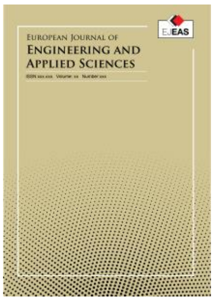
EJEAS

European Journal of
Engineering and Applied Sciences

ISSN: 2651-3412 (Print) & 2667-8454 (Online)
Journal homepage: http://dergipark.gov.tr/EJEAS
Published by Çorlu Faculty of Engineering, Tekirdağ Namık Kemal University

European J. Eng. App. Sci. 6(2), 56-60, 2023



Research Article

Unconditional Optimization of Cylinder-Plane Corona Electrode System

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Received: 26.10.2023 *Accepted: 07.12.2023* *DOI: 10.55581/ejeas.1381865*

Abstract: In this study, based on the Newton-Raphson method, the unconditional optimization of the “thin cylinder-plane” corona electrode system of high-voltage devices with a non-uniform or weakly uniform electric field, widely used in various technological processes of electron-ion technology, is analyzed. For this purpose, the condition of a self-sustaining electric discharge was used for the electrode system under consideration.

Keywords: *Corona discharge, Newton Raphson method, Self-sustained condition, Unconditional optimization.*

Silindir-Düzlem Korona Elektrot Sisteminin Koşulsuz Optimizasyonu

Özet: Bu çalışmada, çeşitli teknolojik işlemlerde yaygın olarak kullanılan, düzgün olmayan veya zayıf düzgün bir elektrik alanına sahip yüksek gerilim cihazlarının “ince silindir-düzlem“ korona elektrot sistemi için, Newton-Raphson yöntemine dayalı olarak koşulsuz optimizasyonu analiz edilmektedir. Bu amaçla, söz konusu elektrot sistemi için, elektrik deşarjın kendi-kendini besleme(self-sustained) koşulu kullanılmıştır

Anahtar kelimeler: *Kendi-kendini besleme koşulu, Korona deşarjı, Koşulsuz optimizasyon, Newton Raphson yöntemi.*

1. Introduction

As it is known, in various high-voltage technological processes, in order to implement a barrier discharge, mainly one of four different structures of electrode systems is used, respectively: “needle-plane”, “conductor-plane”, “plane-plane”, and “sphere-plane [1-3]. It should be noted that in both the “needle-plane” and “conductor-plane” electrode systems, the electric field in the interelectrode gap is non-uniform, and for the “needle-plane” electrode system the degree of field non-uniformity is higher. Whereas the electric field in the interelectrode gap in the “sphere-plane” electrode system is weakly inhomogeneous.

Unlike a homogeneous electric field, where all field lines are identical, in inhomogeneous fields the distributions of field strength along different field lines can differ from each other. In this regard, in inhomogeneous fields, the time of application of voltage is of significant importance. In particular, if the time of application of voltage is unlimitedly long, then at the point of exit from the electrode of the central field line, an electron necessarily appears and the initial voltage will be determined by the condition for the development of an avalanche along this field line, since in this case the condition for self-sustaining discharge is satisfied at the lowest voltage [1-6]. If

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the voltage is applied for a limited time, then the electron may not appear at the central point, and the initial avalanche will be formed by the electron that appeared closest to the central point. Therefore, with short-term exposure, the initial voltage loses its definition and will change from experience to experience. This circumstance is one of the reasons for the scatter of initial stresses, which are the greatest during short-term stress effects [3,5]. In this work, we will assume that the voltage is applied for an indefinitely long time.

If in a uniform electric field, the initial voltages obey Paschen's law, according to which $V_0 = f(pd)$ then in inhomogeneous fields a generalization of this law is the law of similarity of discharges [2, 3, 5]. According to this law, in inhomogeneous fields, the static initial stress is a function of the product of the gas pressure or its relative density by one of the geometric dimensions of the gap and the ratio of all other defining geometric dimensions to this size.

Finding potentials and electric fields for a given distribution of electric charges in space is easily solved only in the simplest cases, when the positions of all charges are fixed. Solving problems of electrostatics in this case comes down to the application of Coulomb's law and several integrations [7-12]. But for real practical electrostatic problems, the actual distribution of charges is not always known. The position that the charges take depends on the electric field strength E, and it, in turn, is determined by the charge distribution itself. The introduction of any conductor or insulator into the field of other charges, the location of which is known, leads to the movement of all charges and, thereby, to a change in the charge distribution density on each body, regardless of whether it is a conductor or a dielectric. For practical problems, special methods for approximate calculation of fields, graphical and semi-graphical methods have been developed. More complex problems can be solved by using a number of techniques and a special mathematical apparatus, which in some cases makes it possible to obtain a solution in analytical form [7-9].

In reality, we mainly have to deal with solving problems, for which the charge distributions are not known, but the shapes of the conductors, their relative location in space and potentials are given. Finding a solution in analytical form for such systems is usually a complex mathematical problem

In this study, based on solving the Laplace equation for the "cylinder-plane" electrode system, taking into account the image charges and using the condition of self-sufficiency of the electric discharge, a nonlinear equation for the potential on the electrode surface with the smallest radius of curvature is obtained. Next, to optimize this nonlinear equation, the Newton Raphson numerical method is used, as a result of which an expression is obtained for determining the initial discharge voltage of the corona discharge ignition. This approach to determining the initial ignition discharge voltage for the "cylinder-plane" electrode system can be considered as a scientific novelty of the presented work.

1.1. Statement of the Problem

For the "thin cylinder-plane" electrode system, using the Newton-Raphson numerical method, it is required to find with sufficient accuracy the extremum of the function $V_0 = f(r_0)$.

In this case, it is assumed that above the surface of a grounded flat ideally conducting electrode (thin cylinder), parallel to it at a height h, an infinite conductor of circular cross-section with radius r_0 (see Fig. 1) is suspended, the potential of which is equal to V and the relative density of the medium in the interelectrode gap is equal to δ. It is necessary to determine the radius r_0, of the cylinder at which the initial voltage of the electric discharge V_0 around the conductor would be maximum.

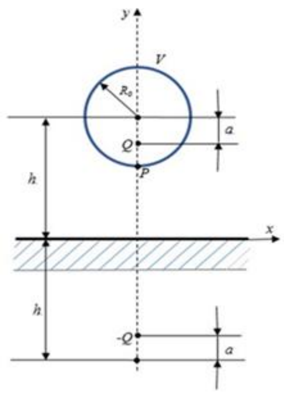

Figure 1. *"Thin cylinder-plane" electrode system*

1.2. The Approach to Solving the Problem

In this work, within the framework of the stated problem, we have investigated the "cylinder-plane" electrode system, which is widely used in the implementation of various technical problems of electron-ion technology. At the same time, it should be noted that the solution to a similar problem for other electrode systems can be implemented using similar reasoning, but taking into account the determination of the potential and electric field strength for a given geometric configuration of the electrode system. For example, for the "sphere-plane" electrode system, Laplace's equation is solved in a spherical coordinate system, and for the "plane-plane" electrode system in a Cartesian coordinate system, etc. And then, the implementation of the problem is carried out similarly to the case considered.

Note that the mentioned electrode systems were not part of the purpose of the task and therefore were not analyzed within the framework of this work.

Obviously, a self-sustaining electric discharge near a round conductor begins when the maximum electric field strength E_{max} on its surface is equal to the initial strength E_0.

Since the value of E_{max} depends on the cylinder potential V in a linear way, then from the equation $E_{max} = E_0$ it is easy to find the dependence of the initial voltage V_0 on the radius of the conductor R_0. After this, using the Newton Raphson method, it is easy to find the extremum of the function $V_0 = f(r_0)$, which will be the solution to the problem.

The conditions of the problem do not stipulate in any way the relationship between the values of h and r_0, so we will consider them comparable. In this case, it is necessary to take into account the displacement of the electrical axis of the cylinder relative to its geometric axis. To do this, we use the simplest system of equivalent charges, constructed in accordance with the image method in a cylinder at a distance

$a = r_0^2/2h$ under the geometric axis of the conductor, i.e. At a height $(h - a)$ above a grounded flat ideally conducting electrode we will place an infinite uniformly charged axis with a charge density Q and below it at a depth $(h - a)$under a grounded flat ideally conducting electrode an axis with a charge density $-Q$.

Thus, the system of equivalent charges that replace the real charges distributed over the surfaces of the conductor and the grounded flat ideally conducting electrode, in this case, consists of the two mentioned uniformly charged axes. In accordance with the principle of superposition, their total charge is zero.

Since a circular wire of radius R_0 has a potential $V > 0$ and a grounded flat ideally conducting electrode has zero potential, the maximum electric field strength on the surface of the cylinder will be achieved at its point P which is closest to the grounded electrode (see Fig. 1)

2. Theory

To achieve the goal, the problem is solved step by step. Initially, for the system under consideration, based on the Laplace equation in a cylindrical coordinate system, the potential on the surface of the electrode with the smallest radius of curvature (cylinder) is analytically determined, taking into account the charge of the image. However, this expression of the potential for a given electrode system does not allow us to determine the initial voltage of the occurrence of a corona discharge due to the fact that it includes optimization parameters characterizing a given electrode system that must be determined. For this reason, further using the relationship between the electric field strength and the potential, as well as the condition of independence of the electric discharge, a nonlinear equation for the potential of a cylindrical electrode is obtained that can be optimized. At the final stage of the algorithm, in order to optimize this nonlinear equation, the Newton-Raphson numerical method was used, for implementation of which a program was compiled in MATLAB.

As it is known, the most general method for calculating electric fields is the direct integration of the Laplace or Poisson equations, under given boundary conditions. [9,11,13].

$$-\nabla^2(\varepsilon V) = 0 \tag{1a}$$

$$E = -\nabla V \tag{1b}$$

where E and V are the electric field and potential respectively, and $\varepsilon = \varepsilon_0 \varepsilon_r$ is permittivity coefficient.

To find the parameters of the electrostatic field of the “thin cylinder-plane” electrode system, it is convenient to use a cylindrical coordinate system. Then Laplace's equation will take the form.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right) = 0 \tag{1c}$$

Consequently, for the case under consideration, the fundamental solution of Laplace's equation will have the form:

$$\varphi_p = \frac{Q}{2\pi\varepsilon_0}\log\left(\frac{2h - r_0 - a}{R_0 - a}\right) \tag{2}$$

Based on this expression, if we accept that $\varphi_P = V$i.e. the given potential of the cylinder V, then it is easy to determine the values of the unknown charge Q, therefore:

$$Q = \frac{2\pi\varepsilon_0 V}{\log\left(\frac{2h - r_0 - a}{r_0 - a}\right)} \tag{3}$$

The maximum value of the electric field strength E_{max} on the surface of the cylinder is obviously achieved at point P. It is equal to:

$$E_{max} = \frac{Q}{\pi\varepsilon_0}\cdot\frac{(h-a)}{(r_0 - a)(2h - r_0 - a)} \tag{4}$$

Substituting the previously obtained expression for Q into this formula, we obtain the functional dependence $E_{max}(R_0)$,

$$E_{max}(r_0) = \frac{V}{\log\left(\frac{2h - r_0 - a}{r_0 - a}\right)}\cdot\frac{(h-a)}{(r_0 - a)(2h - r_0 - a)} \tag{5}$$

The condition for the independence of the electric discharge will be satisfied under the condition $E_{max} = E_0$, where E_0 is the initial field strength, which for the case of a smooth round cylindrical conductor with radius r_0 can be determined by the empirical formula [1,3].

$$E_0(r_0) = 24.5\cdot 10^3\delta\left(1 + \frac{0.65}{(r_0\delta)^{0.38}}\right) \quad \text{V/cm} \tag{6}$$

where δ -is the relative density of air.

Assuming the equality $E_{max}(r_0) = E_0(r_0)$ we obtain the following equation for the unknown value of the initial voltage $V_0(r_0)$:

$$V_0(r_0) = 24.5\cdot 10^3\delta\left(1 + \frac{0.65}{(100r_0\delta)^{0.38}}\right)\cdot \log\left(\frac{2h - r_0 - a}{r_0 - a}\right)\cdot\frac{(r_0 - a)(2h - r_0 - a)}{(h-a)} \tag{7}$$

Finding the extremum of the function $V_0 = V(r_0)$, described by expression (7), by the Newton-Raphson method is organized in the form of a sequence of approximations $r_0^{(k)}$, $k = 0,1,2, \ldots$ calculated by the formula [14]:

$$r_0^{(k)} = r_0^{(k-1)} - \frac{V_0'\left(r_0^{(k-1)}\right)}{V_0''\left(r_0^{(k-1)}\right)} \tag{8}$$

where $V_0\left(r_0^{(k)}\right)$ values of the right hand side of expression (7) at points $r_0^{(k)}$, $V_0'\left(r_0^{(k-1)}\right)$ values of the first derivative of the function $f(r_0)$ at points $r_0^{(k-1)}$, $V_0''\left(r_0^{(k-1)}\right)$ - the values of its second derivative at points $r_0^{(k-1)}$. Often, both the first and the second derivatives of a function look unwieldy. In this case, the derivatives can be replaced by the corresponding finite differences. For the first derivative, we choose the central difference, and for the second one we choose the symmetrical second difference. Due to the complexity of the function $V_0(r_0)$ on the right hand side of expression (7), it is advisable to calculate its first and second derivatives approximately, using the central finite difference of the second order of accuracy.

$$V_0{}'(r_0) = \frac{V_0(r_0 + \Delta r_0) - V_0(r_0 - \Delta r_0)}{2\Delta r_0} \tag{9a}$$

$$V_0{}''(r_0) = \frac{V_0{}'(r_0 + \Delta r_0) - V_0{}'(r_0 - \Delta r_0)}{2\Delta r_0} \tag{9b}$$

where Δr_0 is the final increment of the value of r_0, which was taken equal to $\Delta r_0/r_0 = 10^{-6}$.

3. Analysis of the Results

Based on formulas (5) and (7) for the considered cylinder-plane electrode system, Fig. 2 and Fig. 3 show the results of calculating the electric field strength $E_{max}(r_0)$ and the initial voltage $V_0(r_0)$ at $h = 0.1\ m\ ,\ \delta = 1\ , V = 1kV$, respectively.

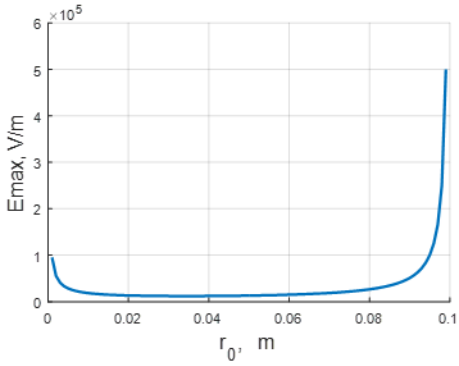

***Figure 2**. Dependence of the maximum electric field strength on the electrode radius r_0 for the "thin cylinder-plane" electrode system $(h = 0.1\ m\ ,\ \delta = 1\ , V = 1kV)$.*

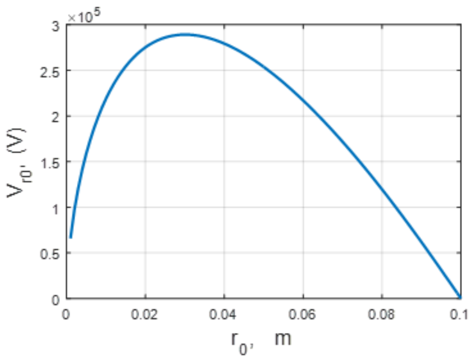

***Figure 3.** Dependence of the initial ignition voltage on the electrode radius r_0 for the "thin cylinder-plane" electrode system $(h = 0.1\ m\ , \delta = 1)$*

Analyzing the nature of the change in Fig. 3, we see that the maximum value of the curve $V_0(r_0)$ is in the range of values $0.02 \leq\ r_0 \leq\ 0.04\ m$. Therefore, as the initial value of r_0 we will take the obviously larger value $r_0 = 0.05\ m$. The sequence of values, calculated using iterative formula (8) $r_0(k),\quad k = 1,2,\ldots,10$. In this case, the relative calculation error $\delta r_0^{(k)}$ is calculated using the formula [14],

$$\delta r_0^{(k)} = \left| \frac{r_0^{(k)} - r_0^{(k-1)}}{r_0^{(k)}} \right| \tag{10}$$

By using the Newton-Raphson method in Matlab, for different gap values of the "cylinder-plane" electrode system, the calculated optimization values $V_0(k)$ and $r_0(k)$ are presented in Fig. 4 and fig. 5, respectively. In this case, the calculation continues until $\delta r_0^{(k)} \leq \varepsilon = 10^{-6}$. Taking these circumstances into account, it is easy to show that the convergence of the Newton-Raphson method for optimizing the values of V_{0max} and r_{0max} according to Eqs. (7), (8), (9a), and (9b) for various values of h at a relative air density δ is completed in six or seven steps. These results are shown in Figures 6 and 7 respectively.

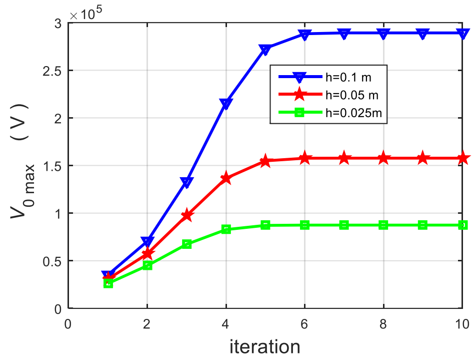

***Figure 4.** Convergence of the Newton-Raphson method for optimization values of V_{0max} by using formula (7) for the three different values of h and the relative air density $\delta = 1$.*

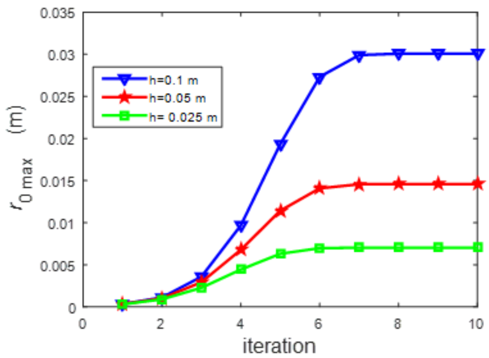

***Figure 5.** Convergence of the Newton-Raphson method for the optimization of r_{0max} by using formula (7) for the three different values of h and the relative air density $\delta = 1$.*

In conclusion, it should be noted that such an expression method for determining the initial ignition voltage of a corona discharge is of practical importance for solving a number of technical problems in electron-ion technology, for example, in the development of a discharge cell for modifying the surface properties of materials, in nanotechnology, in the creation of electrets, etc. For this reason, this work is expected to stimulate the development of various in-demand electrode systems that are used in a wide variety of engineering fields.

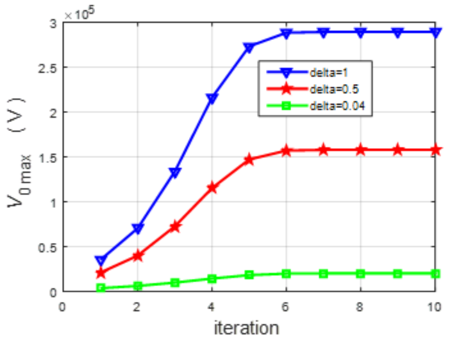

Figure 6. *Convergence of the Newton-Raphson method for the optimization of* V_{0max} *for the three different values of the relative air density* δ *at h= 0.1 m.*

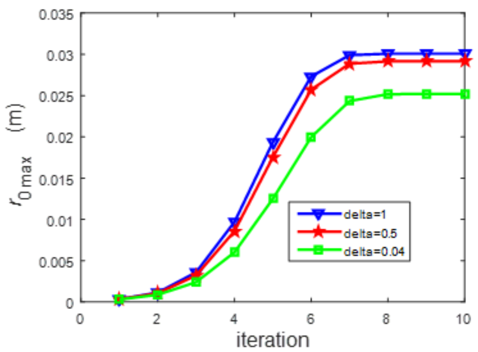

Figure 7. *Convergence of the Newton-Raphson method for the optimization of* r_{0max} *for the three different values of the relative air density* δ *at* $h = 0.1\ m$.

4. Conclusion

Based on the condition of self-sufficiency of the electric discharge, an expression has been found to determine the initial ignition voltage for the “cylinder-plane” electrode system.

By using the Newton-Raphson method in symbolic programming language Matlab for the “cylinder-plane” electrode system, the convergence of the method was analyzed for varying degrees of electric field inhomogeneity in the interelectrode gap.

Author Contribution

Conceive-G.A., H.A.; Design-H.A, N.Ö, L.U; Supervision-HA; Experimental Performance, Data Collection and/or Processing-N.Ö.,L.U.,G.A; Analysis and/or Interpretation-G.A., H.A; Literature Review-L.U.,G.A.,H.A; Writer-L.U. N.Ö.; Critical Reviews –G.A., H.A.

Declaration of Conflict of Interest

The authors have declared no conflicts of interest.

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