European Journal of Technique

journal homepage: https://dergipark.org.tr/en/pub/ejt

Vol.13, No.2 ,2023

Research Article

Twin Rotor Control via Second Order Sliding Modes

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ARTICLE INFO

Received: Oct., 26. 2023
Revised: Nov., 19. 2023
Accepted: Nov, 20. 2023

Keywords:
Second Order Sliding Mode Control
Twin rotor model
Numerical Results
Equivalent Control Estimation

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ISSN: *2536-5010* | e-ISSN: *2536-5134*

DOI: https://doi.org/10.36222/ejt.1381813

ABSTRACT

The control problem of a twin rotor system is considered in this study. Since the twin rotor system has highly nonlinear and coupled dynamics, a second order sliding mode controller is proposed which reduces chattering. An estimation for the equivalent part of the controller is also proposed. The classical sliding controller is also calculated and adapted to the twin-rotor system to compare.

The numerical results presented that the designed controller achieved superior performance than the classical sliding mode controller in terms of reference tracking and chatter attenuation.

1. INTRODUCTION

The twin rotor system exhibits similarities to a helicopter in various aspects when it comes to its dynamics. [1]. Thus during the last decade, studies concerning twin rotor, have increased. Juang et. al. [2] implemented a hybrid proportional integral derivative (PID) controller to a twin rotor system. Taskin [3] investigated the performance of twin rotor system under hovering conditions with fuzzy logic controller (FLC). Aras and Kaynak [4] developed a neural fuzzy controller for the twin rotor system. The designed controller was compared with a traditional neuro-fuzzy structure and an interval type-2 fuzzy neural system. Hacioglu [5] proposed a new Multi Input Multı Output PIPD type fuzzy logic controller for experimental setup. Juang et al. [6] designed a fuzzy PID control algortihm with a real-valued optimization for twin rotor. Mondal and Mahanta proposed a second order sliding mode (SOSM) controller to experimental twin-rotor system. [7]. The simulation results of that controller showed sufficient tracking performance and robustness to external disturbances. To improve controller performance, a new sliding surface has been developed by Ahmed et al. to handle transverse effects in those system. [8]. Ilyas et al. [9] designed a hybrid controller with backstepping-sliding mode and validated their performances on a twin-rotor system via simulations. Raghavan and Thomas [10] proposed an implementable control design for a twin rotor system.
Sliding mode control (SMC) is known for its robust behavior. For nonlinear and uncertain systems, it may be modified. [11-14]. According to this control method, the control system is not sensitive to outside sources and parameter changes when the system is on the sliding surface. [12]. Alternatively, the control signal and system states may experience rapid oscillations at high frequencies, known as chattering. The components of the dynamic systems, such as the servomotors, may be damaged. Various approaches have been suggested to prevent this problem [15-16]. Huseyinoglu and Abut [17] proposed a SMC using a saturation function to prevent chattering for a two-degree-of-freedom (2-DOF) robotic arm. Aydin et al. [18] used a sigmoid function in sliding mode observer based control method to reduce chattering effect on a permanent magnet synchronous motor.
Nevertheless, the motion that occurs is not an ideal sliding motion. Thus, higher order sliding mode controller techniques have been proposed in the research area to suppress or almost eliminate chattering [19, 20]. In particular, the Super Twisting Algorithm (STA), a famous Second Order Sliding Mode method, is described by Levant [20] and has been applied for this area [20-24].

In this paper, a second-order SMC with estimation (SOSMCE) via the super twisting algorithm is designed for a twin-rotor model. The designed controller adopts an estimation law for the equivalent part of the control input, which separates it from previous high order sliding mode controllers in the current research. The coefficients are selected by multi objective genetic algorithm (MOGA) optimization. To verify the capability and chattering suppression of the designed controller, a classical sliding mode controller is also proposed, then implemented on the twin rotor model for validation. The capabilities of the proposed controllers are validated by numerical results.

2. CONTROLLER DESIGN

2.1. Twin Rotor Model

An attempt is made to control the position, speed [25], torque production [26] and energy consumption [27] values of dynamic electromechanical systems and their interactions. Due to its simple structure and good representation of the behaviour of cross-coupled axial motions, the twin rotor system is widely preferred by researchers.

Figure 1 shows the twinrotor model where m_{hel} stands for the mass of the helicopter, l is the distance from the center of mass to the pitch axis along the helicopter body, B_{pitch}, B_{yaw} are the equivalent viscous damping, I_{pitch}, I_{yaw} are the total moment of inertia, respectively. Here, θ and ψ are angular displacements and $\overline{\tau_\theta}, \overline{\tau_\psi}$ represent moments. The mathematical model is obtained using Lagrange's method, as shown below:

$$m_{hel}l^2\ddot{\theta} + I_{pitch}\ddot{\theta} + m_{hel}l^2(\dot{\psi})^2\cos(\theta)\sin(\theta) + B_{pitch}\dot{\theta} + m_{hel}glcos(\theta) = \overline{\tau_\theta} + \widetilde{d_\theta} \quad (1)$$

$$m_{hel}l^2\ddot{\psi}cos^2(\theta) - 2m_{hel}l^2\dot{\theta}(\dot{\psi})\cos(\theta)\sin(\theta) + I_{yaw}\ddot{\psi} + B_{yaw}\dot{\psi} = \overline{\tau_\psi} + \widetilde{d_\psi} \quad (2)$$

$\widetilde{d_\theta}$, $\widetilde{d_\psi}$ are the outside effects that are applied to the model.

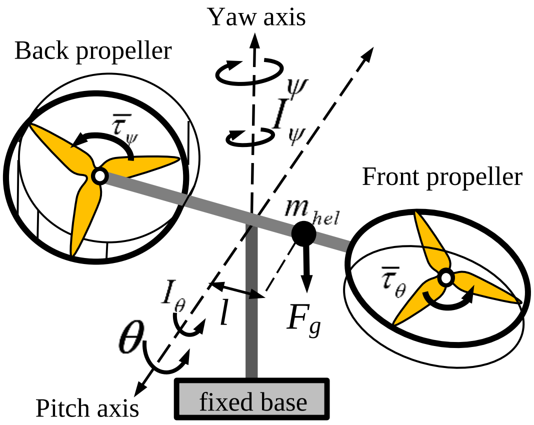

Figure 1. The physical of the twin rotor

Using the state variables

$$[x_1 \quad x_2 \quad x_3 \quad x_4]^T = [\theta \quad \dot{\theta} \quad \psi \quad (\dot{\psi})]^T \quad (3)$$

$$\dot{x_1} = x_2 \quad (4)$$

$$\dot{x_2} = \frac{-m_{hel}l^2(x_4)^2\cos(x_1)\sin(x_1)}{(m_{hel}l^2+I_{pitch})} + \frac{-B_{pitch}x_2 - m_{hel}glcos(x_1)}{(m_{hel}l^2+I_{pitch})} + \frac{\overline{\tau_\theta}}{(m_{hel}l^2+I_{pitch})} + \widetilde{d_\theta} \quad (5)$$

$$\dot{x_3} = x_4 \quad (6)$$

$$\dot{x_4} = \frac{2m_{hel}l^2(x_2)(x_4)\cos(x_1)\sin(x_1)}{m_{hel}l^2cos^2(x_1)+I_{yaw}} + \frac{-B_{yaw}(x_4)}{m_{hel}l^2cos^2(x_1)+I_{yaw}} + \frac{\overline{\tau_\psi}}{m_{hel}l^2cos^2(x_1)+I_{yaw}} + \widetilde{d_\psi} \quad (7)$$

2.2. The designed Second Order Sliding Mode Controller with Estimation (SOSMCE)

In this section, a SOSMC based on the super twisting algorithm is presented. This controller design is different from other studies in that it includes the estimation of equivalent control. That estimation will be emphasized along with the controller design.

SOSMC can be described as the motion on nonempty set $\sigma = \dot{\sigma} = 0$ consisting of Filippov trajectories. $\sigma = \dot{\sigma}$ denote continuous functions of the closed-system state parameters [20, 22].

$$\dot{\eta_1} = \eta_2 \quad (8)$$

$$\dot{\eta_2} = f(\eta_1, \eta_2) + g(\eta_1, \eta_2)\bar{u} + \bar{d} \quad (9)$$

where η_1 and η_2 are states, $\bar{u}$: control input and $\bar{d}$: limited disturbance. $g(\eta_1, \eta_2)$ is a known control input function, $f(\eta_1, \eta_2)$ may contain undefined or uncertain conditions, which are later taken into account in the estimation of the equivalent control. The sliding surface is described as

$$\sigma = \alpha(\eta_{1r} - \eta_1) + (\dot{\eta}_{1r} - \dot{\eta_1}) \quad (10)$$

where the sliding surface parameter is $\alpha > 0$. Through the determination of the time derivative of the sliding surface and the use of the Eqs. (8) - (9)

$$\dot{\sigma} = \alpha(\eta_{2r} - \eta_2) + \dot{\eta}_{2r} - f(\eta_1, \eta_2) - g(\eta_1, \eta_2)\bar{u} - \bar{d} \quad (11)$$

$$\phi(\eta_1, \eta_2) = \alpha(\eta_{2r} - \eta_2) + \dot{\eta}_{2r} - f(\eta_1, \eta_2) \quad (12)$$

$$u = -g(\eta_1, \eta_2)\bar{u} \quad (13)$$

$$d = -\bar{d} \quad (14)$$

$$\dot{\sigma} = \phi(\eta_1, \eta_2) + u + d \quad (15)$$

The disturbance is estimated to be limited as $|d| \leq \Delta\sqrt{|\sigma|}$, $\Delta > 0$. Bound condition $\dot{\sigma} = 0$ and nominal system that is $d = 0$, the equivalent control u_{eq}:

$$u_{eq} = -\phi(\eta_1, \eta_2) \quad (16)$$

We propose the super twisting method introduced in Levant [20] for the discontinuous part of the control rule.

$$u_{dc} = -k_1|\sigma|^{1/2} sign(\sigma) + v \quad (17)$$

$$\dot{v} = -k_2 sign(\sigma) \quad (18)$$

Hence the cumulative control rule is,

$$u = u_{dc} + u_{eq} \quad (19)$$

The stability of the control algorithm will be ensured by this Lyapunov function [28],

$$V = 2k_2|\sigma| + \frac{1}{2}v^2 + \frac{1}{2}(k_1|\sigma|^{1/2} sign(\sigma) - v)^2 \quad (20)$$

$$V = \xi^T \mathbf{P}\, \xi \quad (21)$$

where

$$\xi^T = [|\sigma|^{1/2} sign(\sigma) \quad v] \quad (22)$$

$$\mathbf{P} = \begin{bmatrix} 2k_2 + \frac{{k_1}^2}{2} & \frac{-k_1}{2} \\ \frac{-k_1}{2} & 1 \end{bmatrix} \quad (23)$$

$$\dot{V} = \dot{\xi}^T \mathbf{P}\, \xi + \xi^T \mathbf{P}\, \dot{\xi} = \dot{\sigma}\, sign(\sigma)\left(2k_2 + \frac{1}{2}{k_1}^2\right) - k_1 \dot{v}|\sigma|^{1/2}\, sign(\sigma) - \frac{k_1 v \dot{\sigma}}{2|\sigma|^{1/2}} + 2\dot{v}v \quad (24)$$

By using Eqs. (15) - (19)

$$\begin{aligned} \dot{V} = {} & [-k_1|\sigma|^{1/2} sign(\sigma) + v + d]\, sign(\sigma)\left(2k_2 + \frac{1}{2}{k_1}^2\right) \\ & - k_1[-k_2 sign(\sigma)]|\sigma|^{1/2}\, sign(\sigma) \\ & - \frac{k_1 v[-k_1|\sigma|^{1/2} sign(\sigma) + v + d]}{2|\sigma|^{1/2}} \\ & + 2[-k_2 sign(\sigma)]v \\ = {} & -\left(k_1 k_2 + \frac{{k_1}^3}{2}\right)|\sigma|^{1/2} + {k_1}^2 v\, sign(\sigma) - \frac{k_1 v^2}{2|\sigma|^{\frac{1}{2}}} \\ & - d(t,\sigma)\frac{k_1 v}{2|\sigma|^{\frac{1}{2}}} \\ & + d(t,\sigma)\left[\left(2k_2 + \frac{1}{2}{k_1}^2\right) sign(\sigma)\right] \\ \leq {} & -\left(k_1 k_2 + \frac{{k_1}^3}{2}\right)|\sigma|^{1/2} + {k_1}^2 v\, sign(\sigma) - \frac{k_1 v^2}{2|\sigma|^{1/2}} \\ & + \Delta\left(2k_2 + \frac{1}{2}{k_1}^2\right)|\sigma|^{1/2} - \frac{k_1 v}{2}\Delta sign(\sigma) \end{aligned} \quad (25)$$

$$\dot{V} \leq \frac{-k_1}{2|\sigma|^{1/2}} \xi^T \mathbf{Q}\, \xi \quad (26)$$

$$\mathbf{Q} = \begin{bmatrix} 2k_2 + {k_1}^2 - \left(\frac{4k_2}{k_1} + k_1\right)\Delta & -k_1 + \frac{\Delta}{2} \\ -k_1 + \frac{\Delta}{2} & 1 \end{bmatrix} \quad (27)$$

If k_1 and k_2 provide,

$$k_1 > 2\Delta \quad (28)$$

$$k_2 > \frac{k_1 \Delta^2}{8\,(k_1 - 2\Delta)} \quad (29)$$

$\dot{V} < 0$ will be negative definite, and attaining to the sliding surface is ensured. $\phi(\eta_1, \eta_2)$ can be undetermined or unidentified. Hence, the developed equivalent control signals can be different from the actual equivalent control signals. In this way, an estimated equivalent control $\hat{u}_{eq}$ is used in this research, which is obtained by filtering the cumulative control input signal through a low pass filter. The main concept of applying a low pass filter is that the low frequencies define the characteristics of the input and the high frequencies arise from unknown sources. The estimated equivalent control law:

$$\hat{u}_{eq} = \frac{\varepsilon}{s + \varepsilon} u \quad (30)$$

The output of the low-pass filter tends towards equivalent control if the cut-off frequency ε is large enough to preserve the slow component undistorted, but small enough to eliminate the high-frequency component [11]. The system stability is therefore guaranteed. The cumulative control signal rule for the designed SOSMC is described:

$$\bar{u} = -g^{-1}(\eta_1, \eta_2)u \tag{31}$$

$$u = \hat{u}_{eq} - k_1|\sigma|^{1/2}sign(\sigma) + \upsilon \tag{32}$$

$$\dot{\upsilon} = -k_2 sign(\sigma) \tag{33}$$

$$\dot{\hat{u}}_{eq} = \varepsilon(u - \hat{u}_{eq}) \tag{34}$$

By using the governing Eqs. (4) - (7) for the axial motions, the control inputs of the designed SOSMCE for the model are obtained:

$$\overline{\tau_\theta} = -(m_{hel}l^2 + I_{pitch})\tau_\theta \tag{35}$$

$$\tau_\theta = \hat{\tau}_{\theta eq} - k_{1\theta}\,|\alpha_\theta(x_{1r} - x_1) + \dot{x}_{1r} - x_2|^{1/2} \times sign(\alpha_\theta(x_{1r} - x_1) + \dot{x}_{1r} - x_2) + \upsilon_\theta \tag{36}$$

$$\dot{\upsilon}_\theta = -k_{2\theta}\,\{sign(\alpha_\theta(x_{1r} - x_1) + \dot{x}_{1r} - x_2)\} \tag{37}$$

$$\dot{\hat{\tau}}_{eq\theta} = \varepsilon(\tau_\theta - \hat{\tau}_{eq\theta}) \tag{38}$$

$$\overline{\tau_\psi} = -(m_{hel}l^2cos^2(x_1) + I_{yaw})\tau_\psi \tag{39}$$

$$\tau_\psi = \hat{\tau}_{\psi eq} - k_{1\psi}\,|\alpha_\psi(x_{3r} - x_3) + \dot{x}_{3r} - x_4|^{1/2} \times sign(\alpha_\psi(x_{3r} - x_3) + \dot{x}_{3r} - x_4) + \upsilon_\psi \tag{40}$$

$$\dot{\upsilon}_\psi = -k_{2\psi}\,\{sign(\alpha_\psi(x_{3r} - x_3) + \dot{x}_{3r} - x_4)\} \tag{41}$$

$$\dot{\hat{\tau}}_{eq\psi} = \varepsilon(\tau_\psi - \hat{\tau}_{eq\psi}) \tag{42}$$

2.3. Search for optimal controller coefficients using multi-objective genetic algorithm

Genetic algorithms (GAs) utilize mechanisms derived by genetical principles found in biology to solve real-world tasks. Genetic algorithms commonly include Reproduction, Crossover and Mutation operators. For each problem to be performed, a fitness function must be designed [29]. The purpose of Multi Objective Optimization with Genetic Algorithm (MOGA) is to minimize several fitness functions at the same time. Used to solve multi objective optimization problems by defining the Pareto front, the set of uniformly distributed, non-dominated optimal solutions [30, 31].

In this study, optimal controller parameters are searched using ten proposed fitness functions. It is aimed to reduce tracking errors and attenuate possible chattering.β_1, β_2 represent reference tracking performance. β_3, β_4 represent fluctuations in control signal. β_5, β_6 represent number of passes from zero for acceleration input. $\beta_7, \beta_8, \beta_9, \beta_{10}$, represent the mean values for variables. The optimum parameters achieved by MOGA are listed in Table 2 in Appendix.

$$\beta_1 = \sum_{i=1}^{n}|\theta_{refi} - \theta_i| \tag{43}$$

$$\beta_2 = \sum_{i=1}^{n}|\psi_{refi} - \psi_i| \tag{44}$$

$$\beta_3 = \frac{1}{\sqrt{n}}[\sum_{i=1}^{n}(\dot{u}_{\theta i})^2]^{1/2} \tag{45}$$

$$\beta_4 = \frac{1}{\sqrt{n}}\left[\sum_{i=1}^{n}(\dot{u}_{\psi i})^2\right]^{1/2} \tag{46}$$

$$\beta_5 = dimension\left[\ddot{\theta}_i \rightarrow \begin{Bmatrix} \ddot{\theta}_i < 0 & and & \ddot{\theta}_{i-1} > 0 \\ \ddot{\theta}_i > 0 & and & \ddot{\theta}_{i-1} < 0 \end{Bmatrix}\right] \tag{47}$$

$$\beta_6 = dimension\left[\ddot{\psi}_i \rightarrow \begin{Bmatrix} \ddot{\psi}_i < 0 & and & \ddot{\psi}_{i-1} > 0 \\ \ddot{\psi}_i > 0 & and & \ddot{\psi}_{i-1} < 0 \end{Bmatrix}\right] \tag{48}$$

$$\beta_7 = \frac{1}{n}\sum_{i=1}^{n}\left|u_{\theta i} - \frac{1}{n}\sum_{i=1}^{n}|u_{\theta i}|\right| \tag{49}$$

$$\beta_8 = \frac{1}{n}\sum_{i=1}^{n}\left|u_{\psi i} - \frac{1}{n}\sum_{i=1}^{n}|u_{\psi i}|\right| \tag{50}$$

$$\beta_9 = \frac{1}{n}\sum_{i=1}^{n}\left|\ddot{\theta}_i - \frac{1}{n}\sum_{i=1}^{n}|\ddot{\theta}_i|\right| \tag{51}$$

$$\beta_{10} = \frac{1}{n}\sum_{i=1}^{n}\left|\ddot{\psi}_i - \frac{1}{n}\sum_{i=1}^{n}|\ddot{\psi}_i|\right| \tag{52}$$

2.4. Performance indicators

In evaluating the results of this study, the following performance indicators will be used.

Integral Time Absolute Error (ITAE) [13, 32]:

$$ITAE = \int_0^t t|e|dt \tag{53}$$

Control effort indicator (CEI) [33]:

$$CEI = \sqrt{\frac{\sum_{i=1}^{n}(u_i)^2}{n}} \tag{54}$$

Chattering indicator (CI) [33]:

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$$CI = \sqrt{\frac{\sum_{i=1}^{n}(\dot{u}_i)^2}{n}} \qquad (55)$$

Here ITAE presents the error value of the angular displacement on time vector and penalizes the errors late in time heavily. The CEI is a measure of the control performance. The CI measures the amount of chattering in the control input.

3. NUMERICAL RESULTS

The simulation results of the twin rotor model are shown in this part for the designed second order sliding mode controller with estimation. To investigate the achievement of the designed controller in the presence of outside effects, the disturbance with 2 Hz and 0.26 Nm ($\tilde{d}_\theta$), 2 Hz and 0.14 Nm ($\tilde{d}_\psi$), as presented in Figure 2, is applied to the control signal. For comparison purpose, the classical sliding mode controller was also developed and adapted to model.

Figure 3 presented the time versus motions. From this figure it is observed that both proposed SOSMCE and first order SMC tracked the reference angles for motions precisely. It is also seen that both controllers well coped with the disturbances since they continued to track the desired motion angles. Figure 4 presents the control signals. For the first order SMC it can be seen that there is chattering. Conversely, the proposed controller produced much smoother control signals.

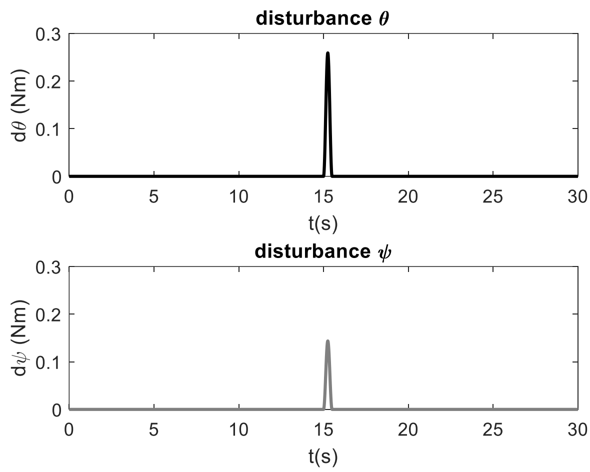
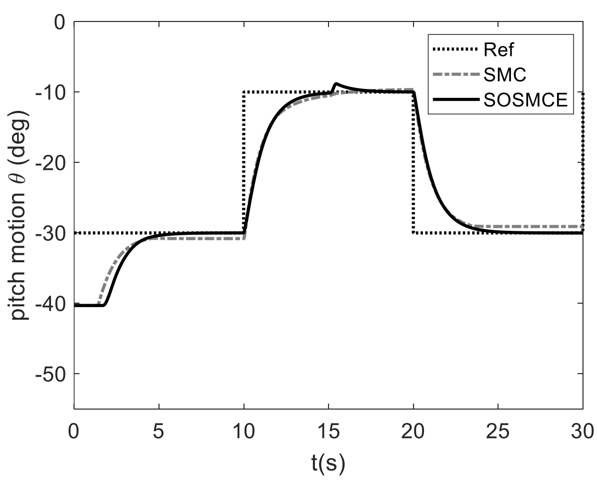
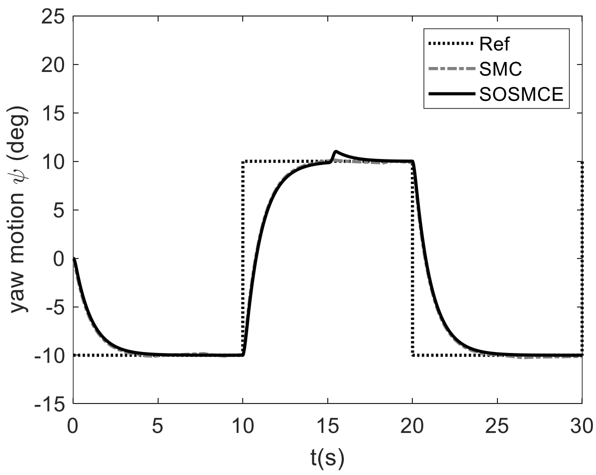


Figure 2. Disturbing torques



(a)



(b)

Figure 3. Tracking reference a) pitch motion, b) yaw motion

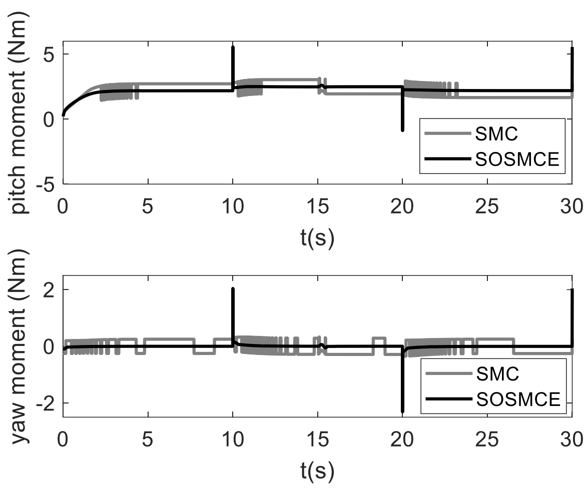


Figure 4. Pitch and yaw control signals

The performance indicators were also calculated and presented in Figure 5. It is seen that the proposed SOSMCE provided lower ITAE for the pitch motion whereas for the yaw motion performances are approximately same. The same figure also shows that although CEI both controllers are almost the same values, the proposed SOSMCE significantly suppresses chattering in the control signal.

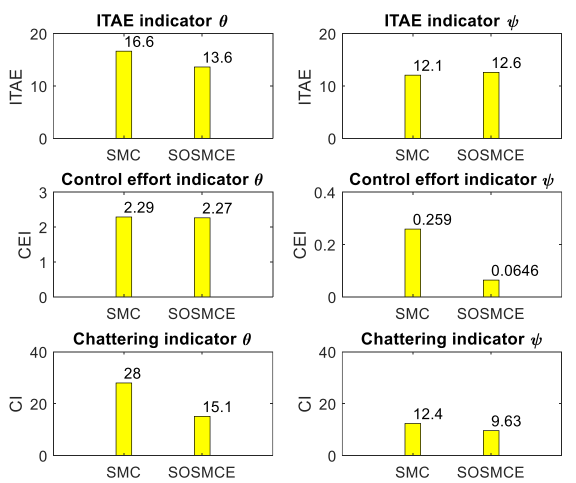


Figure 5. Performance indicators

 ISSN 2536-5010 | **e-ISSN** 2536-5134 https://dergipark.org.tr/en/pub/ejt

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4. CONCLUSION

In this study, a second order sliding mode controller with estimation (SOSMCE) was presented for the twin rotor model. The controller coefficients were provided by performing multi objective genetic algorithm searches. The classical sliding mode controller (SMC) was also provided and implemented to the twin rotor system for validate.

It was observed from Figure 5 that when there is external disturbance the ITAE indicator for SOSMCE *(13.6)* is smaller than the one for classical SMC *(16.6)* for pitch motions. In addition, the ITAE indicator for SOSMCE *(12.6)* is almost same the one for classical SMC *(12.1)* for yaw motions. Furthermore, it is seen that the designed SOSMCE used less control effort than the classical SMC, and the chattering indicator value is bigger for the classical SMC.

It was observed from numerical results that tracking performance was increased while chattering in the control signal was reduced with proposed controller.

For this reason, the proposed SOSMCE may be highly recommended for the control of aerial vehicles.

APPENDIX

Classical Sliding Mode Controller (SMC) Design

$$\dot{\xi}_1 = \xi_2 \qquad (56)$$

$$\dot{\xi}_2 = f(\xi_1,\xi_2) + g(\xi_1,\xi_2)\bar{u} + \bar{d} \qquad (57)$$

$$\sigma = \alpha(\xi_{1r} - \xi_1) + (\dot{\xi}_{1r} - \dot{\xi}_1) \qquad (58)$$

$$V = \frac{1}{2}\sigma^2 \qquad (59)$$

$$\dot{V} = \sigma\dot{\sigma} \qquad (60)$$

with bounded condition the Eq. (60)

$$\dot{\sigma} = \alpha(\dot{\xi}_{1r} - \dot{\xi}_1) + (\ddot{\xi}_{1r} - \ddot{\xi}_1) = 0 \quad (61)$$

Eqs. (56) - (57) and (61), equivalent control $\bar{u}_{eq}$ is defined as:

$$\bar{u}_{eq} = g^{-1}(\xi_1,\xi_2)\{\alpha(\xi_{2r} - \xi_2) + \dot{\xi}_{2r} - f(\xi_1,\xi_2)\} \qquad (62)$$

$$\bar{u} = \bar{u}_{eq} + kg^{-1}(\xi_1,\xi_2)sign(\sigma) \qquad (63)$$

Lyapunov function's derivative:

$$\begin{aligned}\dot{V} = \sigma\dot{\sigma} &= \sigma\{\alpha(\xi_{2r} - \xi_2) \\ &\quad + (\dot{\xi}_{2r} - f(\xi_1,\xi_2) - g(\xi_1,\xi_2)\bar{u} - \bar{d})\} \\ &= -k\sigma sign(\sigma) - \sigma\bar{d} \\ &\leq -|\sigma|(k - \Delta)\end{aligned} \qquad (64)$$

$k > \Delta$, $\dot{V} < 0$, In this way, it is obliged to approach the sliding surface. Control law of is defined as

$$\bar{\tau}_\theta = \widehat{\tau_{eq\theta}} + k_\theta\left(m_{hel}l^2 + I_{pitch}\right) \times sign\{(\alpha_\theta(\xi_{1r} - \xi_1) + \dot{\xi}_{1r} - \xi_2)\} \qquad (65)$$

$$\left(\dot{\widehat{\tau_{eq\theta}}}\right) = \varepsilon\left(\bar{\tau}_\theta - \widehat{\tau_{eq\theta}}\right) \qquad (66)$$

$$\bar{\tau}_\psi = \widehat{\overline{\tau}_{eq\psi}} + k_\psi\left(m_{hel}l^2cos^2(\xi_1) + I_{yaw}\right) \times sign\{(\alpha_\psi(\xi_{3r} - \xi_3) + \dot{\xi}_{3r} - \xi_4)\} \quad (67)$$

$$\left(\dot{\widehat{\overline{\tau}_{eq\psi}}}\right) = \varepsilon\left(\bar{\tau}_\psi - \widehat{\overline{\tau}_{eq\psi}}\right) \qquad (68)$$

TABLE I
THE PARAMETERS OF TWIN ROTOR MODEL

Model Parameters	Symbol	Value	Unit
helicopter mass	m_{hel}	1.3872	kg
helicopter body length of pitch axis	l	0.186	m
pitch damping	B_{pitch}	0.8	N/V
yaw damping	B_{yaw}	0.318	N/V
pitch inertia moment	I_p	0.0384	kgm^2
yaw inertia moment	I_y	0.0432	kgm^2
θ initial value	$\theta(0)$	-40.3	°
ψ initial value	$\psi(0)$	0	°

TABLE II
CONTROLLER WITH MOGA

Controller	Parameter	Value
SMC	k_θ	6.37
	k_ψ	3.20
SOSMCE	$k_{1\theta}$	5.80
	$k_{2\theta}$	0.67
	$k_{1\psi}$	3.83
	$k_{2\psi}$	0.15

ACKNOWLEDGEMENT

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

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