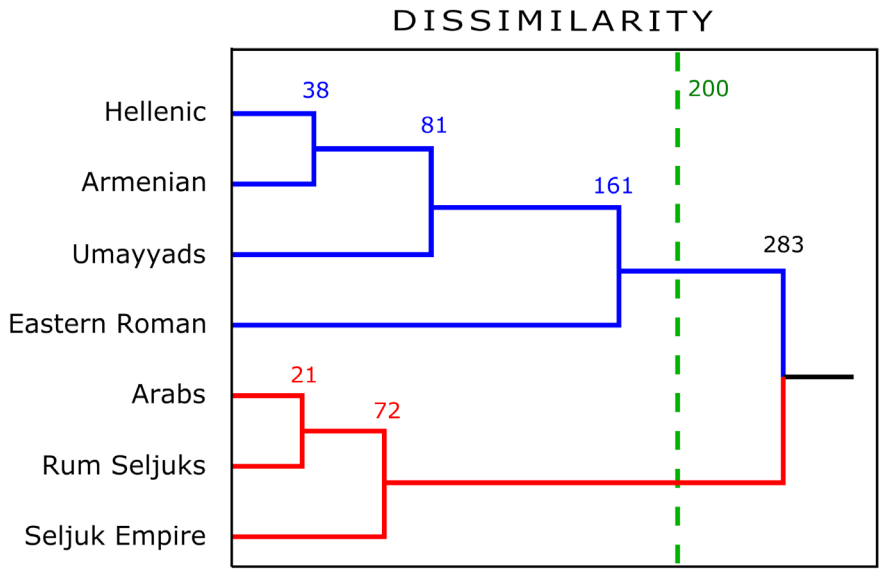


Figure 6
A dendrogram based on dissimilarities between 7 medieval cultures. The lengths of branches are the dissimilarity values from Table 2. A section at $D = 200$ shows the formation of two distinct clusters, indicated by blue and red branches.

The cut through the branches at $D = 200$ of the dendrogram shown in Figure 6 signals the formation of two distinct clusters, the blue and the red. The blue cluster is inspired by the Hellenic art, which originated in Western Asia Minor in pre-Christian times. This cult flourished in neighboring civilizations and was even transferred to the Iberian Peninsula. The Islamic cluster in red is found in the Arab and Turkic civilizations.

4.2. Multidimensional Scaling

The similarity relationships between the seven cultures can also be represented using Multidimensional Scaling, MDS (Kruskal 1964: 1-27). MDS is a visual representation of dissimilarities between groups of objects. Objects that are more similar are closer together on the graph than objects that are less similar. While MDS interprets dissimilarities as distances on a graph, it serves as a dimensionality reduction technique for high-dimensional data, computing a summerizing representation of a dissimilarity matrix on a two-dimensional surface. MDS is performed by iteratively minimizing a function called *stress*. Stress is the squared difference between the coordinates of the objects (output) and the dissimilarity values (input). Stress is thus a measure of goodness-of-fit, based on the differences between predicted and actual distances. The positions are found by minimizing the stress term. MDS is thus a form of nonlinear dimensionality reduction.

Figure 7 shows the results of the MDS calculations, which reduce the data to two arbitrary dimensions given as distances. All resulting values are concentrated in the vertical distance around zero. The group with the negative horizontal distance, the blue group, shows a larger spread. The cultures that make up this group are the Hellenic, Eastern Roman, Armenian, and Umayyads of Andalusia. The red group is on the positive side of the horizontal distance and consists of the Arabs, the Rum Seljuks, and the Seljuk Empire. The formation of the two groups supports the results of the HCA.

5. Conclusions

The distribution of wallpaper use among the Seljuks of Rum and the Arabs of the Middle East, shown in Table 1, already indicates a great similarity in their artistry. Visual inspection of the correlation values from Table 2 clearly shows the

least dissimilarities between these Islamic cultures. One certainly does not need sophisticated statistical calculations such as HCA or MDS to see this obvious similarity. However, less obvious interactions between distant cultures may go undetected by simple observation when statistical methods are unavoidable. In particular, possible clustering of cultures is best revealed by such methods.

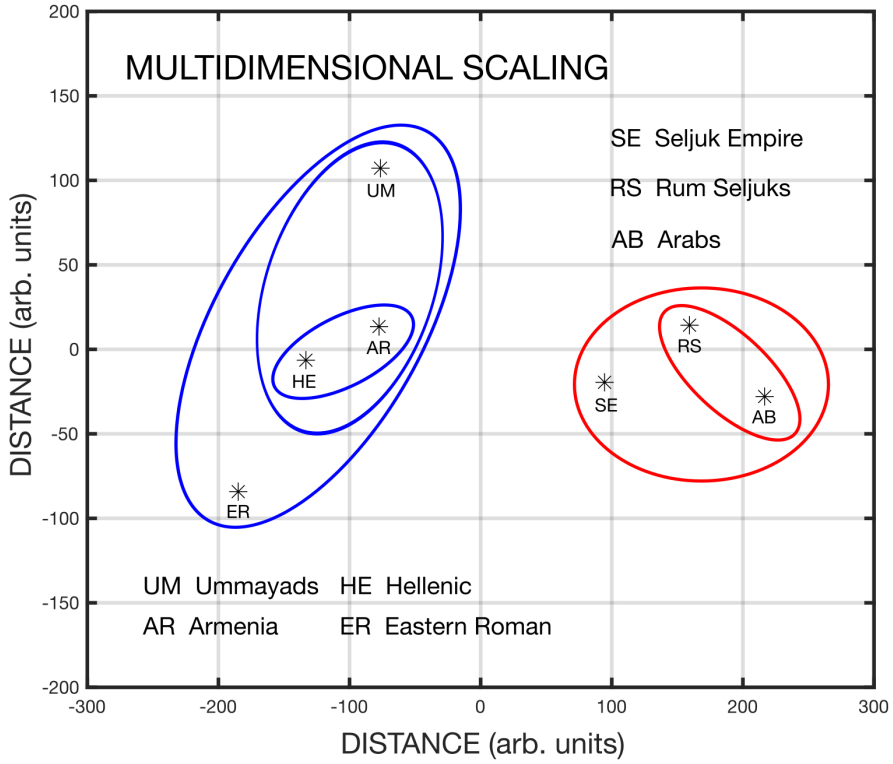


Figure 7
Visualization of the dissimilarity matrix created by calculated MDS in two dimensions. Each point represents a different culture. The distances between the cultures correspond to their dissimilarities. The blue and red ellipses are drawn arbitrarily to highlight the formation of two distinct groups. The results show the seven cultures in two distinct groups, blue and red.

We have treated ornaments and the symmetry they contain as members of the wallpaper group as a fingerprint of the cultural group that created the ornament. We have studied the wallpapers that are present in all available artworks of the particular cultural group. This is the most important information for our starting point: 17 individual values representing the group. We then calculated the overlap of these values for different medieval cultural groups and determined a pairwise correlation value. A large overlap or correlation indicates a high similarity between the two cultural groups and thus a high effective cultural interaction. To evaluate the correlations, we used two widely-accepted statistical methods, HCA and MDS.

The results show that seven cultural groups studied here can be divided into two clusters. The first consists of the Arabs of the Near East and the Seljuks, the empire existed in Persia and the Seljuks of Rum flourished in Asia Minor. The Seljuks invaded Persian territory from the east early in the 11th century. By this time, they had already converted to Islam. About a century later, a group moved further west and established the Seljuks of Rum with Konya as its capital. They brought with them their Perso-Islamic culture mixed with some Central Asian customs. They mixed with the existing Christian tradition of Romans, Greeks, Armenians, and Syriacs in Asia Minor. Therefore, we did not expect such a strong overlap between the Seljuk Rum and Arabic art. The Seljuk motifs could be very similar to their Persian and Christian examples from Anatolia. Unfortunately, our method is insensitive to the local characteristics of the ornaments. The two-dimensional extended symmetry properties used by the Seljuks of Rum are almost identical to those used by the Arabs. It remains a matter of speculation which of the two peoples had a stronger influence.

It is indeed no surprise that the blue group of civilizations can be found in the environment of the Hellenic tradition. Cultural development in Asia Minor began several millennia before our era. Assyrians, Babylonians, Hittites and Sumerians were the first known groups before the Hellenic colonization of Asia Minor. Urartu was about a millennium before Christ. The Persian invasion and subsequent conquest by Alexander the Great opened Asia Minor to Hellenistic civilization. During the Roman conquest, Hellenistic culture prevailed. Around the 8th century AD, Armenians from the east moved into Cilicia. Asia Minor was thus dominated by Hellenistic-Armenian art before the Seljuks arrived in the 11th century. The blue cluster of four civilizations is therefore ambiguous. On the timeline, the first culture in Asia Minor is the Hellenic-Roman group with its polytheistic involvement. The Eastern Roman Empire was Christian with Eastern Orthodox nomination. The Armenians confessed to Oriental Orthodoxy. Under the Umayyads of Andalusia, a true mixture of Islam and Judaism prevailed. Thus, religious conviction was not the driving force that led these cultures to create works of art with similar characteristics.

First, we examine the relations between Armenia and the Eastern Roman Empire. The advance of the Umayyads into Armenian lands triggered the migration of the Armenians westward to the Roman Empire, where they sought a safe haven. The Roman Empire encouraged this migration and included many Armenians in its army, which fought against Slavic tribes on European soil and Arabs in the East. In the 9th and 10th centuries, a large part of the Byzantine army consisted of Armenians. They often became high-ranking military officers, and many Armenians attained intellectual and political positions in the empire. During the Macedonian dynasty, several emperors of the Byzantine Empire were at least partially of Armenian origin. This period, known as the Macedonian Renaissance, was fertile ground for intense cultural interaction between the Eastern Roman Empire and Armenian artistry (Goodyear 2016).

The connection between the Eastern Roman Empire and the Umayyads of Andalusia began in the 9th century. The cultural interaction was initially politically and militarily motivated and was under the influence of the common enemy, the Abbasids. Andalusian civilization was initiated by the Umayyads, who escaped the Abbasid massacre. In the 9th century, the terror of the Abbasids against Byzantium triggered their solidarity with the Umayyads of Andalusia (Wasserstein 1987: 76-101; Cordoso 2015). This connection intensified on a political, military, and cultural level. On the one hand, the Eastern Roman Empire sought military help after the loss of Crete and Sicily, on the other hand, the Umayyads admired the Eastern Roman architectural achievements, which they could see during their visits to Constantinople. Cultural missions were carried out in the 10th century and intensified during the reign of the Calif Al-Hakam II. He requested expertise for the detailed construction of the Great Mosque in Cordoba. Around 965, ceramic tiles and mosaic material arrived in Cordoba, along with skilled craftsmen sent by Emperor Nikephoros Phokas (Lévi-Provençal 1952; Marçais 1965: 147-56).

We realize that there is a close organic relationship between Armenia and the Eastern Roman Empire, while at the same time the Empire actively interacted culturally with the Umayyads of Andalusia. These facts suggest that some Armenian influence may have reached the Iberian Peninsula. This assumption may not fully explain the great Armenian-Andalusian similarity in ornamental symmetries, but it may give an indication of how early Andalusian creation was influenced.

We have demonstrated the possibility of applying mathematical analyses to the cultural treasures of different civilizations. We often find some obvious connections between peoples, but we also see that the methods we use have the potential to uncover hidden connections, such as between Armenia and the Iberian Muslim population, which consists of Arabs, Jews, Berbers, and Spanish Moslems.

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Bibliography – Kaynaklar

Abas - Salman 1995 — S. J. Abas – A. S. Salman, Symmetries of Islamic Geometrical Patterns, World Scientific, Singapore. https://www.worldscientific.com/doi//pdf/101142/9789814335942_fmatter.

Başan 2010 — A. Başan, The Great Seljuks: A History, Routledge, London.

Bijnen 1973 — E. J. Bijnen, Cluster Analysis, Tilburg University Press, Groningen.

Bonner 2017 — J. Bonner, Islamic Geometric Patterns, Springer, New York.

Bourgoin 1879 — J. Bourgoin, Les Élements de L'Arabe, Librairie de Firmin-Didot, Paris; Arabic Geometrical Pattern and Design, Dover, New York.

Brinton et al. 1963 — C. Brinton – J. B. Christopher – R. L. Wolff, A History of Civilization, vol. 1, 2nd ed., Prentice Hall, New Jersey.

Cordoso 2015 — E. R. Fe. Cardoso, Diplomacy and oriental influence in the court of Cordoba, Dissertation in History of Islamic Mediterranean Societies, University of Lisbon. https://repositorio.ul.pt/bitstream/10451/18276/1/ulfl183083.pdf.

Critchlow 1976 — K. Critchlow, Islamic Patterns, London. https://archive.org/details/IslamicPatterns_201805/page/n1.

Dodd 2020 — E. Dodd, "Late Roman viticulture in Rough Cilicia: an unusual wine-press at Antiochia ad Cragum", JRA 33, 467-482.

Erbudak 2020a — M. Erbudak, "Mathematical Determination of the Cultural Interaction between Medieval Groups", JMSM 3, 95-101.

Erbudak 2020b — M. Erbudak, "Mathematical Classification of Rum Seljuk Ornaments", Symmetry: Culture and Science 2, 177-198.

Erbudak - Onat 2020 — M. Erbudak – M. S. Onat, "The Role of the Greco-Roman Practice as a Progenitor of the Armenian and Eastern Roman Ornamental Art", arXiv:2011.10973.

Goodyear 2016 — M. Goodyear, "Inside and Outside the Purple: How Armenians Made Byzantium", Armstrong Undergraduate Journal of History 6, 22-47.

Hammond 2015 — C. Hammond, The Basics of Crystallography and Diffraction, 4th ed., Oxford.

Hattstein - Delius 2000 — M. Hattstein – P. Delius, Islam, Kunst und Architektur, Könemann, Köln.

Hutt - Harrow 1977 — A. Hutt – L. Harrow, Iran I, London.

Kaldellis 2019 — A. Kaldellis, Romanland: Ethnicity and Empire in Byzantium, Cambrigde.

Kruskal 1964 — J. B. Kruskal, "Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis", Psychometrika 29, 1-27.

Kyurkchyan - Khatcherian 2010 — A. Kyurkchyan (Author) – Hrair Hawk Khatcherian (Photographer), Armenian Ornamental Art, Craftology, Yerevan.

Kyurkchyan 2016 — A. Kyurkchyan, Armenian Block Printing Fabric, Kyurkchyan, Yerevan.

Lévi-Provençal 1952 — E. Lévi-Provençal, Histoire l'Espagne musulmane, vol. 2, Le caliphate umaiyade de Cordue (912-1031), 2nd ed., Leiden.

Makowicky 2016 — E. Makowicky, Symmetry, Through The Eyes of Old Masters, Berlin.

Makowicky - Makowicky 1977 — E. Makowicky – M. Makowicky, "Arabic Geometrical Patterns – A Treasury for Crystallographic Teaching", Neues Jahrbuch für Mineralogie Monatshefte 2, 58-66.

Marçais 1965 — G. Marçais, Sur les mosaïques de la Grand Mosquée de Codoue, Islamic Art and Architecture in Honour of Professor K.A.C. Cresswell, Cairo, 147-56.

Mecit 2014 — S. Mecit, The Rum Seljuqs: Evolution of a Dynasty, London.

Müller 1944 — E. A. Müller, Gruppentheoretische und strukturanalytische Untersuchung der Maurischen Ornamente aus der Alhambra in Granada, PhD Thesis, University of Zurich, Zurich.

Payaslian 2007 — S. Payaslian, The History of Armenia: From the Origins to the Present, New York.

Pólya 1924 — G. Pólya, "Über die Analogie der Kristallsymmetrie in der Ebene", Z. Kristall 60, 278-282.

Schneider 1980 — G. Schneider, Geometrische Bauornamente der Seldschuken in Kleinasien, Wiesbaden.

Sokal - Michener 1958 — R. R. Sokal – C. D. Michener, "A Statistical Method for Evaluating Systematic Relationship", University of Kansas Science Bulletin 58, 1409-1438.

Tabanlı 2007 D. Tabanlı, Roma Dönemi Mozaiklerinin Efes Örneğinde İncelenmesi, Yayınlanmamış Yüksek Lisans Tezi, Ege Universitesi, İzmir.

Vasiliev 1952 A. A. Vasiliev, History of the Byzantine Empire, vols. I - II, Wisconsin.

Vio 2012 E. Vio, Il Manto di Pietra della Basilica di San Marco a Venezia, Venice.

Washburn - Crowe 1999 D. K. Washburn – D. W. Crowe, Symmetries of Culture: Theory and Practice of Plane Pattern Analysis, Washington.

Wasserstein 1987 D. Wasserstein, Byzantium and Al-Andalus, MedHistR 2:1, 76-101.

Wichmann 2020 B. Wichmann, https://tilingsearch.mit.edu/FirstUsage.html

Ye 2011 J. Ye, "Cosine similarity measures for intuitionistic fuzzy sets and their applications", Math. Comput. Modelling 53, 91-97.

Zeugma Archeological site of Zeugma: https://whc.unesco.org/en/tentativelists/5726/, https://museumofwander.com/zeugma-mosaic-museum-gaziantep/ and photographs of the author.