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GENERALIZED HEAT POLYNOMIALS

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Abstract. The present study deals with some new properties for the generalized heat polynomials. The results obtained here include various families of multilinear and multilateral generating functions, miscellaneous properties and also some special cases for these polynomials. In addition, we derive a theorem giving certain families of bilateral generating functions for the generalized Heat polynomials and the generalized Lauricella functions. Finally, we get several interesting results of this theorem.

1. Introduction

The generalized Heat polynomials $P_{n,v}(x,u)$ are defined by the generating relation (see, for example,[2], p. 444, Problem 9),

$$\sum_{n=0}^{\infty} P_{n,v}(x,u)\frac{t^n}{n!} = (1-4ut)^{-v-\frac{1}{2}} \exp\left(\frac{x^2 t}{1-4ut}\right). \left(|ut|<\frac{1}{4}\right) \tag{1.1}$$

It is from (1.1) that

$$P_{n,v}(x,u) = \sum_{k=0}^{n} 2^{2k}\binom{n}{k}\frac{\Gamma(v+n+\frac{1}{2})}{\Gamma(v+n-k+\frac{1}{2})}x^{2n-2k}u^k, \tag{1.2}$$

where $\Gamma(z)=\int\limits_0^\infty e^{-t}t^{z-1}dt$ for $Re(z)>0$ is the classical Euler gamma function.

We have the following relationship between the generalized Heat polynomials $P_{n,v}(x,u)$ and the Laguerre polynomials $L_n^{(\alpha)}(x)$(see, for example,[2], p. 426):

$$P_{n,v}(x,u) = (4u)^n n! L_n^{(v-\frac{1}{2})}(\frac{-x^2}{4u})$$

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The following generating function holds true (see, for example,[2], p. 444, Problem 20):

$$(1.3)\qquad \sum_{n=0}^{\infty} P_{n+m,v}(x,u)\frac{t^n}{n!} = (1-4ut)^{-v-m-\frac{1}{2}} \exp\left(\frac{x^2t}{1-4ut}\right) P_{m,v}\left(\frac{x}{\sqrt{1-4ut}},u\right). \left(|ut|<\frac{1}{4}\right)$$

The four Appell functions of two variables, denoted by F_1, F_2, F_3 and F_4, were generalized by Lauricella functions of n variables which are denoted by $F_A^{(n)}$, $F_B^{(n)}$, $F_C^{(n)}$ and $F_D^{(n)}$ [2] and

$$F_A^{(2)} = F_2,\ F_B^{(2)} = F_3,\ F_C^{(2)} = F_4,\ F_D^{(2)} = F_1.$$

A further generalization of the familiar Kampé de Fériet hypergeometric function in two variables is due to Srivastava and Daoust [3] who defined the generalized Lauricella (or the Srivastava-Daoust) function as follows:

$$F_{C:D^{(1)};...;D^{(n)}}^{A:B^{(1)};...;B^{(n)}}\left(\begin{array}{ll} \left[(a):\theta^{(1)},...,\theta^{(n)}\right]: \ \left[(b^{(1)}):\phi^{(1)}\right]; \ ...; \ \left[(b^{(n)}):\phi^{(n)}\right]; & \\ & z_1,...,z_n \\ \left[(c):\psi^{(1)},...,\psi^{(n)}\right]: \ \left[(d^{(1)}):\delta^{(1)}\right]; \ ...; \ \left[(d^{(n)}):\delta^{(n)}\right]; & \end{array}\right)$$

$$= \sum_{m_1,...,m_n=0}^{\infty} \Omega(m_1,...,m_n)\frac{z_1^{m_1}}{m_1!}...\frac{z_n^{m_n}}{m_n!},$$

where, for convenience,

$$\Omega(m_1,...,m_n) := \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j^{(1)}+...+m_n\theta_j^{(n)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j^{(1)}+...+m_n\psi_j^{(n)}}} \frac{\prod_{j=1}^{B^{(1)}}(b_j^{(1)})_{m_1\phi_j^{(1)}}}{\prod_{j=1}^{D^{(1)}}(d_j^{(1)})_{m_1\delta_j^{(1)}}} ... \frac{\prod_{j=1}^{B^{(n)}}(b_j^{(n)})_{m_n\phi_j^{(n)}}}{\prod_{j=1}^{D^{(n)}}(d_j^{(n)})_{m_n\delta_j^{(n)}}}$$

the coefficients

$$\theta_j^{(k)}\ (j=1,...,A;\ k=1,...,n),\ \text{and}\ \phi_j^{(k)}\ (j=1,...,B^{(k)};\ k=1,...,n),$$

$$\psi_j^{(k)}\ (j=1,...,C;\ k=1,...,n),\ \ \text{and}\ \delta_j^{(k)}\ (j=1,...,D^{(k)};\ k=1,...,n)$$

are real constants and $\left(b_{B^{(k)}}^{(k)}\right)$ abbreviates the array of $B^{(k)}$ parameters

$$b_j^{(k)}\ (j=1,...,B^{(k)};\ k=1,...,n)$$

with similar interpretations for other sets of parameters [1]. Here, as usual, $(\lambda)_v$ denotes the Pochhammer symbol.

For a suitably bounded non-vanishing multiple sequence $\{\Omega(m_1;m_2,...,m_s)\}_{m_1,m_2,...,m_s\in\mathbb{N}_0}$ of real or complex parameters, let $\phi_n(u_1;u_2,...,u_s)$ of s (real or complex) variables $u_1;u_2,...,u_s$ defined by

$$(1.4)\qquad \phi_n(u_1;u_2,...,u_s) \ : \ = \sum_{m_1=0}^{n}\ \sum_{m_2,...,m_s=0}^{\infty} \frac{(-n)_{m_1}((b))_{m_1\phi}}{((d))_{m_1\delta}} \times\Omega\left(f(m_1,m_2,...,m_s),m_2,...,m_s\right)\frac{u_1^{m_1}}{m_1!}...\frac{u_s^{m_s}}{m_s!}$$

where, for convenience,

$$((b))_{m_1\phi} = \prod_{j=1}^{B} (b_j)_{m_1\phi_j} \quad \text{and} \quad ((d))_{m_1\delta} = \prod_{j=1}^{D} (d_j)_{m_1\delta_j}.$$

The main object of this paper to study different properties of the generalized Heat polynomials. Various families of multilinear and multilateral generating functions, miscellaneous properties and also some special cases for these polynomials are given. In addition, we derive a theorem giving certain families of bilateral generating functions for the generalized Heat polynomials and the generalized Lauricella functions. Nowadays, there are a lot of works related to generalized Heat polynomials and Lauricella functions theory and its applications (see [11]).

2. Bilinear and Bilateral Generating Functions

We study a number of families of bilinear and bilateral generating fuctions for the generalized Heat polynomials $P_{n,v}(x,u)$ which are produce by (1.1) and given by (1.2) using the similar method considered in (see [1],[5]-[10]).

We begin by stating the following theorem.

Theorem 2.1. *Corresponding to an identically non-vanishing function* $\Omega_\mu(y_1, ..., y_r)$ *of* r *complex variables* $y_1, ..., y_r$ $(r \in \mathbb{N})$ *and of complex order* μ*, let*

$$\Lambda_{\mu,\psi}(y_1, ..., y_r; \zeta) \quad : \quad = \sum_{k=0}^{\infty} a_k \Omega_{\mu+\psi k}(y_1, ..., y_r) \zeta^k$$
$$(a_k \quad \neq \quad 0\,,\ \mu, \psi \in \mathbb{C})$$

and

$$\Theta_{n,p}^{\mu,\psi}\,(x, u; y_1, ..., y_r; \xi) := \sum_{k=0}^{[n/p]} a_k P_{n-pk,v}(x,u) \Omega_{\mu+\psi k}(y_1, ..., y_r) \frac{\xi^k}{(n-pk)!}.$$

Then, for $p \in \mathbb{N}$; *we have*

$$\sum_{n=0}^{\infty} \Theta_{n,p}^{\mu,\psi}\left(x, u; y_1, ..., y_r; \frac{\eta}{t^p}\right) t^n = (1-4ut)^{-v-\frac{1}{2}} \exp\left(\frac{x^2 t}{1-4ut}\right) \Lambda_{\mu,\psi}(y_1, ..., y_r; \eta), \tag{2.1}$$

provided that each member of (2.1) exists.

Proof. For convenience, let S denote the first member of the assertion (2.1) of Theorem 2.2 Then,

$$S = \sum_{n=0}^{\infty} \sum_{k=0}^{[n/p]} a_k P_{n-pk,v}(x,u) \Omega_{\mu+\psi k}(y_1, ..., y_r) \eta^k \frac{t^{n-pk}}{(n-pk)!}.$$

Replacing n by $n+pk$, we may write that

$$\begin{aligned} S &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_k\, P_{n,v}(x,u)\Omega_{\mu+\psi k}(y_1,...,y_r)\eta^k \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} P_{n,v}(x,u)\frac{t^n}{n!}\sum_{k=0}^{\infty} a_k \Omega_{\mu+\psi k}(y_1,...,y_r)\eta^k \\ &= (1-4ut)^{-v-\frac{1}{2}} \exp\left(\frac{x^2 t}{1-4ut}\right)\Lambda_{\mu,\psi}(y_1,...,y_r;\eta), \end{aligned}$$

which completes the proof. □

By using a similar idea, we also get the next result immediately.

Theorem 2.2. *Corresponding to an identically non-vanishing function* $\Omega_\mu(y_1,...,y_r)$ *of* r *complex variables* $y_1,...,y_r$ $(r\in\mathbb{N})$ *and of complex order* μ, *let*

$$\Lambda_{\mu,p,q}\left[x,u;y_1,...,y_r;t\right] := \sum_{n=0}^{\infty} a_n P_{m+qn,v}(x,u)\Omega_{\mu+\psi n}(y_1,...,y_r)\frac{t^n}{(nq)!}$$

where $a_n \neq 0\,,\ \ \mu,\psi\in\mathbb{C}$ *and*

$$\theta_{n,p,q}(y_1,...,y_r;z) := \sum_{k=0}^{[n/q]}\binom{n}{n-qk} a_k\Omega_{\mu+pk}(y_1,...,y_r)z^k.$$

Then, for $p\in\mathbb{N}$; *we have*

$$\sum_{n=0}^{\infty} P_{m+n,v}(x,u)\theta_{n,p,q}(y_1,...,y_r;z)\frac{t^n}{n!} =$$

$$(2.2)\quad (1-4ut)^{-v-m-\frac{1}{2}} \exp\left(\frac{x^2 t}{1-4ut}\right)\times\Lambda_{\mu,p,q}\left[\frac{x}{\sqrt{1-4ut}},u;y_1,...,y_r;z(\frac{t}{1-t})^q\right],$$

provided that each member of (2.2) exists.

Proof. For convenience, let T denote the first member of the assertion (2.2) of Theorem 2.2. Then,

$$T = \sum_{n=0}^{\infty} P_{m+n,v}(x,u)\sum_{k=0}^{[n/q]}\binom{n}{n-qk} a_k\Omega_{\mu+pk}(y_1,...,y_r)z^k\frac{t^n}{n!}.$$

Replacing n by $n+qk$, we may write that

$$\begin{aligned} T &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\binom{n+qk}{n}P_{m+n+qk,v}(x,u)a_k\,\Omega_{\mu+pk}(y_1,...,y_r)z^k\frac{t^{n+qk}}{(n+qk)!} \\ &= \sum_{k=0}^{\infty}\left(\sum_{n=0}^{\infty}P_{m+n+qk,v}(x,u)\frac{t^n}{n!}\right)a_k\Omega_{\mu+pk}(y_1,...,y_r)\frac{(zt^q)^k}{(qk)!} \\ &= \sum_{k=0}^{\infty}(1-4ut)^{-v-m-qk-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ut}\right)\times P_{m+qk,v}\left(\frac{x}{\sqrt{1-4ut}},u\right)a_k\Omega_{\mu+pk}(y_1,...,y_r)\frac{(zt^q)^k}{(qk)!} \\ &= (1-4ut)^{-v-m-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ut}\right) \\ &\quad\times\sum_{k=0}^{\infty}(1-4ut)^{-qk}P_{m+qk,v}\left(\frac{x}{\sqrt{1-4ut}},u\right)a_k\Omega_{\mu+pk}(y_1,...,y_r)\frac{(zt^q)^k}{(qk)!} \\ &= (1-4ut)^{-v-m-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ut}\right)\Lambda_{\mu,p,q}\left(\frac{x}{\sqrt{1-4ut}},u;y_1,...,y_r;z\left(\frac{t}{1-4ut}\right)^q\right), \end{aligned}$$

which completes the proof. □

3. Special Cases

As an application of the above theorems, when the multivariable function $\Omega_{\mu+\psi k}(z_1,...,z_r)$, $k\in\mathbb{N}_0$, $r\in\mathbb{N}$, is expressed in terms of simpler functions of one and more variables, then we can give further applications of the above theorems. We first set

$$\Omega_{\mu+\psi k}(z_1,...,z_r)=u_{\mu+\psi k}^{(\alpha_1,...,\alpha_r)}(z_1,...,z_r)$$

in Theorem 2.1 , where the Erkus–Srivastava polynomials $u_{\mu+\psi k}^{(\alpha_1,...,\alpha_r)}(x_1,...,x_r)$ [8], generated by

$$\prod_{j=1}^{r}\left\{(1-x_jt^{m_j})^{-\alpha_j}\right\}=\sum_{n=0}^{\infty}u_n^{(\alpha_1,...,\alpha_r)}(x_1,...,x_r)t^n \tag{3.1}$$

$$(\alpha_j\in\mathbb{C}\quad(j=1,...,r);\quad |t|<\min\left\{|x_1|^{-1/m_1},...,|x_r|^{-1/m_r}\right\}.$$

We are thus led to the following result which provides a class of bilateral generating functions for the Erkus–Srivastava polynomials and generalized Heat polynomials.

Corollary 3.1. *If*

$$\Lambda_{\mu,\psi}(z_1,...,z_r;w):=\sum_{k=0}^{\infty}a_ku_{\mu+\psi k}^{(\alpha_1,...,\alpha_r)}(z_1,...,z_r)w^k\quad(a_k\neq 0,\ \mu,\psi\in\mathbb{C}),$$

then, we have

$$\begin{aligned} &\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]}a_kP_{n-pk}(x,u)u_{\mu+\psi k}^{(\alpha_1,...,\alpha_r)}(z_1,...,z_r)\frac{w^k}{(n-pk)!}t^{n-pk} \\ &\quad= (1-4ut)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ut}\right)\Lambda_{\mu,\psi}(z_1,...,z_r;w), \end{aligned} \tag{3.2}$$

provided that each member of (3.2) exists.

Remark 3.1. Using the generating relation (3.1) for the Erkus-Srivastava polynomials and getting $a_k = 1$, $\mu = 0$, $\psi = 1$, we find that

$$\begin{aligned} &\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} P_{n-pk}(x,u)u_k^{(\alpha_1,...,\alpha_r)}(z_1,...,z_r)w^k\frac{t^{n-pk}}{(n-pk)!} \\ = \quad &(1-4ut)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ut}\right)\prod_{j=1}^{r}\left\{(1-z_jw^{m_j})^{-\alpha_j}\right\}. \end{aligned}$$

$$\left(|ut| < \frac{1}{4}, \quad |w| < \min\left\{|z_1|^{-1/m_1}, ..., |z_r|^{-1/m_r}\right\}\right)$$

If we set

$$s = r \text{ and } \Omega_{\mu+\psi k}(y_1,...,y_r) = g_{\mu+\psi k}^{(\alpha_1,...,\alpha_r)}(y_1,...,y_r)$$

in Theorem 2.2, where the Chan-Chyan-Srivastava polynomials defined in [7] and the generalized Heat polynomials $P_{n-pk}(x,u)$ given explicitly by (1.3).

Corollary 3.2. *If*

$$\Lambda_{\mu,p,q}(x,u;y_1,...,y_r;z) \quad : \quad = \sum_{n=0}^{\infty} a_n P_{m+qn}(x,u) g_{\mu+\psi n}^{(\alpha_1,...,\alpha_r)}(y_1,...,y_r)\frac{t^n}{(nq)!}$$
$$(a_n \neq 0, \ m \in \mathbb{N}_0, k \neq 0)$$

and

$$\theta_{n,m,q}^{p,\mu}(y_1,...,y_r;z) := \sum_{k=0}^{[n/q]}\binom{n}{n-qk}a_k g_{\mu+\psi k}^{(\alpha_1,...,\alpha_r)}(y_1,...,y_r)z^k,$$

where $n, p \in \mathbb{N}$, then we have

$$\begin{aligned} (3.3) \qquad &\sum_{n=0}^{\infty} P_{m+n}(x,u)\theta_{n,m,q}^{p,\mu}(y_1,...,y_r;z)\frac{t^n}{n!} \\ = \quad &(1-4ut)^{-v-m-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ut}\right) \\ &\times\Lambda_{\mu,p,q}\left[\frac{x}{\sqrt{1-4ut}}, u; y_1,...,y_r; z\left(\frac{t}{1-4ut}\right)^q\right], \end{aligned}$$

provided that each member of (3.3) exists.

Furthermore, for every suitable choice of the coefficients a_k (or a_n) $k, n \in \mathbb{N}_0$), if the multivariable functions $\Omega_{\mu+\psi k}(y_1,...,y_r)$, $(r \in \mathbb{N})$ and $P_{n-pk}(x,u)$ are expressed as an appropriate product of several simpler functions, the assertions of Theorem 2.1, Theorem 2.2, can be applied in order to derive various families of multilinear and multilateral generating functions for the family of generalized Heat polynomials given explicitly by (1.2).

4. Miscellaneous Properties

We now discuss some miscellaneous recurrence relations of the generalized Heat polynomials $P_{n,v}(x,u)$ given by (1.2).

By differentiating each member of the generating function relation (1.1) with respect to x and using

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A(k,n)=\sum_{n=0}^{\infty}\sum_{k=0}^{n}A(k,n-k), \tag{4.1}$$

we arrive at the following (differential) recurrence relation for the generalized Heat polynomials $P_n(x,u)$:

$$\frac{\partial}{\partial x}P_{n,v}(x,u)=\sum_{m=0}^{n-1}2x(4u)^m n!\frac{P_{n-m-1,v}(x,u)}{(n-m-1)!}. \tag{4.2}$$

By differentiating each member of the generating function relation (1.1) with respect to u and using (4.1) we arrive at the following (differential) recurrence relation for the generalized Heat polynomials $P_n(x,u)$:

$$\begin{aligned}\frac{\partial}{\partial u}P_{n,v}(x,u) &= (4v+2)\left(\sum_{m=0}^{n-1}(4u)^m\frac{P_{n-m-1,v}(x,u)}{(n-m-1)!}\right)\\ &\quad +(4x^2)\left(\sum_{p=0}^{n-2}(p+1)(4u)^p\frac{P_{n-p-2,v}(x,u)}{(n-p-2)!}\right).\end{aligned} \tag{4.3}$$

If we consider (4.2) and (4.3), we can easily get the following recurrence relation for the generalized Heat polynomials :

$$\frac{\partial}{\partial u}P_{n,v}(x,u)-\frac{(4v+2)}{2x.n!}\left[\frac{\partial}{\partial x}P_{n,v}(x,u)\right]=(4x^2)\left(\sum_{p=0}^{n-2}(p+1)(4u)^p\frac{P_{n-p-2,v}(x,u)}{(n-p-2)!}\right).$$

Besides, by differentiating each member of the generating function relation (1.2) with respect to t, we have

$$P_{n+1,v}(x,u)=\sum_{m=0}^{n}\left[\left(v+\frac{1}{2}\right)(4u)^{m+1}+x^2(m+1)(4u)^m\right]P_{n-m,v}(x,u).$$

5. BILATERAL GENERATING FUNCTIONS

Now, we derive various families of bilateral generating functions for the generalized Lauricella (or the Srivastava-Daoust) functions and the generalized Heat polynomials.

Theorem 5.1. *The following bilateral generating function holds true*:

$$\begin{aligned}&\sum_{n=0}^{\infty}P_{n,v}(x,h)\phi_n(u_1;u_2,...,u_s)\frac{t^n}{n!}\\ &=(1-4ht)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ht}\right)\sum_{k,m_1,m_2,...,m_s=0}^{\infty}\frac{(v+\frac{1}{2})_{(m_1+k)}}{(v+\frac{1}{2})_{m_1}}\frac{((b))_{(m_1+k)\phi}}{((d))_{(m_1+k)\delta}}\\ &\times\Omega\left(f(m_1+k,m_2,...,m_s),m_2,...,m_s\right)\frac{(\frac{4u_1t}{4ht-1})^k}{k!}\frac{(-\frac{u_1x^2t}{(1-4ht)^2})^{m_1}}{m_1!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!},\end{aligned}$$

where $\phi_n(u_1;u_2,...,u_s)$ is given by (1.4).

Proof. By using the relationship (1.3), it is easily observed that

$$\begin{aligned}
&\sum_{n=0}^{\infty} P_{n,v}(x,h)\phi_n(u_1;u_2,...,u_s)\frac{t^n}{n!}\\
= &\sum_{n=0}^{\infty} P_{n,v}(x,h)\sum_{m_1=0}^{n}\sum_{m_2,...,m_s=0}^{\infty}\frac{(-n)_{m_1}((b))_{m_1\phi}}{((d))_{m_1\delta}}\times\Omega\left(f(m_1,m_2,...,m_s),m_2,...,m_s\right)\frac{u_1^{m_1}}{m_1!}\cdots\frac{u_s^{m_s}}{m_s!}\frac{t^n}{n!}\\
= &\sum_{m_1,m_2,...,m_s=0}^{\infty}\left(\sum_{n=0}^{\infty}P_{n+m,v}(x,h)\frac{t^n}{n!}\right)\frac{((b))_{m_1\phi}}{((d))_{m_1\delta}}\\
&\times\Omega\left(f(m_1,m_2,...,m_s),m_2,...,m_s\right)\frac{(-u_1t)^{m_1}}{m_1!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!}\\
= &\sum_{m_1,m_2,...,m_s=0}^{\infty}\left((1-4ht)^{-v-m_1-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ht}\right)P_{m_1,v}\left(\frac{x}{\sqrt{1-4ut}},h\right)\right)\frac{((b))_{m_1\phi}}{((d))_{m_1\delta}}\\
&\times\Omega\left(f(m_1,m_2,...,m_s),m_2,...,m_s\right)\frac{(-u_1t)^{m_1}}{m_1!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!}\\
= &(1-4ht)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ht}\right)\sum_{m_1,m_2,...,m_s=0}^{\infty}P_{m_1,v}\left(\frac{x}{\sqrt{1-4ut}},h\right)\frac{((b))_{m_1\phi}}{((d))_{m_1\delta}}\\
&\times\Omega\left(f(m_1,m_2,...,m_s),m_2,...,m_s\right)\frac{(-\frac{u_1t}{1-4ht})^{m_1}}{m_1!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!}\\
= &(1-4ht)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ht}\right)\\
&\times\sum_{m_1,m_2,...,m_s=0}^{\infty}\left\{\sum_{k=0}^{m_1}2^{2k}\binom{m_1}{k}\frac{\Gamma(v+m_1+\frac{1}{2})}{\Gamma(v+m_1-k+\frac{1}{2})}\left(\frac{x}{\sqrt{1-4ht}}\right)^{2m_1-2k}h^k\right\}\\
&\times\frac{((b))_{m_1\phi}}{((d))_{m_1\delta}}\Omega\left(f(m_1,m_2,...,m_s),m_2,...,m_s\right)\frac{(-\frac{u_1t}{1-4ht})^{m_1}}{m_1!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!}\\
= &(1-4ht)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ht}\right)\sum_{k,m_1,m_2,...,m_s=0}^{\infty}\left\{\frac{(v+\frac{1}{2})_{(m_1+k)}}{(v+\frac{1}{2})_{m_1}}\frac{((b))_{(m_1+k)\phi}}{((d))_{(m_1+k)\delta}}2^{2k}\right.\\
&\times\left.\left(\frac{x}{\sqrt{1-4ht}}\right)^{2m_1}h^k\right\}\Omega\left(f(m_1+k,m_2,...,m_s),m_2,...,m_s\right)\frac{(-\frac{u_1t}{1-4ht})^{m_1+k}}{m_1!k!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!}\\
= &(1-4ht)^{-v-\frac{1}{2}}\exp\left(\frac{x^2t}{1-4ht}\right)\sum_{k,m_1,m_2,...,m_s=0}^{\infty}\frac{(v+\frac{1}{2})_{(m_1+k)}}{(v+\frac{1}{2})_{m_1}}\frac{((b))_{(m_1+k)\phi}}{((d))_{(m_1+k)\delta}}\\
&\times\Omega\left(f(m_1+k,m_2,...,m_s),m_2,...,m_s\right)\frac{(\frac{4u_1t}{4ht-1})^k}{k!}\frac{(-\frac{u_1x^2t}{(1-4ht)^2})^{m_1}}{m_1!}\frac{u_2^{m_2}}{m_2!}\cdots\frac{u_s^{m_s}}{m_s!}
\end{aligned}$$

□

By appropriately choosing the multiple sequence $\Omega(m_1,m_2,...,m_s)$ in Theorem 5.1, we obtain several interesting results as follows which give bilateral generating functions for the generalized Heat polynomials $P_{n,v}(x,h)$ and the generalized Lauricella (or the Srivastava-Daoust) functions.

Upon setting

$$\Omega\left(f(m_1, m_2, ..., m_s), m_2, ..., m_s\right) = \frac{(a)_{m_1+...+m_s}(b_2)_{m_2}...(b_s)_{m_s}}{(v+\frac{1}{2})_{m_1}(c_2)_{m_2}...(c_s)_{m_s}}$$

and

$$\phi = \delta = 0 \ \ \text{(that is, } \phi_1 = ... = \phi_B = \delta_1 = ... = \delta_D = 0)$$

in Theorem 5.1, we obtain the following result.

Corollary 5.1. *The following bilateral generating function holds true:*

$$\sum_{n=0}^{\infty} P_{n,v}(x,h) F_A^{(s)}\left[a, -n, b_2, ..., b_s; (v+\frac{1}{2}), c_2, ..., c_s; u_1, u_2, ..., u_s\right]\frac{t^n}{n!}$$

$$= (1-4ht)^{-v-\frac{1}{2}} \exp\left(\frac{x^2 t}{1-4ht}\right) F^{1:0;1;1;...;1}_{0:0;1;1;...;1}$$

$$\left(\begin{array}{l} \left[(a):\psi^{(1)},...,\psi^{(s+1)}\right]: \ -; \quad -; \quad [b_2:1]; \ ...; \ [b_s:1]; \\ \left[\ \ -\ \ \right]: \quad -; \ \left[v+\frac{1}{2}:1\right]; \ [c_2:1]; \ ...; \ [c_s:1]; \\ \qquad (\frac{4hu_1t}{4ht-1})(-\frac{u_1x^2t}{(1-4ht)^2}), u_2, ..., u_s \end{array}\right),$$

where the cofficients $\psi^{(n)}$ *are given by* $\psi^{(1)} = ... = \psi^{(s+1)} = 1.$

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